

Distributed Energy Trading in Microgrids: A Game-Theoretic Model and Its Equilibrium Analysis

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Abstract—This paper proposes a distributed mechanism for energy trading among microgrids in a competitive market. We consider multiple interconnected microgrids in a region where, at a given time, some microgrids have superfluous energy for sale or to keep in storage facilities, whereas some other microgrids wish to buy additional energy to meet local demands and/or storage requirements. Under our approach, sellers lead the competition by independently deciding the amount of energy for sale subject to a tradeoff between the attained satisfaction from the received revenue and that from the stored energy. Buyers follow the sellers' actions by independently submitting a unit price bid to the sellers. Correspondingly, the energy is allocated to the buyers in proportion to their bids, whereas the revenue is allocated to the sellers in proportion to their sales. We study the economic benefits of such an energy trading mechanism by analyzing its hierarchical decision-making scheme as a multileader–multifollower Stackelberg game. We show that distributing the energy based on a well-defined utility function converges to a unique equilibrium solution for maximizing the payoff of all participating microgrids. This game-theoretic study provides an incentive for energy trading among microgrids in future power grids.

Index Terms—Energy trading, equilibrium analysis, microgrids, Stackelberg game.

I. INTRODUCTION

NEXT-GENERATION power grids are envisioned to be a smart-grid architecture driven by a growing demand for higher energy efficiency, reduced greenhouse gas emissions, and improved power quality and reliability [1]–[4]. This evolution is enabled by innovations and continuing developments in distributed generation and energy storage [5]–[7], advanced power electronics [8], [9], and modern communication technologies [10], [11]. To manage and operate such a critical and complex infrastructure efficiently and reliably, an important building block, which is known as a *microgrid* [12]–[14], has emerged as a promising platform for a smart grid to integrate and coordinate a potentially huge number of distributed energy resources (DERs) in a decentralized way [15]–[19].

Microgrids are small-scale power systems that can distribute energy in small geographic areas more flexibly and reliably. They typically utilize DERs, including distributed generation units [20] and energy storage facilities [21], for meeting local demands. Thus, they can reduce reliance on the conventional centralized power grid (also called a macrogrid or a main grid in the literature of power systems) that typically uses large central station generation. In addition to environmental benefits in terms of utilizing locally available renewable energy sources such as photovoltaic panels, fuel cells, or wind turbines, microgrids can reduce the transmission and distribution loss because of the physical proximity of DERs and loads.

Various forms of demand and supply problems have been studied in literature, aiming for the optimal operation of a microgrid taking into account load demand forecasting, prediction for power generation, and/or energy storage capacities [22]–[28]. One important feature of renewable energy sources such as wind and solar power is that their supply is, by nature, intermittent and highly variable. Distributed generation using such unpredictable energy sources is, in general, hard to control. With the ongoing large-scale deployment of renewable energy sources, it is challenging to balance between the energy demand and supply while fully utilizing the capacity of renewable energy sources.

This concern has recently motivated studies of market-based energy trading mechanisms among microgrids that enable a more effective utilization of DERs across a distribution network [29]–[33]. Consider a situation where, at a given time, several microgrids have superfluous energy that they wish to sell to the market, whereas some other microgrids do not have sufficient

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energy to support their local loads and need to buy the shortfall from the market. One way to deal with this situation is for microgrids to exclusively trade with a macrogrid. Specifically, microgrids as sellers feed the excess into the macrogrid, and those as buyers purchase the shortfall from the macrogrid. Another way is to encourage energy trading among interconnected microgrids within a vicinity. The latter option is more cost effective because it is more reliable and can reduce the energy loss incurred in long-distance transmission. It may also yield savings for microgrids by avoiding the selling/buying spread of the macrogrid [34].

Several game-theoretic approaches were proposed in [30]–[33] for energy trading among microgrids. Saad *et al.* [30] applied the coalitional game theory and developed a cooperative strategy for trading energy among microgrids with the aim of minimizing the average power loss over distribution lines. The cooperative strategy proposed by Matamoros *et al.* [31] aims to minimize the total cost resulting from energy generation and transportation subject to each microgrid satisfying its local demands. The leader–follower strategy presented by Asimakopoulou *et al.* [32] considers the competitive situations of hierarchical decision-making between an energy service provider representing several cooperative microgrids and a large central production unit. The work of Nunna and Doolla [33] is the closest to this paper where the focus is to study noncooperative strategies for trading energy among microgrids in a competitive market. Their approach is to formulate a matching game using an auction algorithm. As discussed in [33], such an auction-based trading mechanism only works for the case where there is an equal number of buyers and sellers in a market. In addition, it is a centralized approach that requires a centralized auctioneer.

In this paper, we propose a fully distributed energy trading mechanism. Under our approach, sellers lead the competition by independently deciding the amount of energy for sale subject to a tradeoff between the attained satisfaction from the received revenue and that from the stored energy. Buyers follow the sellers' actions by independently submitting a unit price bid to the sellers. Proportional sharing applies to both sides of the competition, where the energy is allocated to the buyers in proportion to their bids and where the revenue is allocated to the sellers in proportion to their sales. For the purpose of studying the economic benefits of such a distributed energy trading mechanism with a hierarchical decision-making structure, we analyze it using the framework of Stackelberg games [35]. Through rigorous game-theoretic analysis, we prove that the proposed approach converges to a unique equilibrium solution, and we show that distributing the energy based on a well-defined utility function can maximize the payoff for all participating microgrids at the equilibrium of the game. This provides an incentive for energy trading among microgrids in future power grids.

The rest of this paper is organized as follows. In Section II, we describe the model, and we provide the details of the proposed energy trading mechanism and the design of utility functions for microgrids. In Section III, we formulate the problem as a Stackelberg game, and we provide a rigorous analysis of the existence and uniqueness of the equilibrium solution for

TABLE I
SUMMARY OF MAJOR SYMBOLS

Symbol	Definition
\mathcal{I}	Set of microgrids as buyers
k	Number of buyers, i.e., $k = \mathcal{I} $
\mathcal{J}	Set of microgrids as sellers
\hat{E}_j	Superfluous energy generated by seller j
w_j	Proportion of \hat{E}_j that seller j decides to sell
R_j	Revenue received by seller j
c_i	Unit price offered by buyer i
E_i	Amount of energy allocated to buyer i
U_i	Utility function of buyer i
\hat{U}_j	Utility function of seller j

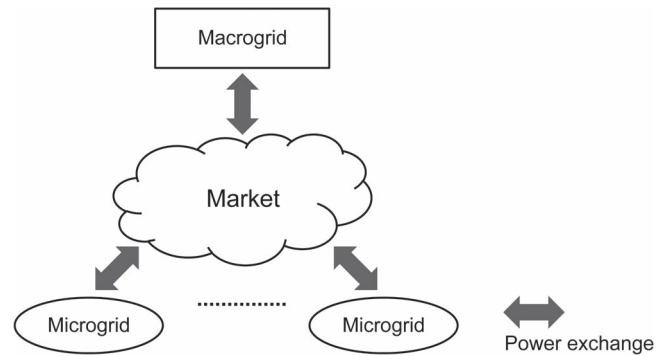


Fig. 1. Distributed energy trading among multiple interconnected microgrids.

such a game. Numerical results are presented in Section IV. Finally, we conclude this paper in Section V.

