| | |
|---|---|
| ***Mechanics*** | The Study of Motion of Objects |
| ***Kinematics*** | Description of how objects move in a straight line |
| ***Dynamics*** | Description of why objects move as they do |

| | |
|---|---|
| ***Position*** | The position of an object at any moment is given by its coordinate axis. Horizontal ***x*** Vertical ***y*** |
| ***Frame of Reference*** | Any measurement of position made with respect to a reference frame, usually origin x = 0. |

| | |
|---|---|
| ***Distance*** | A scalar distance traveled from start to finish. Path dependent. |
| ***Speed*** | A scalar quantity (no direction specified) that shows the rate that distance ***d*** is covered. |
| ***Instantaneous*** | The speed at an instant in time. Right now. Your speedometer reading when you glance it at. |
| ***Average*** | The total distance divided by the total time for the entire trip. |
| ***Constant*** | If the same speed is maintained over the entire trip. |
| $\bar{s} = \frac{d}{t}$ | the bar above the quantity indicates that it is the average value of that quantity. |

| | |
|---|---|
| ***Displacement*** | A vector distance between the some initial point and final point of the event. Path independent. |
| $\Delta x = x - x_0$ | |

| | |
|---|---|
| ***Velocity*** | A vector quantity consisting of a change in displacement by change in time. |
| ***Average*** | The total displacement divided by the total time for the entire trip. |
| ***Instantaneous*** | The velocity at an instant in time. Right now. |
| ***Constant*** | If the same velocity is maintained over the entire trip. |
| $\bar{\mathbf{v}} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0}$ | $\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ $\quad \bar{v} = \frac{\vec{v}_0 + \vec{v}}{2}$ (alternate way at looking at average velocity.) |
| | the bar above the quantity indicates that it is the average value of that quantity. |

| | |
|---|---|
| ***Acceleration*** | A vector quantity consisting a change in velocity by change in time. |
| ***Average*** | The total accelerations divided by the total time for the entire trip. |
| ***Instantaneous*** | The acceleration at an instant in time. Right now. |
| ***Constant*** | If the same acceleration is maintained over the entire trip. |
| $\bar{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{t - t_0}$ | $\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$ |

$x_o$ initial position, $x$ final position, $v_o$ initial velocity, $v$ final velocity, $a$ acceleration, $t$ time
the bar above the quantity indicates that it is the average value of that quantity.

### *The Kinematic Equations*

You can only use the average velocity equation when there is ***no acceleration***. $\bar{v} = \frac{\vec{v}_0 + \vec{v}}{2}$ (an alternate way at looking at average velocity, but mostly used in conjunction with the definitions of speed, velocity and acceleration to derive the next kinematic equations. Occasionally it is used in problems.)

If acceleration is present then you must use one of the appropriate kinematic equations below.

| | | |
|---|---|---|
| ***Velocity*** | $\vec{\mathbf{v}} = \vec{v}_0 + \vec{\mathbf{a}}\mathbf{t}$ | Rearranged acceleration equation from above. Useful for determining *v*, when *a* and *t* are given. However, if any three variables are available and the fourth is needed, rearrange as necessary. |
| ***Position*** | $x = x_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ | Key equation to determine displacement when *a* is involved. Used extensively in falling body problems. Its derivative is the velocity equation above. |
| ***No Time*** | $\vec{v}^2 = \vec{\mathbf{v}}_0^2 + 2\mathbf{a}(\mathbf{x} - \mathbf{x}_0)$ | When *v*, *a*, and/or *Δx* are known, but no information is given about *t*. |
| ***Quadratic Functions*** | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | Quadratic functions have two solutions. Sometimes only one corresponds to reality, sometimes both, and a sometimes does not apply to the actual physical conditions of the event, (ie. negative time). |

## *GUESS [A Problem Solving Strategy]*

1. Draw a picture (mental or on paper)
2. **(GU)** List known and unknown variables.
   a) Caution; some may be extraneous, and are not necessary to solve the problem.
   b) Often either the starting or ending point is at rest, meaning a value of zero.
   c) Do necessary conversions.
3. **(E)** Choose an equation that can be solved with the known variables.
   a) This equation may or may not be the answer you are looking for.
   b) It may provide a new variable for use in another equation.
   c) This may lead to a succession of equations.
4. **(S)** Substitute the given variables into the equation chosen to solve the problem.
5. **(S)** Solve the problem to include the units of measurement (SI)

**+ or —** **????:** " + " & "– " can be used to indicate direction, and/or pos acceleration (+) or neg acceleration (–).

**–9.8 *m/s*²** be careful here. Does this mean the object is slowing down or does it mean that the object is moving along a negative direction (perhaps the *y*) axis? It would depend on the event. For an object moving on the ***x*** axis it would mean the object is slowing down. For an object falling along the ***y*** axis, due to gravity, it means the object is accelerating, but in the downward direction (— 9.8). **In forces it is easier to use 9.8 *m/s*² as a positive number**.

## *Freely Falling Objects*

| | | |
|---|---|---|
| **Displacement:** | $y_0 = 0$ | Initial position. We can choose the reference frame / coordinate axis. |
| | $y = 0$ | If the object ends the event at the same elevation it started at. |
| | $y = +$ | If the object ends the event at a higher elevation than it started. |
| | $y = -$ | If the object ends the event at a lower elevation than it started. |
| **Velocity, initial:** | $\vec{v}_0 = 0$ | If it is dropped from rest. |
| | $\vec{v}_0 = +$ | If fired upward. |
| | $\vec{v}_0 = -$ | If fired downward. |
| **Velocity, final:** | $\vec{v} = 0$ | At the moment it reaches maximum altitude, right before falling back to earth. |
| | $\vec{v} = +$ | If it hits something on the way up and never reaches max altitude (Rare problem). |
| | $\vec{v} = -$ | On the return trip. |
| | $\vec{v} = v_0$ | If it lands at the same elevation that the event began at. |
| **Acceleration *due to gravity*:** | | $\vec{g} = 9.8\frac{m}{s^2}$ |

*Projectile Motion*

Motion in two dimensions that happens simultaneously.

In the *x* direction the velocity is constant, with no acceleration occurring in this dimension.

In the *y* direction the acceleration of gravity slows upward motion and enhances downward motion.

Both happen simultaneously, however they can be analyzed separately using vector components.

> The following review of variables can be overwhelming to memorize.
> It is much easier if you think it through or draw a pictorial representation.

**Angles** All angles are measured from the horizontal. Above the horizon is positive, below negative.

| | | |
|---|---|---|
| **Displacement** | $x_0 = 0$ | |
| | $y_0 = 0$ | |
| | $x = +$ | the *x* is always positive |
| | $y = 0$ | if the object ends the event at the same elevation it began with. |
| | $y = +$ | if the object ends the event at a higher elevation it began with. |
| | $y = -$ | if the object ends the event at a lower elevation it began with. |
| **Velocity, initial** | $\vec{v}_0$ | Splits into components, $\vec{v}_{0x} = \vec{v}_0 \cos\theta$ or $\vec{v}_{0x} = \vec{v}_0 \sin\theta$ |
| | $\vec{v}_{ox} = +$ | In every problem, we choose to fire it in the positive x direction. |
| | $\vec{v}_{oy} = 0$ | If fired horizontally. |
| | $\vec{v}_{oy} = +$ | If fired at a positive angle (above the horizon). |
| | $\vec{v}_{oy} = -$ | If fired at a negative angle (below the horizon). |
| **Velocity, final** | $\vec{v}_x = v_{0x}$ | Since there is constant velocity in the x direction, initial and final are the same. |
| | $\vec{v}_{oy} = 0$ | At the top of the trajectory |
| | $\vec{v}_{oy} = +$ | If the object hits something on the way up. Not used in problems very often. |
| | $\vec{v}_{oy} = -$ | On the return trip. |
| | $\vec{v}_R$ | Resultant from adding vectors $\vec{v}_x$ and $\vec{v}_y$. Has an angle not a + or – |
| Acceleration | $-\vec{g} =$ | $9.8 \frac{m}{s^2}$ |

### General Kinematic Equations for Constant Acceleration in Two Dimensions

| **X component (horizontal)** | **Y component (vertical)** |
|---|---|
| $\vec{v}_x = \vec{v}_{0x} + \vec{a}_x t$ | $\vec{v}_y = \vec{v}_{0y} + \vec{a}_y t$ |
| $x = x_o + \vec{v}_{ox} t + \frac{1}{2}\vec{a}t^2$ | $y = y_o + \vec{v}_{oy} t + \frac{1}{2}\vec{a}t^2$ |
| $\vec{v}_x^2 = \vec{v}_{0x}^2 + 2\vec{a}_x(x - x_0)$ | $\vec{v}_x^2 = \vec{v}_{0x}^2 + 2\vec{a}_x(y - y_0)$ |

### General Kinematic Equations for Constant Acceleration in Two Dimensions

| **X component (horizontal)** | **Y component (vertical)** |
|---|---|
| $(\vec{a}_x = 0, \vec{v}_x = \text{constant})$ | $(\vec{a}_y = -g = \text{constant})$ |
| $x = x_o + \vec{v}_{0x} t$ | $y = \vec{v}_{oy} t - \vec{g}t^2$ |
| $\vec{v}_x = \vec{v}_{0x} = \text{constant}$ | $\vec{v}_y = \vec{v}_{oy} - \vec{g}t)$ |
| | $\vec{v}_y^2 = \vec{v}_{0y}^2 - 2\vec{g}(y - y_0)$ |

| *X component variables* | *Common to both X & Y* | *Y component variables* |
|---|---|---|
| $x_0 = 0$ | $\vec{v}_0 = 0$ | $y_0 = 0$ |
| $x =$ | $\theta =$ | $y =$ |
| $\vec{v}_{0x} = \vec{v}_0 \cos\theta$ | $t =$ | $\vec{v}_{0y} = \vec{v}_0 \sin\theta$ |
| $\vec{v}_x = \vec{v}_{0x} = \vec{v}_0 \cos\theta$ | $\vec{v} = \sqrt{\vec{v}_x^2 + \vec{v}_y^2}$ | $\vec{v}_y =$ |
| $\vec{a}_x = 0$ | | $\vec{g} = 9.8$ |

***Strategies that work most of the time.***

*When no time is given. Finding time is the key to all falling body or projectile motion problems

1st $\quad \mathbf{\vec{v}_y^2 = \vec{v}_{0y}^2 - 2\vec{g}(y - y_0)}$ $\quad$ solve for $\mathbf{\vec{v}_y}$ this can be +/- , but is usually minus

2nd $\quad \mathbf{\vec{v}_y = \vec{v}_{oy} - \vec{g}t}$ $\quad$ use $\mathbf{\vec{v}_y}$ from above to get *t*

3rd $\quad \mathbf{x} = R = \mathbf{\vec{v}_{ox}t}$ $\quad$ use t from above to solve for range, *x*

(Alternative: $\mathbf{y = \vec{v}_{oy}t - \vec{g}t^2}$, and the quadratic, followed by $\boldsymbol{x = R = \vec{v}_{ox}t}$

*When time or range *x* is given. This makes the problem easy since velocity is constant in the *x* direction.

