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## AP EXAM 2020

## Notes

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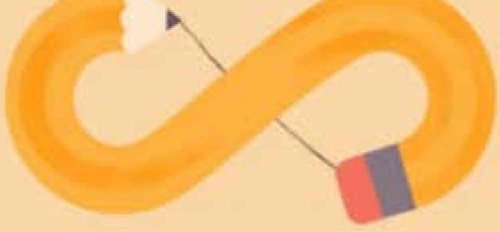

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# Unit 1: Kinematics

## 1.1: Position, Velocity, and Acceleration

| Scalar quantities (magnitude only) | Vector quantities (magnitude and direction) |
|---|---|
| Distance: The length of the path traveled. | Displacement: The change in position of an object in motion. |
| Speed: total distance traveled over a period of time. | Velocity: The rate which position changes (change in position over time). |
| | Acceleration: The rate at which velocity changes (change in velocity over time). |

## 1.2: Representations of Motion

Kinematic Problems

- 5 kinematic quantities describe motion between an initial and final position.
  - Initial velocity, final velocity, time, displacement, acceleration
- The cart's position (x) tells where the cart is on a field, line, etc. Displacement (Δx) tells how far the object ends up away from its starting point, regardless of the path taken between starting and ending positions.
- Speed (v) tells how fast an object is moving (no direction, just magnitude).
- Acceleration (a) tells how much the object's speed changes in one second.
  - If acceleration and velocity are in the same direction, the object will speed up.
  - If acceleration and velocity are in opposite directions, the object will slow down.
  - Acceleration doesn't say anything about which way something is moving, unless you know whether the thing is speeding up or is slowing down.
- Free-fall: the only force acting is the force of gravity (always 9.81 $m/s^2$ downwards)
- If there are multiple vectors acting on an object, the sum of vectors is where net acceleration or velocity will act. (same for acceleration w/forces)
- To apply kinematic equations, you need 3 known quantities. There are 5 equations that each contain 4 of the quantities. They can be named by the missing quantity:
  - No acceleration: $\Delta x = \frac{1}{2}(v_0 + v_f)t$
  - No displacement: $v_f = v_0 + at$
  - No time: $v_f^2 = v_0^2 + 2a\Delta x$
  - No final velocity: $\Delta x = v_0 t + \frac{1}{2}at^2$
  - No initial velocity: $\Delta x = v_f t + \frac{1}{2}at^2$

Kinematics Derived Equations:

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Time: Start from $\Delta x = v_0 t + \frac{1}{2}at^2$

$$t^2 = \frac{2\Delta x}{a} \quad t = \sqrt{\frac{2\Delta x}{a}}$$

Max Height $(when\ v_f = 0)$ :

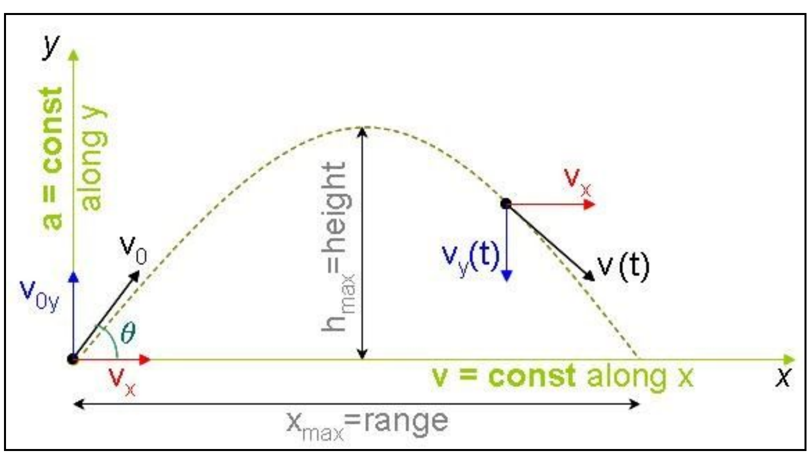
$$v_f^2 = v_0^2 + 2a\Delta x \qquad 0 = v_0^2 + 2a\Delta x \qquad \Delta x = \frac{-v_0^2}{2a}$$ (a = -g if in free-fall)

Kinematics Graphs:

| | Position vs. Time | Velocity vs. Time | Acceleration vs. Time |
|---|---|---|---|
| Area Under Curve | N/A | Displacement | Change in velocity |
| Slope | Velocity | Acceleration | N/A |
| y-axis | Distance from detector | Speed of object | Acceleration |

- Position-time graph: the object's speed is the slope of the graph. The steeper the slope, the faster the object moves. If the slope is a front slash (/), the movement is in the positive direction; if the slope is a backslash (\), the movement is in the negative direction.
- Velocity-time graph: object's speed is read from the vertical axis. The direction of motion is indicated by the sign on the vertical axis.
- Acceleration is the slope of a Velocity-time graph.
- Displacement is area between the graph and the horizontal axis. The location of the object can't be determined from a velocity-time graph; only how far it ended up from its starting point can be determined.
- Acceleration-time graph: How fast or slow the velocity is changing.
  - If velocity is constant, then acceleration is 0

1.3: Projectile Motion



For projectile motion problems:

- A projectile's x and y directional motion are independent and are related through time.
- Separate a projectile's motion into horizontal and vertical components using trigonometry.
- Then use UAM equations to solve for wanted variables.
- $V_y$ is zero at the top of a projectile's parabolic path
- Pay attention to initial velocities, positions in x and y direction.

| Horizontal (x-direction) | Vertical (y-Direction) |
|---|---|
| $V_x$=V cos( $\theta$ ) | $V_y$=V sin( $\theta$ ) |
| X=$V_x$t | |
| $V_x = V_{0x}$ (Velocity is always constant) | $V_y$ = $V_o$sin( $\theta$ )-gt |
| $A_x$=0 | $A_y$ = -g = -10m/s$^2$ |

Projectile Motion Derived Equations

Projectile Max Height (Max height):

$$0 = v_0 \sin(\theta) - gt \qquad t_{top} = \frac{v_0 \sin(\theta)}{g} \leftarrow t_{up}$$

[when $V_y$ = 0]

Then use $\Delta y = \frac{1}{2}at^2 + v_0 t$ to get ymax:

$$\Delta y = v_0 \sin(\theta)\left(\frac{v_0 \sin(\theta)}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin(\theta)}{g}\right)^2 \qquad \Delta y_{max} = \frac{v_0^{\,2} sin^2(\theta)}{2g}$$

Projectile Range and Time:

$$\Delta x = v_{0x} + \frac{1}{2}a_x t^2 \qquad \Delta y = v_{0y} - \frac{1}{2}gt^2$$

If projectile returns to 0, Δy = 0

$$0 = -\frac{1}{2}gt^2 + v_0 \sin(\theta)\, t \qquad \frac{1}{2}gt^2 = v_0 \sin(\theta)\, t \qquad t_{total} = \frac{2v_0 \sin(\theta)}{g}$$

Let R be the horizontal range of the projectile (R = Δx):

$$R = v_0 \cos(\theta)\, t \qquad R = \frac{2v_0^{\,2} \sin(\theta)\cos(\theta)}{g} = \frac{v_0^{\,2} \sin(2\theta)}{g}$$

# Unit 2: Dynamics

## 2.1: Systems

- A system is an object or a collection of two or more objects that is treated as having no internal structure.
  - A system approach is often used to find the acceleration (*F = ma,* according to Newton's Second Law of Motion) in which the total mass of the system is used.

In the following image, the rope holding the system together is representative of a force known as tension. The tension between objects remains the same regardless of friction or not.


- In systems, internal interactions, such as Tension between block A and B, have little changes or none at all to the system's acceleration. They exhibit Newton's 3rd Law, and balance out due to being equal and opposite.

