Interval Graph Coloring related to Optical Network Planning

Mikhail Kharitonov

www.huawei.com

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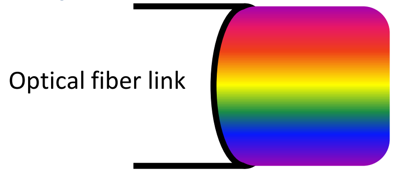
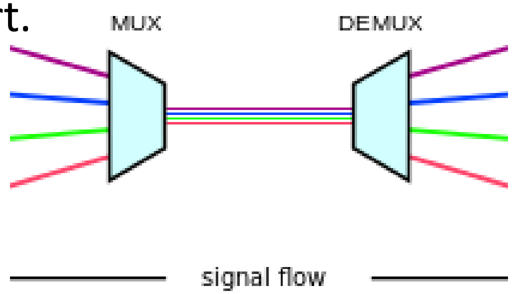
Huawei,
Mathematical Modeling and Optimization Algorithm Competence Center,
Moscow Research Center

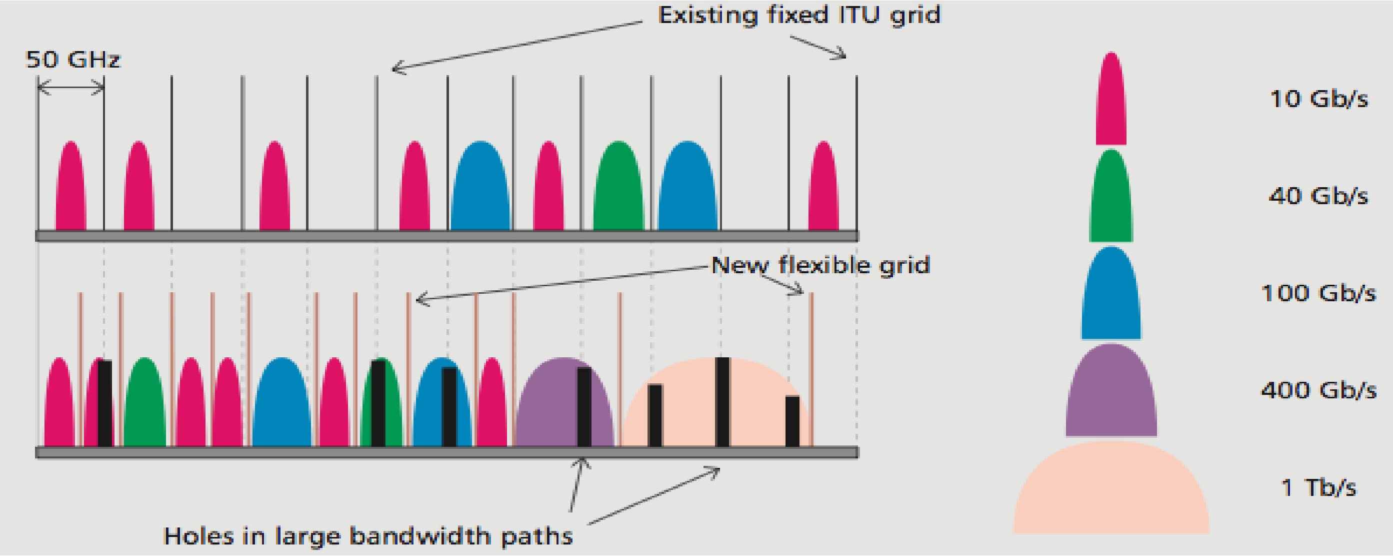
WDM: Wavelength Division Multiplexing

WDM multiplexes a number of optical carrier signals onto a single optical fiber by using different wavelengths (i.e., colors) to increase capacity.
WDM uses a multiplexer (MUX) to combine several signals together, and a de-multiplexer (DEMUX) at the receiver to split them apart.