1st $\quad \mathbf{x} = R = \mathbf{\vec{v}_{ox}t}$ $\quad$ Given time solve for *x*. Given *x* solve for time.

2nd $\quad \mathbf{y = \vec{v}_{oy}t - \vec{g}t^2}$ $\quad$ Once you have time this is easy, and you don't need the quadratic.

*When an object is dropped or fired horizontally $y_{0_y} = 0$ and $y_0 = 0$.

$\mathbf{\vec{v}_y = \vec{v}_{oy} - \vec{g}t}$ becomes $\mathbf{\vec{v}_y = -\vec{g}t}$

$\mathbf{y = \vec{v}_{oy}t - \vec{g}t^2}$ becomes $\mathbf{y = -\vec{g}t^2}$

> These versions are time savers, particularly the last one, since it now no longer requires the quadratic formula.

## *PROJECTILE MOTION*

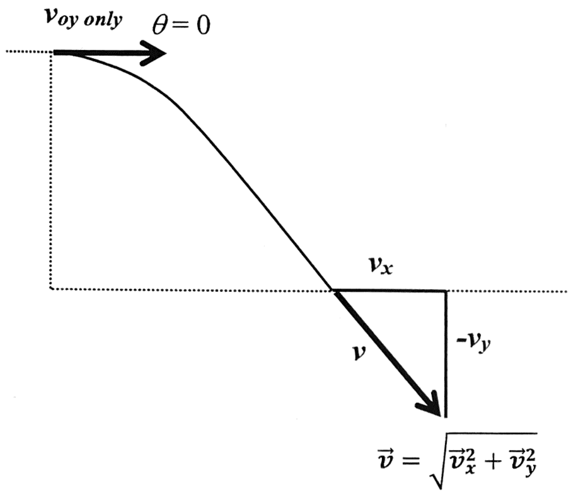
Time is ruled by gravity and height. Most problems require $y$ variables and $y$ equations to solve for time. From time distance in the $x$ direction and the final $v$ can be determined.

$\vec{v}_{0x} = \vec{v}_x$ Holds true for all projectile motion problems.

### *Horizontal*

$v_{0x} = v_0 = v_x$

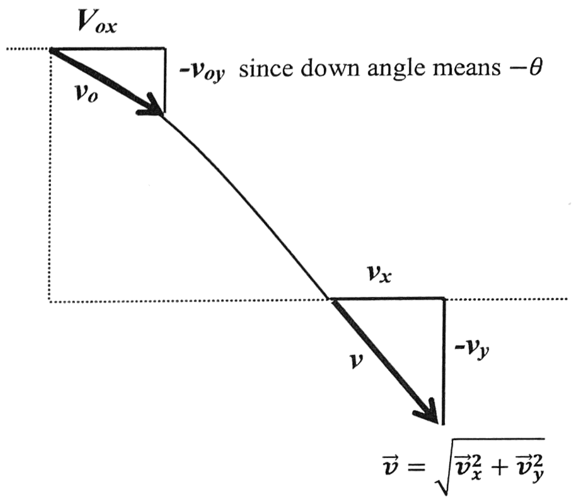
$\vec{v}_{0y} = 0$




### *Vertical*

$\vec{v}_{0x} = \vec{v}_0 \cos\theta$

$\vec{v}_{0y} = \vec{v}_0 \sin\theta$



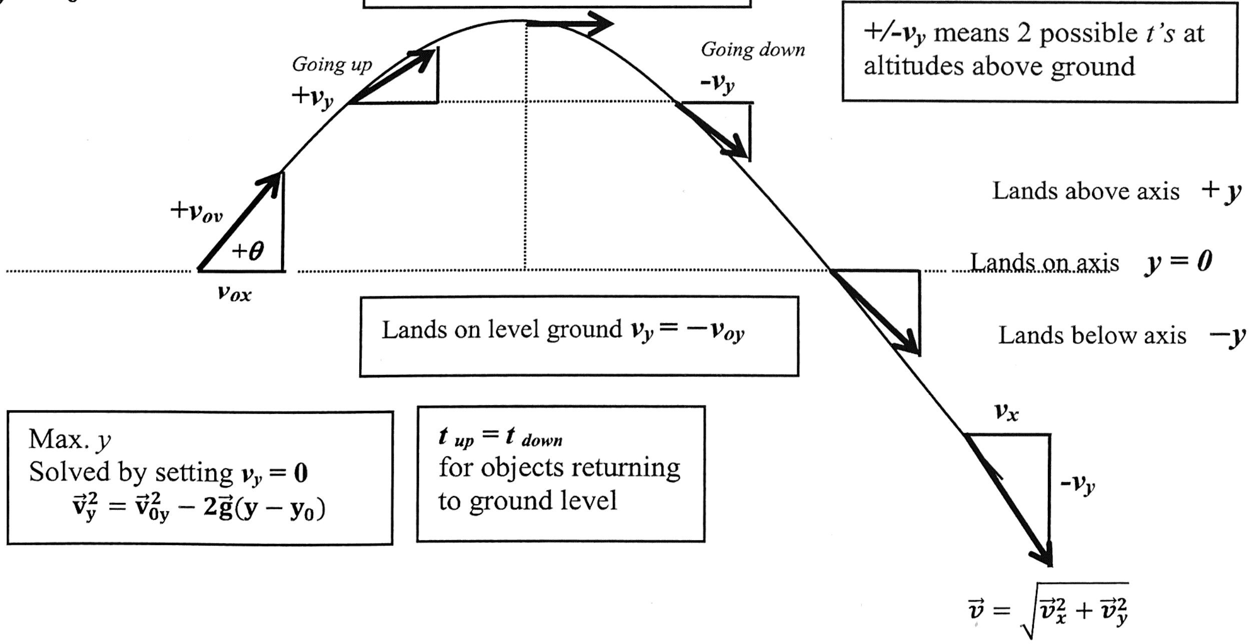

### *Type A and B Projectile*

$\vec{v}_{0x} = \vec{v}_0 \cos\theta$

$\vec{v}_{0y} = \vec{v}_0 \sin\theta$

At top: $v_y = 0$, $v = v_x = v_{ox}$

$+/-v_y$ means 2 possible $t$'s at altitudes above ground




Lands on level ground $v_y = -v_{oy}$

Max. $y$
Solved by setting $v_y = 0$
$$\vec{v}_y^2 = \vec{v}_{0y}^2 - 2\vec{g}(y - y_0)$$

$t_{up} = t_{down}$
for objects returning to ground level

### *Force*

Any push or pull

**Newton's 1st Law** Law of inertia (Restatement of Galileo's principle of inertia)

**Newton's 2nd Law** $\sum \vec{F} = m\vec{a}$ or $\vec{F}_{net} = m\vec{a}$

**Newton's 3rd Law** Equal and opposite forces. For every action force there is an equal & opposite reaction force. Forces come in action - reaction pairs. Force acts on two different objects.

> $\Sigma F$ Key to all problems.
> $\Sigma Fx$ is the sum of all forces in $x$ direction on traditional coordinate axis.
> $\Sigma$Fy is the sum of all forces in $y$ direction.

Sum of force is *Net Force*. You may need to solve for a using the kinematic equations, then solve for the force, or given a force you solve for acceleration and then use it in the kinematic equations to find $v$, $x$, or $t$.

### *Strategy on Force events*

1. Draw FBD.
2. Set direction of motion. *What would the object do if it could*? Considered this the positive direction.
3. Using the Forces listed below write the $\Sigma$ F equations relevant to the problem. In what direction is the object moving? What matters, the x or the y direction? Any force vectors in the FBD pointing in the direction of motion are positive while any vectors the other way are negative.
4. Substitute known equation, (Forces like *Fg* become *mg*).
5. Substitute for $\Sigma F$ for *Fnet*. Ask yourself what the sum of Force should be based on the chart below. Is the object standing still, moving at constant velocity, or accelerating? Substitute zero or *ma* for *Fnet*.

| | | | |
|---|---|---|---|
| **v = 0** | **Δ v** = 0 | **a** = 0 | **F net** = 0 |
| **v** = +/–<br>a constant value | **Δ v** = 0 | **a** = 0 | **F net** = 0 |
| **v** increasing or decreasing | **Δ v** = +/–<br>a constant value | **a** = +/–<br>a constant value | **F net** = **m a** |

6. Plug in and solve. (All values including 9.8 are entered as positives. The negative signs were decided when setting up the sum of force equation. Plugging in – 9.8 will just turn a vector assigned as – *Fg* into a positive. You decided its sign based on the way it was pointing relative to the problem's direction of motion. Don't reverse it now!)

### *Force Nomenclature*

**$F_P$ or $F_A$** Push or a Pull.

**$F_g$** Force due to acceleration of gravity. $F_g = mg$

**$F_T$** Tension is a rope, string, etc. This force has no equation. You either solve for it, or it cancels, or it's given.

**$F_N$** Force Normal. A contact force, always perpendicular to the surface.

**$FF_s$, $FF_k$** Friction force. $\boldsymbol{F_F = \mu F_N}$ Always resists motion. Static friction: not moving. Kinetic friction: object moving.

**$F_S$** Force due to a spring

**$F_{air}$** Force of air resistance. This force has no equation. You either solve for it, or it cancels, or it's given.

**$F_c$** Force Centripetal. It is the $\Sigma$ F in circular motion problems. So Fc can be any force that keeps an object in circular motion. $F_C = F_N$ $F_C = F_g$ $F_C = F_T$ $F_C = F_{F,S}$ $F_C = F_{F,K}$ $F_C = F_B$ *etc.*
It can also be two or more of these added together. The direction of motion is toward the center. So any force directed toward the center is positive and any force directed outward is negative. The key in using any of these equations are to ask yourself:
1. What is causing the circular motion?
2. Then set up the equality.
3. Substitute known equations.
4. Solve.

**$F_{mag}$** Force due to a magnetic field. This force is perpendicular to the field and perpendicular to the velocity of the particle. So, any charged particle will move in a circle. Use the right-hand rule for positive charges or positive current and use the left hand for negative charges or electron current.

**$F_B$** Force due to buoyancy. Objects submerged in a fluid appear to weigh less due to when an object is immersed in a fluid is equal to the weight of the fluid displaced by that object.

### *Friction*

Retard's motion. Motion is always parallel to a surface, so friction always acts parallel to resist.