## 2.2: The Gravitational Field

- All objects in the universe exert an attractive force of gravity on each other. The formula for the force of gravity between 2 objects is: $F_g = \frac{GMm}{r^2}$
  - r denotes distance between the centers of the objects, not the height from surfaces or radius
  - G = 6.674 * $10^{-11}$ $m^3$ / kg $\cdot$ $s^2$
- The formula for  Force of gravity can be used to derive the expression for acceleration due to gravity on a planet of mass M:
  - g=GM/r2
  - On earth the value is 9.81 m/s2
- Gravity is a non-contact force.
  - The force of gravity at any given point is the vector sum of all forces of gravity (due to every object in the universe) at that point.
- If two objects or planets have the same radius, they must have the same volume
  - If the density of one planet is less, the mass must be less as well

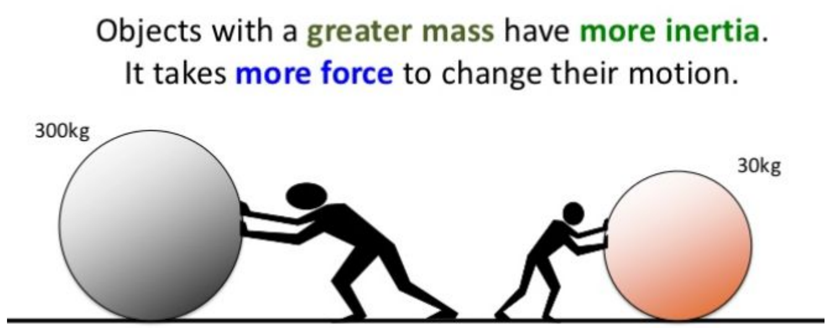
## 2.3: Contact Forces

- Contact forces are action-reaction pairs of forces produced by physical contact of two objects.
  - Examples Include: Friction, Normal, Tension, Applied, and Spring
- Normal force, N , is perpendicular to the contact surface along which an object moves or is capable of moving. Thus, for an object on a level surface, N and W are equal in size but opposite in direction.
  - However, for an object on a ramp, this statement is not true because N  is perpendicular to the surface of the ramp. The W value will only contain a component that is opposing the Normal force.

- Tension, T , is the force transmitted through a string. The tension is the same throughout the length of an ideal string.
- Applied force, Fapp , is a force that is applied to an object by a person or another object. If a person is pushing a desk across the room, then there is an applied force acting upon the object. The applied force is the force exerted on the desk by the person.
- Spring force, Fspring , is the force exerted by a compressed or stretched spring upon any object that is attached to it. The force of an ideal spring stretched or compressed by an amount x is given by Hooke's Law, $F_x = -kx$ .
  - Note that if we are only interested in magnitude, we use F kx = where k is the spring or force constant. Hooke's Law is also used for rubber bands, bungee cords, etc.
- Force of friction, Ff, is the force exerted by a surface as an object moves across it or makes an effort to move across it. There are at least two types of friction force - kinetic and static friction.
  - Though it is not always the case, the friction force often opposes the motion of an object. Friction results from the two surfaces being pressed together closely, causing intermolecular attractive forces between molecules of different surfaces.

2.4: Newton's First Law

- An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by a net external force.
  - Newton's laws are only valid in inertial reference frames that are non-accelerating.
- The ability to resist changes in state of motion is called inertia, which is dependent on mass.
  - Objects with more mass have more inertia.

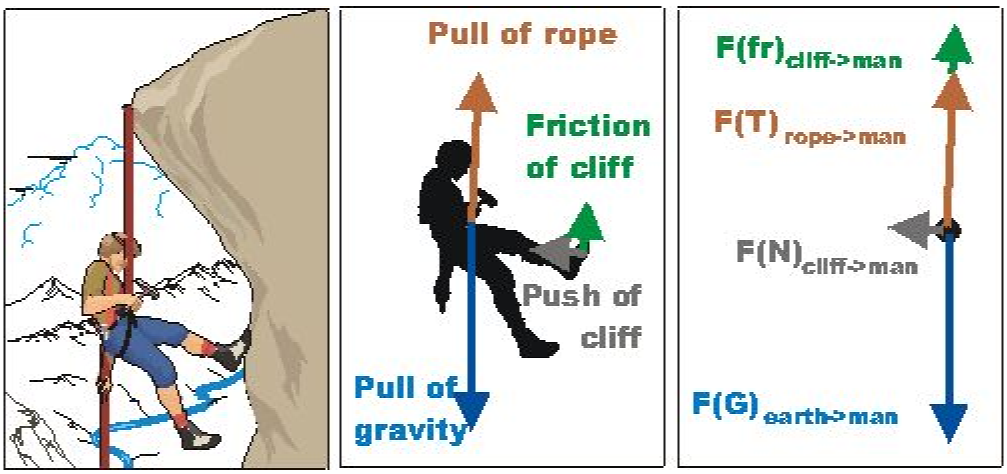



2.5: Newton's Third Law and Free Body Diagrams

- For every action, there is an equal and opposite reaction.
- Forces occur in action/reaction pairs.
- TIPS FOR 3RD LAW:
  - There are no unpaired forces in the universe. Recall that all forces arise from an interaction between two objects.

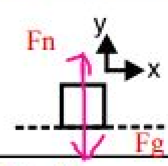
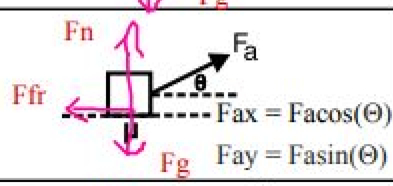
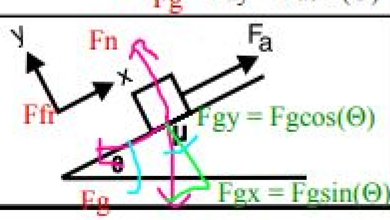
- Action and reaction forces are applied to two different objects. The pair of forces do not cancel each other for this reason.
- One force does not cause the other. Which force is the action and which is the reaction is a matter of perspective. They happen simultaneously.
- Action-reaction force pairs are the same type of force. In other words, the reaction to a normal force must be another normal force, in the opposite direction, on the other surface. For example, a normal force would never be the reaction force to a gravitational force

Ex. What’s the action-reaction pair of a person's weight standing on a horizontal floor?
The normal force of the floor pushing back on the person




A free body diagram consists of the objects in motion with arrows drawn to represent the direction of each force (the direction of the total force, resolving the vectors, or separating each force into components is a pseudo-free body diagram). Ideally, the relative length of each arrow should represent the relative magnitudes of the forces.

<u>Common one body configurations:</u>

| **I. ONE-BODY CONFIGURATIONS** | F-B Diagram (Show only *x*- and *y*-components of all forces acting ON the body) | $\Sigma F_x =$ | $\Sigma F_y =$ |
|---|---|---|---|
| **1.** Frictionless level surface. |  | | Fn - Fg = 0 |
| **2.** Level surface with friction. Applied force at an angle $\theta$. |  | Fax - Ffr = ma<br>*while object is moving | Fn - Fg + Fay = 0 |
| **3.** Incline with friction. Applied force parallel to incline. $F_a > W_{//}$ |  | Ffr + Fgx - Fa = ma<br>or Fa-Ff-Fgx = ma<br>or Fg - Ffr - Fgx = 0 | Fn - Fgy = 0<br>Fn = Fgy |

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**On the AP exam DO NOT DRAW COMPONENTS ON FBD, draw them in a separate FBD!!

## 2.6: Newton's Second Law

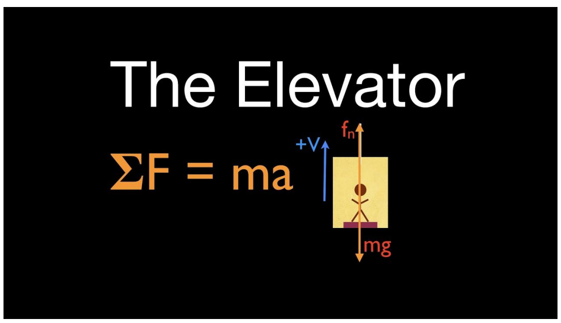
$F_{Net} = ma$

- Force and acceleration are directly proportional. Mass and acceleration are inversely proportional while force is held constant.

If a graph has force on the y-axis and acceleration on the x-axis, the slope is mass.

