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Report about Neural Networks

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Fall 2024

1 Perceptron

1.1 Overview

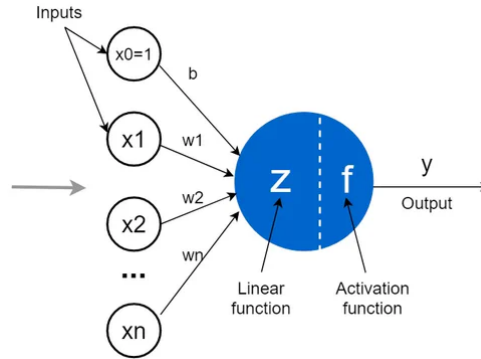


Figure 1 Perceptron schematics

- x_1, x_2, \dots, x_n are inputs;
- w_1, w_2, \dots, w_n are weights;
- Z is a linear combination of x_i s: $Z = w_0 + w_1 * x_1 + w_2 * x_2 + \dots w_n * x_n$;
- ϕ is an activation function;
- w_0 is a bias;
- \hat{y} is output in binary form.

1.2 Linear function

We can represent our x_i s and w_i s as vectors X and W respectively (1xN matrixes). Then linear function can be represented as $Z = X^T W + w_0$

1.3 Activation function and predictions

$$\phi = \begin{cases} 1, & z \geq 0 \\ -1, & z < 0 \end{cases}$$

Prediction (\hat{y}) = $\phi(z)$, where z - linear combination.

1.4 Loss function

Using perceptron we update weights based on a number of wrong predictions. This model has the same input and output, but uses a different activation function and loss calculation.

$$L = \sum_{i=1}^n \delta(\hat{y} \neq y)$$

2 Logistic Regression

2.1 Overview

The schematic representation of a logistic regression is depicted in figure 2.

- x_1, x_2, \dots, x_n are inputs;
- w_1, w_2, \dots, w_n are weights;
- $Z(\sum)$ is a linear combination of x_i s: $Z = w_0 + w_1 * x_1 + w_2 * x_2 + \dots w_n * x_n$;

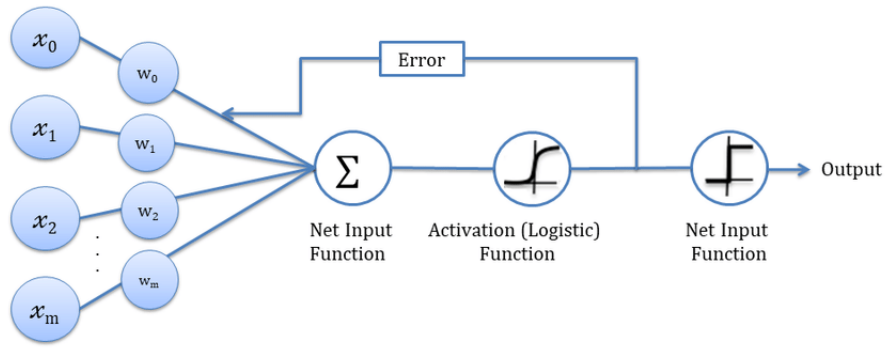


Figure 2 Logistic regression

- ϕ is an activation function;
- w_0 is a bias;
- \hat{y} is output in binary form.

2.2 Activation function

For activation function we use sigmoid:

$$\phi(z) = \frac{1}{1 + e^{-z}}$$

Graphic of sigmoid function is depicted in figure 3.

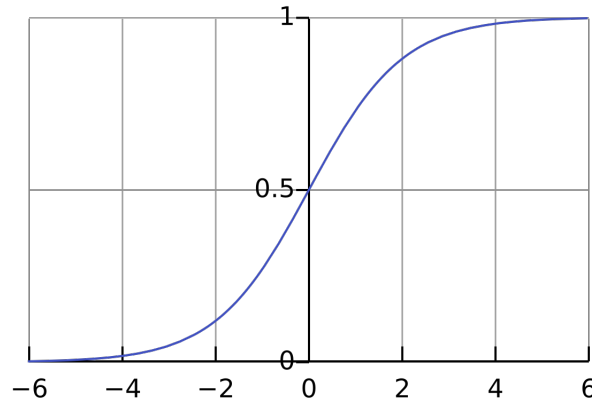


Figure 3 Sigmoid function

2.3 Loss function

For loss function we use cross-entropy loss:

$$L(y, \hat{y}) = -\frac{1}{n} \sum_{i=1}^N (y \log(\hat{y}) + (1 - y) \log(1 - \hat{y}))$$

Here \hat{y} are our predictions $= \phi(Z)$

2.4 Updating weights

$$\frac{\delta L}{\delta w_j} = \frac{1}{m} \sum_{i=1}^m (\hat{y} - y_i) x_j^{(i)}$$

$$\frac{\delta L}{\delta b} = \frac{1}{m} \sum_{i=1}^m (\hat{y} - y_i)$$

Updating weights:

$$w_j := w_j - \eta \frac{\delta L}{\delta w_j}$$

Here η represents learning rate.

For calculating we use so-called rule of chain:

$$\frac{\delta L}{\delta w_j} = \frac{\delta L}{\delta \hat{y}} \frac{\delta \hat{y}}{\delta z} \frac{\delta z}{\delta w_j}$$

3 Multilayer perceptron

3.1 Overview

Multilayer perceptron is a combination of regression model and a perceptron on a bigger scale. Instead of one single neuron, we can have multiple layers of many neurons. The schematics is shown in figure 3.