**Static Friction**:

Friction that will prevent an object from moving. As long as the object is standing still the force of friction must be equal to the push, pull, and component of gravity or other force that attempts to move the object. (If there is no force attempting to cause motion, then there can be no friction). Static friction is the strongest type of friction since the surfaces have a stronger adherence when stationary.

**Kinetic Friction:**

Friction for moving objects. Once an object begins to move breaking static frictions hold, then the friction is termed kinetic. Kinetic friction is not as strong as static friction, but it still resists motion.

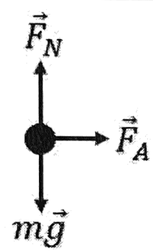
**Coefficient of friction**: $\mu$ a value of the adherence or strength of friction. $\mu_s$ for static friction. $\mu_k$ for kinetic friction.

## *Free Body Diagrams or Force Diagrams*

### Forces in the Horizontal Plane

**Frictionless Horizontal Surfaces**

$mg$ = weight
$F_N$ = Normal Force
$F_A$ = Pulling Force

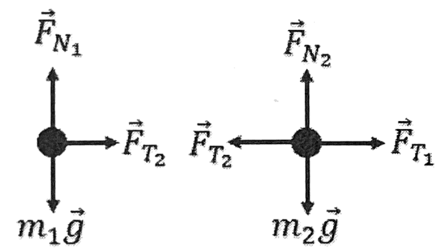



$$F_{net\,y} = F_N - mg$$
$$ma = F_N - mg$$
$$0 = F_N - mg$$
$$F_N = mg$$

$$F_{net\,x} = F_P$$
$$ma = F_P$$
$$a = \frac{F_P}{m}$$

**Frictionless Horizontal Surfaces (Tension)**

$mg$ = weight
$F_N$ = Normal Force
$F_T$ = Tensional Force




$$F_{net\,y} = F_N - mg$$
$$ma = F_N - mg$$
$$0 = F_N - mg$$
$$F_N = mg$$

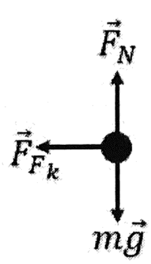
$$F_{net\,x\,2} = F_{T\,2}$$
$$m_2 a = F_{T\,2}$$

$$F_{net\,x\,1} = F_{T\,1} - F_{T\,2}$$
$$m_1 a = F_{T\,1} - m_2 a$$
$$F_{T\,1} = m_2 a + m_1 a$$
$$a = \frac{F_{T\,1}}{m_1 + m_2}$$

**Friction on horizontal surfaces**

1. Friction is the only force in the horizontal direction

$F_g$ = mg
$F_N$ = Normal Force
$F_{F\,k}$ = Frictional Force kinetic
μ = coefficient of friction



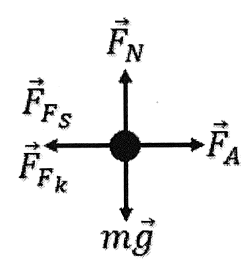

$$F_{net\,y} = F_N - mg$$
$$ma = F_N - mg$$
$$0 = F_N - mg$$
$$F_N = mg$$

$$F_{net\,x} = F_{F\,k}$$
$$F_{net\,x} = \mu\, F_N$$
$$ma = \mu\, mg$$
$$a = \mu\, g$$

**Friction on horizontal surfaces**

2. When friction and forward force are equal. Object may be at rest or moving with constant velocity.

$F_g$ = mg
$F_N$ = Normal Force
$F_P$ = Pulling Force
$F_{F\,S}$ = Frictional Force static
$F_{F\,k}$ = Frictional Force kinetic
μ = coefficient of friction

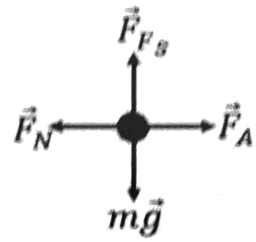



$$F_{net\,y} = F_N - mg$$
$$ma = F_N - mg$$
$$0 = F_N - mg$$
$$F_N = mg$$

$$F_{net\,x} = F_P - F_F$$
$$ma = F_P - \mu\, F_N$$
$$0 = F_P - \mu\, mg$$
$$F_P = \mu\, mg$$

**Box against a wall**

$F_g = mg$
$F_N$ = Normal Force
$F_P$ = Pulling Force
$F_F$ = Frictional Force




$$F_{net\ y} = F_P - mg$$
$$ma = F_P - mg$$
$$0 = F_P - mg$$
$$F_P = mg$$

$$F_{net\ x} = F_P + F_N$$
$$ma = F_P + F_N$$
$$0 = F_P + F_N$$
$$F_P = F_N$$

## Forces in the Vertical Plane

**Force Normal**

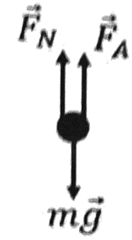
$F_g = mg$
$F_N$ = Normal Force


$$F_{net\ y} = F_N - mg$$
$$ma = F_N - mg$$
$$0 = F_N - mg$$
$$F_N = mg$$

The Normal Force on the box exerted by the table is equal in magnitude to the box's weight.

**Force Normal and pull**

$F_g = mg$
$F_N$ = Normal Force
$F_P$ = Pulling Force

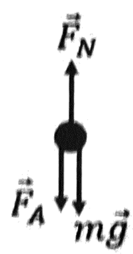



$$F_{net\ y} = F_N + F_P - mg$$
$$ma = F_N + F_P - mg$$
$$0 = F_N + F_P - mg$$
$$F_N = F_P - mg$$

The table pushes back with less force because of the applied force.

**Force Normal and push**

$F_g = mg$
$F_N$ = Normal Force
$F_P$ = Pushing Force

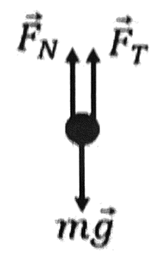



$$F_{net\ y} = F_N - F_P - mg$$
$$ma = F_N - F_P - mg$$
$$0 = F_N - F_P - mg$$
$$F_N = F_P + mg$$

The table pushes back with more force because of the applied force.

**Force Normal and Tensional Force**

$F_g = mg$
$F_N$ = Normal Force
$F_T$ = Tension Force




$$F_{net\ y} = F_N + F_T - mg$$
$$ma = F_N + F_T - mg$$
$$0 = F_N + F_T - mg$$
$$-F_N = F_T - mg$$

The table does not push against the full weight of the box because of the upward pull of the tensional force

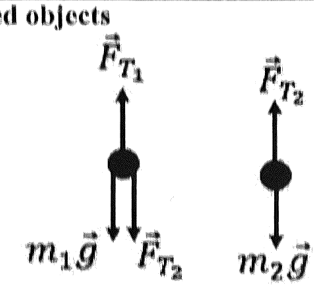
**Tensional Force, suspended object**

$F_g = mg$
$F_T$ = Tensional Force upward


$$F_{net\ y} = F_T - mg$$
$$ma = F_T - mg$$
$$0 = F_T - mg$$
$$F_T = mg$$

**Net Tensional Force, suspended objects**

$F_g = mg$
$F_{T1}$ = Tensional Force
$F_{T2}$ = Tensional Force




$$F_{net\ y\,2} = F_{T\,2} - m_2 g$$
$$m_2 a = F_{T\,2} - m_2 g$$
$$0 = F_{T\,2} - m_2 g$$
$$F_{T\,2} = m_2 g$$

$$F_{net\ y\,1} = F_{T\,1} - F_{T\,2} - m_1 g$$
$$m_1 a = F_{T\,1} - F_{T\,2} - m_1 g$$
$$0 = F_{T\,1} - m_2 g - m_1 g$$
$$F_{T\,1} = (m_2 + m_1) g$$

**Net Tensional Force, suspended object**

$F_g = mg$

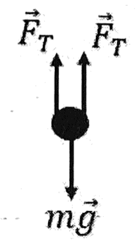
$F_T$ = Tensional Force

$F_{net\,y} = F_T + F_T - mg$

$ma = 2F_T - mg$

$0 = 2F_T - mg$

$F_T = \frac{mg}{2}$




**Force Tension with upward acceleration**

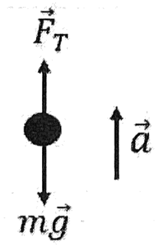
$F_g = mg$

$F_T$ = Tensional Force upward

$F_{net\,y} = F_T - mg$

$ma = F_T - mg$

$F_T = mg + ma$




**Force Tension with downward acceleration**

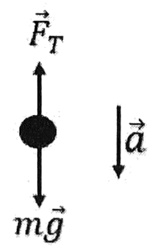
$F_g = mg$

$F_T$ = Tensional Force downward

$F_{net\,y} = F_T - mg$

$-ma = F_T - mg$

$-F_T = ma - mg$




**Apparent weight in elevators with downward acceleration**

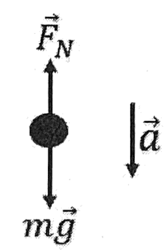
$F_g = mg$

$F_N$ = Normal Force

$F_{net\,y} = F_N - mg$

$-ma = F_N - mg$

$F_N = mg - ma$

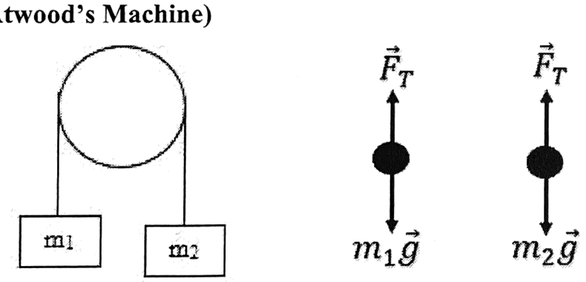



True mass doesn't change as a result of the downward acceleration.

## Complex Force Problems

Set direction of motion as positive. If you are not sure what the direction of motion will be take a guess. If the problem returns negative values for the final result, you were wrong, the problem went the opposite of your prediction.

**Force Tension (Atwood's Machine)**




It really doesn't say which block is more massive, pick one. I picked m2 as the heavier object to set the direction of motion. Rearrange to set equations in terms of tension equal to themselves, substitute, and solve for acceleration.