II. MODEL

For the reader's convenience, we provide in Table I a list of major symbols that we shall define and use in this paper.

We consider multiple microgrids that are deployed in a region. Each microgrid consists of DERs that are capable of generating and storing energy and loads that have demands for power. As illustrated in Fig. 1, the microgrids are connected to each other and can exchange energy with each other using a possible architecture such as the architecture discussed in [3] that interconnects microgrids through dedicated power exchange highways. The microgrids are also connected to the macrogrid and thus can exchange energy with the macrogrid.

Let the time be divided into consecutive fixed-length intervals. In each time interval, some microgrids have superfluous energy that they wish to sell to the market, whereas some other microgrids do not have sufficient energy to support their local demands and need to buy the shortfall from the market. To model this, we define \mathcal{I} as the set of buyers and \mathcal{J} as the set of sellers, where \mathcal{I} and \mathcal{J} are two disjoint sets. Let $k = |\mathcal{I}|$ denote the total number of buyers.

In a market with fluctuating energy prices [36], one can expect that microgrids have motivations and benefits to store energy. In such a context, those microgrids as sellers may

choose to keep part of the superfluous energy. Likewise, those microgrids as buyers may wish to buy and store extra energy if available because they can use or resell it at a later time. In our model, we only consider storage costs in terms of investment costs for energy storage systems, and we treat the investment costs as sunk costs. Studies of financial incentives for investments in energy storage systems can be found in, e.g., [37] and [38].

For each $j \in \mathcal{J}$, we define \hat{E}_j as the amount of superfluous energy generated by seller j . Let w_j be the proportion of \hat{E}_j that seller j decides to sell to the market. The set of available strategies for seller j , which is denoted as \mathcal{W}_j , is a continuum given by $\mathcal{W}_j = [0, 1]$. Thus, the amount of energy available for sale from seller j is $\hat{E}_j w_j$, and the amount of energy that is kept in seller j 's storage facility is $\hat{E}_j(1 - w_j)$. Let $\mathbf{w} = (w_j)_{j \in \mathcal{J}}$ be a profile of strategies for the sellers. Let \mathbf{w}_{-j} denote the profile of strategies for the sellers except j , i.e., $\mathbf{w}_{-j} = (w_l)_{l \in \mathcal{J} \setminus \{j\}}$. By definition, we have $\mathbf{w} = (w_j, \mathbf{w}_{-j})$.

For each $i \in \mathcal{I}$, we define c_i as the unit price bid submitted by buyer i . Let $c_S > 0$ and $c_B > 0$ be the selling price and the buying price, respectively, set by the macrogrid. The set of available strategies for buyer i , which is denoted as \mathcal{C}_i , is a continuum given by $\mathcal{C}_i = [c_B, c_S]$. Note that we expect $c_i \geq c_B$ to hold for all $i \in \mathcal{I}$ in this context because otherwise the sellers would achieve a higher revenue if they directly trade the energy with the macrogrid. On the other hand, we require $c_i \leq c_S$ to hold for all $i \in \mathcal{I}$ since otherwise the buyers would pay less by directly obtaining the same amount of energy from the macrogrid. Let $\mathbf{c} = (c_i)_{i \in \mathcal{I}}$ be a profile of strategies for the buyers. Let \mathbf{c}_{-i} denote the profile of strategies for the buyers except i , i.e., $\mathbf{c}_{-i} = (c_l)_{l \in \mathcal{I} \setminus \{i\}}$. By definition, we have $\mathbf{c} = (c_i, \mathbf{c}_{-i})$.

Following the principle of proportional sharing, for each $j \in \mathcal{J}$, the amount of energy $\hat{E}_j w_j$ from seller j is allocated to the buyers in proportion to their bids. For convenience, let

$$\hat{E} = \sum_{j \in \mathcal{J}} \hat{E}_j w_j.$$

Thus, the total amount of energy E_i allocated to buyer i by the sellers is

$$E_i = \sum_{j \in \mathcal{J}} \hat{E}_j w_j \frac{c_i}{\sum_{l \in \mathcal{I}} c_l} = \hat{E} \frac{c_i}{\sum_{l \in \mathcal{I}} c_l}. \quad (1)$$

Note that, in our model, if E_i is not sufficient for buyer i to support its local demands in the current time interval, buyer i would make up the shortfall with energy from the macrogrid. On the other hand, if E_i is larger than the local demands, we assume that buyer i has sufficient capacity to store the excess. Since the amount of superfluous energy generated by the sellers during each trading interval in our model depends on the length of the trading interval, this latter assumption is reasonable if the length of the trading interval is set to be sufficiently small.

Following the principle of proportional sharing, for each $i \in \mathcal{I}$, a payment in the amount of $E_i c_i$ from buyer i is allocated to

the sellers in proportion to their sales. For convenience, let

$$R = \sum_{i \in \mathcal{I}} E_i c_i. \quad (2)$$

Thus, the total revenue R_j received by seller j from the buyers is

$$R_j = \sum_{i \in \mathcal{I}} E_i c_i \frac{\hat{E}_j w_j}{\hat{E}} = R \frac{\hat{E}_j w_j}{\hat{E}}. \quad (3)$$

A. Utility Function of Seller

Our design of the utility function for seller j , $j \in \mathcal{J}$, considers two terms. The first term represents the satisfaction attained by seller j from the stored energy $\hat{E}_j(1 - w_j)$. The second term represents the satisfaction attained by seller j from the received revenue R_j . The utility function designed in this way aims for a balance between the attained satisfaction from storing the energy and that from selling the energy since the two terms are conflicting with each other.

