

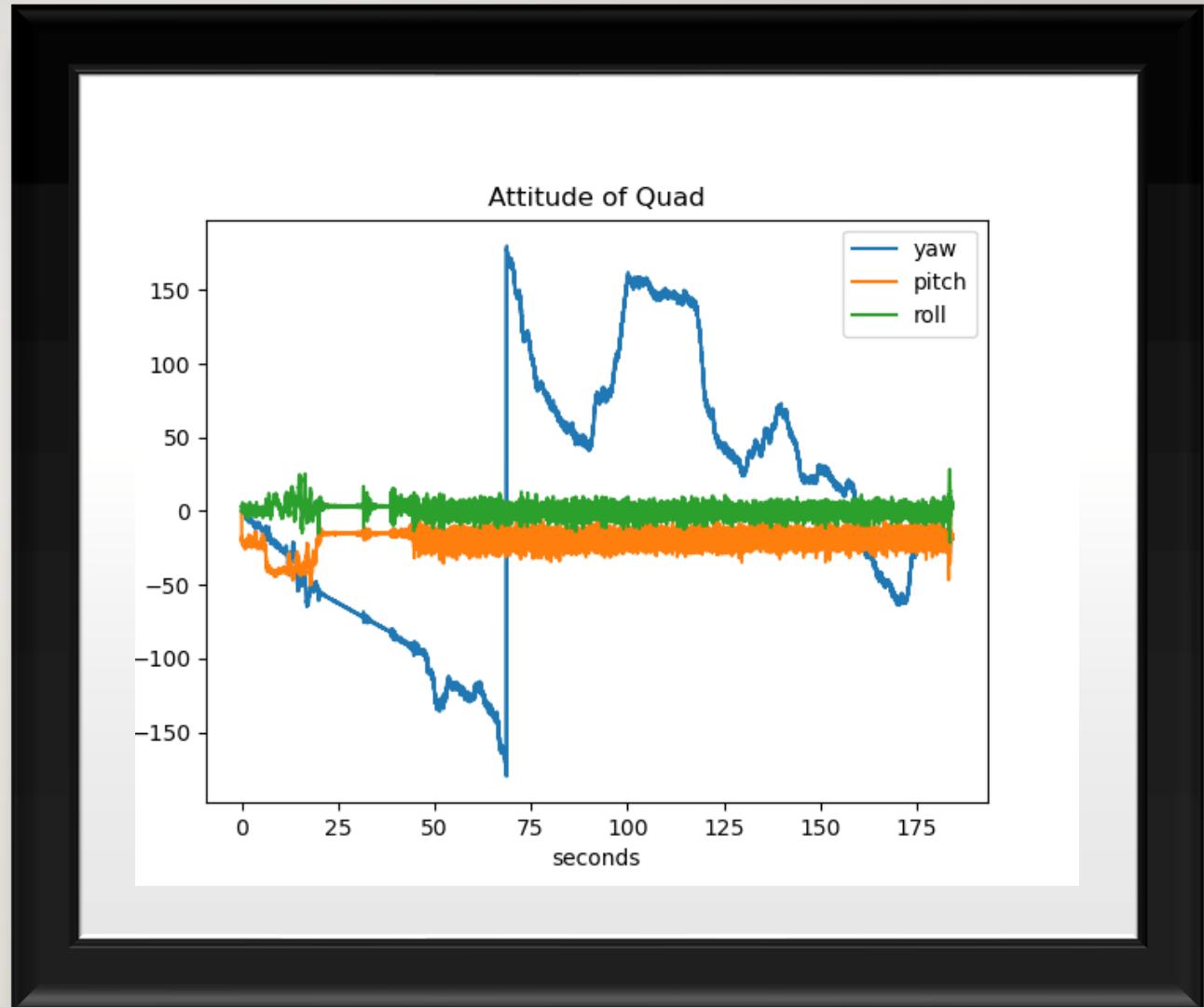
COMPLEMENTARY\_FILTER.PY OUTPUT

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IKENNA ACHILIHU

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PROJECT II PHASE I



(1)

$$a_x = e_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \text{Version 1}$$

$$R_x e_x = g' \Rightarrow \|g'\| = 1$$

$$g' \times e_x = \begin{pmatrix} 0 & -g_2 & g_1 \\ g_2 & 0 & -g_1 \\ -g_1 & g_2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ g_2 \\ -g_1 \end{pmatrix}$$

$$g' \times e_x = \|g'\| \|e_x\| \cos \theta \hat{\omega}$$

$$\sqrt{g_{xx}^2 + g_{yy}^2 + g_{zz}^2} = \|g'\| = 1$$

$$\|e_x\| = 1$$

$$\Rightarrow \hat{\omega} \sin \theta = \begin{pmatrix} 0 \\ g_2 \\ -g_1 \end{pmatrix}$$