$F_{net\,y} = F_T - m_A g$ $\quad F_{net\,y} = F_T - m_2 g$

$m_1 a = F_T - m_1 g$ $\quad -m_2 a = F_T - m_2 g$

$F_T = m_1 a + m_1 g$ $\quad F_T = m_B g - m_B a$

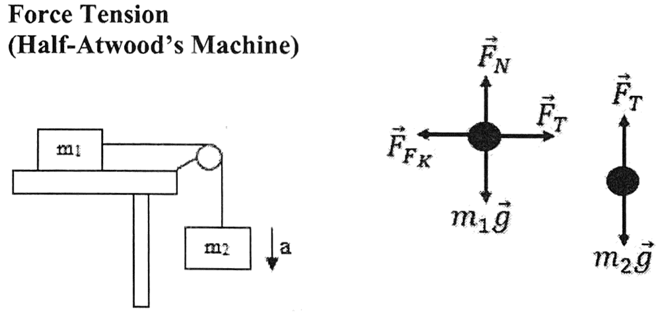
$$m_1 a + m_2 a = m_2 g - m_1 g$$

$$(m_1 + m_2)a = (m_2 - m_1)g$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

Other variables to solve for; $F_T, V_X, V_Y, \Delta x, \Delta y$

**Force Tension (Half-Atwood's Machine)**




Tension is the same between both blocks. Rearrange to set equations in terms of tension equal to themselves, substitute, and solve for acceleration.

$F_{net\,y} = F_N - m_1 g$ $\quad F_{net\,x} = F_T - F_{F\,k}$ $\quad F_{net\,y} = F_T - m_2 g$

$m_1 a = F_N - m_1 g$ $\quad m_1 a = F_T - \mu F_N$ $\quad m_2 a = F_T - m_2 g$

$0 = F_N - m_1 g$ $\quad F_T = m_1 a + \mu F_N$ $\quad F_T = m_2 g - m_2 a$

$F_N = m_1 g$

$$m_1 a + \mu F_N = m_2 g - m_2 a$$

$$(m_1 + m_2)a = m_2 g - \mu F_N$$

$$a = \frac{m_2 g - \mu F_N}{m_1 + m_2}$$

Other variables to solve for; $F_T, V_X, V_Y, \Delta x, \Delta y$

*Apparent Weight*

This is a consequence of your inertia. When an elevator, jet airplane, rocket, etc. accelerates upward the passenger wants to stay put due to inertia and is pulled down by gravity. The elevator pushes up and you feel heavier.

Add the acceleration of the elevator to the acceleration of gravity $F_{g\ apparent} = mg + ma$.

If the elevator is going down subtract $mg_{apparent} = mg - ma$

If the elevator is falling you will feel weightless $g = a$ so $mg_{apparent} = 0$

***Force***

Any push or pull

| | |
|---|---|
| **Newton's 1st Law** | Law of inertia (Restatement of Galileo's principle of inertia) |
| **Newton's 2nd Law** | $\sum \boldsymbol{F} = \boldsymbol{ma}$ or $\boldsymbol{F_{net}} = \boldsymbol{ma_{net}}$ |
| **Newton's 3rd Law** | Equal and opposite forces. For every action force there is an equal & opposite reaction force. Forces come in action - reaction pairs. Force acts on two different objects. |

$\boldsymbol{\Sigma F}$ Key to all problems.

$\boldsymbol{\Sigma Fx}$ is the sum of all forces in $x$ direction on traditional coordinate axis.

$\boldsymbol{\Sigma}$Fy is the sum of all forces in $y$ direction.

Sum of force is Net Force. You may need to solve for a using the kinematic equations, then solve for the force, or given a force you solve for acceleration and then use it in the kinematic equations to find *v*, *x*, or *t*.

***Strategy on Force events***

1. Draw FBD.
2. Set direction of motion. *What would the object do if it could*? Considered this the positive direction.
3. Using the Forces listed below write the Σ F equations relevant to the problem. In what direction is the object moving? What matters, the x or the y direction? Any force vectors in the FBD pointing in the direction of motion are positive while any vectors the other way are negative.
4. Substitute known equation, (Forces like *Fg* become *mg*).
5. Substitute for $\Sigma F$ for *Fnet*. Ask yourself what the sum of Force should be based on the chart below. Is the object standing still, moving at constant velocity, or accelerating? Substitute zero or *ma* for *Fnet.*

| | | | |
|---|---|---|---|
| **v = 0** | **Δ** v = 0 | **a** = 0 | **F net** = 0 |
| v = +/— a constant value | **Δ** v = 0 | **a** = 0 | **F net** = 0 |
| **v** increasing or decreasing | **Δ** v = +/— a constant value | **a** = +/— a constant value | **F net** = **m a** |

6. Plug in and solve. (All values including 9.8 are entered as positives. The negative signs were decided when setting up the sum of force equation. Plugging in – 9.8 will just turn a vector assigned as – *Fg* into a positive. You decided its sign based on the way it was pointing relative to the problems direction of motion. Don't reverse it now!)

***Force Nomenclature***

| | |
|---|---|
| **$F_P$** or **$F_A$** | Push or a Pull. |
| **$F_g$** | Force due to acceleration of gravity. $F_g = mg$ |
| **$F_T$** | Tension is a rope, string, etc. This force has no equation. You either solve for it, or it cancels, or it's given. |
| **$F_N$** | Force Normal. A contact force, always perpendicular to the surface. |
| **$FF_s$, $FF_k$** | Friction force. $\boldsymbol{F_F} = \boldsymbol{\mu F_N}$ Always resists motion. Static friction: not moving. Kinetic friction: object moving. |
| **$F_s$** | Force due to a spring |
| **$F_{air}$** | Force of air resistance. This force has no equation. You either solve for it, or it cancels, or it's given. |
| **$F_c$** | Force Centripetal. It is the Σ F in circular motion problems. So Fc can be any force that keeps an object in circular motion. $F_C = F_N$ $F_C = F_g$ $F_C = F_T$ $F_C = F_{F,s}$ $F_C = F_{F,k}$ $F_C = F_B$ *etc.* It can also be two or more of these added together. The direction of motion is toward the center. So any force directed toward the center is positive and any force directed outward is negative. The key in using any of these equations is to ask yourself: 1. What is causing the circular motion? 2. Then set up the equality. 3. Substitute known equations. 4. Solve. |
| **$F_{mag}$** | Force due to a magnetic field. This force is perpendicular to the field and perpendicular to the velocity of the particle. So any charged particle will move in a circle. Use the right hand rule for positive charges or positive current, and use the left hand for negative charges or electron current. |
| **$F_B$** | Force due to buoyancy. Objects submerged in a fluid appear to weigh less due to when an object is immersed in a fluid is equal to the weight of the fluid displaced by that object. |

### *Friction*

Retards motion. Motion is always parallel to a surface, so friction always acts parallel to resist.

**Static Friction:**

Friction that will prevent an object from moving. As long as the object is standing still the force of friction must be equal to the push, pull, and component of gravity or other force that attempts to move the object. (If there is no force attempting to cause motion, then there can be no friction).Static friction is the strongest type of friction since the surfaces have a stronger adherence when stationary.

**Kinetic Friction:**

Friction for moving objects. Once an object begins to move breaking static frictions hold, then the friction is termed kinetic. Kinetic friction is not as strong as static friction, but it still resists motion.

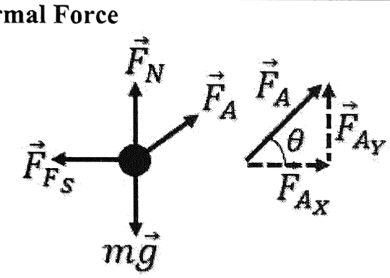
**Coefficient of friction:** $\mu$ a value of the adherence or strength of friction. $\mu_s$ for static friction. $\mu_k$ for kinetic friction.

## *Free Body Diagrams or Force Diagrams*

### Horizontal and Vertical Motion

**Force Applied and Normal Force**

$F_g$ = mg
$F_N$ = Normal Force
$F_A$ = Applied Force
$F_F$ = Frictional Force



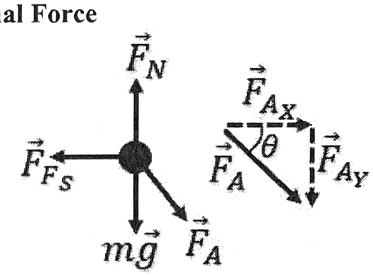

$F_A \sin\theta = F_{A\,y}$
$F_A \cos\theta = F_{A\,x}$

$F_{net\,y} = F_N + F_{A\,y} - mg$
$ma = F_N + F_{A\,y} - mg$
$0 = F_N + F_{A\,y} - mg$
$F_N = F_{A\,y} - mg$

$F_{net\,x} = F_{A\,x} - F_{F\,s}$
$ma = F_{A\,x} - \mu F_N$
$0 = F_{A\,x} - \mu F_N$
$F_{A\,x} = \mu F_N$

**Force Applied and Normal Force**

$F_g$ = mg
$F_N$ = Normal Force
$F_A$ = Applied Force
$F_F$ = Frictional Force




$F_P \sin\theta = F_{p\,y}$
$F_P \cos\theta = F_{p\,x}$

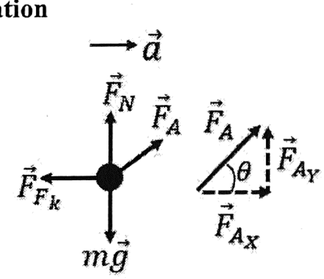
$F_{net\,y} = F_N - F_{p\,y} - mg$
$ma = F_N - F_{p\,y} - mg$
$0 = F_N - F_{p\,y} - mg$
$F_N = F_{p\,y} + mg$

$F_{net\,x} = F_{A\,x} - F_{F\,s}$
$ma = F_{A\,x} - \mu F_N$
$0 = F_{A\,x} - \mu F_N$
$F_{A\,x} = \mu F_N$

The table pushes back with more force due to the pushing force.

