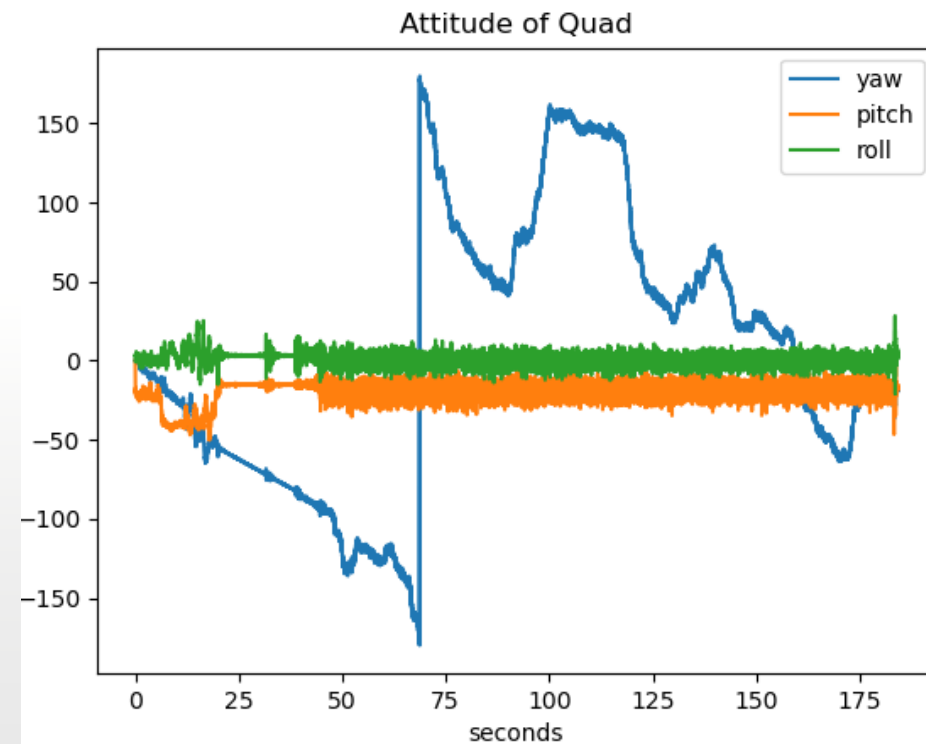


COMPLEMENTARY_FILTER.PY OUTPUT

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PROJECT II PHASE I



①

Version 1

$$a_1 = e_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad g' = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix}$$

$$R_{x\theta} = g' \Rightarrow \|g'\| = 1$$

$$g' x e_x = \begin{pmatrix} 0 & -g_z & g_y \\ g_z & 0 & -g_x \\ -g_y & g_x & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ g_z \\ -g_y \end{pmatrix}$$

$$g' x e_x = \|g'\| \|e_x\| \cos \theta \hat{\omega}$$

$$\sqrt{g_x^2 + g_y^2 + g_z^2} = \|g'\| = 1$$

$$\|e_x\| = 1$$

$$\Rightarrow \hat{\omega} \sin \theta = \begin{pmatrix} 0 \\ g_z \\ -g_y \end{pmatrix}$$

$$\Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$\Rightarrow \hat{\omega} \sqrt{1 - \cos^2 \theta} = \begin{pmatrix} 0 \\ g_z \\ -g_y \end{pmatrix}$$

$$\Rightarrow \hat{\omega} = \frac{1}{\sqrt{1 - \cos^2 \theta}} \begin{pmatrix} 0 \\ g_z \\ -g_y \end{pmatrix}$$

$$\Rightarrow g' \cdot e_x = \|g'\| \|e_x\| \cos \theta$$

$$\Rightarrow g' \cdot e_x = g_x$$

$$\Rightarrow \cos \theta = g_x$$

$$\Rightarrow \hat{\omega} = \frac{1}{\sqrt{(1-g_x)(1+g_x)}} \begin{pmatrix} 0 \\ g_z \\ -g_y \end{pmatrix}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$= \sqrt{\frac{1 - g_x}{2}}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - g_x}{2}}$$

