IME ACM-ICPC Team Notebook

Contents

1 Flags + Template + vimrc

1.1 Flags

```
g++ -fsanitize=address,undefined -fno-omit-frame-pointer -g -Wall -Wshadow -std=c++17 -Wno-unused-result -Wno
      -sign-compare -Wno-char-subscripts
```

1.2 Template

```
#include <bits/stdc++.h>
using namespace std;

#define st first
#define nd second
#define mp make_pair
#define cl(x, v) memset((x), (v), sizeof(x))
#define gcd(x,y) __gcd((x),(y))

#ifndef ONLINE_JUDGE
  #define db(x) cerr << #x << " == " << x << endl
  #define dbs(x) cerr << x << endl
  #define _ << ", " <<
#else
  #define db(x) ((void)0)
  #define dbs(x) ((void)0)
#endif

typedef long long ll;
typedef long double ld;

typedef pair<int, int> pii;
typedef pair<int, pii> piii;
typedef pair<ll, ll> pll;
typedef pair<ll, pll> plll;

const ld EPS = 1e-9, PI = acos(-1.);
const ll LINF = 0x3f3f3f3f3f3f3f3f;
const int INF = 0x3f3f3f3f, MOD = 1e9+7;;
const int N = 1e5+5;

int main() {
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  //freopen("in", "r", stdin);
  //freopen("out", "w", stdout);
  return 0;
}
```

1.3 vimrc

```
syntax on
set et ts=2 sw=0 sts=-1 ai nu hls cindent
nnoremap ; :
vnoremap ; :
noremap <c-j> 15gj
noremap <c-k> 15gk
nnoremap <s-k> i<CR><ESC>
inoremap ,. <esc>
vnoremap ,. <esc>
nnoremap ,. <esc>
```

2 Data Structures

2.1 Bit Binary Search

```
// --- Bit Binary Search in o(log(n)) ---
const int M = 20
const int N = 1 << M

int lower_bound(int val){
  int ans = 0, sum = 0;
  for(int i = M - 1; i >= 0; i--){
    int x = ans + (1 << i);
    if(sum + bit[x] < val)
      ans = x, sum += bit[x];
  }

  return ans + 1;
}
```

2.2 Bit Range

```
struct BIT {
  ll b[N]={};
  ll sum(int x) {
    ll r=0;
    for(x+=2;x;x-=x&-x)
      r += b[x];
    return r;
  }
  void upd(int x, ll v) {
    for(x+=2;x<N;x+=x&-x)
      b[x]+=v;
  }
};
struct BITRange {
  BIT a,b;
  ll sum(int x) {
    return a.sum(x)*x+b.sum(x);
  }
  void upd(int l, int r, ll v) {
    a.upd(l,v), a.upd(r+1,-v);
    b.upd(l,-v*(l-1)), b.upd(r+1,v*r);
  }
};
```

2.3 Bit

```
// Fenwick Tree / Binary Indexed Tree
ll bit[N];

void add(int p, int v) {
  for (p += 2; p < N; p += p & -p) bit[p] += v;
}

ll query(int p) {
  ll r = 0;
  for (p += 2; p; p -= p & -p) r += bit[p];
  return r;
}
```

2.4 Bit 2D

```
// Fenwick Tree 2D / Binary Indexed Tree 2D
int bit[N][N];

void add(int i, int j, int v) {
  for (; i < N; i+=i&-i)
    for (int jj = j; jj < N; jj+=jj&-jj)
      bit[i][jj] += v;
}

int query(int i, int j) {
  int res = 0;
  for (; i; i-=i&-i)
    for (int jj = j; jj; jj-=jj&-jj)
      res += bit[i][jj];
  return res;
}

// Whole BIT 2D set to 1
void init() {
  cl(bit,0);
  for (int i = 1; i <= r; ++i)
    for (int j = 1; j <= c; ++j)
      add(i, j, 1);
}

// Return number of positions set
int query(int imin, int jmin, int imax, int jmax) {
  return query(imax, jmax) - query(imax, jmin-1) - query(imin-1, jmax) + query(imin-1, jmin-1);
}

// Find all positions inside rect (imin, jmin), (imax, jmax) where position is set
void proc(int imin, int jmin, int imax, int jmax, int v, int tot) {
  if (tot < 0) tot = query(imin, jmin, imax, jmax);
  if (!tot) return;

  int imid = (imin+imax)/2, jmid = (jmin+jmax)/2;
  if (imin != imax) {
    int qnt = query(imin, jmin, imid, jmax);
    if (qnt) proc(imin, jmin, imid, jmax, v, qnt);
    if (tot-qnt) proc(imid+1, jmin, imax, jmax, v, tot-qnt);
  } else if (jmin != jmax) {
    int qnt = query(imin, jmin, imax, jmid);
    if (qnt) proc(imin, jmin, imax, jmid, v, qnt);
    if (tot-qnt) proc(imin, jmid+1, imax, jmax, v, tot-qnt);
  } else {
    // single position set!
    // now process position!!!
    add(imin, jmin, -1);
  }
}
```

2.5 Centroid Decomposition

```
// Centroid decomposition

vector<int> adj[N];
int forb[N], sz[N], par[N];
int n, m;
unordered_map<int, int> dist[N];

void dfs(int u, int p) {
  sz[u] = 1;
  for(int v : adj[u]) {
    if(v != p and !forb[v]) {
      dfs(v, u);
      sz[u] += sz[v];
    }
  }
}

int find_cen(int u, int p, int qt) {
  for(int v : adj[u]) {
    if(v == p or forb[v]) continue;
    if(sz[v] > qt / 2) return find_cen(v, u, qt);
  }
  return u;
}

void getdist(int u, int p, int cen) {
  for(int v : adj[u]) {
    if(v != p and !forb[v]) {
      dist[cen][v] = dist[v][cen] = dist[cen][u] + 1;
      getdist(v, u, cen);
    }
  }
}


void decomp(int u, int p) {
  dfs(u, -1);

  int cen = find_cen(u, -1, sz[u]);
  forb[cen] = 1;
  par[cen] = p;
  dist[cen][cen] = 0;
  getdist(cen, -1, cen);

  for(int v : adj[cen]) if(!forb[v])
    decomp(v, cen);
}

// main
decomp(1, -1);
```

2.6 Color Update

```
// Color Update - O(q log n)
// Heavily inspired by Um_nik's implementation
// q -> number of inserts

struct ColorUpdate {
  struct Seg {
    int l, r, c;
    Seg(int _l = 0, int _r = 0, int _c = 0) : l(_l), r(_r), c(_c) {};
    bool operator<(const Seg& b) const { return l < b.l; }
  };

  set<Seg> segs;

  void cut(int x) {
    auto it = segs.lower_bound({ x, 0, 0 });
    if (it == segs.begin()) return;
    it--;
    if (it->r == x - 1) return;
    Seg s = *it;
    segs.erase(it);
    segs.insert(Seg(s.l, x - 1, s.c));
    segs.insert(Seg(x, s.r, s.c));
  }

  void add(int l, int r, int c) {
    cut(l), cut(r + 1);
    Seg s(l, r, c);
    auto it = segs.lower_bound(s);
    while (it != segs.end() and it->l <= s.r) {
      auto it2 = it++;
      segs.erase(it2);
    }
    segs.insert(s);
  }
};
```