**Force Applied with acceleration**

$F_g$ = mg
$F_N$ = Normal Force
$F_A$ = Applied Force
$F_F$ = Frictional Force




$F_A \sin\theta = F_{A\,y}$
$F_A \cos\theta = F_{A\,x}$

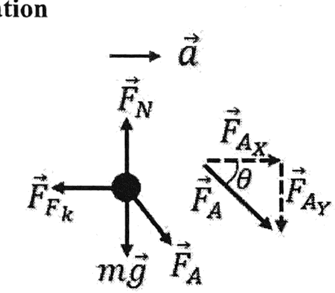
$F_{net\,y} = F_N + F_{A\,y} - mg$
$ma = F_N + F_{A\,y} - mg$
$0 = F_N + F_{A\,y} - mg$
$F_N = F_{A\,y} - mg$

$F_{net\,x} = F_{A\,x} - F_F$
$ma = F_{A\,x} - \mu F_N$

$a = \frac{F_{A\,x} - \mu F_N}{m}$

**Force Applied with acceleration**

$F_g$ = mg
$F_N$ = Normal Force
$F_A$ = Applied Force
$F_F$ = Frictional Force




$F_A \sin\theta = F_{A\,y}$
$F_A \cos\theta = F_{A\,x}$

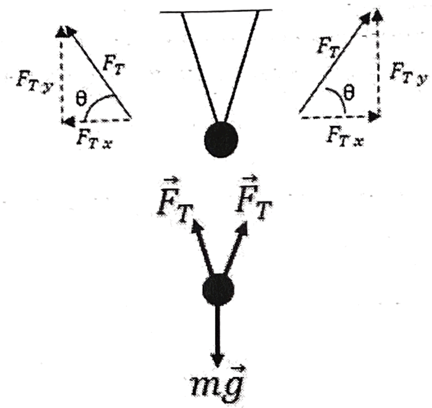
$F_{net\,y} = F_N - F_{A\,y} - mg$
$ma = F_N - F_{A\,y} - mg$
$0 = F_N - F_{A\,y} - mg$
$F_N = F_{A\,y} + mg$

$F_{net\,x} = F_{A\,x} - F_F$
$ma = F_{A\,x} - \mu F_N$

$a = \frac{F_{A\,x} - \mu F_N}{m}$

## Tensional Force, suspended object, angles equal

$F_g = mg$
$F_T$ = Tensional Force upward




$F_{T1}\sin\theta_1 = F_{T\,y1}$
$F_{T1}\cos\theta_1 = F_{T\,x1}$

$F_{T2}\sin\theta_2 = F_{T\,y2}$
$F_{T2}\cos\theta_2 = F_{T\,x2}$

$$F_{T\,1} = F_{T\,2}$$

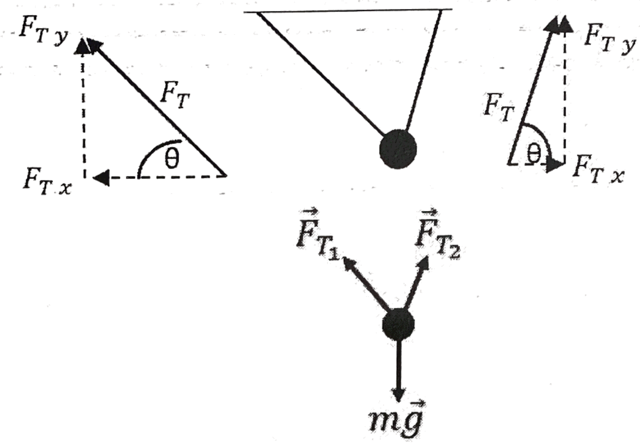
$$F_{net\,y} = F_{T\,y1} + F_{T\,y2} - mg$$
$$ma = 2\,F_{T\,Y} - mg$$
$$0 = 2F_{T\,y} - mg$$

$$2\ F_T\sin\theta = mg$$

$$F_T = \frac{mg}{2\sin\theta}$$

## Tensional Force, suspended object, angles unequal

$F_g = mg$
$F_T$ = Tensional Force upward




$F_{T1}\sin\theta_1 = F_{T\,y1}$
$F_{T1}\cos\theta_1 = F_{T\,x1}$

$F_{T2}\sin\theta_2 = F_{T\,y2}$
$F_{T2}\cos\theta_2 = F_{T\,x2}$

$$F_{net\,x} = F_{T\,x2} - F_{T\,x1}$$
$$ma = F_{T2}\cos\theta_2 - F_{T1}\cos\theta_1$$
$$0 = F_{T2}\cos\theta_2 - F_{T1}\cos\theta_1$$

$$F_{net\,y} = F_{T\,y1} + F_{T\,y2} - mg$$
$$ma = F_{T1}\sin\theta_1 + F_{T2}\sin\theta_2 - mg$$
$$0 = F_{T1}\sin\theta_1 + \frac{F_{T1}\cos\theta_1}{\cos\theta_2}\sin\theta_2 - mg$$

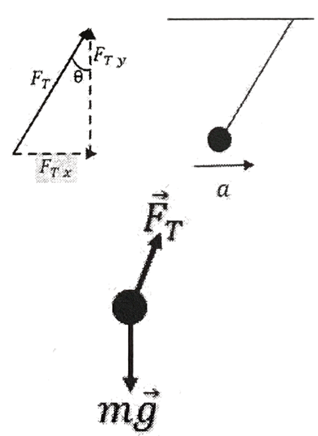
$$F_{T2}\cos\theta_2 = F_{T1}\cos\theta_1$$

$$F_{T1} = \frac{\frac{F_{T1}\cos\theta_1}{\cos\theta_2}\sin\theta_2}{\sin\theta_1} - mg$$

$$F_{T2} = \frac{F_{T1}\cos\theta_1}{\cos\theta_2}$$

## Accelerometer, suspended object

$F_g = mg$
$F_T$ = Tensional Force
$\theta = 8°$




$$F_T\cos\theta = F_{T\,y}$$
$$F_T\sin\theta = F_{T\,x}$$

$$F_{net\,y} = F_{T\,y} - F_g$$
$$ma = ma\cos\theta - mg$$
$$0 = ma\cos\theta - mg$$

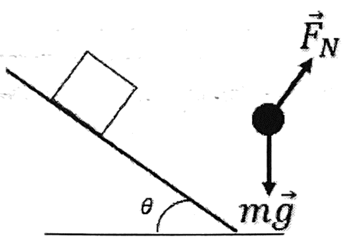
$$a = \frac{g}{\cos\theta}$$

$$F_{net\,x} = F_{T\,x}$$
$$ma = ma\sin\theta$$
$$a = \frac{g}{\cos 8}\sin\theta$$

$$a = 1.38\,\frac{m}{s^2}$$

## *Inclines*

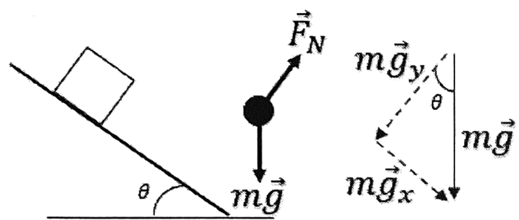
### Normal Force




Gravity pulls the object **down the slope and into the slope.** It we only consider the motion into the slope (perpendicular), the object has no perpendicular velocity. So $\boldsymbol{F\ net} \perp = \mathbf{0}$. Then the surface must push upward, equal and opposite to the perpendicular gravity component called Normal Force. Recall that the Normal Force is a surface contact force and operates perpendicular to any surface.

$\boldsymbol{F_N = mg\ \cos\theta}$ Where theta, $\theta$ is the angle between $mg$ and $mg\cos\theta$. It is also the tilt angle of the surface measured from the ground.

### Force Parallel to the slope




$\boldsymbol{mg_x = mg\ \sin\theta}$

Motion on the slope is parallel to the slope. *mg* and $F_N$ are at an angle to each other leaving a gap of magnitude $\boldsymbol{mg_x\ sin\ \theta}$ when these two vectors are added tip to tail. $\boldsymbol{mg_x\ sin\ \theta}$ is not a force by itself, it is the sum of force when *mg* and $F_N$ are added together. It is not part of the FBD. $\boldsymbol{mg_y}$ describes the motion of the object parallel to the slope if no other forces are acting on the object. What if we sum the forces in the direction of motion (which is parallel to the slope)? $\boldsymbol{mg_x\ sin\ \theta}$ is down the slope and positive, and objects generally want to go downhill (direction of natural motion is positive). Any force opposing the natural downward motion is a retarding force and is identified as negative. So uphill is negative.

The overall $\boldsymbol{F_{net\,x} = mg_x\ sin\ \theta - F_{retarding}}$

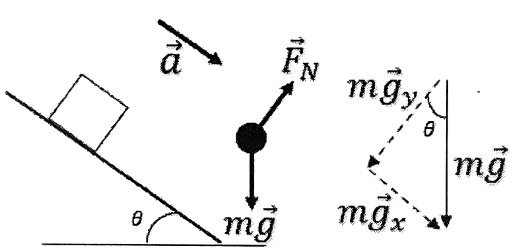
*What would you use for Force retarding*? It could be:

Force Friction $\boldsymbol{F_F}$ Force Tensional $\boldsymbol{F_T}$ Force applied $\boldsymbol{F_A}$ Force air resistance $\boldsymbol{F_{air}}$

or a combination of forces. Substitute the appropriate F and solve.

## *Friction on the slope*

Friction is the retarding force in the scenarios discussed above.




**1. No Friction**

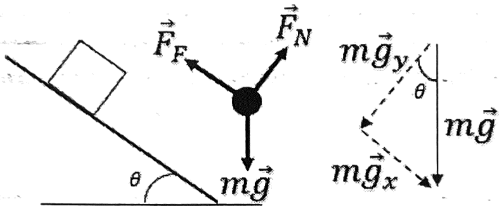
Accelerates $\boldsymbol{F_{net} = ma}$

$mg_y = mg\cos\theta$

$F_{net\,y} = F_N - mg_y$
$ma = F_N - mg_y$
$0 = F_N - mg_y$
$F_N = mg_y$

$mg_x = mg\sin\theta$

$F_{net\,x} = mg_x$
$ma_x = mg_x$
$a_x = g_x$




**2. v = 0 or v is constant.**
No acceleration $\boldsymbol{F_{net}} = \mathbf{0}$

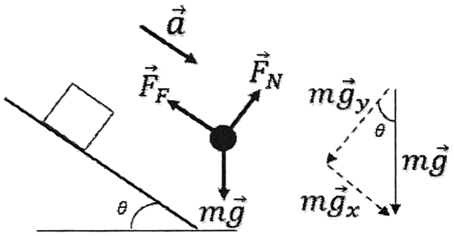
$mg_y = mg\cos\theta$

$F_{net\,y} = F_N - mg_y$
$ma = F_N - mg_y$
$0 = F_N - mg_y$
$F_N = mg_y$

$mg_x = mg\sin\theta$

$F_{net\,x} = mg_x - F_F$
$ma_x = mg_x - \mu F_N$
$0 = mg_x - \mu F_N$
$\mu F_N = mg_x$

