COMPLEMENTARY_FILTER.PY OUTPUT

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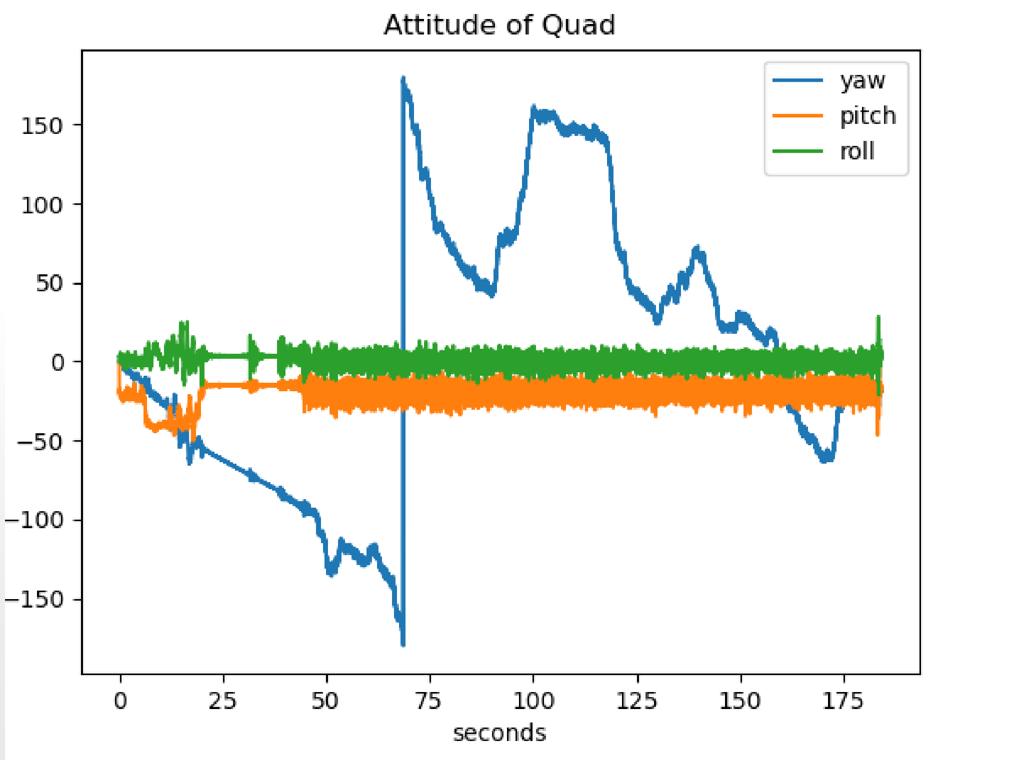
MEAM 620, SPRING 2021

PROJECT II PHASE 1



Version 1

$$a_1 = e_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \; ; \; g' = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix}$$

$$R_{1x} a_x = g' \quad \Rightarrow \|g'\| = 1$$

$$g' \times e_x = \begin{pmatrix} 0 & -g'_z & g'_y \\ g'_z & 0 & -g'_x \\ -g'_y & g'_x & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ g'_z \\ -g'_y \end{pmatrix}$$

$$g' \times e_x = \|g'\| \, \|e_x\| \sin\theta \, \hat{\omega}$$

$$\sqrt{g_x'^2 + g_y'^2 + g_z'^2} = \|g'\| = \underline{1}$$

$$\|e_x\| = 1$$

$$\Rightarrow \hat{\omega} \sin\theta = \begin{pmatrix} 0 \\ g'_z \\ -g'_y \end{pmatrix}$$

$$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta}$$

$$\Rightarrow \hat{\omega} \sqrt{1-\cos^2\theta} = \begin{pmatrix} 0 \\ g'_z \\ -g'_y \end{pmatrix}$$

$$\Rightarrow \hat{\omega} = \frac{1}{\sqrt{1-\cos^2\theta}} \begin{pmatrix} 0 \\ g'_z \\ -g'_y \end{pmatrix}$$

