

ESTIMATE_POSE_RANSAC.PY OUTPUT

IKENNA ACHILIHU

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PROJECT II PHASE II

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Derivation

①

$$\underline{\text{Eq 1:}} \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \left[\left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right] - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} u_1' \\ v_1' \\ 1 \end{pmatrix} \right]$$

$$\underline{\text{Eq 2:}} \quad S = \begin{pmatrix} 1 & 0 & -u_1' \\ 0 & 1 & -v_1' \end{pmatrix} \left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right]$$

sol'n

$$\Rightarrow \underline{R}_{1/2} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right]$$

Substitute

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \left[\underbrace{\left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right]}_{3 \times 1} - \underbrace{\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right]}_{1 \times 3} \begin{pmatrix} u_1' \\ v_1' \\ 1 \end{pmatrix} \right]$$

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$\Rightarrow \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} = \begin{pmatrix} r_{11}u_2' + r_{12}v_2' + r_{13} \\ r_{21}u_2' + r_{22}v_2' + r_{23} \\ r_{31}u_2' + r_{32}v_2' + r_{33} \end{pmatrix} = R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix}$$

$$d_2' T = d_2' \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \begin{pmatrix} d_2' t_1 \\ d_2' t_2 \\ d_2' t_3 \end{pmatrix} = d_2' T$$

$$\Rightarrow R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T = \begin{pmatrix} r_{11}u_2' + r_{12}v_2' + r_{13} + d_2' t_1 \\ r_{21}u_2' + r_{22}v_2' + r_{23} + d_2' t_2 \\ r_{31}u_2' + r_{32}v_2' + r_{33} + d_2' t_3 \end{pmatrix} = a'$$

②

$$\Rightarrow \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} (a')$$

$$= \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11}u_2' + r_{12}v_2' + r_{13} + d_2' t_1 \\ r_{21}u_2' + r_{22}v_2' + r_{23} + d_2' t_2 \\ r_{31}u_2' + r_{32}v_2' + r_{33} + d_2' t_3 \end{pmatrix}$$

$$= r_{31}u_2' + r_{32}v_2' + r_{33} + d_2' t_3 \begin{pmatrix} u_1' \\ v_1' \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} [r_{31}u_2' + r_{32}v_2' + r_{33} + d_2' t_3] u_1' \\ [r_{31}u_2' + r_{32}v_2' + r_{33} + d_2' t_3] v_1' \\ [r_{31}u_2' + r_{32}v_2' + r_{33} + d_2' t_3] 1 \end{pmatrix} = b$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \left[\left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right] - b \right]$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} b = r_{31}u_2' + r_{32}v_2' + r_{33} + d_2' t_3 \begin{pmatrix} u_1' \\ v_1' \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} a = \begin{pmatrix} r_{11}u_2' + r_{12}v_2' + r_{13} + d_2' t_1 \\ r_{21}u_2' + r_{22}v_2' + r_{23} + d_2' t_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} r_{11}u_2' + r_{12}v_2' + r_{13} + d_2' t_1 \\ r_{21}u_2' + r_{22}v_2' + r_{23} + d_2' t_2 \end{pmatrix} - \begin{pmatrix} r_{31}u_2' + r_{32}v_2' + r_{33} + d_2' t_3 \\ r_{31}u_2' + r_{32}v_2' + r_{33} + d_2' t_3 \end{pmatrix}$$

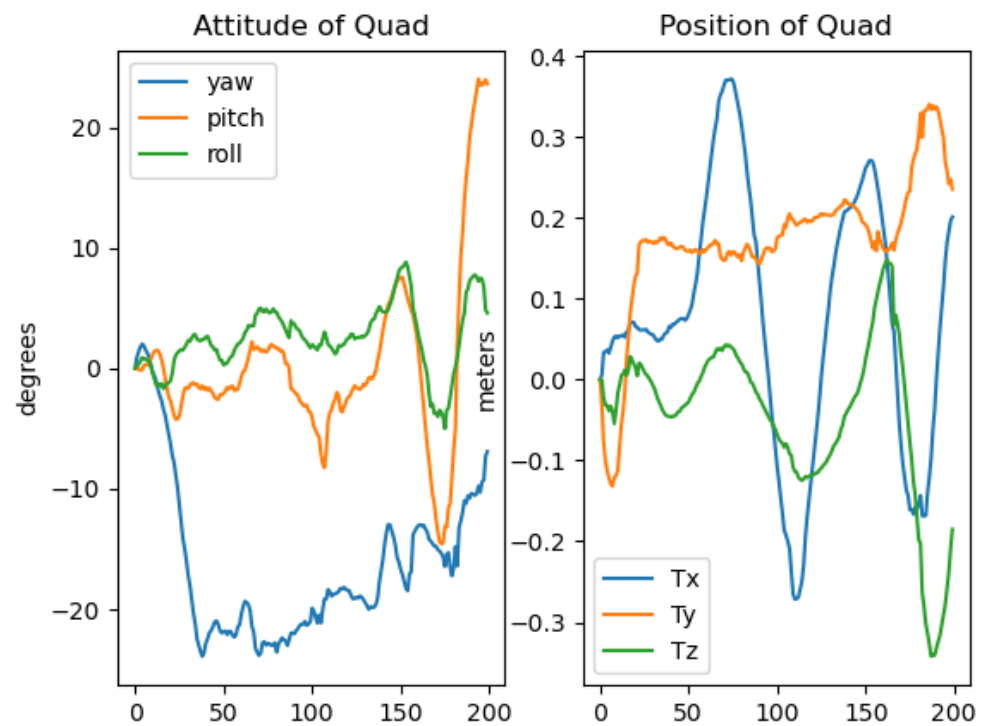
Expanding Eq 2.