2.7 Heavy-Light Decomposition (new)

```
vector<int> adj[N];
int sz[N], nxt[N];
int h[N], par[N];
int in[N], rin[N], out[N];
int t;

void dfs_sz(int u = 1){
  sz[u] = 1;
  for(auto &v : adj[u]) if(v != par[u]) {
    h[v] = h[u] + 1;
    par[v] = u;

    dfs_sz(v);
    sz[u] += sz[v];
    if(sz[v] > sz[adj[u][0]])
      swap(v, adj[u][0]);
  }
}

void dfs_hld(int u = 1){
  in[u] = t++;
  rin[in[u]] = u;
  for(auto v : adj[u]) if(v != par[u]) {
    nxt[v] = (v == adj[u][0] ? nxt[u] : v);
    dfs_hld(v);
  }

  out[u] = t - 1;
}

int lca(int u, int v){
  while(nxt[u] != nxt[v]){
    if(h[nxt[u]] < h[nxt[v]]) swap(u, v);
    u = par[nxt[u]];
  }

  if(h[u] > h[v]) swap(u, v);
  return u;
}

int query_up(int u, int v) {
  if(u == v) return 1;
  int ans = 0;
  while(1){
    if(nxt[u] == nxt[v]){
      if(u == v) break;
      ans = max(ans, query(1, 0, n - 1, in[v] + 1, in[u]));
      break;
    }

    ans = max(ans, query(1, 0, n - 1, in[nxt[u]], in[u]));
    u = par[nxt[u]];
  }

  return ans;
}

int hld_query(int u, int v) {
  int l = lca(u, v);
  return mult(query_up(u, l), query_up(v, l));
}
```

2.8 Heavy-Light Decomposition

```
// Heavy-Light Decomposition
vector<int> adj[N];
int par[N], h[N];

int chainno, chain[N], head[N], chainpos[N], chainsz[N], pos[N], arrsz;
int sc[N], sz[N];

void dfs(int u) {
  sz[u] = 1, sc[u] = 0; // nodes 1-indexed (0-ind: sc[u]=-1)
  for (int v : adj[u]) if (v != par[u]) {
    par[v] = u, h[v] = h[u]+1, dfs(v);
    sz[u]+=sz[v];
    if (sz[sc[u]] < sz[v]) sc[u] = v; // 1-indexed (0-ind: sc[u]<0 or ...)
  }
}

void hld(int u) {
  if (!head[chainno]) head[chainno] = u; // 1-indexed
  chain[u] = chainno;
  chainpos[u] = chainsz[chainno];
  chainsz[chainno]++;
  pos[u] = ++arrsz;

  if (sc[u]) hld(sc[u]);

  for (int v : adj[u]) if (v != par[u] and v != sc[u])
    chainno++, hld(v);
}

int lca(int u, int v) {
  while (chain[u] != chain[v]) {
    if (h[head[chain[u]]] < h[head[chain[v]]]) swap(u, v);
    u = par[head[chain[u]]];
  }
  if (h[u] > h[v]) swap(u, v);
  return u;
}

int query_up(int u, int v) {
  if (u == v) return 0;
  int ans = -1;
  while (1) {
    if (chain[u] == chain[v]) {
      if (u == v) break;
      ans = max(ans, query(1, 1, n, chainpos[v]+1, chainpos[u]));
      break;
    }

    ans = max(ans, query(1, 1, n, chainpos[head[chain[u]]], chainpos[u]));
    u = par[head[chain[u]]];
  }
  return ans;
}

int query(int u, int v) {
  int l = lca(u, v);
  return max(query_up(u, l), query_up(v, l));
}
```

2.9 Lichao Tree (ITA)

```
#include <cstdio>
#include <vector>
#define INF 0x3f3f3f3f3f3f3f3f
#define MAXN 1009
using namespace std;

typedef long long ll;

/*
 * LiChao Segment Tree
 */

class LiChao {
        vector<ll> m, b;
        int n, sz; ll *x;
#define gx(i) (i < sz ? x[i] : x[sz-1])
        void update(int t, int l, int r, ll nm, ll nb) {
                ll xl = nm * gx(l) + nb, xr = nm * gx(r) + nb;
                ll yl = m[t] * gx(l) + b[t], yr = m[t] * gx(r) + b[t];
        if (yl >= xl && yr >= xr) return;
                if (yl <= xl && yr <= xr) {
                        m[t] = nm, b[t] = nb; return;
                }
                int mid = (l + r) / 2;
                update(t<<1, l, mid, nm, nb);
                update(1+(t<<1), mid+1, r, nm, nb);
        }
public:
        LiChao(ll *st, ll *en) : x(st) {
                sz = int(en - st);
                for(n = 1; n < sz; n <<= 1);
                m.assign(2*n, 0); b.assign(2*n, -INF);
        }
        void insert_line(ll nm, ll nb) {
                update(1, 0, n-1, nm, nb);
        }
        ll query(int i) {
                ll ans = -INF;
                for(int t = i+n; t; t >>= 1)
                        ans = max(ans, m[t] * x[i] + b[t]);
                return ans;
        }
};

/*
 * UVa 12524
 */

ll w[MAXN], x[MAXN], A[MAXN], B[MAXN], dp[MAXN][MAXN];

int main(){
        int N, K;
        while(scanf("%d %d", &N, &K)!=EOF) {
                for(int i=0; i<N; i++){
                        scanf("%lld %lld", x+i, w+i);
                        A[i] = w[i] + (i>0 ? A[i-1] : 0);
                        B[i] = w[i]*x[i] + (i>0 ? B[i-1] : 0);
                        dp[i][1] = x[i]*A[i] - B[i];
                }
                for(int k=2; k<=K; k++){
                        dp[0][k] = 0;
            LiChao lc(x, x+N);
                        for(int i=1; i<N; i++){
                                lc.insert_line(A[i-1], -dp[i-1][k-1]-B[i-1]);
                                dp[i][k] = x[i]*A[i] - B[i] - lc.query(i);
                        }
                }
                printf("%lld\n", dp[N-1][K]);
        }
        return 0;
}
```

2.10 Merge Sort Tree

```
// Mergesort Tree - Time <O(nlogn), O(log^2n)> - Memory O(nlogn)
// Mergesort Tree is a segment tree that stores the sorted subarray
// on each node.
vi st[4*N];

void build(int p, int l, int r) {
  if (l == r) { st[p].pb(s[l]); return; }
  build(2*p, l, (l+r)/2);
  build(2*p+1, (l+r)/2+1, r);
  st[p].resize(r-l+1);
  merge(st[2*p].begin(), st[2*p].end(),
        st[2*p+1].begin(), st[2*p+1].end(),
        st[p].begin());
}

int query(int p, int l, int r, int i, int j, int a, int b) {
  if (j < l or i > r) return 0;
  if (i <= l and j >= r)
    return upper_bound(st[p].begin(), st[p].end(), b) -
           lower_bound(st[p].begin(), st[p].end(), a);
  return query(2*p, l, (l+r)/2, i, j, a, b) +
         query(2*p+1, (l+r)/2+1, r, i, j, a, b);
}
```

2.11 Minimum Queue

```
// O(1) complexity for all operations, except for clear,
// which could be done by creating another deque and using swap

struct MinQueue {
  int plus = 0;
  int sz = 0;
  deque<pair<int, int>> dq;

  bool empty() { return dq.empty(); }
  void clear() { plus = 0; sz = 0; dq.clear(); }
  void add(int x) { plus += x; } // Adds x to every element in the queue
  int min() { return dq.front().first + plus; } // Returns the minimum element in the queue
  int size() { return sz; }

  void push(int x) {
    x -= plus;
    int amt = 1;
    while (dq.size() and dq.back().first >= x)
      amt += dq.back().second, dq.pop_back();
    dq.push_back({ x, amt });
    sz++;
  }

  void pop() {
    dq.front().second--, sz--;
    if (!dq.front().second) dq.pop_front();
  }
};
```

