

Calculus for Artificial Intelligence



1. Power Rule

For a function $f(x,y)=x^n$ or $f(x,y)=y^n$:

$$\frac{\partial}{\partial x}(x^n) = nx^{n-1}$$

$$rac{\partial}{\partial x}(x^n)=nx^{n-1} \ rac{\partial}{\partial y}(y^n)=ny^{n-1}$$

Example:

 $f(x,y) = x^3$

 $\frac{\partial f}{\partial x} = 3x^2$

 $g(x,y) = y^4$

 $rac{\partial g}{\partial u}=4y^3$



2. Constant Multiple Rule

For a function $f(x,y) = c \cdot g(x,y)$, where c is a constant:

$$rac{\partial}{\partial x}(c \cdot g(x,y)) = c \cdot rac{\partial g(x,y)}{\partial x} \ rac{\partial}{\partial y}(c \cdot g(x,y)) = c \cdot rac{\partial g(x,y)}{\partial y}$$

$$egin{aligned} f(x,y) &= 5x^2 \ rac{\partial f}{\partial x} &= 5 \cdot 2x = 10x \ g(x,y) &= 7y^3 \ rac{\partial g}{\partial y} &= 7 \cdot 3y^2 = 21y^2 \end{aligned}$$



3. Sum Rule

For a function
$$f(x,y)=g(x,y)+h(x,y)$$
: $\frac{\partial}{\partial x}(g(x,y)+h(x,y))=\frac{\partial g(x,y)}{\partial x}+\frac{\partial h(x,y)}{\partial x}$ $\frac{\partial}{\partial y}(g(x,y)+h(x,y))=\frac{\partial g(x,y)}{\partial y}+\frac{\partial h(x,y)}{\partial y}$

$$egin{aligned} f(x,y) &= x^2 + y^2 \ rac{\partial f}{\partial x} &= rac{\partial}{\partial x}(x^2) + rac{\partial}{\partial x}(y^2) = 2x + 0 = 2x \ g(x,y) &= 3x^3 + 4y \ rac{\partial g}{\partial y} &= rac{\partial}{\partial y}(3x^3) + rac{\partial}{\partial y}(4y) = 0 + 4 = 4 \end{aligned}$$



4. Product Rule

For a function $f(x,y) = g(x,y) \cdot h(x,y)$: $rac{\partial}{\partial x}(g(x,y)\cdot h(x,y)) = g(x,y)\cdot rac{\partial h(x,y)}{\partial x} + h(x,y)\cdot rac{\partial g(x,y)}{\partial x} \qquad rac{\partial f}{\partial x} = u\cdot rac{\partial v}{\partial x} + v\cdot rac{\partial u}{\partial x}$ $rac{\partial x}{\partial u}(g(x,y)\cdot h(x,y)) = g(x,y)\cdot rac{\partial h(x,y)}{\partial y} + h(x,y)\cdot rac{\partial g(x,y)}{\partial y} \qquad rac{\partial f}{\partial x} = u\cdot \left(rac{\partial}{\partial x}v
ight) + v\cdot \left(rac{\partial}{\partial x}u
ight)$

Simplified,

$$egin{align} rac{\partial f}{\partial x} &= u \cdot rac{\partial v}{\partial x} + v \cdot rac{\partial u}{\partial x} \ rac{\partial f}{\partial x} &= u \cdot \left(rac{\partial}{\partial x}v
ight) + v \cdot \left(rac{\partial}{\partial x}u
ight) \end{aligned}$$

$$egin{aligned} f(x,y) &= (x^2) \cdot (y^3) \ rac{\partial f}{\partial x} &= (x^2) \cdot rac{\partial (y^3)}{\partial x} + (y^3) \cdot rac{\partial (x^2)}{\partial x} = (x^2) \cdot 0 + (y^3) \cdot (2x) = 2xy^3 \ g(x,y) &= (3x) \cdot (4y^2) \ rac{\partial g}{\partial y} &= (3x) \cdot rac{\partial (4y^2)}{\partial y} + (4y^2) \cdot rac{\partial (3x)}{\partial y} = (3x) \cdot (8y) + (4y^2) \cdot 0 = 24xy \end{aligned}$$