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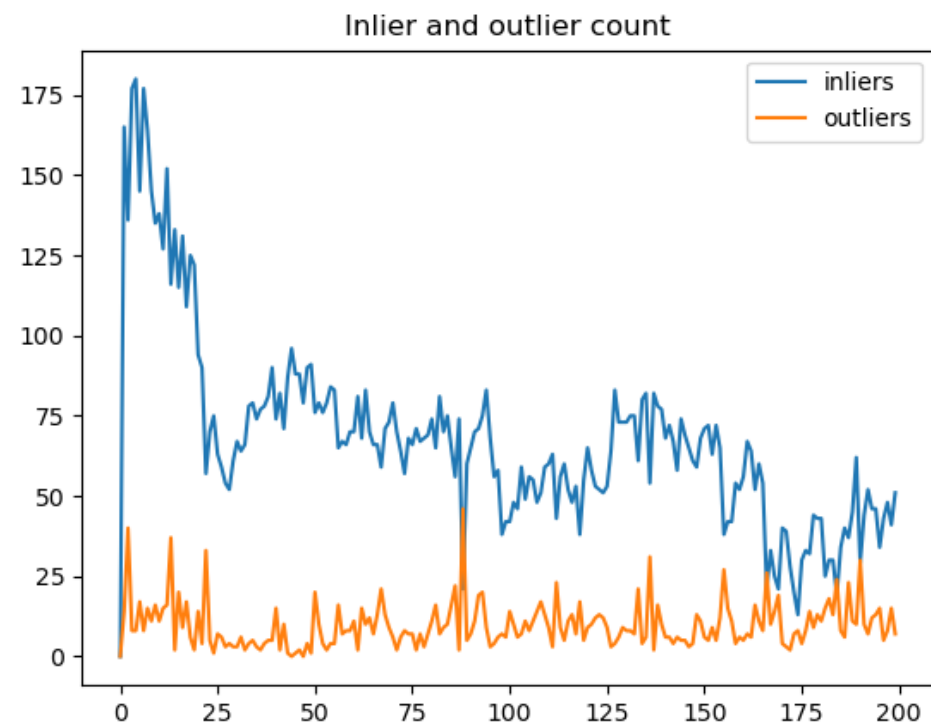
$$\Rightarrow \begin{pmatrix} 1 & 0 & -u_1 \\ 0 & 1 & -v_1 \end{pmatrix} \begin{bmatrix} r_{11}u_2' + r_{12}v_2' + r_{13} + d_2't_1 \\ r_{21}u_1' + r_{22}v_2' + r_{23} + d_2't_2 \\ r_{31}u_2' + r_{32}v_2' + r_{33} + d_2't_3 \end{bmatrix}$$

$$\Rightarrow \begin{pmatrix} r_{11}u_2' + r_{12}v_2' + r_{13} + d_2't_1 \\ r_{21}u_1' + r_{22}v_2' + r_{23} + d_2't_2 \end{pmatrix} - \begin{pmatrix} r_{31}u_2'u_1' + r_{32}v_2'u_1' + r_{33}u_1' + d_2't_3u_1' \\ r_{31}u_2'v_1' + r_{32}v_2'v_1' + r_{33}v_1' + d_2't_3v_1' \end{pmatrix}$$