$\mu = \frac{mg_x}{F_N}$




**3. Accelerating (friction present).**
Accelerates $\boldsymbol{F_{net}} = \boldsymbol{ma}$

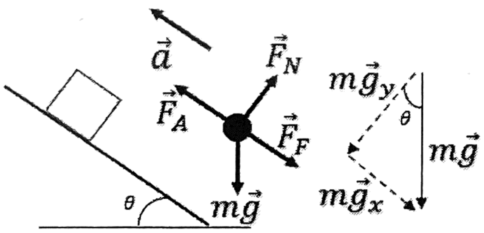
$mg_y = mg\cos\theta$

$F_{net\,y} = F_N - mg_y$
$ma = F_N - mg_y$
$0 = F_N - mg_y$
$F_N = mg_y$

$mg_x = mg\sin\theta$

$F_{net\,x} = mg_x - F_F$
$ma_x = mg_x - \mu F_N$
$ma_x = mg_x - \mu F_N$

$a_x = \frac{mg_x - \mu F_N}{m}$




**4. Pushed/Pulled up the slope (friction present).**
Accelerates $\boldsymbol{F_{net}} = \boldsymbol{ma}$

$mg_y = mg\cos\theta$

$F_{net\,y} = F_N - mg_y$
$ma = F_N - mg_y$
$0 = F_N - mg_y$
$F_N = mg_y$

$mg_x = mg\sin\theta$

$F_{net\,x} = F_F + mg_x - F_P$
$-ma_x = \mu F_N + mg_x - F_P$

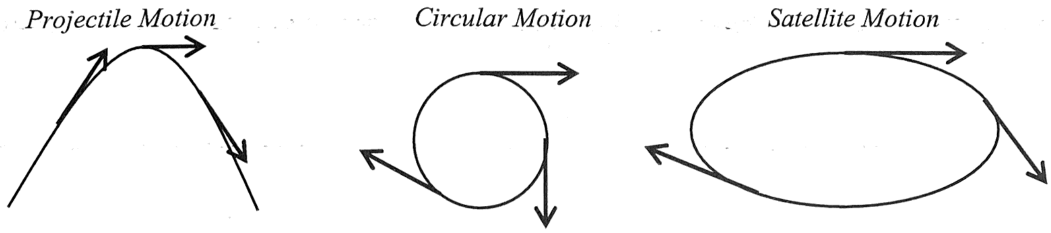
$a_x = \frac{mg_x + \mu F_N - F_P}{-m}$

***Frequency*** How often a repeating event happens. Measured in revolutions per second. Time is in the denominator.

***Period*** The time for one complete revolution. $T = \frac{1}{f}$ Time is in the numerator. It is the inverse of frequency.

***Speed*** Traveling in circles requires speed since direction is changing.

***Velocity*** However, you can measure instantaneous velocity for a point on the curve. Instantaneous velocity in any type of curved motion is tangent to the curve. ***Tangential Velocity.***




| The equation for speed and tangential velocity is the same $v = \frac{2\pi r}{T}$ |
|---|

***Acceleration*** Centripetal Acceleration. Due to inertia objects would follow the tangential velocity. But, they don't. The direction is being changed toward the center of the circle, or to the foci. In other words they are being accelerated toward the center. $a_r = \frac{v^2}{r}$ Centripetal means center seeking.

***Force*** ***Centripetal Force***. If an object is changing direction (accelerating) it must be doing so because a force is acting. Remember objects follow inertia (in this case the tangential velocity) unless acted upon by an external force. If the object is changing direction to the center of the circle or to the foci it must be forced that way. $F_C = ma_r$ $F_C = m\frac{v^2}{r}$

1. ***As always, ask what the object is doing.*** Changing direction, accelerating, toward the center, force centripetal.
2. ***Set the direction of motion as positive.*** Toward the center is positive, since this is the desired outcome.
3. ***Identify the net force equation.*** In circular motion $F_c$ is the net force. Therefore, $F_C$ can be any of the previous forces. If gravity is causing circular motion then $F_C = F_g$. If friction is present then, $F_C = F_{Fk}$. If a surface is present, then $F_C = F_N$.
4. ***Substitute the relevant force equations and solve***. For $F_c$ substitute $m\frac{v^2}{r}$

***Gravitation*** is $F_g = G\frac{m_1m_2}{r^2}$ and $F_g = mg$ then combined is $mg = G\frac{m_1m_2}{r^2}$ and simplified $g = G\frac{m}{r^2}$
$r$ is not a radius, but is the distance between attracting objects measured from center to center. Is the problem asking for the height of a satellite above earth's surface? After you get $r$ from the equation subtract earth's radius. Are you given height above the surface? Add the earth's radius to get $r$ and then plug this in. Think center to center.

***Inverse Square Law***

If $r$ doubles (x2), invert to get ½ and then square it to get ¼. Gravity is ¼ its original value so $F_g$ is ¼ of what it was and $g$ is ¼ of what it was. So multiply $F_g$ by ¼ to get the new weight, or multiply $g$ by ¼ to get the new acceleration of gravity. If $r$ is cut to a (x 1/3), invert it to get 3 and square it to get 9. Multiply $F_g$ or $g$ by 9.

***Apparent Weight***

This is a consequence of your inertia. When an elevator, jet airplane, rocket, etc. accelerates upward the passenger wants to stay put due to inertia and is pulled down by gravity. The elevator pushes up and you feel heavier.
Add the acceleration of the elevator to the acceleration of gravity $F_{g\ apparent} = mg + ma$.
If the elevator is going down subtract $F_{g\ apparent} = mg - ma$.
If the elevator is falling you will feel weightless, $g = a$ so, $F_{g\ apparent} = 0$

This same phenomenon works in circular motion. Your inertia wants to send you flying at the tangential velocity. You feel pressed up against the side of the car on the outside of the turn. So you think there is a force directed outward. This false non-existent force is really your inertia trying to send you out of the circle. The side of the car keeps you in moving in a circle just as the floor of the elevator moves you up. The car is forced to the center of the turn. No force exists to the outside. However, it feels like gravity, just like your inertia in the accelerating elevator makes you feel heavier. You are feeling g's similar to what fighter pilots feel when turning hard. It is not your real weight, but rather what you appear to weight, ***apparent weight***.

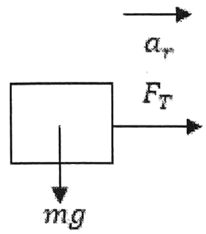
## *Free Body Diagrams or Force Diagrams*

### *Horizontal Circular Motion*

**Tension Force**
**(no surface)**

$F_g = mg$
$F_T$ = Tensional Force

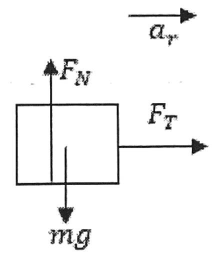
$F_{net\,x} = F_T$
$ma_r = F_T$
$F_T = m(\frac{v^2}{r})$




**Tension Force**
**(on surface)**

$F_g = mg$
$F_N$ = Normal Force
$F_T$ = Tensional Force

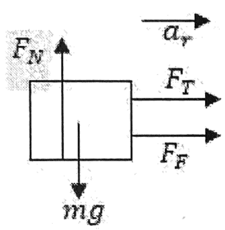
$F_{net\,x} = F_T$
$ma_r = F_T$
$F_T = m(\frac{v^2}{r})$




**Tension Force and**
**Force Friction**

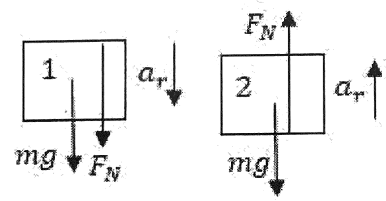
$F_g = mg$
$F_N$ = Normal Force
$F_T$ = Tensional Force

$F_{net\,x} = F_T + F_F$
$ma_r = F_T + \mu F_N$
$F_T = m\left(\frac{v^2}{r}\right) - \mu F_N$




**Normal Force and**
**Weight**

$F_g = mg$
$F_N$ = Normal Force




**1**
$F_{net\,y} = -F_N - mg$
$-ma_r = -F_N - mg$

$F_N = m(\frac{v^2}{r} - g)$

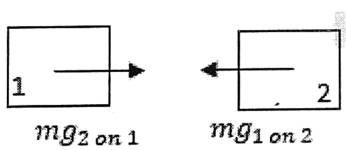
**2**
$F_{net\,y} = F_N - mg$
$ma_r = F_N - mg$

$F_N = m(g + \frac{v^2}{r})$

**Gravitation**

$F_g = mg$

$F_g = G\frac{m_1 m_2}{r^2}$



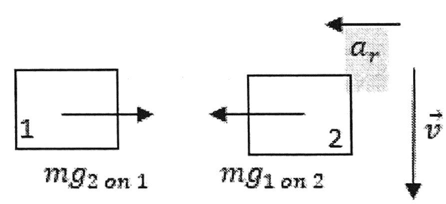

**Planetary Gravity and**
**Orbital Motion**

$F_g = mg$

$F_{net\,y} = F_G$

$ma_r = G\frac{m_1 m_2}{r^2}$

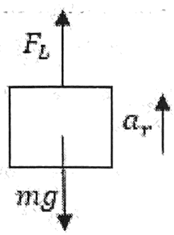
$m\frac{v^2}{r} = G\frac{m_1 m_2}{r^2}$



## *Vertical Circular Motion*

**Lift Force and Weight**

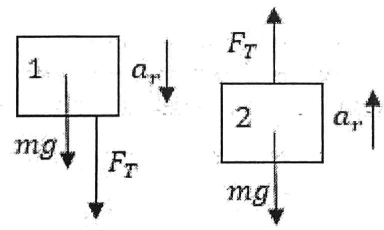
$F_g$ = mg
$F_L$ = Lift force




$F_{net\,y} = F_L - mg$
$ma_r = F_L - mg$
$F_L = m(g + \frac{v^2}{r})$

**Tension Force and Weight**

$F_g$ = mg
$F_T$ = Tensional Force

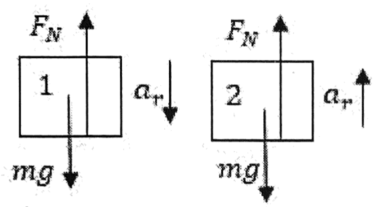



**1**
$F_{net\,y} = -F_T - mg$
$-ma_r = -F_T - mg$
$F_T = m(g - \frac{v^2}{r})$

**2**
$F_{net\,y} = F_T - mg$
$ma_r = F_T - mg$
$F_T = m(g + \frac{v^2}{r})$

**Normal Force and Weight**

$F_g$ = mg
$F_N$ = Normal Force

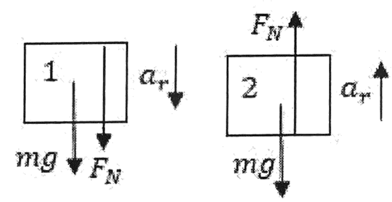



**1**
$F_{net\,y} = F_N - mg$
$-ma_r = F_N - mg$
$F_N = m(g - \frac{v^2}{r})$

**2**
$F_{net\,y} = F_N - mg$
$ma_r = F_N - mg$
$F_N = m(g + \frac{v^2}{r})$

**Normal Force and Weight**

$F_g$ = mg
$F_N$ = Normal Force



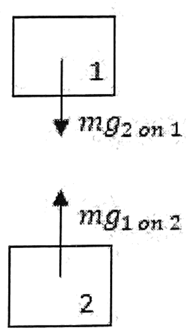

**1**
$F_{net\,y} = -F_N - mg$
$-ma_r = -F_N - mg$
$F_N = m(\frac{v^2}{r} - g)$

**2**
$F_{net\,y} = F_N - mg$
$ma_r = F_N - mg$
$F_N = m(g + \frac{v^2}{r})$

**Gravitation**

$F_g$ = mg

$$F_g = G\frac{m_1 m_2}{r^2}$$



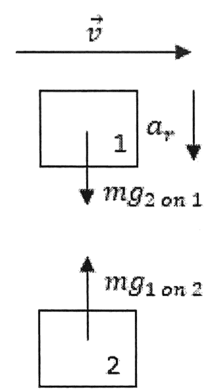

**Planetary Gravity and Orbital Motion**

$F_g$ = mg

$F_{net\,y} = F_G$

$$ma_r = G\frac{m_1 m_2}{r^2}$$

$$m\frac{v^2}{r} = G\frac{m_1 m_2}{r^2}$$



***Understanding the relationships between All Forms of Energy, Conservation of Energy, and Work Energy Theorem are extremely essential for success on the AP Exams. The summary here is very limited, since this critical information is given substantial emphasis in the course overview. Often energy is either the only way to progress in an AP Free Response problem, or it is the easiest (quickest) way to solve the problem. Students who have a thorough understanding of energy will achieve success on the AP Exam and arrive at college as a more accomplished physics student.***