Total frequency bandwidth is then divided into slots



Optimization Problem

Input:

Network G=(V,E)

Commodities $D=\{D_i, i=1,..K\}$, a single path P_i for each commodity

Resource of each edge is the spectrum interval, say, $[0,W) \subseteq \mathbb{N}$

Spectrum needed for each commodity $w: D \to \mathbb{N}$

Decision variables:

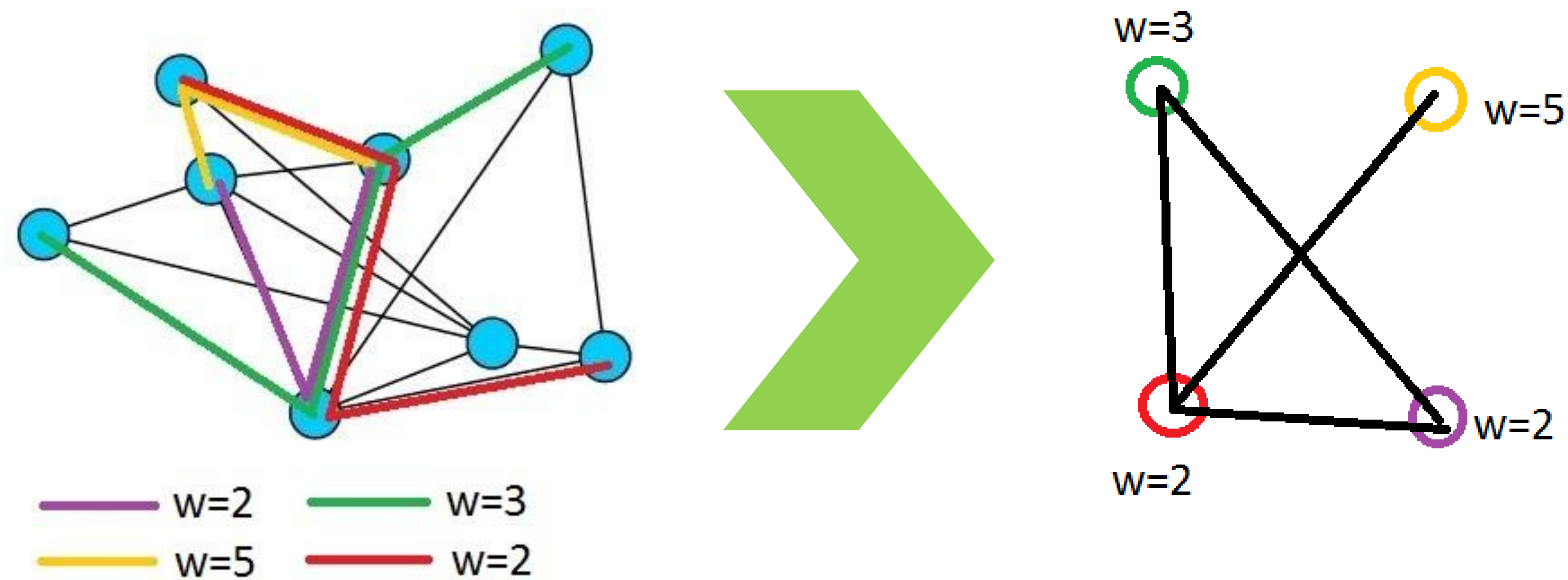
$\forall d \in D$, a spectrum interval $[l_d, r_d) \subseteq [0,W)$, such that $r_d - l_d = w(d)$ and $[l_d, r_d) \cap [l_e, r_e) = \emptyset$ if $P_d \cap P_e \neq \emptyset$ (*intervals are disjoint)*

Objective:

(A) Fix W, and find a feasible subset $F \subseteq D$ with maximal $|F|$ or $\sum_{d \in F} w(d)$

(B) Find the minimal W such that a feasible solution for the whole D exists

Graph Interval Coloring Approach



commodities >> nodes, paths intersection >> edges , bandwidth >> weight of the node

Particular case: string graph A_n

Consider a string graph • — • — • — • … — •
All paths are substrings

Assume all commodities have the same weight.