$$\Rightarrow \cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + g_x}{2}}$$

$$= \sqrt{\frac{1 + g_x}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) \hat{\omega} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{(1-g_x)(1+g_x)}} \begin{pmatrix} 0 \\ g_z \\ -g_y \end{pmatrix}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right) \hat{\omega} = \frac{1}{\sqrt{2(g_x+1)}} \begin{pmatrix} 0 \\ g_z \\ -g_y \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \frac{g_z}{\sqrt{2(g_x+1)}} \\ \frac{-g_y}{\sqrt{2(g_x+1)}} \end{pmatrix}$$

①

$$\Delta q_{acc} = \left(\underbrace{\sqrt{\frac{1+g_x}{2}}}_{\omega}, \underbrace{0}_{\dot{c}}, \underbrace{\frac{g_z}{\sqrt{2(g_x+1)}}}_{j}, \underbrace{\frac{-g_y}{\sqrt{2(g_x+1)}}}_{k} \right)$$

Delta q_acc derivation

⑦

$$\Rightarrow q = \left(\cos \frac{\theta}{2}, \sin \frac{\theta}{2} \hat{\omega} \right)$$

$$\Rightarrow H(u_0, u) = (u_0^T - u^T u) I + 2u_0 u^T + 2u u^T$$

$$\Rightarrow \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \quad \| \omega \| = 1$$

$$\Rightarrow R = \exp(\hat{\omega} \theta) = I + \sin \theta \hat{\omega} + (1 - \cos \theta) \hat{\omega}^2$$

Proof

$$\Rightarrow u_0 = \cos\left(\frac{\theta}{2}\right) \quad u = \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \quad \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\Rightarrow H\left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}\right) = \cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\Rightarrow (u_0^T - u^T u) I = \left(\cos\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) \right) I$$

$$\Rightarrow \boxed{= I} = I$$

$$\Rightarrow 2u_0 u^T = 2 \left(\cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \hat{\omega} \right)$$

$$= 2 \left[\sqrt{\frac{1 + \cos \theta}{2}} \cdot \sqrt{\frac{1 - \cos \theta}{2}} \right] \hat{\omega}$$

$$= 2 \left[\sqrt{\frac{(1 + \cos \theta)(1 - \cos \theta)}{2}} \right] \hat{\omega}$$

⑧

$$\Rightarrow 2 \left[\frac{\sqrt{(1 - \cos^2 \theta)}}{\sqrt{4}} \right] \hat{\omega}$$

$$\Rightarrow = \sqrt{\sin^2 \theta} \hat{\omega}$$

$$= \sin \theta \hat{\omega} = \sin \theta \hat{\omega}$$

$$\Rightarrow 2u u^T = 2 \left[\sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 & \omega_2 & \omega_3 \end{pmatrix} \right]$$

$$= 2 \left(\sin^2\left(\frac{\theta}{2}\right) \begin{bmatrix} \omega_1^2 & \omega_1 \omega_2 & \omega_1 \omega_3 \\ \omega_2 \omega_1 & \omega_2^2 & \omega_2 \omega_3 \\ \omega_3 \omega_1 & \omega_3 \omega_2 & \omega_3^2 \end{bmatrix} \right)$$

$$= 2 \left(\sin^2\left(\frac{\theta}{2}\right) \hat{\omega}^2 \right)$$

$$= 2 \left(\left(\sqrt{\frac{1 - \cos \theta}{2}} \right)^2 \hat{\omega}^2 \right)$$

$$= \boxed{(1 - \cos \theta) \hat{\omega}^2 = 1 - \cos \theta \hat{\omega}^2}$$

H(u_0, u) => exp(\hat{w} * theta) proof