In this context, one can expect that the satisfaction attained from storing the energy obeys the law of diminishing returns [39]. Correspondingly, a concave, strictly increasing, and continuously differentiable function may represent this term. In this paper, we will use a logarithmic utility function for this purpose because it is widely used in literature for quantifying user satisfaction with diminishing returns (see, e.g., [40] and [41] and the references therein).

Thus, for each $j \in \mathcal{J}$, the utility function of seller j is defined as

$$\hat{U}_j(w_j, \mathbf{w}_{-j}, \mathbf{c}) = \ln \left[1 + \hat{E}_j(1 - w_j) \right] + \gamma_j R_j \quad (4)$$

where $\gamma_j \geq 0$ is the weighting factor of the second term. For a given profile of strategies \mathbf{w} and \mathbf{c} , the social welfare of the sellers is defined as

$$\hat{U}(\mathbf{w}, \mathbf{c}) = \sum_{j \in \mathcal{J}} \hat{U}_j(w_j, \mathbf{w}_{-j}, \mathbf{c}).$$

B. Utility Function of Buyer

For each $i \in \mathcal{I}$, we consider the following utility function:

$$U_i(c_i, \mathbf{c}_{-i}, \mathbf{w}) = E_i c_S - E_i c_i \quad (5)$$

to quantify the satisfaction of buyer i attained from the energy trading in the current time interval. For a given profile of strategies \mathbf{c} and \mathbf{w} , the social welfare of the buyers is defined as

$$U(\mathbf{c}, \mathbf{w}) = \sum_{i \in \mathcal{I}} U_i(c_i, \mathbf{c}_{-i}, \mathbf{w}). \quad (6)$$

Note in the right-hand side of (5) that the first term $E_i c_S$ represents the amount of money that buyer i needs to pay if it directly obtains the amount of energy E_i from the macrogrid. The second term $E_i c_i$ is the amount of money that buyer i is willing to pay for buying E_i from the sellers. Thus, the utility

function defined in (5) quantifies the satisfaction of each buyer in terms of the amount of money it would save in the current time interval by obtaining E_i from the sellers with a cheaper unit price than c_S .

III. GAME-THEORETIC ANALYSIS

Distributed energy trading among microgrids in each particular time interval using our proposed mechanism is a hierarchical noncooperative decision problem that can be analyzed as a two-level continuous-kernel Stackelberg game [35].

A. Preliminaries

Stackelberg games are a special class of noncooperative games in which there exists a hierarchy among players. In a two-level game, players are classified as leaders and followers. In the original work of H. von Stackelberg, the Stackelberg model applies to monopolistic conditions where a number of small firms follow the behavior of a large dominant firm, which is the case with one single leader and multiple followers. For examples of the applications of single-leader-multifollower Stackelberg solutions to hierarchical decision problems in a smart grid, see [42]–[44]. Note that, in the literature of game theory, the Stackelberg model has been also extended to the case with multiple leaders and multiple followers [45]. The work in [46] is an example of applying a multileader-multifollower Stackelberg solution to the demand response management in a smart grid.

In general, each player in a Stackelberg game is rational and selfish, and each aims to maximize its utility. In addition, the leaders are in a position to enforce their strategies on the followers. Therefore, in this leader–follower competition, the leaders choose their strategies before the followers decide their strategies so that the followers can observe the strategies of the leaders and adapt their own strategies accordingly. In particular, since each follower has the information of the strategy adopted by each leader, it will choose its best strategy, which is also known as the *best response*, given the strategies of the leaders. On the other hand, the leaders are aware of the fact that each follower will choose its best response to the leaders' strategies. Therefore, the leaders are able to maximize their utilities based on the best responses of the followers. The solution of the game is known as the *Stackelberg equilibrium*.

In our problem, we consider the sellers taking the role of the leaders and the buyers taking the role of the followers. In the buyer-level game, each buyer independently chooses its own strategy. In particular, buyer i submits its unit price bid c_i to the sellers. Knowing the strategies of each seller, i.e., the value of w_j for all $j \in \mathcal{J}$, it aims to maximize utility function $U_i(c_i, \mathbf{c}_{-i}, \mathbf{w})$ defined in (5). Since the buyers do not know the strategies of each other, a Nash equilibrium gives the set of strategies with the property that none of the buyers can increase its own utility by choosing a different strategy given the strategies of the other buyers and those of the sellers.

On the other hand, in the seller-level game of our problem, each seller also independently chooses its own strategy. In particular, seller j decides the value of w_j , aiming to maximize

its utility. We will show in Section III-D that, based on utility function $\hat{U}_j(w_j, \mathbf{w}_{-j}, \mathbf{c})$ defined in (4), we can obtain an alternative form of $\hat{U}_j(w_j, \mathbf{w}_{-j}, \mathbf{c})$ that depends on the strategy of seller j and those of the buyers only. In this way, the seller-level game reduces to a game where the utility of seller j can be maximized by simply finding the optimal strategy of seller j that maximizes its utility for a given profile of strategies for the buyers. As a result, to solve for the Stackelberg equilibrium in our problem, we can use the *backward induction* technique [47]. According to the backward induction principle, we first find the best response of each buyer from the buyer-level game, and then, we plug it into the utility function of each seller and optimize it correspondingly.

B. Noncooperative Game Among Buyers

Definition 1: The best response function $B_i(\mathbf{c}_{-i}, \mathbf{w})$ of buyer i as a follower is the best strategy for buyer i given the other buyers' strategies \mathbf{c}_{-i} and the sellers' strategies \mathbf{w} . By definition, we have

$$B_i(\mathbf{c}_{-i}, \mathbf{w}) = \arg \max_{c_i} U_i(c_i, \mathbf{c}_{-i}, \mathbf{w}) \quad \forall i \in \mathcal{I}. \quad (7)$$

Definition 2: A Nash equilibrium of the noncooperative game among the buyers is a profile of strategies $\mathbf{c}^* = (c_i^*)_{i \in \mathcal{I}}$ with the property that, given the sellers' strategies \mathbf{w} , we have

$$c_i^* = B_i(\mathbf{c}_{-i}^*, \mathbf{w}) \quad \forall i \in \mathcal{I} \quad (8)$$

where $\mathbf{c}_{-i}^* = (c_l^*)_{l \in \mathcal{I} \setminus \{i\}}$.

Lemma 1: The utility function $U_i(c_i, \mathbf{c}_{-i}, \mathbf{w})$ of buyer i is strictly concave on \mathcal{C}_i .