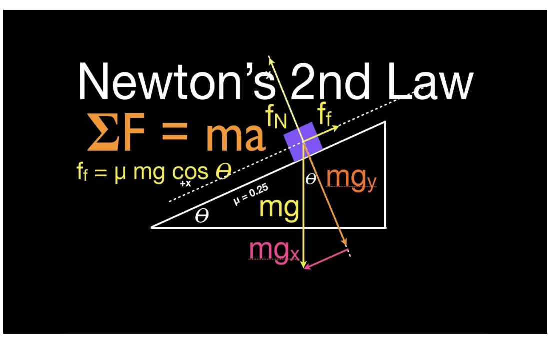
## 2.7: Applications of Newton's Second Law

### Elevators




Scale Reading: the scale reads the normal force.
Upward acceleration means higher normal force, therefore higher scale reading
Apparent Weight on an elevator! Which is also the Normal Force: m(a-g) or m(a+g)

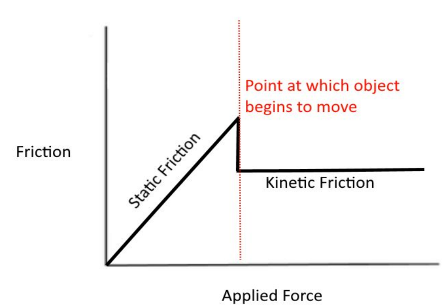
### Inclined Planes




- An object on an inclined plane does not press fully on the plane with its weight. Instead one component of its weight acts parallel to the incline and another component acts down the incline (perpendicular into the incline). To find the normal force, use $F_g cos\theta = F_N$, where theta is the angle between the inline and the ground. $F_g sin\theta = F$ gives the force of gravity acting down the incline. Subtract the frictional force from the Force of gravity
- Steeper angles of inclination will cause more of the force to act down the incline.
  - On the actual AP Exam, do not include components of force in any diagrams.

### Friction

- Friction is caused by microscopic irregularities and opposes motion when objects slide against each other.
- The force of friction is $F = \mu F_N$
  - where Mu (μ) is the coefficient of friction (property of the materials in contact) and $F_N$ is the normal force (the force presses the objects against each other).



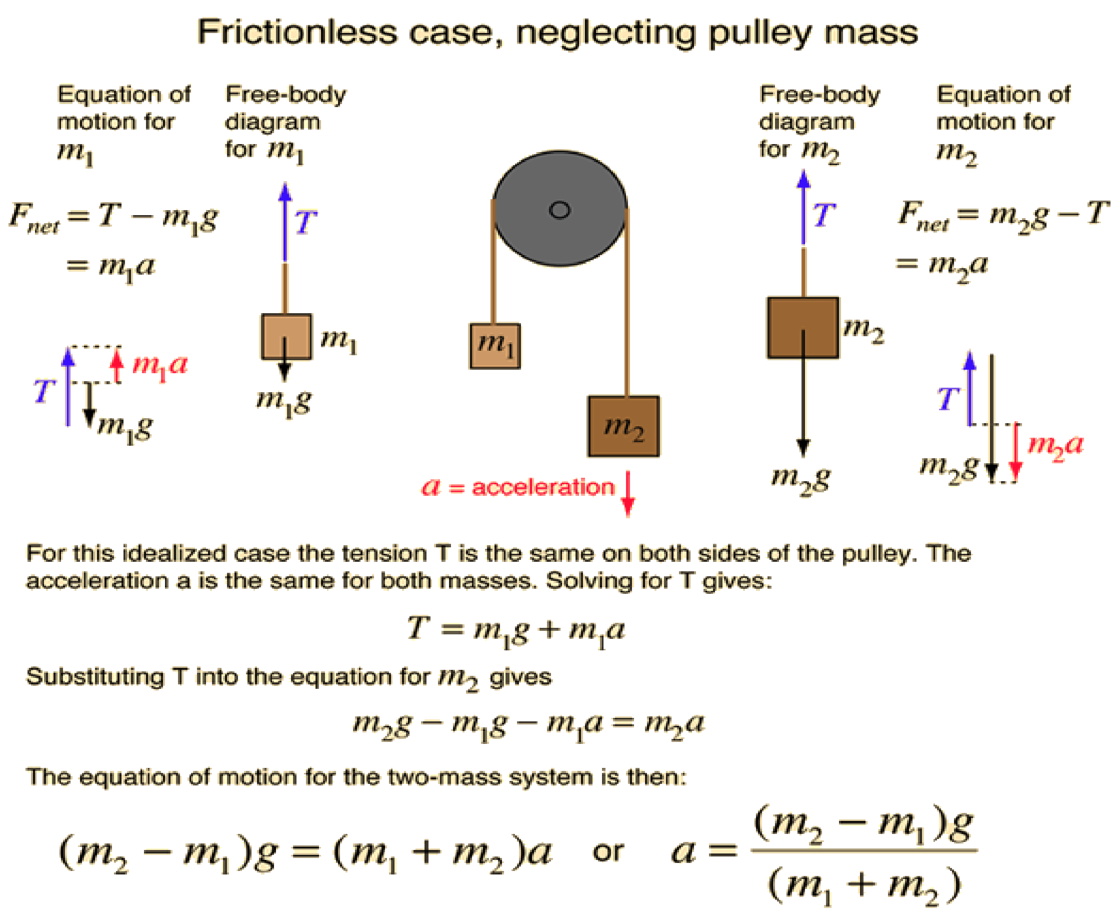
  - Note that μ varies depending on if the object is in motion or not
- In order for an object to move under conditions of friction, the force applied must be large enough in magnitude to overcome the static friction.

<u>Static Equilibrium</u>

- Either the object has no forces acting upon it, or the forces are perfectly balanced and in equilibrium
- Objects in static equilibrium are in both translational equilibrium and rotational equilibrium.
  - Equilibrium can be at rest or moving/rotating with constant velocity.

<u>Atwood Machines</u>

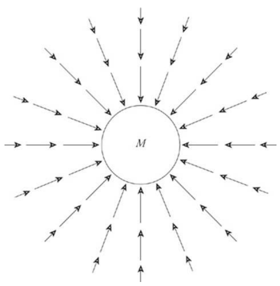
- An Atwood Machine is a basic physics laboratory device often used to demonstrate basic principles of dynamics and acceleration. The machine typically involves a pulley, a string, and a system of masses.
  - Keys to solving Atwood Machine problems are recognizing that the force transmitted by a string or rope, known as tension, is constant throughout the string, and choosing a consistent direction as positive.
- The net forces within the system divided by the total masses present can allow you to solve for the acceleration of the system (See example below)



# Unit 3: Circular Motion & Gravitation

## 3.1: Vector Fields

- Vector fields are a function of position
- Use vectors in a field to describe a physical quantity with direction and magnitude




- In physics I, refers to gravitational fields, so all of the field vectors will be pointing in towards the central mass since gravity is always attractive (as seen above)
- If there's multiple source objects creating fields, the net field can be determined by vector addition
- Vector fields can be used to study the size, location, and number of sources
- Magnitude of force based off of size of the arrow (larger arrow=larger force)

## 3.2: Fundamental Forces

- Fundamental forces: forces that cannot be reduced to more basic interactions
    - 4 known fundamental forces: gravitational, electromagnetic, strong, weak
    - Physics I focuses on gravitational
- Gravitational forces are exerted no matter how small the mass(es) are and no matter how far away
- On astronomical scales (where masses are large and their distances are far apart), gravitational forces dominate

## 3.3: Gravitational and Electric Forces

- Gravitational force is the force of attraction that one object of mass $m_1$ has with another object of mass $m_2$ where these objects have centers that are $r$ apart
- Gravitational force formula:

$$\left|F_g\right| = G\frac{m_1 m_2}{r^2}$$

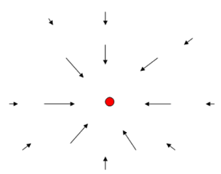
- Gravitational field formula:

$$\vec{g} = \frac{\vec{F}_g}{m}$$

this is equivalent to $\vec{g} = \frac{-GM_{object}}{r^2}\hat{r}$ where $\hat{r}$ is the direction from the object ($M_{object}$) to your test mass $m$. Thus this can also be $\vec{g} = \frac{GM_{object}}{r^2}\hat{r}$ where $\hat{r}$ is in the opposite direction.
- In theory, gravitational field should change with distance, but for certain ranges of heights above Earth's surface, $g$ can assumed to be approximately constant

## 3.4: Gravitational Field/Acceleration Due to Gravity on Different Planets

- Gravitational field g at the location of mass m will cause a force of mg to act on the object through $\vec{g} = \frac{\vec{F}_g}{m}$
  - On earth, this is weight
- Gravitational field in space would be found by using a test mass and recording the force it experiences and using g = F/m
- Gravitational field caused by a spherically symmetric object with mass is radial
  - It weakens and is inversely proportional to the square of the radial distance (as we know from the F = Gmm/$r^2$) equation



## 3.5: Inertial vs. Gravitational Mass

- Gravitational mass determines the force of gravity on an object/system.
  - $m$ is defined so that $\vec{F}_g = m\vec{g}$
- Inertial mass is the object's resistance to acceleration
  - $m$ is defined so that $\vec{F} = m\vec{a}$
- Amazingly, these two things are the same!