2.12 Ordered Set

```
//#include <ext/pb_ds/assoc_container.hpp>
//#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
using namespace __gnu_pbds;

typedef tree<int,null_type,less<int>,rb_tree_tag,
tree_order_statistics_node_update> ordered_set;

ordered_set s;
s.insert(2);
s.insert(3);
s.insert(7);
s.insert(9);

//find_by_order returns an iterator to the element at a given position
auto x = s.find_by_order(2);
cout << *x << "\n"; // 7

//order_of_key returns the position of a given element
cout << s.order_of_key(7) << "\n"; // 2

//If the element does not appear in the set, we get the position that the element would have in the set
cout << s.order_of_key(6) << "\n"; // 2
cout << s.order_of_key(8) << "\n"; // 3
```

2.13 Dynamic Segment Tree (Lazy Update)

```
#include <bits/stdc++.h>

/* tested:
    https://www.spoj.com/problems/BGSHOOT/
   ref:
    https://maratona.ic.unicamp.br/MaratonaVerao2022/slides/AulaSummer-SegmentTree-Aula2.pdf
*/
vector<int> e, d, mx, lazy;
//begin creating node 0, then start your segment tree creating node 1
int create(){
    mx.push_back(0);
    lazy.push_back(0);
    e.push_back(0);
    d.push_back(0);
    return mx.size() - 1;
}

void push(int pos, int ini, int fim){
    if(pos == 0) return;
    if (lazy[pos]) {
        mx[pos] += lazy[pos];
        // RMQ (max/min)   -> update: = lazy[p],         incr: += lazy[p]
        // RSQ (sum)       -> update: = (r-l+1)*lazy[p], incr: += (r-l+1)*lazy[p]
        // Count lights on -> flip:   = (r-l+1)-st[p];
        if (ini != fim) {
            if(e[pos] == 0){
                int aux = create();
                e[pos] = aux;
            }
            if(d[pos] == 0){
                int aux = create();
                d[pos] = aux;
            }
            lazy[e[pos]] += lazy[pos];
            lazy[d[pos]] += lazy[pos];
            // update:    lazy[2*p] = lazy[p],  lazy[2*p+1] = lazy[p];
            // increment: lazy[2*p] += lazy[p], lazy[2*p+1] += lazy[p];
            // flip:      lazy[2*p] ^= 1,       lazy[2*p+1] ^= 1;
        }
        lazy[pos] = 0;
    }
}

void update(int pos, int ini, int fim, int p, int q, int val){
    if(pos == 0) return;

    push(pos, ini, fim);

    if(q < ini || p > fim) return;

    if(p <= ini and fim <= q){
        lazy[pos] += val;
        // update:    lazy[p] = k;
        // increment: lazy[p] += k;
        // flip:      lazy[p] = 1;
        push(pos, ini, fim);
        return;
    }

    int m = (ini + fim) >> 1;
    if(e[pos] == 0){
        int aux = create();
        e[pos] = aux;
    }
    update(e[pos], ini, m, p, q, val);
    if(d[pos] == 0){
        int aux = create();
        d[pos] = aux;
    }
    update(d[pos], m + 1, fim, p, q, val);
    mx[pos] = max(mx[e[pos]], mx[d[pos]]);
}

int query(int pos, int ini, int fim, int p, int q){
    if(pos == 0) return 0;

    push(pos, ini, fim);

    if(q < ini || p > fim) return 0;

    if(p <= ini and fim <= q) return mx[pos];

    int m = (ini + fim) >> 1;
    return max(query(e[pos], ini, m, p, q) , query(d[pos], m + 1, fim, p, q));
}
```

2.14 Dynamic Segment Tree

```
#include <bits/stdc++.h>

/* tested:
    https://www.spoj.com/problems/ORDERSET/
    https://www.eolymp.com/en/contests/8463/problems/72212
    https://codeforces.com/contest/474/problem/E
    https://codeforces.com/problemset/problem/960/F
   ref:
    https://maratona.ic.unicamp.br/MaratonaVerao2022/slides/AulaSummer-SegmentTree-Aula2.pdf
*/

vector<int> e, d, mn;
//begin creating node 0, then start your segment tree creating node 1
int create(){
    mn.push_back(0);
    e.push_back(0);
    d.push_back(0);
    return mn.size() - 1;
}

void update(int pos, int ini, int fim, int id, int val){
    if(id < ini || id > fim) return;

    if(ini == fim){
        mn[pos] = val;
        return;
    }

    int m = (ini + fim) >> 1;
    if(id <= m){
        if(e[pos] == 0){
            int aux = create();
            e[pos] = aux;
        }
        update(e[pos], ini, m, id, val);
    }
    else{
        if(d[pos] == 0){
            int aux = create();
            d[pos] = aux;
        }
        update(d[pos], m + 1, fim, id, val);
    }

    mn[pos] = min(mn[e[pos]], mn[d[pos]]);
}

int query(int pos, int ini, int fim, int p, int q){
    if(q < ini || p > fim) return INT_MAX;

    if(pos == 0) return 0;

    if(p <= ini and fim <= q) return mn[pos];

    int m = (ini + fim) >> 1;
    return min(query(e[pos], ini, m, p, q), query(d[pos], m + 1, fim, p, q));
}
```

2.15 Iterative Segment Tree

```
#include <bits/stdc++.h>

/* tested:
    https://www.spoj.com/problems/ORDERSET/
    https://www.eolymp.com/en/contests/8463/problems/72212
    https://codeforces.com/contest/474/problem/E
    https://codeforces.com/problemset/problem/960/F
   ref:
    https://maratona.ic.unicamp.br/MaratonaVerao2022/slides/AulaSummer-SegmentTree-Aula2.pdf
*/

vector<int> e, d, mn;
//begin creating node 0, then start your segment tree creating node 1
int create(){
    mn.push_back(0);
    e.push_back(0);
    d.push_back(0);
    return mn.size() - 1;
}

void update(int pos, int ini, int fim, int id, int val){
    if(id < ini || id > fim) return;

    if(ini == fim){
        mn[pos] = val;
        return;
    }

    int m = (ini + fim) >> 1;
    if(id <= m){
        if(e[pos] == 0){
            int aux = create();
            e[pos] = aux;
        }
        update(e[pos], ini, m, id, val);
    }
    else{
        if(d[pos] == 0){
            int aux = create();
            d[pos] = aux;
        }
        update(d[pos], m + 1, fim, id, val);
    }

    mn[pos] = min(mn[e[pos]], mn[d[pos]]);
}

int query(int pos, int ini, int fim, int p, int q){
    if(q < ini || p > fim) return INT_MAX;

    if(pos == 0) return 0;

    if(p <= ini and fim <= q) return mn[pos];

    int m = (ini + fim) >> 1;
    return min(query(e[pos], ini, m, p, q), query(d[pos], m + 1, fim, p, q));
}
```