$$\Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$\Rightarrow \hat{\omega} \sqrt{1 - \cos^2 \theta} = \begin{pmatrix} 0 \\ g_2 \\ -g_1 \end{pmatrix}$$

$$\Rightarrow \hat{\omega} = \frac{1}{\sqrt{1 - \cos^2 \theta}} \begin{pmatrix} 0 \\ g_2 \\ -g_1 \end{pmatrix}$$

$$\Rightarrow g' \cdot e_x = \|g'\| \|e_x\| \cos \theta$$

$$\Rightarrow g' \cdot e_x = g' x$$

$$\Rightarrow \cos \theta = g' x$$

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$$\Rightarrow \hat{\omega} = \frac{1}{\sqrt{(1-g_{xx})(1+g_{xx})}} \begin{pmatrix} 0 \\ g_2 \\ -g_1 \end{pmatrix}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$$

$$= \frac{\sqrt{1-\cos\theta}}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-g_{xx}}{2}}$$

$$\Rightarrow \cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}}$$

$$= \frac{\sqrt{1+g_{xx}}}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right) \hat{\omega} = \frac{\sqrt{1-g_{xx}}}{\sqrt{2}} \cdot \frac{1}{\sqrt{(1-g_{xx})(1+g_{xx})}} \begin{pmatrix} 0 \\ g_2 \\ -g_1 \end{pmatrix}$$

$$\Rightarrow \sin\left(\frac{\theta}{2}\right) \hat{\omega} = \frac{1}{\sqrt{2(g_{xx}+1)}} \begin{pmatrix} 0 \\ g_2 \\ -g_1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \frac{g_2}{\sqrt{2(g_{xx}+1)}} \\ \frac{-g_1}{\sqrt{2(g_{xx}+1)}} \end{pmatrix}$$

$$(1) \Delta q_{acc} = \left( \sqrt{\frac{1+g_{xx}}{2}}, \downarrow \omega, \downarrow \zeta, \downarrow \sin\left(\frac{\theta}{2}\right), \downarrow \cos\left(\frac{\theta}{2}\right) \right)$$

Delta q\_acc derivation

$$\Rightarrow \hat{\omega} = \left( \cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \right) \omega$$

$$\Rightarrow H(u_0, u) = (u_0^2 - u_u^T) I + 2u_0 u + 2u u^T$$

$$\Rightarrow \omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \Rightarrow \|\omega\| = 1$$

$$\Rightarrow R = \exp(\hat{\omega} \theta) = I + \sin \theta \hat{\omega} + (1 - \cos \theta) \hat{\omega}^2$$

Proof

$$\Rightarrow u_0 = \cos\left(\frac{\theta}{2}\right) \quad j \quad M = \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}^T \quad \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\Rightarrow H\left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}\right) = \cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}}$$

$$\Rightarrow (M_0^2 - M_u^T) I = \left( \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) \right) I$$

$$\Rightarrow [I] = I$$

$$\Rightarrow 2u_0 u = 2 \left( \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \right) \hat{\omega}$$

$$= 2 \left[ \sqrt{\frac{1+\cos\theta}{2}} \cdot \sqrt{\frac{1-\cos\theta}{2}} \right] \hat{\omega}$$

$$= 2 \left[ \sqrt{\frac{(1+\cos\theta)(1-\cos\theta)}{2}} \right] \hat{\omega}$$

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$$\Rightarrow 2 \left[ \frac{\sqrt{(1-\cos^2\theta)}}{\sqrt{4}} \right] \hat{\omega}$$

$$= \sqrt{\sin^2\theta} \hat{\omega}$$

$$= \sin\theta \hat{\omega} \stackrel{!}{=} \sin\theta \hat{\omega}$$

$$\Rightarrow 2u u^T = 2 \left[ \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \sin\left(\frac{\theta}{2}\right) (\omega_1, \omega_2, \omega_3) \right]$$

$$= 2 \left( \sin^2\left(\frac{\theta}{2}\right) \begin{bmatrix} \omega_1^2 & \omega_1 \omega_2 & \omega_1 \omega_3 \\ \omega_2 \omega_1 & \omega_2^2 & \omega_2 \omega_3 \\ \omega_3 \omega_1 & \omega_3 \omega_2 & \omega_3^2 \end{bmatrix} \right)$$

$$= 2 \left( \sin^2\left(\frac{\theta}{2}\right) \hat{\omega}^2 \right)$$

$$= 2 \left( \left( \sqrt{\frac{1-\cos\theta}{2}} \right)^2 \hat{\omega}^2 \right)$$

$$= \boxed{(1 - \cos\theta) \hat{\omega}^2 = \hat{\omega}^2}$$

 $H(u_0, u) \Rightarrow \exp(\hat{\omega} \cdot \theta)$  proof