## 3.6: Centripetal Acceleration and Centripetal Force

- Centripetal acceleration is need to keep an object in circular motion
- This is always pointing towards the center ⇒ doesn't change the object's speed, only direction
- Given by $a_c = \frac{v^2}{r} = \omega^2 r$ where $\omega = \frac{v}{r}$ the second is useful when given period→ $\omega = \frac{2\pi}{T}$
- Since $F = ma$ , $F_{a_c} = ma_c = m\frac{v^2}{r} = m\omega^2 r$
- v=2πrf(revolutions per second)

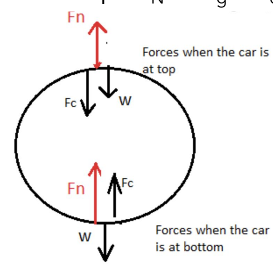
## 3.7: Free-Body Diagrams for Objects in Uniform Circular Motion

- The only force if the object is in <u>uniform</u> circular motion should be one of centripetal acceleration
- Make sure to say what force it is,(eg. $F_g$ not $F_{a_c}$)
- Centripetal acceleration always points to the center.

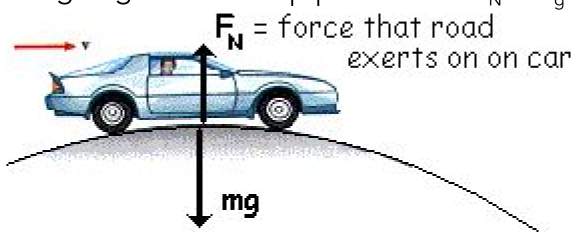
## 3.8: Applications of Circular Motion and Gravitation

- Ferris wheel problem ⇒ at the bottom, $F_N = F_g + F_c = mg + mv^2/r$

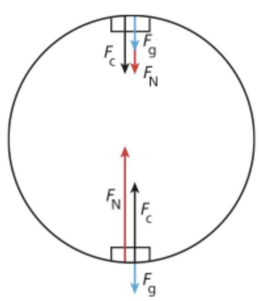
- at the top, $F_N = F_g - F_c = mg - mv^2/r$

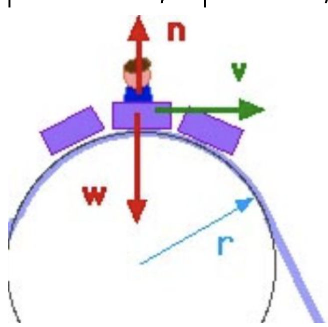



- Car going over a bump problem ⇒ $F_N = F_g + F_c = mg - mv^2/r$

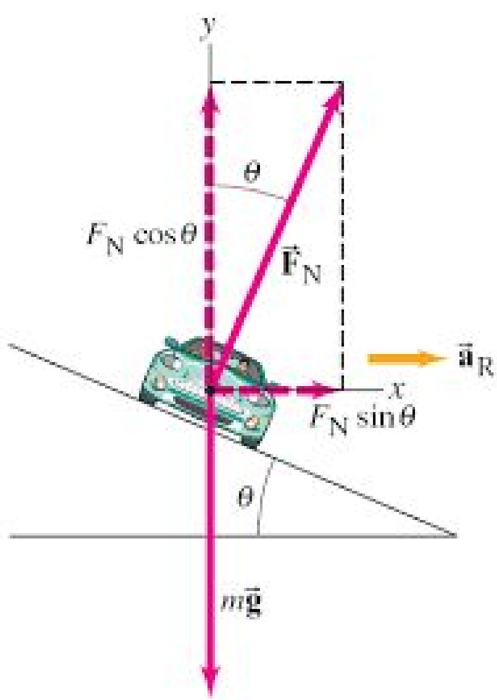



- Rollercoaster problem ⇒ see diagram for the forces and substitute accordingly depending on where the rollercoaster is (upside down, top of a hill, etc)

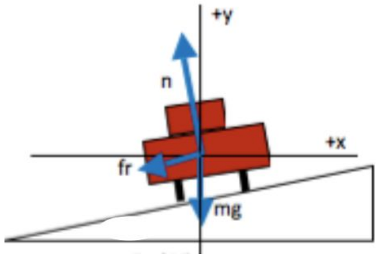


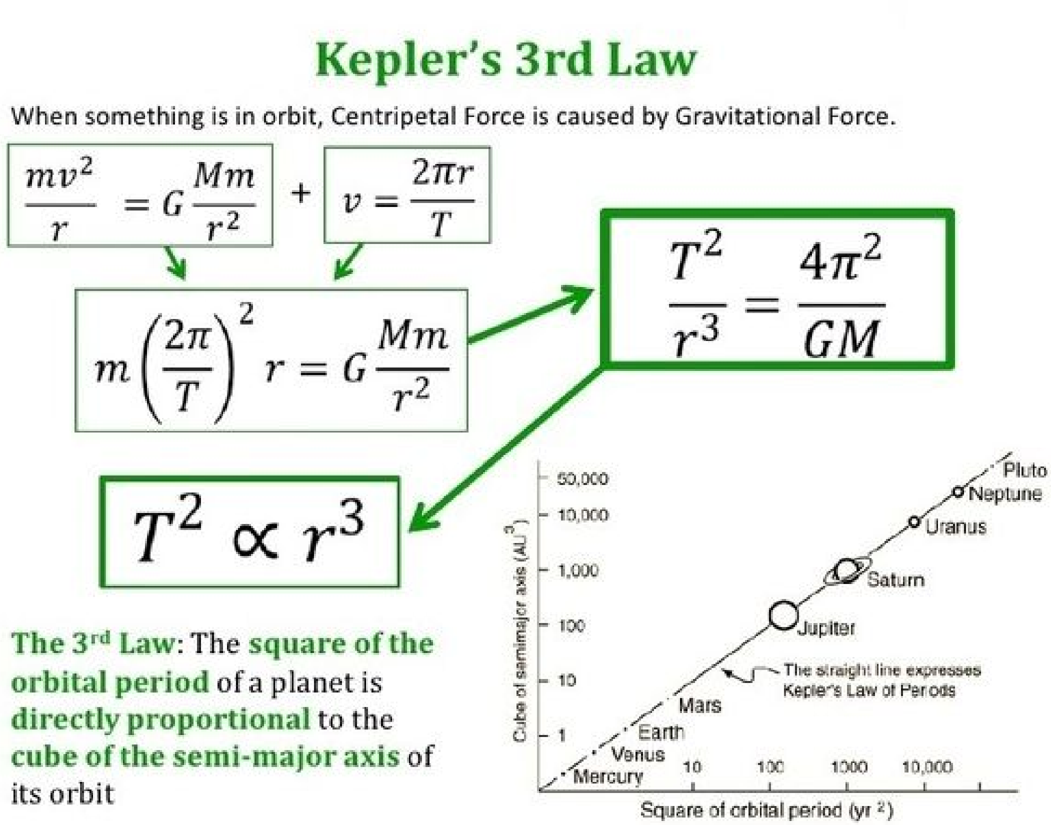

- Banked road WITHOUT FRICTION ⇒ $F_c = F_N * \sin\theta$ ; $F_N * \cos\theta = F_g$



- Banked road WITH FRICTION ⇒ $F_c = F_N * \sin\theta + F_f * \cos\theta$ ; $F_N * \cos\theta = F_g$




- Satellite example yay ⇒ set centripetal force and gravitational force equal to each other
  - $F_{a_c} = ma_c = G\frac{mM_E}{r^2} \Rightarrow a_c = G\frac{M_E}{r^r} = \frac{v^2}{r} \Rightarrow v = \sqrt{G\frac{M_E}{r}}$ so also $\omega = \sqrt{G\frac{M_E}{r^3}}$

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## Unit 4: Energy

### 4.1: Open and Closed Systems

- This is a smaller portion but it is very important
- Systems are a group of objects
- Closed systems have quantities that are constant
- Open systems are able to exchange quantities with their surroundings
- Energy transfer depends on the system

## 4.2: Work and Mechanical Energy

- Kinetic Energy - object in motion.
  - Translational Kinetic Energy $KE_{trans} = \frac{1}{2}mv^2$
  - Rotational Kinetic Energy $KE_{rot} = \frac{1}{2}I\omega^2$
- Gravitational Potential Energy - Two situations
  - $PE_{grav} = mgh$ for an object close to the surface of the earth
  - $PE_{grav} = \frac{GmM_E}{r}$ for an object r away from the center of the earth
    - Usually looked at for large differences in r
- Spring Potential Energy
  - $PE_{spring} = \frac{1}{2}kx^2$ - energy in spring during compression or extension from either the natural length or the point where force is zero
  - You can use the point where force is zero since you are only comparing the two values. ⇒ useful for vertical spring problems
- Mechanical Energy is energy derived from KE or PE. It is also the ability to do work. The energy can be converted to displace another object. Ex: A wrecking ball is brought to a high position and then swung forward to displace a wall.
- Total Mechanical Energy $E = KE_{trans} + KE_{rot} + PE_{grav} + PE_{spring}$ . Friction force is usually an external force and can cause a loss in energy.