Proof: Given (1) and (5), we have

$$U_i(c_i, \mathbf{c}_{-i}, \mathbf{w}) = \hat{E} \frac{c_S c_i - c_i^2}{\sum_{l \in \mathcal{I}} c_l}.$$

Taking the first and second derivatives of $U_i(c_i, \mathbf{c}_{-i}, \mathbf{w})$ with respect to c_i , we have

$$\begin{aligned} \frac{\partial U_i(c_i, \mathbf{c}_{-i}, \mathbf{w})}{\partial c_i} &= \hat{E} \frac{c_S \sum_{l \in \mathcal{I} \setminus \{i\}} c_l - 2c_i \sum_{l \in \mathcal{I}} c_l + c_i^2}{(\sum_{l \in \mathcal{I}} c_l)^2} \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial^2 U_i(c_i, \mathbf{c}_{-i}, \mathbf{w})}{\partial c_i^2} &= -2\hat{E} \frac{c_S \sum_{l \in \mathcal{I} \setminus \{i\}} c_l + (\sum_{l \in \mathcal{I} \setminus \{i\}} c_l)^2}{(\sum_{l \in \mathcal{I}} c_l)^3}. \end{aligned} \quad (10)$$

The right-hand side of (10) is negative. Therefore, the utility function $U_i(c_i, \mathbf{c}_{-i}, \mathbf{w})$ of buyer i is strictly concave on \mathcal{C}_i . ■

Proposition 1: There exists a Nash equilibrium in the noncooperative game among the buyers.

Proof: By definition, for all $i \in \mathcal{I}$, the set \mathcal{C}_i of the payment strategies for buyer i is a closed, bounded, and

convex subset of a finite-dimensional Euclidean space. The utility function $U_i(c_i, \mathbf{c}_{-i}, \mathbf{w})$ of buyer i is continuous and, by Lemma 1, is strictly concave on C_i . Thus, by [35, Th. 1, p. 173] for a maximization problem in our context, the noncooperative game among the buyers has a Nash equilibrium. ■

Now, we will prove that the buyer-level game has indeed a unique Nash equilibrium. The key aspect of the uniqueness proof is to show that the best response function of each buyer defined in (7) is a *standard* function [48].

Definition 3: A function $\mathbf{f}(\mathbf{p}) = (f_1(\mathbf{p}), \dots, f_N(\mathbf{p}))$, where $\mathbf{p} = (p_1, \dots, p_N)$, is said to be standard if the following properties are satisfied for all $\mathbf{p} \geq 0$.

- *Positivity:* $\mathbf{f}(\mathbf{p}) > 0$.
- *Monotonicity:* For all \mathbf{p} and \mathbf{p}' , if $\mathbf{p} \geq \mathbf{p}'$, then $\mathbf{f}(\mathbf{p}) \geq \mathbf{f}(\mathbf{p}')$.
- *Scalability:* For all $\mu > 1$, $\mu \mathbf{f}(\mathbf{p}) > \mathbf{f}(\mu \mathbf{p})$.

Proposition 2: The best response function $B_i(\mathbf{c}_{-i}, \mathbf{w})$ of buyer i is a standard function of \mathbf{c}_{-i} .

Proof: Since the utility function $U_i(c_i, \mathbf{c}_{-i}, \mathbf{w})$ of buyer i is strictly concave on C_i , based on Definition 1 and by (7), the best response function $B_i(\mathbf{c}_{-i}, \mathbf{w})$ of buyer i can be obtained by setting the right-hand side of (9) to zero for maximization. Solving

$$\frac{c_S \sum_{l \in \mathcal{I} \setminus \{i\}} c_l - 2c_i \sum_{l \in \mathcal{I}} c_l + c_i^2}{(\sum_{l \in \mathcal{I}} c_l)^2} = 0$$

we obtain

$$B_i(\mathbf{c}_{-i}, \mathbf{w}) = \sqrt{\sum_{l \in \mathcal{I} \setminus \{i\}} c_l \left(c_S + \sum_{l \in \mathcal{I} \setminus \{i\}} c_l \right)} - \sum_{l \in \mathcal{I} \setminus \{i\}} c_l. \quad (11)$$

Next, we show that $B_i(\mathbf{c}_{-i}, \mathbf{w})$ as a function of \mathbf{c}_{-i} in the form of (11) satisfies the three properties of a standard function described in Definition 3 as follows.

- *Positivity:* Given $c_S > 0$, we have

$$B_i(\mathbf{c}_{-i}, \mathbf{w}) > \sqrt{\sum_{l \in \mathcal{I} \setminus \{i\}} c_l \cdot \sum_{l \in \mathcal{I} \setminus \{i\}} c_l} - \sum_{l \in \mathcal{I} \setminus \{i\}} c_l = 0.$$

- *Monotonicity:* Taking the first derivative of $B_i(\mathbf{c}_{-i}, \mathbf{w})$ with respect to c_l , $l \in \mathcal{I} \setminus \{i\}$, we have

$$\begin{aligned} \frac{\partial B_i(\mathbf{c}_{-i}, \mathbf{w})}{\partial c_j} &= \frac{\sum_{l \in \mathcal{I} \setminus \{i\}} c_l + \frac{1}{2} c_S}{\sqrt{\sum_{l \in \mathcal{I} \setminus \{i\}} c_l (c_S + \sum_{l \in \mathcal{I} \setminus \{i\}} c_l)}} - 1 \\ &> \frac{\sqrt{\sum_{l \in \mathcal{I} \setminus \{i\}} c_l (c_S + \sum_{l \in \mathcal{I} \setminus \{i\}} c_l)}}{\sqrt{\sum_{l \in \mathcal{I} \setminus \{i\}} c_l (c_S + \sum_{l \in \mathcal{I} \setminus \{i\}} c_l)}} - 1 \\ &= 0. \end{aligned}$$

- *Scalability:* Based on (11), we obtain

$$\begin{aligned} \mu B_i(\mathbf{c}_{-i}, \mathbf{w}) &= \mu \sqrt{\sum_{l \in \mathcal{I} \setminus \{i\}} c_l \left(c_S + \sum_{l \in \mathcal{I} \setminus \{i\}} c_l \right)} - \mu \sum_{l \in \mathcal{I} \setminus \{i\}} c_l \\ &= \sqrt{\sum_{l \in \mathcal{I} \setminus \{i\}} c_l \left(\mu^2 c_S + \mu^2 \sum_{l \in \mathcal{I} \setminus \{i\}} c_l \right)} - \mu \sum_{l \in \mathcal{I} \setminus \{i\}} c_l \\ B_i(\mu \mathbf{c}_{-i}, \mathbf{w}) &= \sqrt{\sum_{l \in \mathcal{I} \setminus \{i\}} \mu c_l \left(c_S + \sum_{l \in \mathcal{I} \setminus \{i\}} \mu c_l \right)} - \sum_{l \in \mathcal{I} \setminus \{i\}} \mu c_l \\ &= \sqrt{\sum_{l \in \mathcal{I} \setminus \{i\}} c_l \left(\mu c_S + \mu^2 \sum_{l \in \mathcal{I} \setminus \{i\}} c_l \right)} - \mu \sum_{l \in \mathcal{I} \setminus \{i\}} c_l. \end{aligned}$$

Thus, for all $\mu > 1$, $\mu B_i(\mathbf{c}_{-i}, \mathbf{w}) > B_i(\mu \mathbf{c}_{-i}, \mathbf{w})$. ■

Proposition 3: The noncooperative game among the buyers has a unique Nash equilibrium.

