Derivation

Eq 1: $\delta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \left[\left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right] - \left(\frac{z_1}{z_2} \right) \begin{pmatrix} u_1' \\ v_1' \\ 1 \end{pmatrix} \right]$

Eq 2: $\delta = \begin{pmatrix} 1 & 0 & -u_1' \\ 0 & 1 & -v_1' \end{pmatrix} \left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right]$

Sol'n

$\Rightarrow \frac{z_1}{z_2} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right]$

Substitute

$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \left[\underbrace{\left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right]}_{3\times1 \; + \; 3\times1} - \underbrace{\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}}_{1\times3} \left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right] \begin{pmatrix} u_1' \\ v_1' \\ 1 \end{pmatrix} \right]$

2×3

$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

$\Rightarrow \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} = \begin{pmatrix} r_{11}u_2' + r_{12}v_2' + r_{13} \\ r_{21}u_2' + r_{22}v_2' + r_{23} \\ r_{31}u_2' + r_{32}v_2' + r_{33} \end{pmatrix} = R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix}$

$3\times$

$d_2' T = d_2' \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \begin{pmatrix} d_2' t_1 \\ d_2' t_2 \\ d_2' t_3 \end{pmatrix} = d_2' T$

$\Rightarrow R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T = \begin{pmatrix} r_{11}u_2' + r_{12}v_2' + r_{13} + d_2' t_1 \\ r_{21}u_2' + r_{22}v_2' + r_{23} + d_2' t_2 \\ r_{31}u_2' + r_{32}v_2' + r_{33} + d_2' t_3 \end{pmatrix} = a'$

$$\Rightarrow \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a' \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} u_2' + r_{12} v_2' + r_{13} + d_2' t_1 \\ r_{21} u_2' + r_{22} v_2' + r_{23} + d_2' t_2 \\ r_{31} u_2' + r_{32} v_2' + r_{33} + d_2' t_3 \end{pmatrix}$$

$$= r_{31} u_2' + r_{32} v_2' + r_{33} + d_2' t_3 \begin{pmatrix} u_1' \\ v_1' \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} [r_{31} u_2' + r_{33} v_2' + r_3 + d_2' t_3]\, u_1' \\ [r_{31} u_2' + r_{33} v_2' + r_3 + d_2' t_3]\, v_1' \\ [r_{31} u_2' + r_{32} v_2' + r_{33} + d_2' t_3]\, 1 \end{pmatrix} = b$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \left[\left[R \begin{pmatrix} u_2' \\ v_2' \\ 1 \end{pmatrix} + d_2' T \right] - b \right]$$

$$\Rightarrow \underset{2\times 3}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}} \underset{3\times 1}{b} = r_{31} u_2' + r_{32} v_2' + r_{33} + d_2' t_3 \begin{pmatrix} u_1' \\ v_1' \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} a = \begin{pmatrix} r_{11} u_2' + r_{12} v_2' + r_{13} + d_2' t_1 \\ r_{21} u_2' + r_{22} v_2' + r_{23} + d_2' t_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} r_{11} u_2' + r_{12} v_2' + r_{13} + d_2' t_1 \\ r_{21} u_2' + r_{22} v_2' + r_{23} + d_2' t_2 \end{pmatrix} - \begin{pmatrix} r_{31} u_2' u_1' + r_{32} v_2' u_1' + r_{33} u_1' + d_2' t_3 u_1' \\ r_{31} u_2' v_1' + r_{32} v_2' v_1' + r_{33} v_1' + d_2' t_3 v_1' \end{pmatrix}$$

Expanding Eq 2.

$$\Rightarrow \begin{pmatrix} 1 & 0 & -u_1 \\ 0 & 1 & -v_1 \end{pmatrix} \begin{bmatrix} r_{11} u_2' + r_{12} v_2' + r_{13} + d_2' t_1 \\ r_{21} u_2' + r_{22} v_2' + r_{23} + d_2' t_2 \\ r_{31} u_2' + r_{32} v_2' + r_{33} + d_2' t_3 \end{bmatrix}$$

$$\Rightarrow \begin{pmatrix} r_{11} u_2' + r_{12} v_2' + r_{13} + d_2' t_1 \\ r_{21} u_1' + r_{22} v_2' + r_{23} + d_2' t_2 \end{pmatrix} - \begin{pmatrix} r_{31} u_2' u_1' + r_{32} v_2' u_1' + r_{33} u_1' + d_2' t_3 u_1' \\ r_{31} u_2' v_1' + r_{32} v_2' v_1' + r_{33} v_1' + d_2' t_3 v_1' \end{pmatrix}$$

The expressions are equivalents thus the statement is proved.