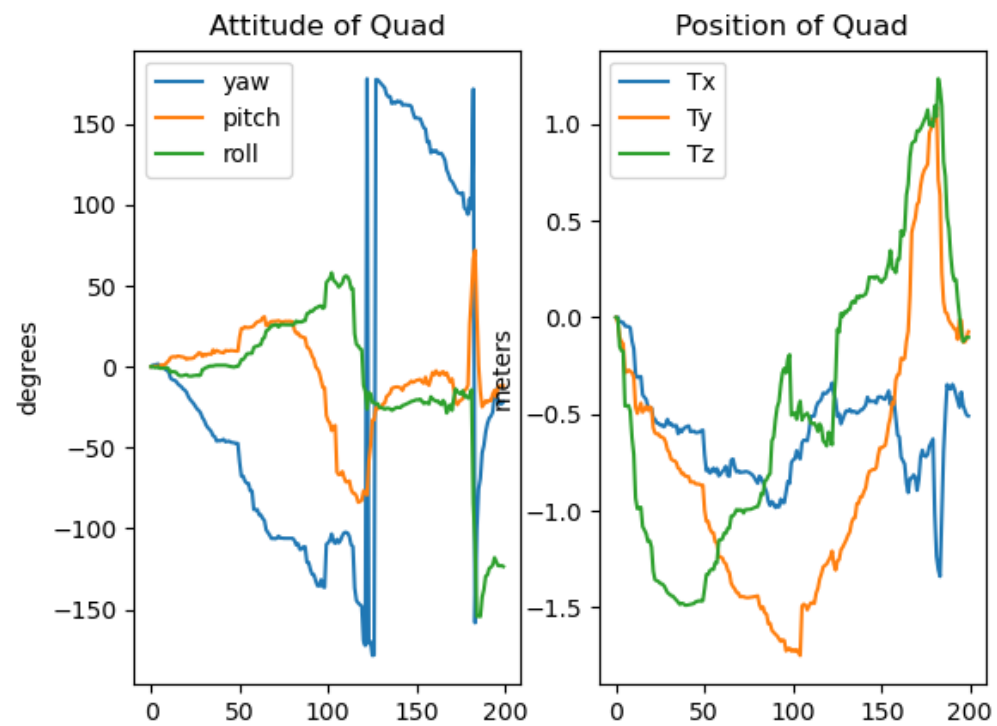
The expressions are equivalent, thus the
statement is proved.



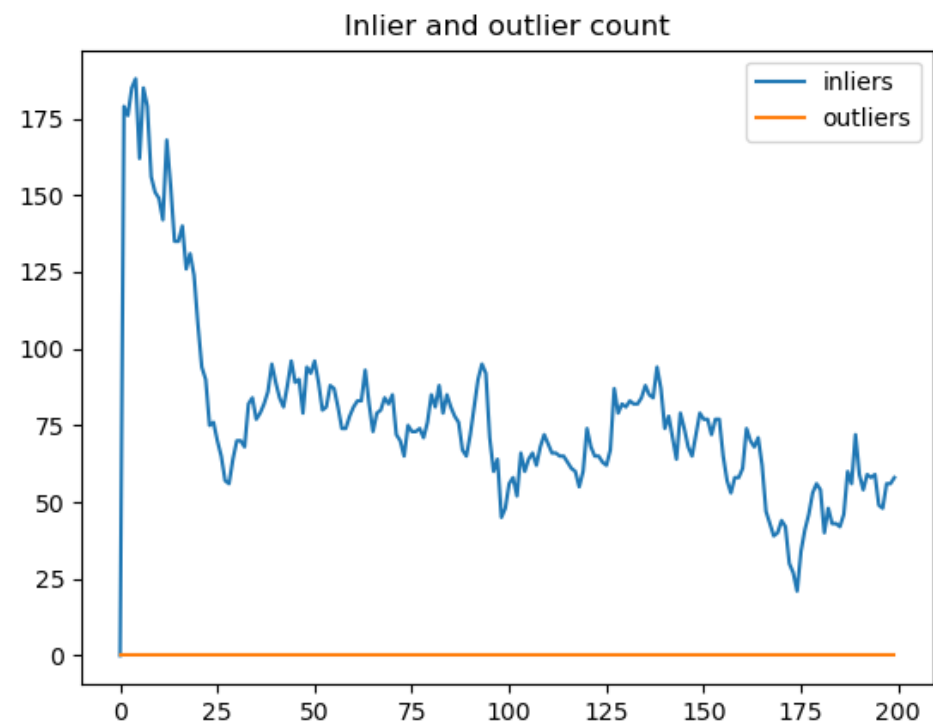
Ransac Iterations = 10



Ransac Iterations = 10



Ransac Iterations = 0



Ransac Iterations = 0

We see that when we use 10 iterations, the position and attitude plots are smooth and continuous. Furthermore, since we are being more stringent in our criteria for outliers, this is represented in the inlier/outlier plot. For 0 iterations, since we treat all correspondence points as inliers, even though they may in fact be outliers, we don't see as smooth position and attitude plots.