Proof: By Proposition 1, we know that there exists a Nash equilibrium in the noncooperative game among the buyers. Since $B_i(\mathbf{c}_{-i}, \mathbf{w}) = B_i(\mathbf{c}^*, \mathbf{w})$ for all $i \in \mathcal{I}$ and letting $\mathbf{B}(\mathbf{c}^*, \mathbf{w}) = (B_i(\mathbf{c}^*, \mathbf{w}))_{i \in \mathcal{I}}$, by Definition 2, the Nash equilibrium must be a fixed point \mathbf{c}^* that satisfies $\mathbf{c}^* = \mathbf{B}(\mathbf{c}^*, \mathbf{w})$. By Proposition 2, we know that $\mathbf{B}(\mathbf{c}^*, \mathbf{w})$ is a standard function. Therefore, by [48, Th. 1], fixed point \mathbf{c}^* must be unique. ■

Proposition 4: For the noncooperative game among the buyers, the unique Nash equilibrium has a closed-form expression given by

$$c_i^* = c_S \frac{k-1}{2k-1} \quad \forall i \in \mathcal{I}. \quad (12)$$

Proof: We observe from (11) that the best response function $B_i(\omega, \mathbf{c}_{-i})$ of buyer i is of the same form for all $i \in \mathcal{I}$. This symmetry implies a Nash equilibrium where $c_i^* = c_j^*$ for all $j \in \mathcal{I} \setminus \{i\}$. Based on Definition 2 and by (8) and (11), we have

$$c_i^* = \sqrt{(k-1)c_i^* [c_S + (k-1)c_i^*]} - (k-1)c_i^* \quad \forall i \in \mathcal{I}. \quad (13)$$

Solving (13) yields (12). We know from Proposition 3 that the noncooperative game among the buyers has a unique Nash equilibrium. Thus, the Nash equilibrium derived in (12) is the unique Nash equilibrium of this game. ■

Clearly, since $k-1 < 2k-1$ for all $k \geq 1$, c_i^* in the form of (12) is guaranteed to satisfy $c_i^* \leq c_S$. On the other hand, for $c_i^* \geq c_B$ to hold, it requires

$$c_S \frac{k-1}{2k-1} \geq c_B. \quad (14)$$

We observe in (14) that the value of $(k-1)/(2k-1)$ monotonically increases as k increases. Therefore, in the worst case where $k = 2$, we need to ensure $c_S \geq 3c_B$. On the other hand, as k tends to infinity, we only require $c_S \geq 2c_B$.

Corollary 1: At the Nash equilibrium given by (12), the amount of energy \hat{E} available for sale from the sellers is equally distributed among the buyers, and we have

$$E_i^* = \frac{\hat{E}}{k} \quad \forall i \in \mathcal{I}.$$

As a result of Proposition 4 and Corollary 1, for a given profile of strategies \mathbf{w} for the sellers, the Nash equilibrium solution to the utility of buyer i , $i \in \mathcal{I}$, is obtained as

$$U_i(c_i^*, c_{-i}^*, \mathbf{w}) = E_i^* c_S - E_i^* c_i^* = \frac{\hat{E} c_S}{2k-1}.$$

The social welfare of the buyers at Nash equilibrium \mathbf{c}^* is obtained as

$$U(\mathbf{c}^*, \mathbf{w}) = \sum_{i \in \mathcal{I}} U_i(c_i^*, c_{-i}^*, \mathbf{w}) = \hat{E} c_S \frac{k}{2k-1}. \quad (15)$$

C. Efficiency of Nash Equilibrium

It is well known that the efficiency of a noncooperative game degrades due to the selfish behavior of its players [49]. The *price of anarchy* [49] is a widely used notion for measuring the inefficiency of equilibrium in noncooperative games. In its general form, the price of anarchy is defined as the ratio between the optimal centralized solution and the worst equilibrium for a game where there exist multiple equilibrium solutions. A related notion is that of the *price of stability* [49], which measures the ratio between the optimal centralized solution and the best equilibrium. Since our problem has a unique Nash equilibrium, the two concepts are equivalent in our context.

In the case where the buyers cooperate with each other aiming to maximize their social welfare $U(\mathbf{c}, \mathbf{w})$ defined in (6), we obtain from (1), (2), (5), and (6) that

$$U(\mathbf{c}, \mathbf{w}) = c_S \sum_{i \in \mathcal{I}} E_i - \sum_{i \in \mathcal{I}} E_i c_i = \hat{E} c_S - R.$$

Given that $c_i \geq c_B$ for all $i \in \mathcal{I}$, we further obtain from (2) that

$$R \geq \hat{E} c_B. \quad (16)$$

Then, using (16), we have

$$U(\mathbf{c}, \mathbf{w}) \leq \hat{E}(c_S - c_B).$$

Thus, for a given profile of strategies \mathbf{w} for the sellers, we have

$$\max_{\mathbf{c}} U(\mathbf{c}, \mathbf{w}) = \hat{E}(c_S - c_B) \quad (17)$$

which can be achieved by letting $c_i = c_B$ for all $i \in \mathcal{I}$.