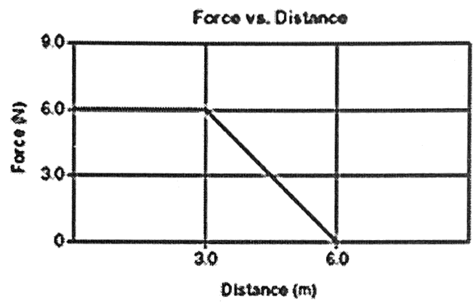
### *Work*

An object or problem has a certain amount of energy starting the problem (potential energy due to position and/or kinetic energy due to motion). Remember were pretending internal energy is zero. Think of work as the energy that is added (+***W***) to the system or subtracted (-***W***) from the system. If you add a force to something that is standing still it will begin to move a distance. This requires positive work, the product of the force used and distance moved.

$W = F_{\parallel} d \cos\theta$ Force applied over a distance. ***Force and distance must be parallel.*** Note: this does not mean the x axis (which cosθ usually goes along with θ ) is the angle between direction of motion and applied force.

### *Work is the area under the Force Distance curve*

This is the integral of the force distance function in a calculus based course. But, our functions will be simple enough to allow us to use geometry to find the area.




W = 27 Joules

### *Work-Energy Theorem*

Work put into a system = the change in energy of the system. If you do work on a system you add energy (+W). If the system moves to a lower energy state (dropping a bowling ball on your toe), then the system does work on the environment (-W). It can transfer energy to the environment. The bowling ball has –W while your toe gets +W (toe gets energy)

$w_{net} = \Delta E$ $\quad W_{net} = \Delta K$ $\quad W = \Delta U_G$ $\quad W = \Delta U_g$ $\quad W = \Delta U_S$ $\quad W = Q_{heat\ energy}$

But, what if the energy changes from zero to some amount or from some amount to zero.

$W = E$ $\quad W = K$ $\quad W = U_G$ $W = U_g$ $W = U_S$ $W = Q_{heat\ energy}$

Work and work-energy theorem are great for changes in energy, when energy moves from one object to another or is added or subtracted. But what if a system doesn't exchange energy with the environment or another system. What if it has certain types of energy in the beginning of the problem, but it has a different amount of each energy at the end?

### *Energy is conserved*

It cannot be created or destroyed, but it can change forms. Can energy be lost?
No! Lost energy goes to the environment. A car (system) loses energy due to air resistance, so air molecules (environment) gain energy and move faster. Energy is conserved.

When we did kinematics, problems might have started at 205.65 m from where we were standing. But, to make it easier we said the problem started at 0 m. For energy, pretend the system has zero internal energy initially. Then only worry about the other forms of energy. We can then solve for how much the internal energy changes in the problem.

| | | | |
|---|---|---|---|
| ***Kinetic Energy*** | $K = \frac{1}{2}mv^2$ | Energy of moving matter. Note that doubling mass doubles kinetic energy, but doubling velocity quadruples kinetic energy. So your car at 60 *mph* is 4 times more lethal than at 30 *mph*. |
| ***Potential Energy Gravitational*** | $U_g = mg\Delta y$ | Depends on height. Consider the lowest point in the problem to be zero height. This isn't correct, but who wants to add the radius of the earth to every number in the problem. Radius factors out at the end anyway. |
| | $U_G = -G\frac{m_1m_2}{r}$ | This expression is useful for the calculation of escape velocity, energy to remove from orbit, etc. However, for objects near the earth the acceleration of gravity g can be considered to be approximately constant and the expression for potential energy relative to the Earth's surface, little g. |
| ***Spring*** | $U_S = \frac{1}{2}kx^2$ | Energy of a compressed spring with spring constant ***k***. |

## *Conservation of Energy*

Energy cannot be created or destroyed, but can change form and be transferred.

**The big picture:** $K_0 + U_{g\,0} + U_{S\,0} + U_{G\,0} \;=\; K + U_g + U_S + U_G$

However, the problem may only talk about two forms of energy.
As an example: If the problem only involves Kinetic Energy and Potential Energy ($g$)

$K_0 + U_{g\,0} \;=\; K + U_g$ then substitute known equations $\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv^2 + mg\Delta y$

Here are some other possibilities: The first is escape velocity, the second is for springs.

$$\frac{1}{2}mv_0^2 + \frac{Gm_1m_2}{r}_0 = \frac{1}{2}mv^2 + \frac{Gm_1m_2}{r} \qquad \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

The following formulas are specific short cuts usually applied when there are two extremes in the problem.

***Gravity*** $mg\Delta y_0 = \frac{1}{2}mv^2$ A mass m starts at the highest point and ends at the lowest point, or vice versa.

***Spring*** $\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$ If a compressed spring extends to the equilibrium position, or vice versa.

***Power*** Rate at which work is done. Powerful machines do more work in the same time, or the same work in less time.

$$P = \frac{\Delta E}{\Delta t} \qquad P = \frac{W}{\Delta t} \qquad P = Fv$$

| Work or Energy delivered as a rate of time. |
|---|

It involves work. Making this another of the very important concepts.

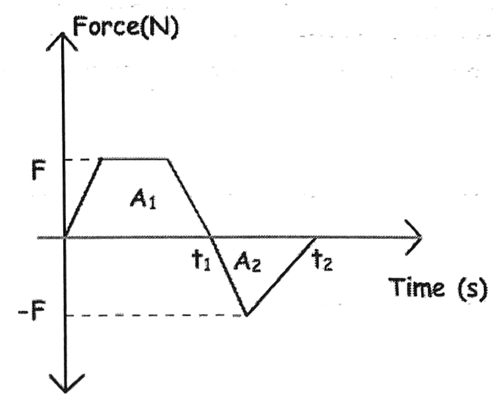
## *Energy and time*

Think Power when you see energy and time, Joules and seconds.

***Momentum*** $p = mv$ inertia in motion, mass on the move. Measure of how difficult it is to stop an object.
$\Delta p = mv - mv_0$ change in momentum.

***Impulse*** $F_{net}\Delta t = \Delta p$ a trade off between time taken to stop and force needed to stop.

***Impulse is the area under the Force Time curve***
Since impulse is equal to the multiplication of force and time then, area under the graph also determines impulse.




### *Conservation of Momentum (CoM)*

Total momentum before a collision must match total momentum after (momentum is conserved.
CoM equations are not given on the AP exam. One object might be standing still at the start or after.

**Elastic Collisions**:
Bounce off completely, No deformations.

Momentum is conserved! $m_1v_{1o} + m_2v_{2o} = m_1v_1 + m_2v_2$

$K$ is conserved! $\frac{1}{2}mv_{1o}^2 + \frac{1}{2}mv_{2o}^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

**Inelastic Collisions**:
Objects bounce of each other and deformed.
Momentum is conserved! $m_1v_{1o} + m_2v_{2o} = m_1v_1 + m_2v_2$

$K$ is **NOT** conserved! $K_{1o} + K_{2o} = K_1 + K_2 + K_{loss}$

**Perfectly Inelastic Collisions**:
The objects stick together, mass adds, one velocity.

Momentum is conserved! $m_1v_{1o} + m_2v_{2o} = (m_1 + m_2)v$

$K$ is **NOT** conserved! $K_{1o} + K_{2o} = K_1 + K_2 + K_{loss}$

### *Energy in Collisions*

$E_{1o} + E_{2o} = E_1 + E_2$ <u>Can be used by itself and with Conservation of Momentum</u>.

$K_{1o} + K_{2o} = K_1 + K_2 + K_{loss}$ In collisions ***total energy is conserved***, but ***kinetic energy <u>is not</u>***.

Unlike momentum, kinetic energy can decrease in collisions which are <u>not</u> ***elastic***. But where does the energy go? The deformation of colliding bodies turns the energy into heat (internal energy) sound, or light. So if you take the kinetic energy at the start, it will equal the kinetic energy at the end plus the amount of kinetic energy loss. The energy loss is conserved.

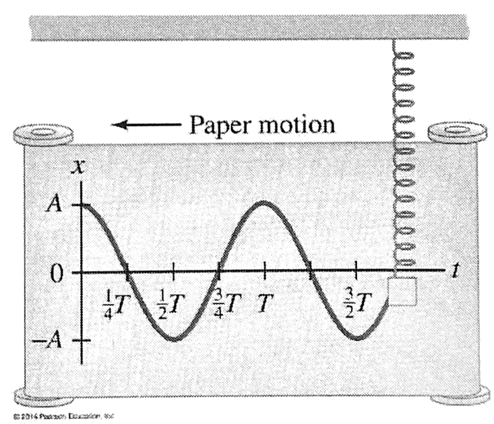
### *Simple Harmonic Motion (SHM)*
SHM refers to periodic vibrations or oscillations that exhibit two characteristics:
(1) the force acting on the object and the magnitude of the object's acceleration are always directly proportional to the displacement of the object from its equilibrium position, and
(2) the force vector (and the acceleration vector) is directed opposite to the displacement vector and therefore inward toward the object's equilibrium position. The force acting an object undergoing SHM follows Hooke's Law $\boldsymbol{F} = -\boldsymbol{k}\,\boldsymbol{x}$, where $F$ is the restoring force acting on the object, $k$ is the proportionality constant called the spring stiffness constant, or spring constant, and $x$ is the magnitude of the object's displacement from equilibrium. At the equilibrium position, the net force on the object equals zero. The negative sign indicates that the force and displacement vectors are oppositely directed.