Mechanical Energy Problems:
- Define the system (define it such that parts of the mechanical energy balance can be 0)
- Set up the mechanical energy equation balance and solve for the unknown
- Note that time CANNOT be found from energy or a mechanical energy balance
  - Energy looks at start and end states
  - Transfer information to kinematics or dynamics when question is asking for time
- When asked about changes in mechanical energy, consider mechanical and non-mechanical forces
  - Conservative forces are forces in which the path does not matter. For example, gravitational potential energy doesn't change if a crane lifts a ball straight up or swings it around while it lifts the ball)
    - Work done by it when moving in a circle is 0
      - Usually a function of position ⇒ gives one force output per one position input
    - Conservative forces DO NOT impact total mechanical energy
  - Non-Conservative forces are path-dependent. Work done by these forces are also path-dependent. Different paths result in different amounts of work via non-conservative forces. For example: friction increases work if a longer path is taken
    - When non-conservative forces are in a mechanical energy balance, the work is included on the "initial side"
    - Non-Conservative forces DO impact total mechanical energy

## 4.3: Conservation of Energy, the Work-Energy Principle, and Power

$$E_i = E_f$$

Total energy in an isolated system must be conserved:

- To solve problems requiring transfer of energy: $PE_i + KE_i + U_{si} = PE_f + KE_f + U_{sf}$

$$\overbrace{mgh}^{\text{Potential Energy}} = \overbrace{\frac{1}{2}mv^2}^{\text{Kinetic Energy}}$$

$$gh = \frac{1}{2}v^2$$

$$2gh = v^2$$

$$v = \sqrt{2gh}$$

Work:

Work is the product of the magnitude of the displacement times the component of the force parallel to the displacement (units: N*m or Joules). Equation: $W = F\Delta x \cos(\theta)$

- Alternatively, the dot product of force and displacement $W = F \bullet d$

- Note here that Work is NOT a vector, and that force in this equation is the magnitude
- Also note that work is done in parallel direction to displacement,
  - Force applied perpendicular to directed motion does ZERO work
- Force can be applied while NO work is being done ($\Delta x = 0$) (e.g: pushing against a wall exerts a force but the wall doesn't move)(cos( $\theta$ )=0, perpendicular to displacement)
- Work can be negative (friction)
- Net work is where the net force and thus net acceleration will be directed. This does not have anything to do with the movement of the object or its velocity, only its acceleration

Work-Energy Theorem:

Work can equal the change in energy (can be kinetic, potential, elastic, etc).

E.g. $W_{net} = \Delta KE = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m{v_1}^2$

- *Remember that it can be equal to any energy and not just W=Fd. If you don't realize this for some scenarios you'll waste time. This is a very common mistake to forget this!*
- Note that the net work *can* affect conservative forces.

$$W_{NC} = \Delta KE + \Delta PE$$

Work Problems:

- Usually these problems require finding the net work at some point… which is done as follows:
  - Determine the net force using a free-body diagram

- Then, multiply the component of the net force that's parallel to the displacement by the displacement, just like you would when finding the work done by any force.
  - If needed, use trigonometry to find parallel displacement
  - Note that work is only done if the object moves parallel to force.
- Plug and chug into work equation
- Using the work-energy theorem is especially useful when force is changing (e.g: a bow is drawing back an arrow or springs). When an outside force acts on a system using a spring or some sort of changing force, ALWAYS use the work-energy theorem
  - Pick the beginning and end points of the balance wisely (probably so that some variable ends up being 0)
  - Solve for the unknown
- Know when to use W=Fd versus the work-energy theorem
  - When velocities and some more info is given (or non-uniform accel) use work-energy theorem
  - Otherwise usually use W=Fd… DON'T waste 10 minutes solving it one way only to realize you need to do it the other way… be aware that both methods exist
  - Some problems may require you to use both,(eg. Find the displacement given the average force per displacement is ___)

Power

Power is the rate at which work gets done or energy gets transferred (units is J/s or Watt)

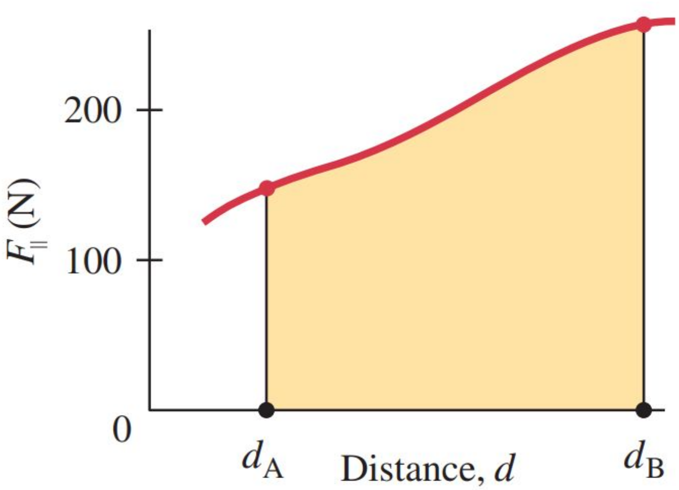
$$P = \frac{W}{\Delta T} = \frac{F\Delta x}{\Delta T} = Fv$$

Where $v$ is velocity

- Note that power is also the change in energy per change in time $P = \frac{\Delta E}{\Delta t}$

Power Problems:
- Identify what you know, and plug it into one of the equivalents of power above
- Sometimes use two equivalents to solve for an unknown instead of power

Force vs. Displacement Graphs:



- Y-intercept: Force when x=0
- Area under curve: amount of work done between two points of displacement. Positive area is positive work, and vice versa.
- For example spring forces where the graph is F=-kx would create a straight line
- If asked for work, split up curve into simple shapes and find individual area (approximate when needed)
- When force is constant, the area under the curve is just a rectangle/square.

## Unit 5: Momentum

### 5.1 Momentum and Impulse

- Linear momentum is defined as the product of its mass and velocity. SI unit = kg x m/s
- Momentum Equation:

$$p = mv$$

- Momentum is a vector, so any change in direction changes momentum
- Impulse-Momentum Theorem:

$$J = F\Delta t = \Delta p = mv_f - mv_0$$

- Change in momentum is equal to impulse. This can be further represented by

$$p_{initial} + J = p_{final}$$

$$mv_0 + F\Delta t = mv_f$$

- Law of Conservation of Momentum: The total linear momentum of an isolated system remains constant.
    - This implies that there is no net change in momentum… this can be represented by:

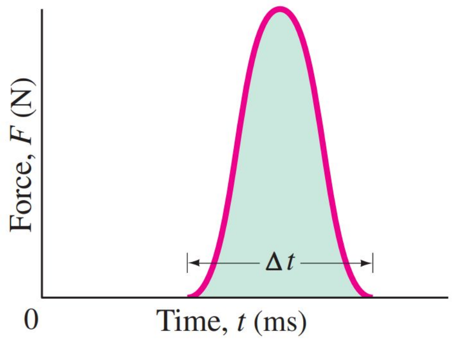
$$p_{initial} = p_{final}$$

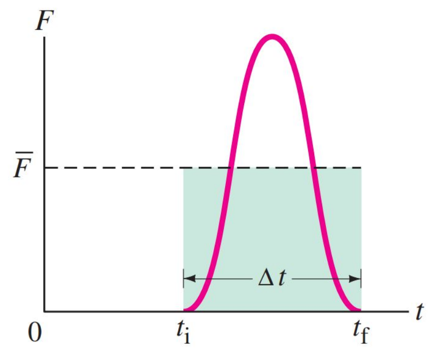
### 5.2 Momentum Problem Approach

1. Define the system:
    - Usually include everything that has quantifiable givens in the problem so that any forces exerted are internal forces
        - E.g two cars collide with each other and the mass & velocities are given . Everything should be included in the system because numbers are known for all objects in the system.
    - Internal forces on a system can occur and still have momentum conserved
    - Or define the equation as one object and have an external force apply the impulse (for example a wall if a ball hits the wall, treat wall as external)
2. Use the Impulse-Momentum Theorem Equation
    - If time is given, then Force can be figured out through impulse
    - Force is ALWAYS in the direction of the impulse and can cause a direction change of the object)