2.16 Mod Segment Tree

```
// SegTree with mod
// op1 (l, r) -> sum a[i], i = { l .. r }
// op2 (l, r, x) -> a[i] = a[i] mod x, i = { l .. r }
// op3 (idx, x) -> a[idx] = x;

const int N = 1e5 + 5;

struct segTreeNode { ll sum, mx, mn, lz = -1; };

int n, m;
ll a[N];
segTreeNode st[4 * N];
```

```
void push(int p, int l, int r) {
  if (st[p].lz != -1) {

    st[p].mx = st[p].mn = st[p].lz;
    st[p].sum = (r - l + 1) * st[p].lz;

    if (l != r) st[2 * p].lz = st[2 * p + 1].lz = st[p].lz;
    st[p].lz = -1;
  }
}

void merge(int p) {
  st[p].mx = max(st[2 * p].mx, st[2 * p + 1].mx);
  st[p].mn = min(st[2 * p].mn, st[2 * p + 1].mn);
  st[p].sum = st[2 * p].sum + st[2 * p + 1].sum;
}

void build(int p = 1, int l = 1, int r = n) {
  if (l == r) {
    st[p].mn = st[p].mx = st[p].sum = a[l];
    return;
  }

  int mid = (l + r) >> 1;
  build(2 * p, l, mid);
  build(2 * p + 1, mid + 1, r);

  merge(p);
}

ll query(int i, int j, int p = 1, int l = 1, int r = n) {
  push(p, l, r);
  if (r < i or l > j) return 0ll;
  if (i <= l and r <= j) return st[p].sum;
  int mid = (l + r) >> 1;
  return query(i, j, 2 * p, l, mid) + query(i, j, 2 * p + 1, mid + 1, r);
}

void module_op(int i, int j, ll x, int p = 1, int l = 1, int r = n) {
  push(p, l, r);
  if (r < i or l > j or st[p].mx < x) return;
  if (i <= l and r <= j and st[p].mx == st[p].mn) {
    st[p].lz = st[p].mx % x;
    push(p, l, r);
    return;
  }
  int mid = (l + r) >> 1;
  module_op(i, j, x, 2 * p, l, mid);
  module_op(i, j, x, 2 * p + 1, mid + 1, r);

  merge(p);
}

void set_op(int i, int j, ll x, int p = 1, int l = 1, int r = n) {
  push(p, l, r);
  if (r < i or l > j) return;
  if (i <= l and r <= j) {
    st[p].lz = x;
    push(p, l, r);
    return;
  }
  int mid = (l + r) >> 1;
  set_op(i, j, x, 2 * p, l, mid);
  set_op(i, j, x, 2 * p + 1, mid + 1, r);

  merge(p);
}
```

2.17 Persistent Segment Tree (Naum)

```
// Persistent Segment Tree
int n;
int rcnt;
int lc[M], rc[M], st[M];

int update(int p, int l, int r, int i, int v) {
  int rt = ++rcnt;
  if (l == r) { st[rt] = v; return rt; }

  int mid = (l+r)/2;
  if (i <= mid) lc[rt] = update(lc[p], l, mid, i, v), rc[rt] = rc[p];
  else          rc[rt] = update(rc[p], mid+1, r, i, v), lc[rt] = lc[p];
  st[rt] = st[lc[rt]] + st[rc[rt]];

  return rt;
}

int query(int p, int l, int r, int i, int j) {
  if (l > j or r < i) return 0;
  if (i <= l and r <= j) return st[p];

  return query(lc[p], l, (l+r)/2, i, j)+query(rc[p], (l+r)/2+1, r, i, j);
}

int main() {
  scanf("%d", &n);
  for (int i = 1; i <= n; ++i) {
    int a;
    scanf("%d", &a);
    r[i] = update(r[i-1], 1, n, i, 1);
  }

  return 0;
}
```

2.18 Persistent Segment Tree

```
// Persistent Segtree
// Memory: O(n logn)
// Operations: O(log n)

int li[N], ri[N]; // [li(u), ri(u)] is the interval of node u
int st[N], lc[N], rc[N]; // Value, left son and right son of node u
int stsz; // Size of segment tree

// Returns root of initial tree.
// i and j are the first and last elements of the tree.
int init(int i, int j) {
  int v = ++stsz;
  li[v] = i, ri[v] = j;

  if (i != j) {
    rc[v] = init(i, (i+j)/2);
    rc[v] = init((i+j)/2+1, j);
    st[v] = /* calculate value from rc[v] and rc[v] */;
  } else {
    st[v] = /* insert initial value here */;
  }

  return v;
}

// Gets the sum from i to j from tree with root u
int sum(int u, int i, int j) {
  if (j < li[u] or ri[u] < i) return 0;
  if (i <= li[u] and ri[u] <= j) return st[u];
  return sum(rc[u], i, j) + sum (rc[u], i, j);
}

// Copies node j into node i
void clone(int i, int j) {
  li[i] = li[j], ri[i] = ri[j];
  st[i] = st[j];
  rc[i] = rc[j], rc[i] = rc[j];
}

// Sums v to index i from the tree with root u
int update(int u, int i, int v) {
  if (i < li[u] or ri[u] < i) return u;

  clone(++stsz, u);
  u = stsz;
  rc[u] = update(rc[u], i, v);
  rc[u] = update(rc[u], i, v);

  if (li[u] == ri[u]) st[u] += v;
  else st[u] = st[rc[u]] + st[rc[u]];

  return u;
}
```

2.19 Struct Segment Tree

```
// Segment Tree (range query and point update)
```

```cpp
// Update - O(log n)
// Query - O(log n)
// Memory - O(n)

struct Node {
  ll val;

  Node(ll _val = 0) : val(_val) {}
  Node(const Node& l, const Node& r) : val(l.val + r.val) {}

  friend ostream& operator<<(ostream& os, const Node& a) {
    os << a.val;
    return os;
  }
};

template <class T = Node, class U = int>
struct SimpleSegTree {
  int n;
  vector<T> st;

  SimpleSegTree(int _n) : n(_n), st(4 * n) {}

  SimpleSegTree(vector<U>& v) : n((int)v.size()), st(4 * n) {
    build(v, 1, 0, n - 1);
  }

  void build(vector<U>& v, int p, int l, int r) {
    if (l == r) { st[p] = T(v[l]); return; }
    int mid = (l + r) / 2;
    build(v, 2 * p, l, mid);
    build(v, 2 * p + 1, mid + 1, r);
    st[p] = T(st[2 * p], st[2 * p + 1]);
  }

  T query(int i, int j, int p, int l, int r) {
    if (l >= i and j >= r) return st[p];
    if (l > j or r < i) return T();
    int mid = (l + r) / 2;
    return T(query(i, j, 2 * p, l, mid), query(i, j, 2 * p + 1, mid + 1, r));
  }

  T query(int i, int j) { return query(i, j, 1, 0, n - 1); }

  void update(int idx, U v, int p, int l, int r) {
    if (l == r) { st[p] = T(v); return; }
    int mid = (l + r) / 2;
    if (idx <= mid) update(idx, v, 2 * p, l, mid);
    else update(idx, v, 2 * p + 1, mid + 1, r);
    st[p] = T(st[2 * p], st[2 * p + 1]);
  }

  void update(int idx, U v) { update(idx, v, 1, 0, n - 1); }
};
```