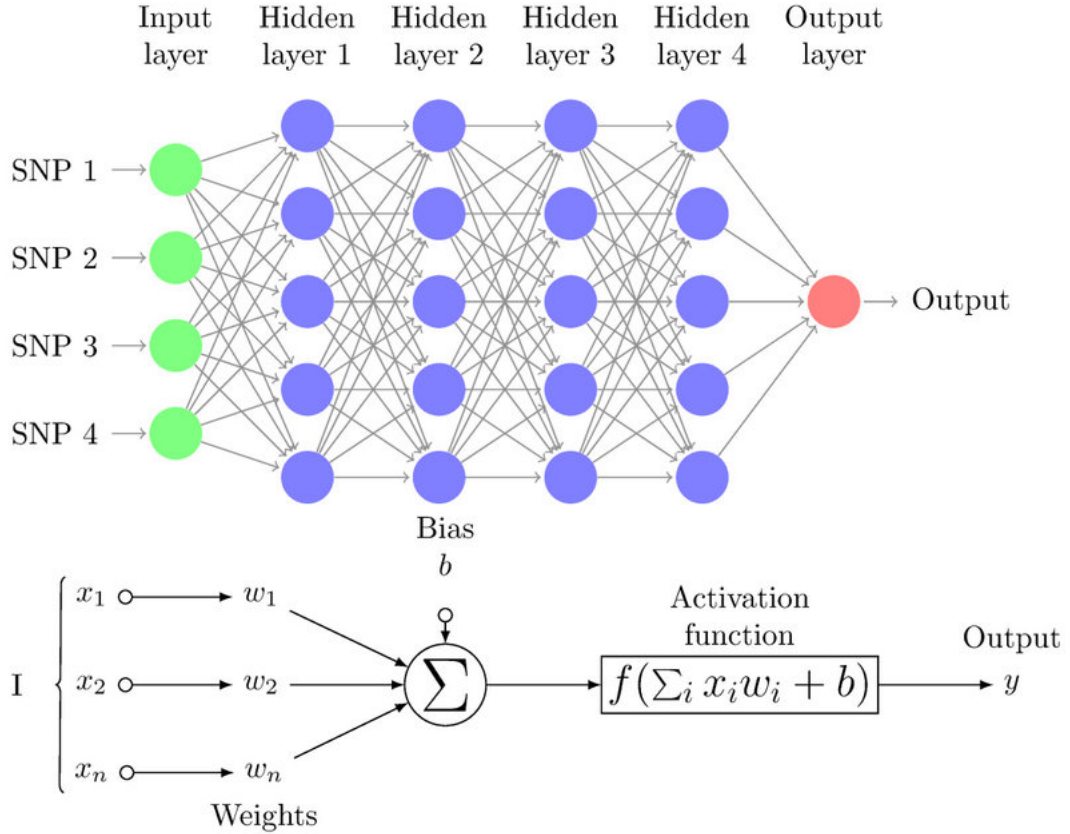


Figure 3 Multilayer perceptron

- x_1, x_2, \dots, x_a are input values;
- h_1, h_2, \dots, h_b are hidden layers;
- ϕ is an activation function;
- \hat{y}_1, \hat{y}_c are predicted values.

Since we have a lot of neurons and many layers, we calculate linear combination for every single neuron. All the layers between the input and output layer are called "hidden" layers.

3.2 Linear combination

In a regular perceptron linear combination can be represented as a multiplication of a column of x_i s and a vector of weights w_i s :

$$Z = X^T W + w_0 = w_1 * x_1 + w_2 * x_2 + ... w_n * x_n + w_0$$

Since now we have multiple layers and neurons, linear combination \sum for layer h can be stored as vector (column):

$$z^{[h]} = W^{[h]} \Phi^{[h-1]} + B^{[h]}$$

where:

$$W^h = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{1a} \\ w_{21} & w_{22} & \dots & w_{2a} \\ \vdots & \vdots & \ddots & \vdots \\ w_{b1} & w_{b2} & \dots & w_{ba} \end{pmatrix}$$

Here we have a weight matrix for one layer from a MLP with b layers and a neurons in each layer;

$$\Phi^{h-1} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_b \end{pmatrix}$$

Here ϕ_i represents a vector of activation function's results for the previous layer. B represents a vector of biases for neurons.

3.3 Activation function

For activation function we use sigmoid:

$$\phi(z) = \frac{1}{1 + e^{-z}}$$

3.4 Prediction

$$\Phi^h = \phi(z^h)$$

3.5 Loss function

We can use cross-entropy loss from perceptron:

$$L(y, \hat{y}) = -\frac{1}{n} \sum_{i=1}^N (y \log(\hat{y}) + (1 - y) \log(1 - \hat{y}))$$

3.6 Weight updates

$$W^{[h]} := W^{[h]} - \eta \frac{\delta L}{\delta W^{[h]}}$$
$$B^{[h]} := B^{[h]} - \eta \frac{\delta L}{\delta B^{[h]}}$$

Conclusion

In this article a mathematical grounding for Perceptron, Logistic Regression and MLP models were observed. This article may serve as an overview to described models for future in-depth study.