3. Remember that momentum is a vector, so even if the speed (mag.) is the same before and after a collision, the direction could be different (that's where impulse matters)

## 5.3 Representations of Changes in Momentum



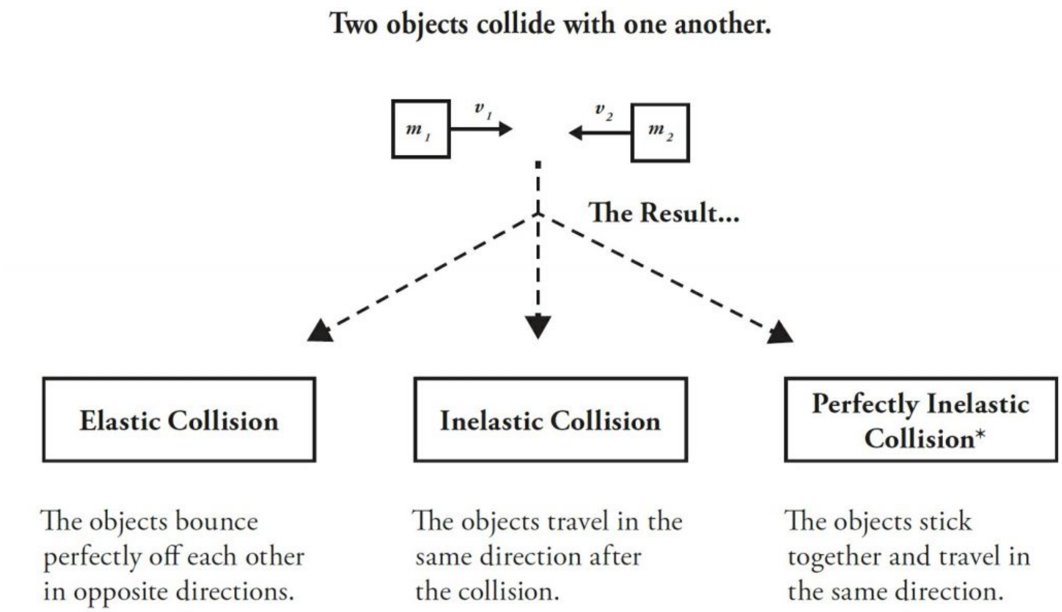

Analyzing Force Vs. Time Graphs:

- Y-intercept: Initial force in system.
- Area under curve represents Impulse. Impulse is the change in momentum, so the area under the curve will give impulse (usually choose $t_1$ and $t_2$ for $\Delta t$ to be when F=0)
- Slope of graph is unimportant
- The Average Force multiplied by the Δt will also give the same answer as impulse, so the graph can be used to find the average force as well

## 5.4 Open and Closed Systems: Momentum

- For closed systems, momentum stays constant because outside factors are kept constant so there is no external force or influence.
- For open systems, momentum can change since outside factors are NOT controlled, so there can be external forces acting.

## 5.5: Collisions




# Collisions

- *Elastic collisions:* $K_1+K_2=K_1'+K_2'$ & $\mathbf{p}_1+\mathbf{p}_2=\mathbf{p}_1'+\mathbf{p}_2'$

$$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2}m_1v_1'^2+\frac{1}{2}m_2v_2'^2 \quad \&$$
$$m_1\mathrm{v}_1+m_2\mathrm{v}_2=m_1\mathrm{v}_1'+m_2\mathrm{v}_2'$$

- *Inelastic collisions:*

$$\mathbf{p}_1+\mathbf{p}_2=\mathbf{p}_1'+\mathbf{p}_2' \quad \text{or}$$
$$m_1\mathbf{v}_1+m_2\mathbf{v}_2=m_1\mathbf{v}_1'+m_2\mathbf{v}_2'$$

- ***Perfectly** inelastic collisions:*

$$\mathbf{p}_1+\mathbf{p}_2=\mathbf{p}_1'+\mathbf{p}_2' \quad \text{or}$$
$$m_1\mathbf{v}_1+m_2\mathbf{v}_2=(m_1+m_2)\mathbf{v}_2'$$

MOMENTUM IS CONSERVED FOR ALL SCENARIOS

Elastic Collision:
- Elastic collisions imply that Kinetic energy is conserved in elastic collisions only
- Because kinetic energy is conserved, set up a system of equations with kinetic energy and momentum to solve for the unknowns

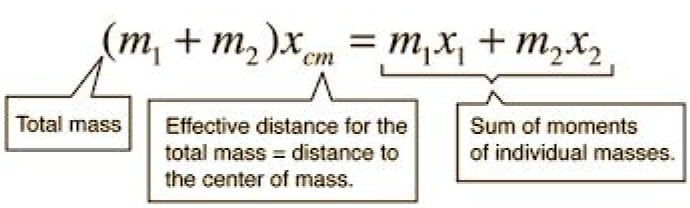
Inelastic Collision:
- KE not conserved

Perfectly Inelastic Collision
- The velocity is the same for BOTH objects after (since they stick together)... so one less value is needed to solve for any other value (also factor out velocity when solving these problems)
- KE not conserved

In general, whenever one cart is at rest, the problem requires 1-2 less knowns to solve for another variable (if there is a way, limit the problem to the scenarios above to solve for knowns) (example: if there is a kinematics problem right after a collision, first solve the collision to find the velocity, then that velocity could be the initial velocity used to find range or friction or something else)

2-D Collisions:
- When objects move in both an x- and a y-direction after a collision, analyze the collision with momentum conservation separately in each direction; break the problem down into x and y components
- **Combine like vectors.** Momentum is conserved in x and y direction independently

## 5.6: Center of Mass



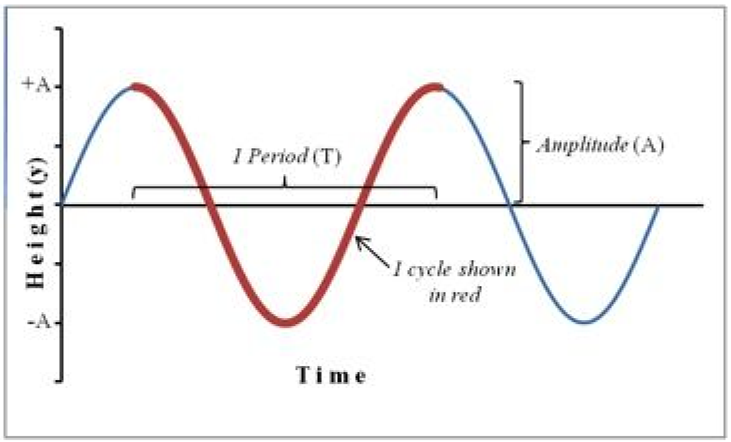

This can be used to find COM, but set it up so that $x_1$ and $x_2$ are from the pivot point.
(Add example from 2018 mc ap Physics C exam (somewhere from 30-35)

# Unit 6: Simple Harmonic Motion

Basics: Simple Harmonic Oscillators are things that move in a back and forth motion that are periodic. Oftentimes, these are in contrast to translational motion because while translational motion is moved away from its resting position by a force, the object continues in the original direction, vibrational motion moves about a fixed point which is it's resting position. SHM is the cause of restoring forces which attempt to bring an object back to equilibrium. Velocity when the object is at zero displacement is at maximum while velocity at zero when the object is displaced the farthest from equilibrium. Acceleration of the object is vice versa.

## 6.1 Period of Simple Harmonic Oscillators

A. Parts of a simple harmonic pendulum




Note: This is called a sine wave.