2.20 Segment Tree

```cpp
// Segment Tree (Range Query and Range Update)
// Update and Query - O(log n)

int n, v[N], lz[4*N], st[4*N];

void build(int p = 1, int l = 1, int r = n) {
  if (l == r) { st[p] = v[l]; return; }
  build(2*p, l, (l+r)/2);
  build(2*p+1, (l+r)/2+1, r);
  st[p] = min(st[2*p], st[2*p+1]); // RMQ -> min/max, RSQ -> +
}

void push(int p, int l, int r) {
  if (lz[p]) {
    st[p] = lz[p];
    // RMQ -> update: = lz[p],         increment: += lz[p]
    // RSQ -> update: = (r-l+1)*lz[p], increment: += (r-l+1)*lz[p]
    if(l!=r) lz[2*p] = lz[2*p+1] = lz[p]; // update: =, increment +=
    lz[p] = 0;
  }
}

int query(int i, int j, int p = 1, int l = 1, int r = n) {
  push(p, l, r);
  if (l > j or r < i) return INF; // RMQ -> INF, RSQ -> 0
  if (l >= i and j >= r) return st[p];
  return min(query(i, j, 2*p, l, (l+r)/2),
             query(i, j, 2*p+1, (l+r)/2+1, r));
  // RMQ -> min/max, RSQ -> +
}

void update(int i, int j, int v, int p = 1, int l = 1, int r = n) {
  push(p, l, r);
  if (l > j or r < i) return;
  if (l >= i and j >= r) { lz[p] = v; push(p, l, r); return; }
  update(i, j, v, 2*p, l, (l+r)/2);
  update(i, j, v, 2*p+1, (l+r)/2+1, r);
  st[p] = min(st[2*p], st[2*p+1]); // RMQ -> min/max, RSQ -> +
}
```

2.21 Segment Tree 2D

```cpp
// Segment Tree 2D - O(nlog(n)log(n)) of Memory and Runtime
const int N = 1e8+5, M = 2e5+5;
int n, k=1, st[N], lc[N], rc[N];

void addx(int x, int l, int r, int u) {
  if (x < l or r < x) return;

  st[u]++;
  if (l == r) return;

  if(!rc[u]) rc[u] = ++k, lc[u] = ++k;
  addx(x, l, (l+r)/2, lc[u]);
  addx(x, (l+r)/2+1, r, rc[u]);
}

// Adds a point (x, y) to the grid.
void add(int x, int y, int l, int r, int u) {
  if (y < l or r < y) return;

  if (!st[u]) st[u] = ++k;
  addx(x, 1, n, st[u]);

  if (l == r) return;

  if(!rc[u]) rc[u] = ++k, lc[u] = ++k;
  add(x, y, l, (l+r)/2, lc[u]);
  add(x, y, (l+r)/2+1, r, rc[u]);
}

int countx(int x, int l, int r, int u) {
  if (!u or x < l) return 0;
  if (r <= x) return st[u];

  return countx(x, l, (l+r)/2, lc[u]) +
         countx(x, (l+r)/2+1, r, rc[u]);
}

// Counts number of points dominated by (x, y)
// Should be called with l = 1, r = n and u = 1
int count(int x, int y, int l, int r, int u) {
  if (!u or y < l) return 0;
  if (r <= y) return countx(x, 1, n, st[u]);

  return count(x, y, l, (l+r)/2, lc[u]) +
         count(x, y, (l+r)/2+1, r, rc[u]);
}
```

2.22 Set Of Intervals

```cpp
// Set of Intervals
// Use when you have disjoint intervals

#include <bits/stdc++.h>
using namespace std;

const int N = 2e5 + 5;

#define pb push_back
#define st first
#define nd second

typedef pair<int, int> pii;
typedef pair<pii, int> piii;

int n, m, x, t;
```

```cpp
set<piii> s;
set<pii> mosq;
vector<piii> frogs;
int c[N], len[N], p, b[N];

void in(int l, int r, int i) {
  vector<piii> add, rem;
  auto it = s.lower_bound({{l, 0}, 0});
  if(it != s.begin()) it--;
  for(; it != s.end(); it++) {
    int ll = it->st.st;
    int rr = it->st.nd;
    int idx = it->nd;

    if(ll > r) break;
    if(rr < l) continue;
    if(ll < l) add.pb({{ll, l-1}, idx});
    if(rr > r) add.pb({{r+1, rr}, idx});
    rem.pb(*it);
  }
  add.pb({{l, r}, i});
  for(auto x : rem) s.erase(x);
  for(auto x : add) s.insert(x);
}

void process(int l, int idx) {
  auto it2 = s.lower_bound({{l, 0}, 0});
  if(it2 != s.begin()) it2--;
  if(it2 != s.end() and it2->st.nd < l) it2++;

  mosq.insert({l, idx});
  if(it2 == s.end() or !(it2->nd)) return;

  vector<pii> rem;
  int ll = it2->st.st, rr = it2->st.nd, id = it2->nd;

  auto it = mosq.lower_bound({ll, 0});
  for(; it != mosq.end(); it++) {
    if(it->st > rr) break;
    c[id]++;
    len[id] += b[it->nd];
    rr += b[it->nd];
    rem.pb(*it);
  }
  for(auto x : rem) mosq.erase(x);
  in(ll, rr, id);
}

int main() {
  ios_base::sync_with_stdio(0), cin.tie(0);
  cin >> n >> m;
  for(int i = 1; i <= n; i++) {
    cin >> x >> t;
    len[i] = t;
    frogs.push_back({{x, x+t}, i});
  }
  s.insert({{0, int(1e9)}, 0});
  sort(frogs.begin(), frogs.end());
  for(int i = frogs.size() - 1; i >= 0; i--)
    in(frogs[i].st.st, frogs[i].st.nd, frogs[i].nd);

  for(int i = 1; i <= m; i++) {
    cin >> p >> b[i];
    process(p, i);
  }

  for(int i = 1; i <= n; i++) cout << c[i] << " " << len[i] << "\n";
}
```

2.23 Sparse Table

```cpp
const int N;
const int M; //log2(N)
int sparse[N][M];

void build() {
  for(int i = 0; i < n; i++)
    sparse[i][0] = v[i];

  for(int j = 1; j < M; j++)
    for(int i = 0; i < n; i++)
      sparse[i][j] =
        i + (1 << j - 1) < n
        ? min(sparse[i][j - 1], sparse[i + (1 << j - 1)][j - 1])
        : sparse[i][j - 1];
}

int query(int a, int b){
  int pot = 32 - __builtin_clz(b - a) - 1;
  return min(sparse[a][pot], sparse[b - (1 << pot) + 1][pot]);
}
```

2.24 Sparse Table 2D

```cpp
// 2D Sparse Table - <O(n^2 (log n) ^ 2), O(1)>
const int N = 1e3+1, M = 10;
int t[N][N], v[N][N], dp[M][M][N][N], lg[N], n, m;

void build() {
    int k = 0;
    for(int i=1; i<N; ++i) {
        if (1<<k == i/2) k++;
        lg[i] = k;
    }

    // Set base cases
    for(int x=0; x<n; ++x) for(int y=0; y<m; ++y) dp[0][0][x][y] = v[x][y];
    for(int j=1; j<M; ++j) for(int x=0; x<n; ++x) for(int y=0; y+(1<<j)<=m; ++y)
        dp[0][j][x][y] = max(dp[0][j-1][x][y], dp[0][j-1][x][y+(1<<j-1)]);

    // Calculate sparse table values
    for(int i=1; i<M; ++i) for(int j=0; j<M; ++j)
        for(int x=0; x+(1<<i)<=n; ++x) for(int y=0; y+(1<<j)<=m; ++y)
            dp[i][j][x][y] = max(dp[i-1][j][x][y], dp[i-1][j][x+(1<<i-1)][y]);
}

int query(int x1, int x2, int y1, int y2) {
    int i = lg[x2-x1+1], j = lg[y2-y1+1];
    int m1 = max(dp[i][j][x1][y1], dp[i][j][x2-(1<<i)+1][y1]);
    int m2 = max(dp[i][j][x1][y2-(1<<j)+1], dp[i][j][x2-(1<<i)+1][y2-(1<<j)+1]);
    return max(m1, m2);
}
```