Calculus for Deep Learning

Partial Derivative



Partial Derivative with Respect to x

First, we compute the partial derivatives of $u(x,y)=x^2+y^2$ and $v(x,y)=x^2y+y^2x$ with respect to x:

$$u(x,y)=x^2+y^2 \ v(x,y)=x^2y+y^2x$$

$$egin{array}{l} rac{\partial u}{\partial x} = 2x \ rac{\partial v}{\partial x} = rac{\partial}{\partial x}(x^2y+y^2x) = 2xy+y^2 \end{array}$$

$$egin{array}{l} rac{\partial f}{\partial x} = u \cdot rac{\partial v}{\partial x} + v \cdot rac{\partial u}{\partial x} & \ rac{\partial f}{\partial x} = (x^2 + y^2)(2xy + y^2) + (x^2y + y^2x)(2x) \end{array}$$

Simplifying this:

$$egin{aligned} rac{\partial f}{\partial x} &= (x^2 + y^2)(2xy + y^2) + 2x(x^2y + y^2x) \ rac{\partial f}{\partial x} &= 2x^3y + x^2y^2 + 2xy^3 + y^4 + 2x^3y + 2xy^3 \ rac{\partial f}{\partial x} &= 4x^3y + x^2y^2 + 4xy^3 + y^4 \end{aligned}$$



Partial Derivative with Respect to y

Now, we compute the partial derivatives of u(x,y) and v(x,y) with respect to y:

$$egin{array}{l} rac{\partial u}{\partial y} = 2y \ rac{\partial v}{\partial y} = rac{\partial}{\partial y}(x^2y+y^2x) = x^2+2yx \end{array}$$

Applying the product rule:

$$egin{array}{c} rac{\partial f}{\partial y} = u \cdot rac{\partial v}{\partial y} + v \cdot rac{\partial u}{\partial y} & rac{\partial f}{\partial y} = x^4 + 2x^3 y \ rac{\partial f}{\partial y} = (x^2 + y^2)(x^2 + 2yx) + (x^2y + y^2x)(2y) \end{array} \ / \ .$$

Simplifying this:

$$egin{align} rac{\partial f}{\partial y} &= (x^2 + y^2)(x^2 + 2yx) + 2y(x^2y + y^2x) \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + x^2y^2 + 2yx^2 + 2y^2x^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + x^2y^2 + 2x^2y + 2x^2y^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2yx^2 + 2y^3x \ rac{\partial f}{\partial y} &= x^4 + 2x^3y + 3x^2y^2 + 2y^2 + 2y^2$$



5. Quotient Rule

For a function $f(x,y)=rac{g(x,y)}{h(x,y)}$:

$$\frac{\partial}{\partial x} \left(\frac{g(x,y)}{h(x,y)} \right) = \frac{h(x,y) \cdot \frac{\partial g(x,y)}{\partial x} - g(x,y) \cdot \frac{\partial h(x,y)}{\partial x}}{[h(x,y)]^2}$$

$$\frac{\partial}{\partial x} \left(g(x,y) \right) = \frac{h(x,y) \cdot \frac{\partial g(x,y)}{\partial x} - g(x,y) \cdot \frac{\partial h(x,y)}{\partial x}}{[h(x,y)]^2}$$

$$\frac{\partial}{\partial y}\left(\frac{g(x,y)}{h(x,y)}\right) = \frac{h(x,y)\cdot\frac{\partial g(x,y)}{\partial y}-g(x,y)\cdot\frac{\partial h(x,y)}{\partial y}}{[h(x,y)]^2}$$