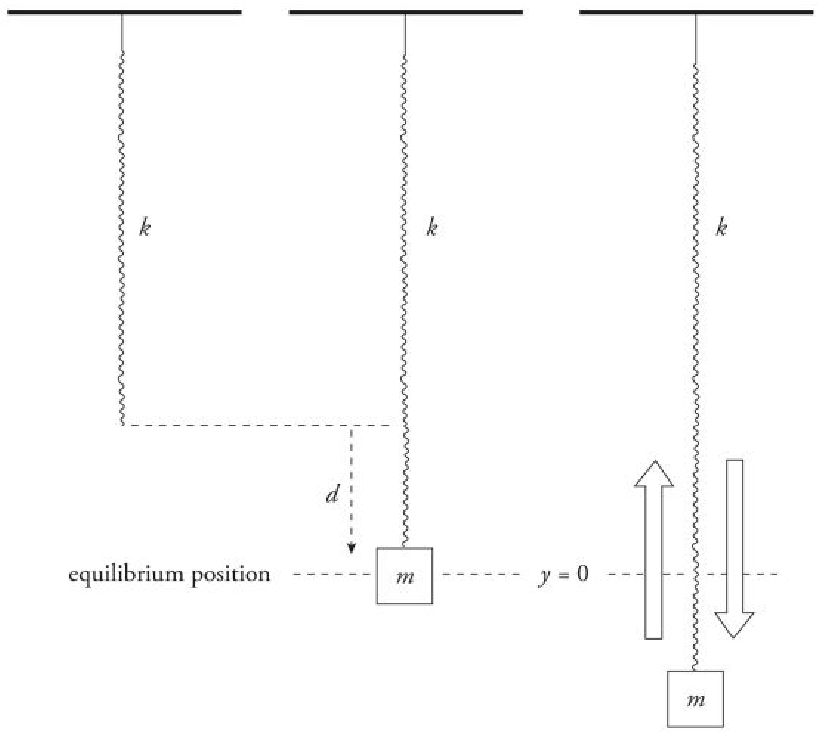
- Period ( $T$ )- The amount of time it takes to complete a cycle is called the period.
    - Represented in seconds (s)
    - Inverse of Frequency ( $f = \frac{1}{T}$ )
- Frequency ( $f$ or sometimes $v$) - The number of cycles that can be completed per second is called the frequency
    - Represented in hertz (hz)
    - Inverse of Period ( $f = \frac{1}{T}$ )
- Wavelength (ƛ) - the distance from any point on a wave to the same point on the next wave
    - Represented in meters (m)
- Amplitude
    - Represented in meters (m)
    - Farthest distance from equilibrium point
- Displacement
    - Represented in meters (m)
    - How far the mass is from the equilibrium point

B. Spring Oscillations

- The force output by a spring is represented by $F = -kx$
    - k is the spring constant
    - x is the distance from the equilibrium point
    - The minus sign on the force indicates that it is a restoring force, in that it wants to go back to the equilibrium point
    - The force isn't constant, so neither is acceleration

- If the spring is hung vertically upside down, then the equilibrium point must change to account for gravity

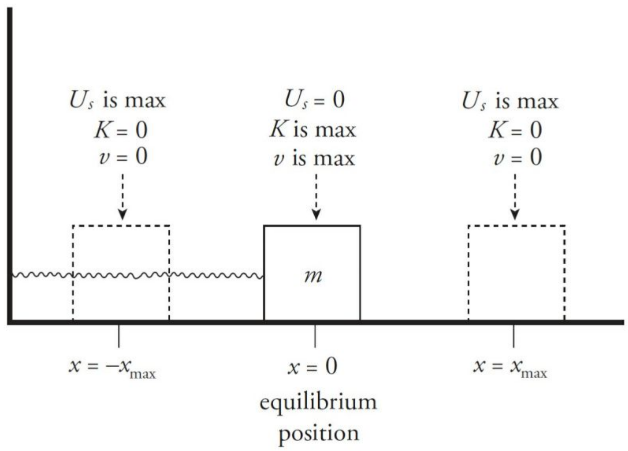
$$kd = mg \quad \Rightarrow \quad d = \frac{mg}{k}$$




  - When gravity is impacting the spring, there is a “new equilibrium” that is simply the length of the spring plus the distance the mass pulls the spring down (equation in pic above)

## 6.2 Energy of a Simple Harmonic Oscillator

- The elastic potential energy equation for a spring is $U_s = \frac{1}{2}kx^2$
  - Max potential energy is when x is the amplitude
- When stretching the block, elastic potential energy is increased. When the block is let go, the potential energy is converted to kinetic energy.
  - At endpoints, KE=0 and PE=max. In equilibrium KE=max and PE=0.




| | $x = -A$ | $x = 0$ | $x = +A$ |
|---|---|---|---|
| Magnitude of Restoring Force | MAX | 0 | MAX |
| Magnitude of Acceleration | MAX | 0 | MAX |
| Potential Energy (U) of Spring | MAX | 0 | MAX |
| Kinetic Energy (K) of Block | 0 | MAX | 0 |
| Speed (v) of Block | 0 | MAX | 0 |

- When calculating maximum velocity, do the following:
    - Max elastic potential energy is converted to max kinetic energy, so find max kinetic energy and find velocity from there:
- If there are multiple springs, draw the FBD, and add the forces together or represent them as distances and add distances together in order to make it seem as if only one spring was present.

$$U_{S,\,max} = \frac{1}{2}kx_{max}{}^2 = \frac{1}{2}kA^2 = K_{max} = \frac{1}{2}mv_{max}{}^2 \qquad v_{max} = \sqrt{\frac{kA^2}{m}}$$

- This gives max KE, PE, velocity, and Total energy of system

Spring Info:
- Frequency and period can be determined from the mass of the block and the force constant of the spring. The equations are as follows:

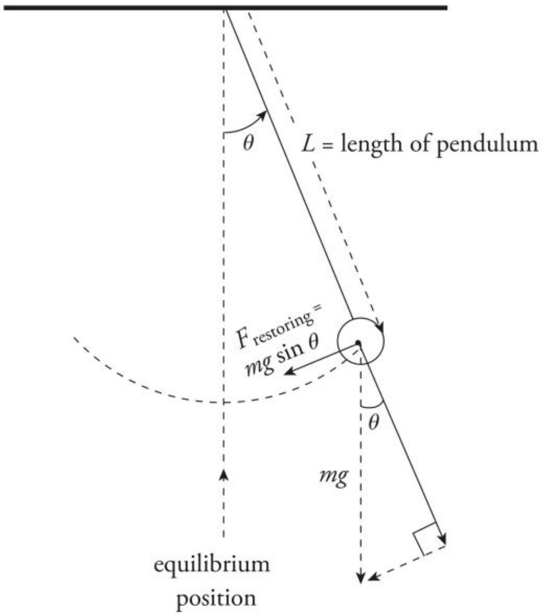
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

-

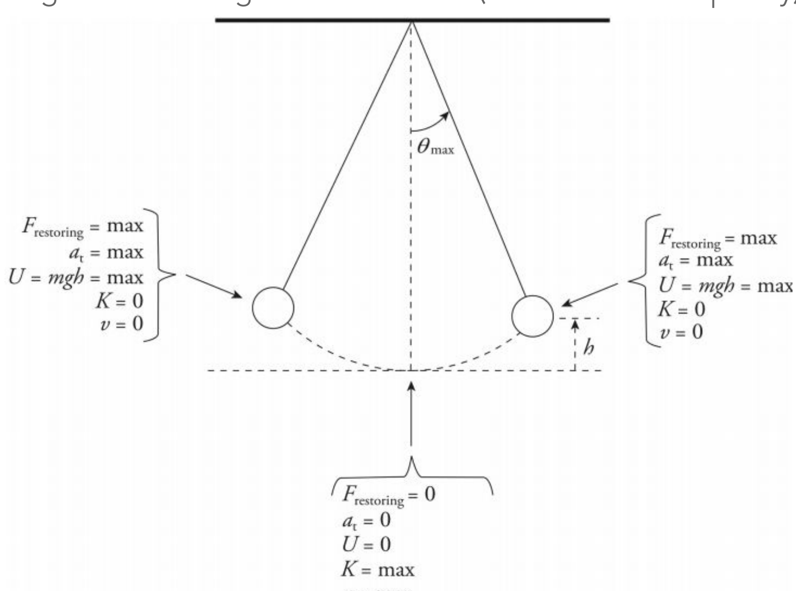
$$T = 2\pi\sqrt{\frac{m}{k}}$$

-
- The period of a mass-spring oscillator increases with mass and decreases with spring stiffness. (vise versa for frequency)