2.25 KD Tree (Stanford)

```cpp
const int maxn=200005;

struct kdtree
{
        int xl,xr,yl,yr,zl,zr,max,flag; // flag=0:x axis  1:y 2:z
} tree[5000005];

int N,M,lastans,xq,yq;
int a[maxn],pre[maxn],nxt[maxn];
int x[maxn],y[maxn],z[maxn],wei[maxn];
int xc[maxn],yc[maxn],zc[maxn],wc[maxn],hash[maxn],biao[maxn];

bool cmp1(int a,int b)
{
        return x[a]<x[b];
}

bool cmp2(int a,int b)
{
        return y[a]<y[b];
}

bool cmp3(int a,int b)
{
        return z[a]<z[b];
}

void makekdtree(int node,int l,int r,int flag)
{
        if (l>r)
        {
                tree[node].max=-maxlongint;
                return;
        }
        int xl=maxlongint,xr=-maxlongint;
        int yl=maxlongint,yr=-maxlongint;
        int zl=maxlongint,zr=-maxlongint,maxc=-maxlongint;
        for (int i=l;i<=r;i++)
                xl=min(xl,x[i]),xr=max(xr,x[i]),
                yl=min(yl,y[i]),yr=max(yr,y[i]),
                zl=min(zl,z[i]),zr=max(zr,z[i]),
                maxc=max(maxc,wei[i]),
                xc[i]=x[i],yc[i]=y[i],zc[i]=z[i],wc[i]=wei[i],biao[i]=i;
        tree[node].flag=flag;
        tree[node].xl=xl,tree[node].xr=xr,tree[node].yl=yl;
        tree[node].yr=yr,tree[node].zl=zl,tree[node].zr=zr;
        tree[node].max=maxc;
        if (l==r) return;
        if (flag==0) sort(biao+l,biao+r+1,cmp1);
        if (flag==1) sort(biao+l,biao+r+1,cmp2);
        if (flag==2) sort(biao+l,biao+r+1,cmp3);
        for (int i=l;i<=r;i++)
                x[i]=xc[biao[i]],y[i]=yc[biao[i]],
                z[i]=zc[biao[i]],wei[i]=wc[biao[i]];
        makekdtree(node*2,l,(l+r)/2,(flag+1)%3);
        makekdtree(node*2+1,(l+r)/2+1,r,(flag+1)%3);
}

int getmax(int node,int xl,int xr,int yl,int yr,int zl,int zr)
{
        xl=max(xl,tree[node].xl);
        xr=min(xr,tree[node].xr);
        yl=max(yl,tree[node].yl);
        yr=min(yr,tree[node].yr);
        zl=max(zl,tree[node].zl);
        zr=min(zr,tree[node].zr);
        if (tree[node].max==-maxlongint) return 0;
        if ((xr<tree[node].xl)||(xl>tree[node].xr)) return 0;
        if ((yr<tree[node].yl)||(yl>tree[node].yr)) return 0;
        if ((zr<tree[node].zl)||(zl>tree[node].zr)) return 0;
        if ((tree[node].xl==xl)&&(tree[node].xr==xr)&&
                (tree[node].yl==yl)&&(tree[node].yr==yr)&&
                (tree[node].zl==zl)&&(tree[node].zr==zr))
        return tree[node].max;
        else
        return max(getmax(node*2,xl,xr,yl,yr,zl,zr),
                               getmax(node*2+1,xl,xr,yl,yr,zl,zr));
}

int main()
{
        // N 3D-rect with weights
        // find the maximum weight containing the given 3D-point
        return 0;
}
```

2.26 Treap

```
// Treap (probabilistic BST)
// O(logn) operations (supports lazy propagation)

mt19937_64 llrand(random_device{}());

struct node {
  int val;
  int cnt, rev;
  int mn, mx, mindiff; // value-based treap only!
  ll pri;
  node* l;
  node* r;

  node() {}
  node(int x) : val(x), cnt(1), rev(0), mn(x), mx(x), mindiff(INF), pri(llrand()), l(0), r(0) {}
};

struct treap {
  node* root;
  treap() : root(0) {}
  ~treap() { clear(); }

  int cnt(node* t) { return t ? t->cnt : 0; }
  int mn (node* t) { return t ? t->mn  : INF; }
  int mx (node* t) { return t ? t->mx  : -INF; }
  int mindiff(node* t) { return t ? t->mindiff : INF; }

  void clear() { del(root); }
  void del(node* t) {
    if (!t) return;
    del(t->l); del(t->r);
    delete t;
    t = 0;
  }

  void push(node* t) {
    if (!t or !t->rev) return;
    swap(t->l, t->r);
    if (t->l) t->l->rev ^= 1;
    if (t->r) t->r->rev ^= 1;
    t->rev = 0;
  }

  void update(node*& t) {
    if (!t) return;
    t->cnt = cnt(t->l) + cnt(t->r) + 1;
    t->mn = min(t->val, min(mn(t->l), mn(t->r)));
    t->mx = max(t->val, max(mx(t->l), mx(t->r)));
    t->mindiff = min(mn(t->r) - t->val, min(t->val - mx(t->l), min(mindiff(t->l), mindiff(t->r))));
  }

  node* merge(node* l, node* r) {
    push(l); push(r);
    node* t;
    if (!l or !r) t = l ? l : r;
    else if (l->pri > r->pri) l->r = merge(l->r, r), t = l;
    else r->l = merge(l, r->l), t = r;
    update(t);
    return t;
  }

  // pos: amount of nodes in the left subtree or
  // the smallest position of the right subtree in a 0-indexed array
  pair<node*, node*> split(node* t, int pos) {
    if (!t) return {0, 0};
    push(t);

    if (cnt(t->l) < pos) {
      auto x = split(t->r, pos-cnt(t->l)-1);
      t->r = x.st;
      update(t);
      return { t, x.nd };
    }

    auto x = split(t->l, pos);
    t->l = x.nd;
    update(t);
    return { x.st, t };
  }

  // Position-based treap
  // used when the values are just additional data
  // the positions are known when it's built, after that you
  // query to get the values at specific positions
  // 0-indexed array!
  /*
  void insert(int pos, int val) {
    push(root);
    node* x = new node(val);
    auto t = split(root, pos);
    root = merge(merge(t.st, x), t.nd);
  }

  void erase(int pos) {
    auto t1 = split(root, pos);
    auto t2 = split(t1.nd, 1);
    delete t2.st;
    root = merge(t1.st, t2.nd);
  }

  int get_val(int pos) { return get_val(root, pos); }
  int get_val(node* t, int pos) {
    push(t);
    if (cnt(t->l) == pos) return t->val;
    if (cnt(t->l) < pos) return get_val(t->r, pos-cnt(t->l)-1);
    return get_val(t->l, pos);
  }
  */
  // -------------------

  // Value-based treap
  // used when the values needs to be ordered
  int order(node* t, int val) {
    if (!t) return 0;
    push(t);
    if (t->val < val) return cnt(t->l) + 1 + order(t->r, val);
    return order(t->l, val);
  }

  bool has(node* t, int val) {
    if (!t) return 0;
    push(t);
    if (t->val == val) return 1;
    return has((t->val > val ? t->l : t->r), val);
  }

  void insert(int val) {
    if (has(root, val)) return; // avoid repeated values
    push(root);
    node* x = new node(val);
    auto t = split(root, order(root, val));
    root = merge(merge(t.st, x), t.nd);
  }

  void erase(int val) {
    if (!has(root, val)) return;

    auto t1 = split(root, order(root, val));
    auto t2 = split(t1.nd, 1);
    delete t2.st;
    root = merge(t1.st, t2.nd);
  }

  // Get the maximum difference between values
  int querymax(int i, int j) {
    if (i == j) return -1;
    auto t1 = split(root, j+1);
    auto t2 = split(t1.st, i);

    int ans = mx(t2.nd) - mn(t2.nd);
    root = merge(merge(t2.st, t2.nd), t1.nd);
    return ans;
  }

  // Get the minimum difference between values
  int querymin(int i, int j) {
    if (i == j) return -1;
    auto t2 = split(root, j+1);
    auto t1 = split(t2.st, i);

    int ans = mindiff(t1.nd);
    root = merge(merge(t1.st, t1.nd), t2.nd);
    return ans;
  }
  // ------------------

  void reverse(int l, int r) {
    auto t2 = split(root, r+1);
    auto t1 = split(t2.st, l);
    t1.nd->rev = 1;
    root = merge(merge(t1.st, t1.nd), t2.nd);
  }

  void print() { print(root); printf("\n"); }
  void print(node* t) {
    if (!t) return;
    push(t);
    print(t->l);
    printf("%d ", t->val);
    print(t->r);
  }
};
```