Let η denote the price of anarchy, which is defined as

$$\eta = \frac{\max_{\mathbf{c}} U(\mathbf{c}, \mathbf{w})}{U(\mathbf{c}^*, \mathbf{w})}.$$

Thus, using (15) and (17), we have

$$\eta = \frac{(c_S - c_B)(2k-1)}{c_S k} = \left(1 - \frac{c_B}{c_S}\right) \left(\frac{2k-1}{k}\right). \quad (18)$$

Proposition 5: For all $k > 1$ and for an arbitrary value of c_B/c_S that satisfies (14), the price of anarchy defined in (18) is bounded as

$$1 \leq \eta < 2. \quad (19)$$

Proof: Because of (14) and by definition $c_B > 0$, we have

$$\begin{aligned} \frac{k-1}{2k-1} &\geq \frac{c_B}{c_S} > 0 \\ \Rightarrow \frac{k}{2k-1} &\leq 1 - \frac{c_B}{c_S} < 1 \\ \Rightarrow 1 &\leq \eta < \frac{2k-1}{k}. \end{aligned}$$

For all $k > 1$, we have

$$\frac{2k-1}{k} = 2 - \frac{1}{k} < 2.$$

Therefore, we obtain (19). ■

Proposition 5 suggests that, when (14) is satisfied, the maximal social welfare of the buyers in such a cooperative game is always larger than or equal to but at most twice of the Nash equilibrium solution to the social welfare of the buyers in the noncooperative game. The equivalence, i.e., $\eta = 1$, holds when

$$\frac{c_B}{c_S} = \frac{k-1}{2k-1}.$$

This is the point where the distributed trading is equivalent to the centralized solution.

D. Utility Maximization for Sellers

We recall that, for each $j \in \mathcal{J}$, the utility function of seller j defined in (4) involves a tradeoff between the attained satisfaction from the stored energy $\hat{E}_j(1 - w_j)$ and that from the received revenue R_j .

From (1)–(3), we have

$$R_j = \sum_{i \in \mathcal{I}} E_i c_i \frac{\hat{E}_j w_j}{\hat{E}} = \hat{E}_j w_j \frac{\sum_{i \in \mathcal{I}} c_i^2}{\sum_{i \in \mathcal{I}} c_i}.$$

For convenience, we define

$$c_N = \frac{\sum_{i \in \mathcal{I}} c_i^2}{\sum_{i \in \mathcal{I}} c_i} \quad (20)$$

as the *normalized* unit price offered by the buyers. Then, we obtain

$$R_j = \hat{E}_j w_j c_N. \quad (21)$$

We observe in (21) that R_j in this alternative form is completely determined by the amount of energy $\hat{E}_j w_j$ that seller j sells to the market and the normalized unit price c_N offered by the buyers. Consequently, utility function $\hat{U}_j(w_j, \mathbf{w}_{-j}, \mathbf{c})$ defined in (4) depends on w_j and \mathbf{c} only. In the remaining analysis of the two-level game, we will drop \mathbf{w}_{-j} and simply use $\hat{U}_j(w_j, \mathbf{c})$ to denote the utility function of seller j , which is given by

$$\hat{U}_j(w_j, \mathbf{c}) = \ln \left[1 + \hat{E}_j(1 - w_j) \right] + \gamma_j \hat{E}_j w_j c_N. \quad (22)$$

By definition, the social welfare of the sellers is obtained by

$$\hat{U}(\mathbf{w}, \mathbf{c}) = \sum_{j \in \mathcal{J}} \hat{U}_j(w_j, \mathbf{c}).$$

Clearly, with the utility function of each seller being in the form of (22), the game among the sellers reduces to a game where the utility of seller j , for each $j \in \mathcal{J}$, can be maximized by simply finding the optimal strategy w_j that maximizes $\hat{U}_j(w_j, \mathbf{c})$ for a given profile of strategies \mathbf{c} for the buyers.

Definition 4: The best response function $\hat{B}_j(\mathbf{c})$ of seller j as a leader is the best strategy for seller j given the buyers' strategies \mathbf{c} . By definition, we have

$$\hat{B}_j(\mathbf{c}) = \arg \max_{w_j} \hat{U}_j(w_j, \mathbf{c}) \quad \forall j \in \mathcal{J}. \quad (23)$$

Definition 5: A Stackelberg equilibrium of the two-level game is a profile of strategies $\mathbf{c}^* = (c_i^*)_{i \in \mathcal{I}}$ for the buyers and a profile of strategies $\mathbf{w}^* = (w_j^*)_{j \in \mathcal{J}}$ for the sellers with the property that

$$\begin{aligned} c_i^* &= B_i(c_{-i}^*, \mathbf{w}^*) \quad \forall i \in \mathcal{I} \\ w_j^* &= \hat{B}_j(\mathbf{c}^*) \quad \forall j \in \mathcal{J}. \end{aligned} \quad (24)$$

Based on Definitions 4 and 5, and by (23) and (24), we have

$$w_j^* = \arg \max_{w_j} \hat{U}_j(w_j, \mathbf{c}^*).$$

At the Nash equilibrium given by (12), we derive the normalized unit price offered by the buyers from (20) as

$$c_N^* = \frac{\sum_{i \in \mathcal{I}} (c_i^*)^2}{\sum_{i \in \mathcal{I}} c_i^*} = c_S \frac{k-1}{2k-1}. \quad (25)$$

Given (22), replacing \mathbf{c} with \mathbf{c}^* , and hence replacing c_N with c_N^* of the form in (25), we obtain

$$\hat{U}_j(w_j, \mathbf{c}^*) = \ln[1 + \hat{E}_j(1 - w_j)] + \gamma_j \hat{E}_j w_j c_N^*.$$

Taking the first and second derivatives of $\hat{U}_j(w_j, \mathbf{c}^*)$ with respect to w_j , we have

$$\frac{\partial \hat{U}_j(w_j, \mathbf{c}^*)}{\partial w_j} = \gamma_j \hat{E}_j c_N^* - \frac{\hat{E}_j}{1 + \hat{E}_j(1 - w_j)} \quad (26)$$

$$\frac{\partial^2 \hat{U}_j(w_j, \mathbf{c}^*)}{\partial w_j^2} = - \frac{\hat{E}_j^2}{[1 + \hat{E}_j(1 - w_j)]^2}. \quad (27)$$

The right-hand side of (27) is negative. Therefore, the utility function $\hat{U}_j(w_j, \mathbf{c}^*)$ of seller j is strictly concave on \mathcal{W}_j . Setting the right-hand side of (26) to zero for maximization, we obtain

$$w_j^* = 1 + \frac{1}{\hat{E}_j} \left(1 - \frac{1}{\gamma_j c_N^*} \right). \quad (28)$$

We recall that, by definition, w_j^* must satisfy $0 \leq w_j^* \leq 1$. Therefore, if w_j^* obtained from (28) turns out to be smaller than 0 or larger than 1, we set $w_j^* = 0$ or $w_j^* = 1$, respectively.

Since the noncooperative game among the buyers has a unique Nash equilibrium \mathbf{c}^* , as shown in Proposition 3, and since, for each $j \in \mathcal{J}$, seller j can always find its optimal strategy w_j^* due to the concavity of $\hat{U}_j(w_j, \mathbf{c}^*)$, we have the following corollary.