### *Amplitude, Frequency, and Period*
**Vibration / oscillation**: Something must be vibrating / oscillating in order to create a wave.
**Medium**: Waves must travel in a medium with one important exception.
Electromagnetic waves are the only type of wave that <u>do not require</u> a medium at all.
**Amplitude**: ***A***, maximum displacement from the equilibrium position (midline on the graph).
**Frequency**: ***f***, (1/T) number of vibrations, oscillations, cycles, revolutions, etc. that take place each second.
**Period**: ***T***, (1/f) time for one complete vibration / oscillation.

### *Energy in the Simple Harmonic Oscillator*
The potential energy stored in a vibrating system undergoing SHM is given by PE = ½ kx². The total energy stored equals the sum of the kinetic potential energies. Assuming that no energy is dissipated due to friction, the total energy is constant: $E = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ at maximum displacement from equilibrium, v = 0 and x = A. At this point, $E = \frac{1}{2}kA^2$. When the displacement equals zero, (x = 0), all the energy is in the form of kinetic energy and $E = \frac{1}{2}mv^2$. Using conservation of energy, it is possible to show that the velocity of the object at any point in its motion can be determined from the equation $v = v_{max}\sqrt{1 - \frac{x^2}{A^2}}$ *or* $v = \sqrt{\left(\frac{m}{k}\right)(A^2 - x^2)}$ where $v$ is the velocity of the object at a displacement $x$ from equilibrium, $v_{max}$ is the maximum velocity of the object, and $A$ is the amplitude of the motion. Maximum velocity occurs as the object is passing through the equilibrium position.

### *The Period of SHM: Reference Circle*
Using the circle as a reference, an analogy can be drawn between the one-dimensional back-and-forth motion of an object undergoing SHM while attached to a spring and one component of the two-dimensional motion exhibited by an object traveling in a circle. We may derive the following equation for the period of an object of mass $m$ attached to a spring that has spring stiffness constant $k$ and is undergoing SHM: $T = 2\pi\sqrt{\frac{m}{k}}$. The larger the object's mass, the greater the inertia and the longer the period. The spring constant $k$ is related to the stiffness of the spring. The stiffer the spring, the greater the magnitude of the restoring force and acceleration at various displacements and the shorter the period.

### *SHM is Sinusoidal*
Using the motion of a spring undergoing SHM as a reference, it is possible to show that the graph of the displacement of the object with time has the form of a sine or cosine wave, that is, the position varies as a function of time. If the displacement of the object equals the amplitude (A) at t = 0, then the equation for the displacement, velocity, and acceleration as a function of time can be written as:




**Displacement:** $x = A\cos\omega t$, where $\omega = 2\pi f$ or $\omega = \frac{2\pi}{T}$

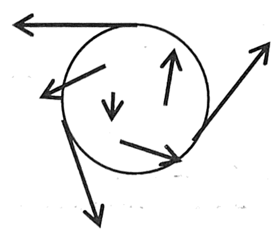
**Velocity:** $v = -\omega A \sin \omega t$

**Acceleration:** $a = -\omega^2 A \cos \omega t$

***Simple Pendulum***

A simple pendulum is assumed to have its entire mass concentrated at the end of a light string. The simple pendulum undergoes SHM if the maximum angle that it is displaced from equilibrium is small (approx. 15° or less). The formula for the period of motion is $T = 2\pi\sqrt{\frac{\ell}{g}}$ where $\ell$ is the length of the pendulum and $g$ is the gravitational acceleration. The period of a simple pendulum depends only on it length and the value of $g$. As a result, it can be used as a timing device. The first pendulum clock was built by Christian Huygens.

### *Rotation*



All parts of an object are rotating around the axis. All parts of the body have the same period of rotation. This means that the parts farther from the central axis of rotation are moving faster. So if we look at some of the tangential velocities diagramed at the right, we see that they are in all directions and vary in magnitude. So we need a new measurement of velocity. Collectively all the velocities are known as the ***angular velocity***, which is a measure of the radians turned by the object per second. Because the period is the same for the various parts of the rotating object, they move through the same angle in the same time. In rotation the parts of a rotating body on the outside move faster. They need to travel through the same number of degree or radians in the same amount of time as the inner parts of the body, but the circumference near the edge of a spinning object is longer than close to the center. So the outer edge must be moving faster to cover the longer distance in the same time interval. (This differs from the circular motion of the planets, which are not attached, and therefore not a single rotating body. The planets move in circular motion individually. Here the inner planets move faster. The planets closer to the sun must move faster in order to escape the gravity of the sun. They also travel a shorter distance and therefore have the shortest period of orbit).

***All the equations for an object in circular motion hold true if we are looking at a single point and only a specific point on a rotating object.***

Rotating objects have ***rotational inertia*** and an accompanying ***angular momentum***, meaning that a rotating object will continue to rotate unless acted upon by an ***unbalanced torque***, & a non-rotating object will not rotate unless acted upon by an ***unbalanced torque***.

***Torque*:** The force that causes rotation. In rotation problems we look at the sum of torque (not the sum of force). But it is exactly the same methodology.

$$\tau = rF\ \sin\theta$$

Strongest when the force is ***perpendicular*** to the lever arm (since sin 90° equals one).

**Balanced Torque:** The sum of torque is zero. No rotation.

**Unbalance Torque:** Adding all the clockwise and counterclockwise torque does not sum to zero. So there is excess torque in either the clockwise or counterclockwise direction. This will cause the object to rotate.

1. **As always, ask what the object is doing.** Is it rotating or is it standing still?
2. **Set the direction of motion as positive.** It will either rotate clockwise (CW −) or counterclockwise (CCW +). If you pick the incorrect direction your final answer will be negative, telling you that you did the process in reverse. But, the answer will be correct nonetheless. If it is not moving pick one direction to be positive, it really doesn't matter. But the other must be negative, so that the torque cancels.
3. **Identify the sum of torque equation.**

$$\tau_{net} = \tau_{ccw} + \tau_{cw}$$

4. **Substitute the relevant torque equalities and solve**

Rotating you get some +/− $\quad \tau_{net} = rF_{ccw} \sin\theta + rF_{cw} \sin\theta$

Not Rotating $\quad 0 = rF_{ccw} \sin\theta + rF_{cw} \sin\theta$

***Rotational Kinetic Energy:*** a rotating object has the ability to do work and therefore has energy. This energy is in the form of rotational kinetic energy and is given by this equation

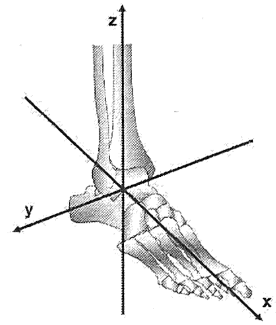
$$k_{rot} = \frac{1}{2} I\omega^2$$

The total kinetic energy of a rotating object equals the sum of its translational kinetic energy and its rotational kinetic energy.

$$k_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

### *Equilibrium*
An object is in static equilibrium if it is a rest. If left undisturbed, such an object will undergo no translational or rotational acceleration.

### *First Condition for Equilibrium*
In order for an object to be in equilibrium, the vector sum of the forces acting on it must be zero. $\boldsymbol{F_{net}} = \mathbf{0}$
To solve problems involving equilibrium, resolve the force vectors into x, y, and z components.
The equation $\boldsymbol{F_{net}} = \mathbf{0}$ is replaced by $\boldsymbol{F_{net\,x}} = \mathbf{0}$ $\boldsymbol{F_{net\,y}} = \mathbf{0}$ $\boldsymbol{F_{net\,z}} = \mathbf{0}$




If the line of action of each force acting on the object passes through a common point, $\overrightarrow{\boldsymbol{F_{net}}}$ is the only condition needed to solve the problem. The diagram to the right is an example of such a situation.

### *Second Condition for Equilibrium*
The 2nd condition states that the sum of all torques acting about any axis is perpendicular to the plane of the forces must be zero: $\boldsymbol{\tau_{net}} = \mathbf{0}$. A torque that causes a CCW rotation is defined as positive, and a CW rotation is negative. Thus, for equilibrium, the sum CCW and CW torques must add to zero.

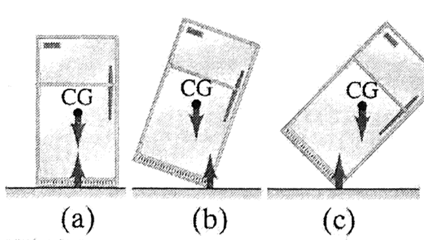
Since no rotation occurs, any point may be selected to the location of the reference axis. To simplify the solution to most problems, you usually take the axis of rotation to be a point through which the line of action of an unknown force passes. In this situation, the lever arm from the rotation point to the line of action of the unknown force is zero. Therefore, the torque produced by the unknown force is zero, and at least one unknown has been eliminated from the torque equation. When the line of action of each force does not pass through a common point, both conditions of equilibrium must usually be applied to solve the problem.

### *Alternate Conditions for Equilibrium*

$F_{net\,up} = F_{net\,down}$ $F_{net\,left} = F_{net\,right}$ $\tau_{CW} = \tau_{CCW}$

### *Stable, Unstable, and Neutral Equilibrium*
An object whose center of gravity (CG) is below its support point is said to be in stable equilibrium. Any displacement from the undisturbed position results in a torque that tends to restore the object to its original position.

If the object's CG is above the support point, stability depends on the position of the CG relative to the support point.




(a) Standing refrigerator
(b) Tipped slightly, 'fridge'will return back to its original position
(c) Tipped too far (critical point) CG no longer above base of support

A pencil standing on its sharpened point is in unstable equilibrium.
A spherical ball on a flat surface is in neutral equilibrium, CG remains over its support base no matter how the ball is displaced.

***Rotational Momentum:*** Depends on mass (like regular momentum) and it also depends on mass distribution. As an ice skater brings their arms closer to the body they begin to spin faster, since the mass has a shorter distance to travel. It is the product of an object's moment of inertia and angular velocity.

$$L = I\omega$$

The units of angular momentum are $kgm^2/s$, and angular momentum is a vector quantity. The direction of the vector is along the axis of rotation, and it is found by the right-hand rule. The right-hand rule states that when the fingers of the right hand curl in the direction in which the object is rotating, the thumb of the right hand points in the direction of the angular momentum vector.

***Rotational momentum is conserved.*** The radius gets smaller, but angular velocity increases (vice versa as the skater moves arms outward). A galaxy, solar system, star, or planet forms from a larger cloud of dust. As the cloud is pulled together by gravity its radius shrinks. So the angular velocity must increase. These objects all begin to spin faster and faster. That is why we have day and night.

$$\tau = \frac{\Delta L}{\Delta t}$$

***Law of Conservation of Rotational Momentum:*** in the absence of a net torque acting on an object, the object's angular momentum must remain constant in both magnitude and direction:

$$L_0 = L$$

$$I_{initial}\omega_{initial} = I_{final}\omega_{final}$$