2.27 Trie

```
// Trie <O(|S|), O(|S|)>
int trie[N][26], trien = 1;

int add(int u, char c){
  c-='a';
  if (trie[u][c]) return trie[u][c];
  return trie[u][c] = ++trien;
}

//to add a string s in the trie
int u = 1;
for(char c : s) u = add(u, c);
```

2.28 Union Find

```
/*********************************************************************************
* DSU (DISJOINT SET UNION / UNION-FIND)                                          *
* Time complexity:  Unite - O(alpha n)                                           *
*                   Find - O(alpha n)                                            *
* Usage: find(node), unite(node1, node2), sz[find(node)]                         *
* Notation: par: vector of parents                                               *
*           sz:  vector of subsets sizes, i.e. size of the subset a node is in   *
*********************************************************************************/

int par[N], sz[N];

int find(int a) { return par[a] == a ? a : par[a] = find(par[a]); }

void unite(int a, int b) {
  if ((a = find(a)) == (b = find(b))) return;
  if (sz[a] < sz[b]) swap(a, b);
  par[b] = a; sz[a] += sz[b];
}

// in main
for (int i = 1; i <= n; i++) par[i] = i, sz[i] = 1;
```

2.29 Union Find (Partial Persistent)

```
/*********************************************************************************
* DSU (DISJOINT SET UNION / UNION-FIND)                                          *
* Time complexity:  Unite - O(log n)                                             *
*                   Find - O(log n)                                              *
* Usage: find(node), unite(node1, node2), sz[find(node)]                         *
* Notation: par: vector of parents                                               *
*           sz:  vector of subsets sizes, i.e. size of the subset a node is in   *
*           his: history: time when it got a new parent                          *
*           t: current time                                                      *
*********************************************************************************/

int t, par[N], sz[N], his[N];

int find(int a, int t){
    if(par[a] == a) return a;
    if(his[a] > t) return a;
    return find(par[a], t);
}

void unite(int a, int b){
    if(find(a, t) == find(b, t)) return;
    a = find(a, t), b = find(b, t), t++;
    if(sz[a] < sz[b]) swap(a, b);
    sz[a] += sz[b], par[b] = a, his[b] = t;
}

//in main
for(int i = 0; i < N; i++) par[i] = i, sz[i] = 1, his[i] = 0;
```

2.30 Union Find (Rollback)

```
/*********************************************************************************
* DSU (DISJOINT SET UNION / UNION-FIND)                                          *
* Time complexity:  Unite - O(alpha n)                                           *
*                   Rollback - O(1)                                              *
*                   Find - O(alpha n)                                            *
* Usage: find(node), unite(node1, node2), sz[find(node)]                         *
* Notation: par: vector of parents                                               *
*           sz:  vector of subsets sizes, i.e. size of the subset a node is in   *
*           sp:  stack containing node and par from last op                      *
*           ss:  stack containing node and size from last op                     *
*********************************************************************************/

int par[N], sz[N];
stack <pii> sp, ss;

int find (int a) { return par[a] == a ? a : find(par[a]); }

void unite (int a, int b) {
    if ((a = find(a)) == (b = find(b))) return;
    if (sz[a] < sz[b]) swap(a, b);
    ss.push({a, sz[a]});
    sp.push({b, par[b]});
    sz[a] += sz[b];
    par[b] = a;
}

void rollback() {
    par[sp.top().st] = sp.top().nd; sp.pop();
    sz[ss.top().st]  = ss.top().nd; ss.pop();
}

int main(){
    for (int i = 0; i < N; i++) par[i] = i, sz[i] = 1;
    return 0;
}
```

3 Dynamic Programming

3.1 Convex Hull Trick (emaxx)

```
struct Point{
  ll x, y;
  Point(ll x = 0, ll y = 0):x(x), y(y) {}
  Point operator-(Point p){ return Point(x - p.x, y - p.y); }
  Point operator+(Point p){ return Point(x + p.x, y + p.y); }
  Point ccw(){ return Point(-y, x); }
  ll operator%(Point p){ return x*p.y - y*p.x; }
  ll operator*(Point p){ return x*p.x + y*p.y; }
  bool operator<(Point p) const { return x == p.x ? y < p.y : x < p.x; }
};

pair<vector<Point>, vector<Point>> ch(Point *v){
  vector<Point> hull, vecs;
  for(int i = 0; i < n; i++){
    if(hull.size() and hull.back().x == v[i].x) continue;

    while(vecs.size() and vecs.back()*(v[i] - hull.back()) <= 0)
      vecs.pop_back(), hull.pop_back();

    if(hull.size())
      vecs.pb((v[i] - hull.back()).ccw());

    hull.pb(v[i]);
  }
  return {hull, vecs};
}

ll get(ll x) {
    Point query = {x, 1};
    auto it = lower_bound(vecs.begin(), vecs.end(), query, [](Point a, Point b) {
        return a%b > 0;
    });
    return query*hull[it - vecs.begin()];
}
```