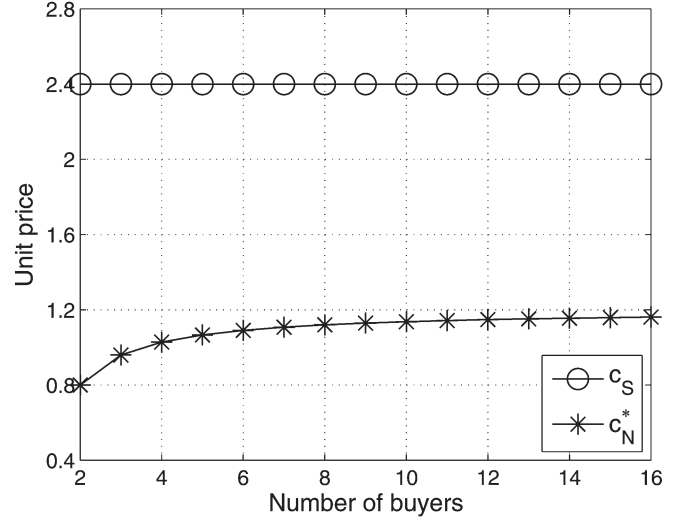


Fig. 2. Nash equilibrium solution of the normalized unit price c_N^* with respect to the number of buyers k .

Corollary 2: A unique Stackelberg equilibrium exists in the proposed two-level energy trading game.

IV. NUMERICAL RESULTS

In this section, we provide the detailed numerical results that we have obtained for illustrating the equilibrium behavior of the system. We are interested in the effect of the various system parameters on the utility of microgrids and their strategies using our proposed distributed energy trading mechanism.

Fig. 2 illustrates the Nash equilibrium solution of the normalized unit price c_N^* with respect to the number of buyers k . For this example, we set $c_S = 2.4/\text{kWh}$. The results are calculated from (25), where we see that the Nash equilibrium of the normalized unit price monotonically increases as the number of buyers in the system increases. In particular, when $k = 2$, we have $c_N^* = 0.8$, which is $(1/3)c_S$. As k tends to infinity, we observe that c_N^* approaches 1.2, which is $(1/2)c_S$. Thus, as a result of our proposed trading mechanism, we have $c_N^* < c_S$ for all $k \geq 2$. We know from (12) and (25) that, in our problem, $c_i^* = c_N^*$ for all $i \in \mathcal{I}$. Therefore, trading with the sellers using our proposed approach ensures that each buyer pays a unit price not more than half of c_S at the Nash equilibrium.

We recall that the Nash equilibrium in our context is the set of strategies where none of the buyers can increase its own utility by choosing a different strategy given the strategies of the other buyers. This effect can be demonstrated by the results plotted in Fig. 3. For this example, we set $c_B = 0.8/\text{kWh}$ and $c_S = 2.4/\text{kWh}$. We consider three buyers and four sellers, where $\hat{E}_j = \{38, 20, 10, 25\}$ kWh, and $\gamma_j = 0.2$ for all j . We fix $c_2 = c_3 = c_N^*$, which is $0.96/\text{kWh}$ in this case, and vary the value of the c_1 of buyer 1 from c_B to c_S . The results in Fig. 3 confirm that buyer 1 cannot increase its own utility at the Stackelberg equilibrium by choosing a different strategy.

For the same example setting in Fig. 3, we demonstrate in Fig. 4 the strategy of each seller at the Stackelberg equilibrium with respect to the number of buyers k . We observe that, for

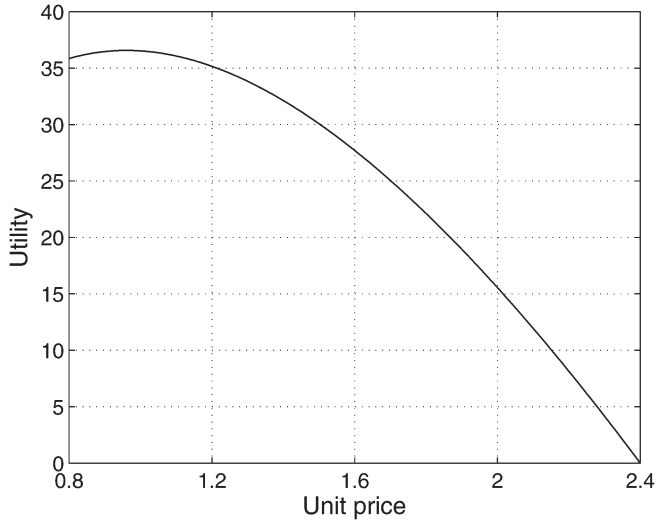


Fig. 3. Utility of buyer 1 with respect to its strategy c_1 , given that each of the two other buyers offers c_N^* .

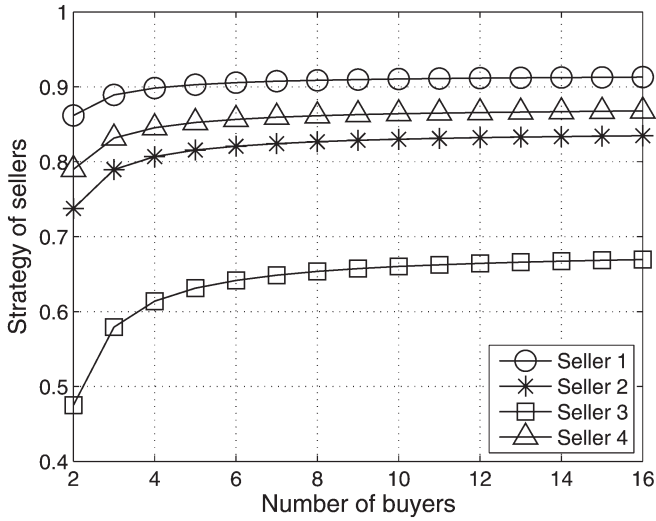


Fig. 4. Strategy of each seller w_j^* with respect to the number of buyers k at the Stackelberg equilibrium.

all j , the value of w_j^* monotonically increases as k increases. This is because as the number of buyers in the system increases, the competition among the buyers becomes more severe so that the unit price offered by each buyer at the equilibrium also increases. As a result, the sellers tend to sell more energy that can increase their utilities. We note that similar results can be also observed in Figs. 5 and 6, where we illustrate the social welfare of the buyers and that of the sellers at the Stackelberg equilibrium, respectively. As the number of buyers in the system increases, the social welfare of the buyers monotonically decreases because of the increasing competition among the buyers. On the other hand, the sellers as the leaders have the advantage by choosing their strategies first and hence can achieve higher utilities.