Example:

$$egin{aligned} f(x,y) &= rac{x^2}{y} \ rac{\partial f}{\partial x} &= rac{y \cdot rac{\partial (x^2)}{\partial x} - x^2 \cdot rac{\partial y}{\partial x}}{y^2} = rac{y \cdot (2x) - x^2 \cdot 0}{y^2} = rac{2xy}{y^2} = rac{2x}{y} \ g(x,y) &= rac{y^2}{x} \ rac{\partial g}{\partial y} &= rac{x \cdot rac{\partial (y^2)}{\partial y} - y^2 \cdot rac{\partial x}{\partial y}}{x^2} = rac{x \cdot (2y) - y^2 \cdot 0}{x^2} = rac{2xy}{x^2} = rac{2y}{x} \end{aligned}$$

Simplified,

$$f(x,y)=rac{u(x,y)}{v(x,y)}$$

$$rac{\partial f}{\partial x} = rac{v \cdot rac{\partial u}{\partial x} - u \cdot rac{\partial v}{\partial x}}{v^2}$$

$$rac{\partial f}{\partial x} = rac{v \cdot \left(rac{\partial}{\partial x} u
ight) - u \cdot \left(rac{\partial}{\partial x} v
ight)}{v^2}$$



6. Chain Rule

For a function z = f(g(x, y), h(x, y)):

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial g} \cdot \frac{\partial g}{\partial x} + \frac{\partial z}{\partial h} \cdot \frac{\partial h}{\partial x}$$
$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial g} \cdot \frac{\partial g}{\partial y} + \frac{\partial z}{\partial h} \cdot \frac{\partial h}{\partial y}$$

$$f(x,y)=e^{x^2+y^2}$$

Let
$$u=x^2+y^2$$
, then $f=e^u$.

$$\frac{\partial u}{\partial x} = 2x$$

$$\frac{\partial f}{\partial u} = e^u$$

$$rac{\partial f}{\partial x} = rac{\partial f}{\partial u} \cdot rac{\partial u}{\partial x} = e^{x^2 + y^2} \cdot 2x = 2xe^{x^2 + y^2}$$

Calculus for Deep Learning

Some Important Calculus Formulas



Derivative

$$\frac{d}{dx}n=0$$

$$\frac{d}{dx}x = 1$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^{x} = e^{x}$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}n^{x} = n^{x} \ln x$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

Integral (Antiderivative)

$$\int 0 dx = C$$

$$\int 1 \, dx = x + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x dx = e^x + C$$

$$\int \frac{1}{x} dx = \ln x + C$$

$$\int n^x dx = \frac{n^x}{\ln n} + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}$$
 sec $x = \sec x \tan x$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}$$
 arctan $x = \frac{1}{1 + x^2}$

$$\frac{d}{dx}\arctan\cot x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx} \arccos x = \frac{1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx} \arccos x = -\frac{1}{x\sqrt{x^2 - x^2}}$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \ dx = -\cot x + C$$

$$\int \tan x \sec x \, dx = \sec x + C$$

$$\int \cot x \csc x \, dx = -\csc x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

$$\int -\frac{1}{\sqrt{1-x^2}} dx = \arccos x + C$$

$$\int \frac{1}{1+x^2} dx = \arctan x + C$$

$$\int -\frac{1}{1+x^2} dx = \operatorname{arc} \cot x + C$$

$$\frac{d}{dx}\operatorname{arc}\sec x = \frac{1}{x\sqrt{x^2 - 1}} \qquad \qquad \int \frac{1}{x\sqrt{x^2 - 1}} \, dx = \operatorname{arc}\sec x + C$$

$$\frac{d}{dx}\operatorname{arc}\operatorname{csc} x = -\frac{1}{x\sqrt{x^2 - 1}} \qquad \int -\frac{1}{x\sqrt{x^2 - 1}} dx = \operatorname{arc}\operatorname{csc} x + C$$



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