$$\Rightarrow g' \cdot e_x = \|g'\| \, \|e_x\| \cos\theta$$

$$\Rightarrow g' \cdot e_x = g'_x$$

$$\Rightarrow \cos\theta = g'_x$$

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$$\Rightarrow \hat{\omega} = \frac{1}{\sqrt{(1-g'_x)(1+g'_x)}} \begin{pmatrix} 0 \\ g'_z \\ -g'_y \end{pmatrix}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}} = \frac{\sqrt{1-\cos\theta}}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{1-g'_x}}{\sqrt{2}}$$

$$\Rightarrow \cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{1+\cos\theta}}{\sqrt{2}} = \frac{\sqrt{1+g'_x}}{\sqrt{2}}$$

$$\sin\left(\frac{\theta}{2}\right)\hat{\omega} = \frac{\sqrt{1-g'_x}}{\sqrt{2}} \cdot \frac{1}{\sqrt{(1-g'_x)(1+g'_x)}} \begin{pmatrix} 0 \\ g'_z \\ -g'_y \end{pmatrix}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right)\hat{\omega} = \frac{1}{\sqrt{2(g_x+1)}} \begin{pmatrix} 0 \\ g'_z \\ -g'_y \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{g'_z}{\sqrt{2(g_x+1)}} \\ \frac{-g'_y}{\sqrt{2(g_x+1)}} \end{pmatrix}$$

Version

$$(1) \quad \Delta q_{acc} = \Big(\underset{\omega}{\underset{\downarrow}{\sqrt{\frac{1+g'_x}{2}}}},\; \underset{i}{\underset{\downarrow}{0}},\; \underset{j}{\underset{\downarrow}{\frac{g'_z}{\sqrt{2(g_x+1)}}}},\; \underset{k}{\underset{\downarrow}{\frac{-g'_y}{\sqrt{2(g_x+1)}}}} \Big)$$

Delta q_acc derivation

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$$\Rightarrow q = \left(\cos\frac{\theta}{2}, \sin\left(\frac{\theta}{2}\right)\omega\right)$$

$$\Rightarrow H(u_0, u) = (u_0^2 - u^T u) I + 2u_0 \hat{u} + 2uu^T$$

$$\Rightarrow \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \; ; \quad \|\omega\| = 1$$

$$\Rightarrow R = \exp(\hat{\omega}\theta) = I + \sin\theta\,\hat{\omega} + (1-\cos\theta)\hat{\omega}^2$$

Proof

$$\Rightarrow u_0 = \cos\left(\frac{\theta}{2}\right) \; ; \; u = \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}^T \quad \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$$

$$\Rightarrow H\left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}\right) =$$

$$\Rightarrow (u_0^2 - u^T u) I = \left(\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right) I$$

$$\Rightarrow \boxed{= I} \overset{\checkmark}{=} I$$

$$\Rightarrow 2u_0\hat{u} = 2\left(\cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right)\right)\hat{\omega}$$

$$= 2\left[\sqrt{\frac{1+\cos\theta}{2}} \cdot \sqrt{\frac{1-\cos\theta}{2}}\right]\hat{\omega}$$

$$= 2\left[\sqrt{\frac{(1+\cos\theta)(1-\cos\theta)}{2}}\right]\hat{\omega}$$

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$$\Rightarrow 2\left[\frac{\sqrt{(1-\cos^2\theta)}}{\sqrt{4}}\right]\hat{\omega}$$

$$\Rightarrow = \sqrt{\sin^2\theta}\,\hat{\omega}$$

$$\boxed{= \sin\theta\,\hat{\omega} \overset{\checkmark}{=} \sin\theta\,\hat{\omega}}$$

$$\Rightarrow 2uu^T = 2\left[\sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 & \omega_2 & \omega_3 \end{pmatrix}\right]$$

$$= 2\left(\sin^2\left(\frac{\theta}{2}\right) \begin{bmatrix} \omega_1^2 & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_2^2 & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_3^2 \end{bmatrix}\right)$$

$$= 2\left(\sin^2\left(\frac{\theta}{2}\right) \hat{\omega}^2\right)$$

$$= 2\left(\left(\sqrt{\frac{1-\cos\theta}{2}}\right)^2 \hat{\omega}^2\right)$$

$$= \boxed{(1-\cos\theta)\,\hat{\omega}^2 \overset{\checkmark}{=} 1-\cos\theta\,\hat{\omega}^2}$$

H(u_0, u) => exp(\hat{w} * theta) proof