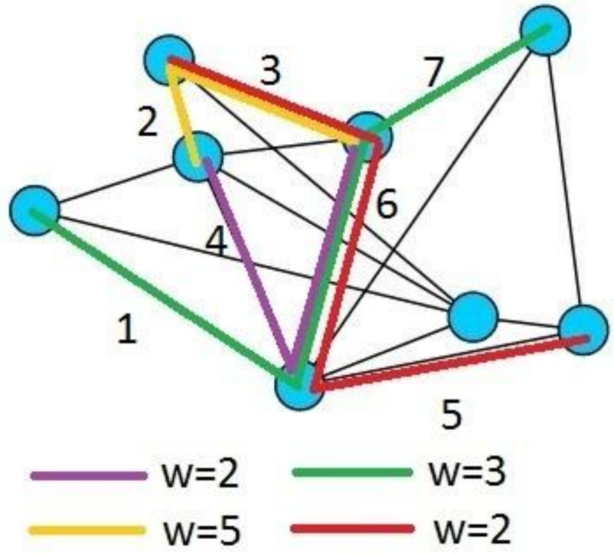
A greedy algorithm of $O(n\ ln(n))$ working time yields the optimum for (B)

1. Sort all commodities by their left end in increasing order $d_1, d_2,\ldots,d_K$.
2. *W := 0*
3. For $j=1$ *to* K do
    1. If d_j spectrum can be allocated into *[0,W)*:
        1. allocate it into *[0,W)*
    2. Else:
        1. assign d_j spectrum as *[W,W+1)*
        2. *W=W+1*

Gene

Difference compared with A_n:

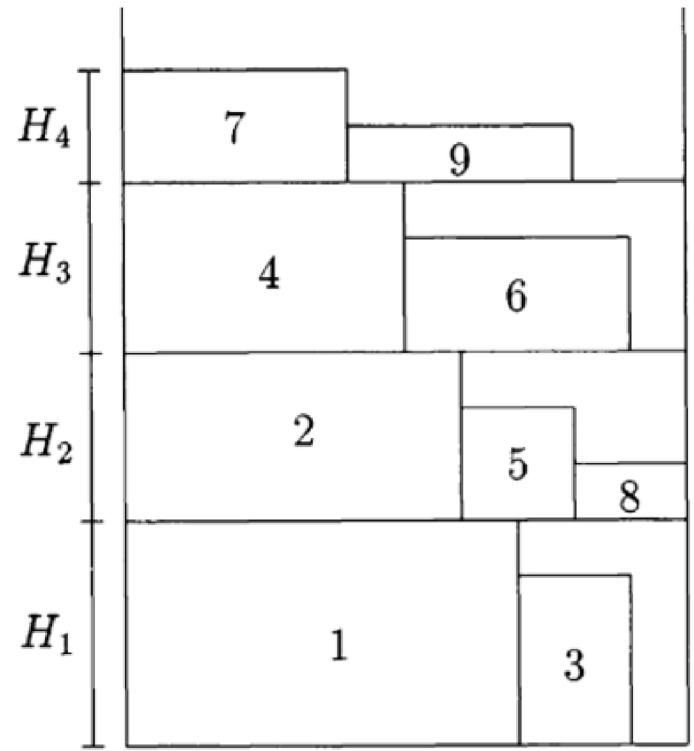
- "Bricks" are not connected anymore
- Commodities have different weights



Max-coloring approach

An interesting model reduces Interval coloring to a specific usual coloring problem:
Take a coloring $c: V\text{-}> \{1,\ldots,M\}$ such that no edge has endpoints of the same color.
Let $v_i \in V(i)=c^{-1}(i)$ be the node with maximal weight: $v_i=argmax_{v \in V(i)}(w(v)),\ i=1,..,M$
Let $W_i = \sum_{j=1,\ldots i-1} w(v_j)$

Then the solution $Int(v)=[W_i,W_i+w(v))$ is feasible.



So an optimization arises, to find a coloring with in an **arbitrary** number M of colors with minimal possible $T=\sum_{i=1,\ldots,M} w(v_i)$

The minimal M will not necessarily yield the minimum of T

Complexity

- Both problems (A), feasibility, and (B) optimality are NP hard
- We need a time efficient method able to work with
  - 10-500 nodes
  - Up to 10000 demands
- For small cases (about 10 nodes) the model should be at 10% from the global optimum (provided e.g. by ILP solvers)
- Please send your solution to mikhail.kharitonov1@huawei.com

THANK YOU