Finally, in Figs. 7 and 8, we demonstrate the efficiency of the Nash equilibrium by illustrating the price of anarchy with respect to the number of buyers k and that with respect to ratio

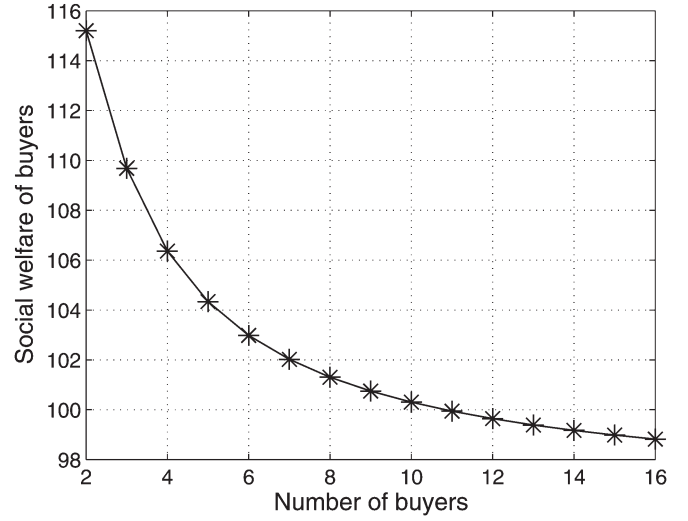


Fig. 5. Social welfare of the buyers $U(c^*, w^*)$ with respect to the number of buyers k at the Stackelberg equilibrium.

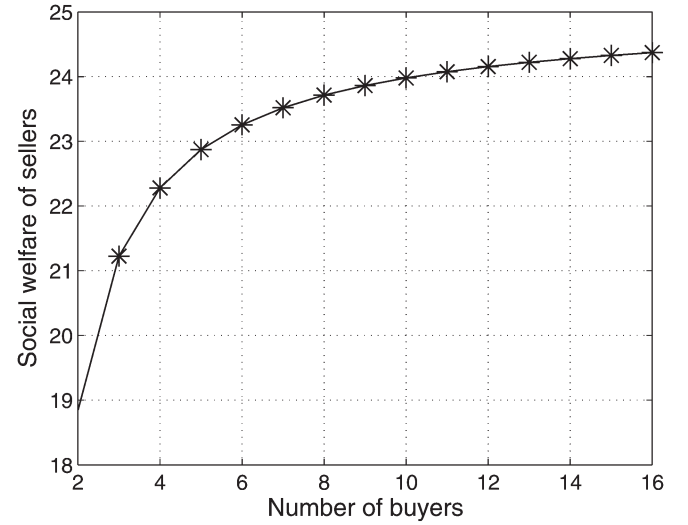


Fig. 6. Social welfare of the sellers $\hat{U}(w^*, c^*)$ with respect to the number of buyers k at the Stackelberg equilibrium.

c_B/c_S , respectively. For the example in Fig. 7, we set $c_B = 0.8/\text{kWh}$ and $c_S = 2.4/\text{kWh}$. For the example in Fig. 8, we fix $k = 16$. The results confirm that the Nash equilibrium solution of the noncooperative game is always lower bounded by half of the optimal centralized solution of the cooperative game, and it is equivalent to the optimal centralized solution when $\eta = 1$.

V. CONCLUSION

We have proposed a fully distributed mechanism for energy trading among microgrids. We have presented a rigorous game-theoretic analysis for deriving the equilibrium solution of the system. The results have demonstrated that our proposed approach is guaranteed to have a unique equilibrium solution that has comparable performance to the optimal centralized solution and thus can maximize the payoff for all participating microgrids. This paper has provided an incentive for energy trading among microgrids in future power grids.

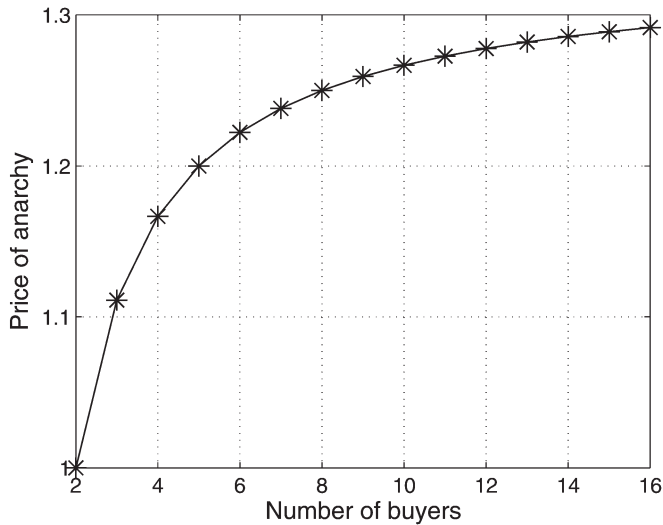


Fig. 7. Price of anarchy η with respect to the number of buyers k for the noncooperative game among the buyers.

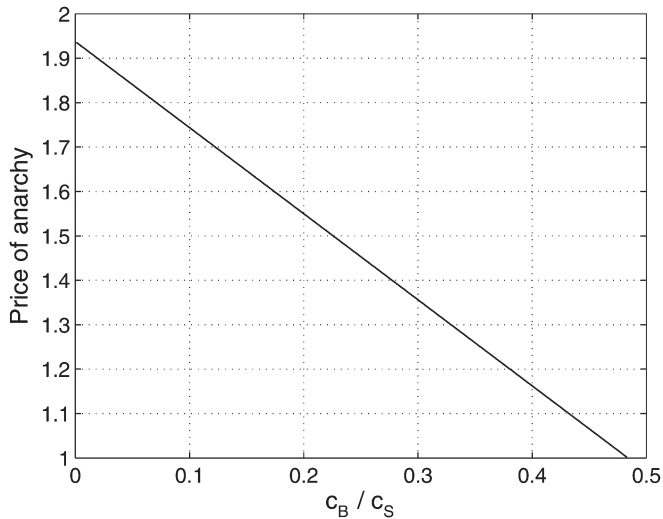


Fig. 8. Price of anarchy η with respect to ratio c_B / c_S for the noncooperative game among the buyers.

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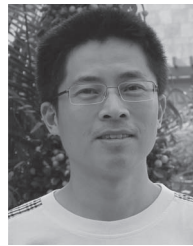
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