Pendulums:
- Pendulums are mainly driven back and forth by the restoring force of gravity




- Displacement is zero at the equilibrium position.
- At the endpoints of the oscillation region (where $\theta = \pm\theta$ max ), the restoring force and the tangential acceleration (at) have their greatest magnitudes, the speed of the pendulum is zero, and the potential energy is maximized.
- As the pendulum passes through the equilibrium position, its kinetic energy and speed are maximized.
- For a simple pendulum, the period increases with the length of the pendulum and decreases with the magnitude of the gravitational field. (vise versa for frequency)




- Frequency and period for Pendulums can be seen with the equation as follows:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

- For a simple pendulum, the period increases with the length of the pendulum and decreases with the magnitude of the gravitational field. (vise-versa for frequency)

Everything they could ask for SHM:
- Minima/Maxima/Zeros
  - Look at diagrams and solve
- Force given displacement
  - Understand where the net force is in a SHM. At the ends, the force is directed towards the opposite end.
- Acceleration given displacement
  - Same direction as force (F=ma).

# Unit 7: Torque & Rotational Motion

## 7.1: Rotational Kinematics

A. Definitions:
   a. Angular displacement $\theta$ indicates the angle through which an object has rotated $[rad]$.
      i. 1 rev = 360° = 2π radians ≈ 6.28 radians
      ii. Be familiar with switching between them!
   b. Average angular velocity $\omega$ is angular displacement divided by the time interval over which that angular displacement occurred $[rad/s]$.
   c. Angular acceleration $\alpha$ tells how much an object's angular speed changes in one second $[rad/s^2]$
   d. Angular acceleration and centripetal acceleration $a_c$ are independent. $\alpha$ changes an object's rotational speed, while $a_c$ changes an object's direction of motion.

B. Analogous Relationship between U$\alpha$M and UAM
   a. Kinematic Equations $[\alpha = constant,\ a = constant]$

| | |
|---|---|
| $v_f = v_0 + at$ | $\theta_f = \theta_0 + \alpha t$ |
| $\Delta x = v_0 t + \frac{1}{2}at^2$ | $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ |
| $v_f^2 = v_0^2 + 2a\Delta x$ | $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$ |
| $\Delta x = \frac{1}{2}(v_f + v_0)t$ | $\Delta\theta = \frac{1}{2}(\omega_f + \omega_0)t$ |

   b. Tangential displacement, speed, and acceleration
      i. For any angular displacement $\Delta\theta$, where $s$ is the linear displacement, $s = r\Delta\theta\ [m]$
      ii. The linear (tangential) speed of an object rotating w/o slipping is given by $v = r\omega\ [m/s]$
      iii. The linear acceleration of an object rotating is given by $a = r\alpha\ [m/s^2]$

## 7.2: Torque and Angular Acceleration

A. Torque [N*m] is more commonly known as the product of the lever arm *r* and the force *F*, provided that both the force and lever arm have a component perpendicular to each other.
   a. Can be expressed as: $\tau = rF_{\perp}$ or $\tau = r_{\perp}F$ or $\tau = rF\sin\theta$ (where $\theta$ is the angle between the radial arm and the force vector)
   b. The further the force is applied from the axis of rotation the greater the torque and vice versa
   c. Torque can be applied to Newton's Second Law For Rotation to find either angular acceleration or moment of inertia $\tau = I\alpha$
   d. In order for an object to be in rotational equilibrium, the net torque must be equal to 0 (similar to net forces)
   e. SI units for torque are N*m (NOT J = N*m because a joule (J) is only for work/energy, never torque)
   f. The direction for angular (vector) quantities is determined by the "right-hand rule"
   g. If $\Sigma\tau \neq 0$ the object will (angularly) accelerate in the direction of the greater torque

B. Rotational Inertia (I)
   a. A quantity expressing an object's ability to resist angular acceleration $[kg * m^2]$
   b. For a point mass, the rotational inertia is given by the particle mass times it's distance from its axis of rotation squared, or $I = mr^2$.
   c. For systems of point masses, the rotational inertia is simply the sum of the rotational inertia of each particle, relative to the center of mass $(I = \Sigma mr^2)$

C. Angular Acceleration $(\alpha)$
   a. The rate at which the angular speed changes in one second $[rad/s^2]$
   b. Refer to angular kinematics equations above to see relationships with other variables
   c. Angular acceleration can be translated to linear acceleration by using the equation $a = r\alpha$
   d. Can be used to find net torque or rotational inertia of an object using $\Sigma\tau = I\alpha$
   e. All points on an object have the same angular acceleration but DO NOT HAVE THE SAME LINEAR ACCELERATION
      i. The linear velocity of a rotating object changes with the radius

D. Rotational Kinetic Energy $(KE_{rotational})$
   a. Calculated with $KE_{rot.} = \frac{1}{2}I\omega^2$ [J]
   b. Objects with greater rotational inertia will convert more of their potential energy into rotational kinetic energy rather than translational KE
      i. Objects rolling down an incline: an object with a larger rotational inertia will convert more potential energy into $KE_{rot.}$ instead of $KE_{translational}$, which results in a lower linear velocity.

## 7.3: Angular Momentum and Torque

A. Angular Momentum *L* is quantity of rotation something has around a body
   a. Equations

| | |
|---|---|
| $L = mr$ | $v\perp = \omega r$ |
| $L = m\omega r^2$ | $\omega = \frac{v\perp}{r}$ |
| $L = P\perp r$ | $\Delta t = \Delta L$ |
| $L = I\omega$ use this for non point mass systems always. The formula for I will change depending on the object | |

b. If there is no net torque acting on a system ($\Sigma = 0$) then there can be no change in angular momentum

c. If it is not a point mass system, all points on the object must be calculated together.

i. USE $L = I\omega$ TO CALCULATE THIS

1. $I$ will change depending on the mass distribution of the object. Objects with mass further from the axis of rotation have a greater value for $I$ and thus have a greater tendency to oppose a change in motion. On the other hand, objects with most of their mass at the center of rotation have a smaller value of $I$ and are less likely to oppose a change in motion.

a.

| | |
|---|---|
| Thin Hoop | $MR^2$ |
| Solid Cylinder | $\frac{1}{2}MR^2$ |
| Uniform Sphere | $\frac{2}{5}MR^2$ |
| Rod (axis through the center) | $\frac{1}{12}ML^2$ |
| Rod (axis at one end) | $\frac{1}{3}ML^2$ |
| Satellite (point away from center) | $MR^2$ |

B. To get the angular momentum of a point mass

| | |
|---|---|
| $L = mvr * sin\theta$ | $L = mvr$ |

a. r is the distance from the point mass to an axis (the point of closest approach).

## 7.4: Conservation of Angular Momentum

A. For collisions between a point mass and a rod
   a. Find the angular momentum of the point mass (see 7.3, B)
   b. If the rod is stationary at the beginning, it can be assumed it has 0 torque because no force is being exerted on it
      i. This means angular momentum has to be the same at the end of the system
         1. (see 7.3, A point b)
         2. $L_i = L_f$
   c. If the ball stops on contact
      i. $mvr = I\omega$
         1. Because all the momentum of the point mass is being transferred to the other object, the ball no longer has any angular momentum
   d. If the ball bounced back
      i. You have to add the angular momentum of the ball after contact to the equation
         1. $mvr = I\omega + m(-v)R$

# Resources

AP Teachers Youtube
JacobsPhysics Fact Sheet
MIT Practice
Practice Exams
Giant list of physics resources
Fat pdf of physics notes
AP Physics 1 Formula Sheet
PhET simulations (simulates labs)
Physics B Questions (Solutions) (Only do questions marked 1)
5 steps to a 5 pdf
PRINCETON REVIEW PDF 2020 edition!
Physlets
edX Physics
The Physics Classroom

## Free Youtube Channels

Flipping Physics
Bozeman Science
Organic Chemistry tutor
Khan Academy Physics

## Review of all topics (includes video lectures):

https://bit.ly/2JpM2EN

## Previous year FRQs:

2019 FRQ Questions
2018 FRQ Questions
2017 FRQ Questions
2016 FRQ Questions
2015 FRQ Questions