3.2 Convex Hull Trick

```
// Convex Hull Trick

// ATTENTION: This is the maximum convex hull. If you need the minimum
// CHT use {-b, -m} and modify the query function.

// In case of floating point parameters swap long long with long double
typedef long long type;
struct line { type b, m; };

line v[N]; // lines from input
int n; // number of lines
// Sort slopes in ascending order (in main):
sort(v, v+n, [](line s, line t){
     return (s.m == t.m) ? (s.b < t.b) : (s.m < t.m); });

// nh: number of lines on convex hull
// pos: position for linear time search
// hull: lines in the convex hull
int nh, pos;
line hull[N];

bool check(line s, line t, line u) {
  // verify if it can overflow. If it can just divide using long double
  return (s.b - t.b)*(u.m - s.m) < (s.b - u.b)*(t.m - s.m);
}

// Add new line to convex hull, if possible
// Must receive lines in the correct order, otherwise it won't work
void update(line s) {
  // 1. if first lines have the same b, get the one with bigger m
  // 2. if line is parallel to the one at the top, ignore
  // 3. pop lines that are worse
  // 3.1 if you can do a linear time search, use
  // 4. add new line

  if (nh == 1 and hull[nh-1].b == s.b) nh--;
  if (nh > 0  and hull[nh-1].m >= s.m) return;
  while (nh >= 2 and !check(hull[nh-2], hull[nh-1], s)) nh--;
  pos = min(pos, nh);
  hull[nh++] = s;
}

type eval(int id, type x) { return hull[id].b + hull[id].m * x; }

// Linear search query - O(n) for all queries
// Only possible if the queries always move to the right
type query(type x) {
  while (pos+1 < nh and eval(pos, x) < eval(pos+1, x)) pos++;
  return eval(pos, x);
  // return -eval(pos, x);    ATTENTION: Uncomment for minimum CHT
}

// Ternary search query - O(logn) for each query
/*
type query(type x) {
  int lo = 0, hi = nh-1;
  while (lo < hi) {
    int mid = (lo+hi)/2;
    if (eval(mid, x) > eval(mid+1, x)) hi = mid;
    else lo = mid+1;
  }
  return eval(lo, x);
  // return -eval(lo, x);     ATTENTION: Uncomment for minimum CHT
}

// better use geometry line_intersect (this assumes s and t are not parallel)
ld intersect_x(line s, line t) { return (t.b - s.b)/(ld)(s.m - t.m); }
ld intersect_y(line s, line t) { return s.b + s.m * intersect_x(s, t); }
*/
```

3.3 Divide and Conquer Optimization

```
// Divide and Conquer DP Optimization - O(k*n^2) => O(k*n*logn)
//
// dp[i][j] = min k<i { dp[k][j-1] + C[k][i] }
//
// Condition: A[i][j] <= A[i+1][j]
// A[i][j] is the smallest k that gives an optimal answer to dp[i][j]
//
// reference (pt-br): https://algorithmmarch.wordpress.com/2016/08/12/a-otimizacao-de-pds-e-o-garcom-da-
      maratona/

int n, maxj;
int dp[N][J], a[N][J];

// declare the cost function
int cost(int i, int j) {
  // ...
}

void calc(int l, int r, int j, int kmin, int kmax) {
  int m = (l+r)/2;
  dp[m][j] = LINF;

  for (int k = kmin; k <= kmax; ++k) {
    ll v = dp[k][j-1] + cost(k, m);

    // store the minimum answer for d[m][j]
    // in case of maximum, use v > dp[m][j]
    if (v < dp[m][j]) a[m][j] = k, dp[m][j] = v;
  }

  if (l < r) {
    calc(l,    m, j, kmin,    a[m][k]);
    calc(m+1, r, j, a[m][k], kmax    );
  }
}

// run for every j
for (int j = 2; j <= maxj; ++j)
  calc(1, n, j, 1, n);
```

3.4 Knuth Optimization

```cpp
// Knuth DP Optimization - O(n^3) -> O(n^2)
//
// 1) dp[i][j] = min i<k<j { dp[i][k] + dp[k][j] } + C[i][j]
// 2) dp[i][j] = min k<i { dp[k][j-1] + C[k][i] }
//
// Condition: A[i][j-1] <= A[i][j] <= A[i+1][j]
// A[i][j] is the smallest k that gives an optimal answer to dp[i][j]
//
// reference (pt-br): https://algorithmmarch.wordpress.com/2016/08/12/a-otimizacao-de-pds-e-o-garcom-da-
      maratona/
//

// 1) dp[i][j] = min i<k<j { dp[i][k] + dp[k][j] } + C[i][j]
int n;
int dp[N][N], a[N][N];

// declare the cost function
int cost(int i, int j) {
  // ...
}

void knuth() {
  // calculate base cases
  memset(dp, 63, sizeof(dp));
  for (int i = 1; i <= n; i++) dp[i][i] = 0;

  // set initial a[i][j]
  for (int i = 1; i <= n; i++) a[i][i] = i;

  for (int j = 2; j <= n; ++j)
    for (int i = j; i >= 1; --i)
      for (int k = a[i][j-1]; k <= a[i+1][j]; ++k) {
        ll v = dp[i][k] + dp[k][j] + cost(i, j);

        // store the minimum answer for d[i][k]
        // in case of maximum, use v > dp[i][k]
        if (v < dp[i][j])
          a[i][j] = k, dp[i][j] = v;
      }
}


// 2) dp[i][j] = min k<i { dp[k][j-1] + C[k][i] }
int n, maxj;
int dp[N][J], a[N][J];

// declare the cost function
int cost(int i, int j) {
  // ...
}

void knuth() {
  // calculate base cases
  memset(dp, 63, sizeof(dp));
  for (int i = 1; i <= n; i++) dp[i][1] = // ...

  // set initial a[i][j]
  for (int i = 1; i <= n; i++) a[i][1] = 1, a[n+1][i] = n;

  for (int j = 2; j <= maxj; j++)
    for (int i = n; i >= 1; i--)
      for (int k = a[i][j-1]; k <= a[i+1][j]; k++) {
        ll v = dp[k][j-1] + cost(k, i);

        // store the minimum answer for d[i][k]
        // in case of maximum, use v > dp[i][k]
        if (v < dp[i][j])
          a[i][j] = k, dp[i][j] = v;
      }
}
```

3.5 Longest Increasing Subsequence

```cpp
// Longest Increasing Subsequence - O(nlogn)
//
// dp(i) = max j<i { dp(j) | a[j] < a[i] } + 1
//

// int dp[N], v[N], n, lis;

memset(dp, 63, sizeof dp);
for (int i = 0; i < n; ++i) {
  // increasing: lower_bound
  // non-decreasing: upper_bound
  int j = lower_bound(dp, dp + lis, v[i]) - dp;
  dp[j] = min(dp[j], v[i]);
  lis = max(lis, j + 1);
}
```

3.6 SOS DP

```cpp
// O(N * 2^N)
// A[i] = initial values
// Calculate F[i] = Sum of A[j] for j subset of i
for(int i = 0; i < (1 << N); i++)
        F[i] = A[i];
for(int i = 0; i < N; i++)
        for(int j = 0; j < (1 << N); j++)
                if(j & (1 << i))
                        F[j] += F[j ^ (1 << i)];
```

3.7 Steiner tree

```cpp
// Steiner-Tree O(2^t*n^2 + n*3^t + APSP)

// N - number of nodes
// T - number of terminals
// dist[N][N] - Adjacency matrix
// steiner_tree() = min cost to connect first t nodes, 1-indexed
// dp[i][bit_mask] = min cost to connect nodes active in bitmask rooting in i
// min{dp[i][bit_mask]}, i <= n if root doesn't matter

int n, t, dp[N][(1 << T)], dist[N][N];

int steiner_tree() {
  for (int k = 1; k <= n; ++k)
    for (int i = 1; i <= n; ++i)
      for (int j = 1; j <= n; ++j)
        dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);

  for(int i = 1; i <= n; i++)
    for(int j = 0; j < (1 << t); j++)
      dp[i][j] = INF;
  for(int i = 1; i <= t; i++) dp[i][1 << (i-1)] = 0;

  for(int msk = 0; msk < (1 << t); msk++) {
    for(int i = 1; i <= n; i++) {
      for(int ss = msk; ss > 0; ss = (ss - 1) & msk)
        dp[i][msk] = min(dp[i][msk], dp[i][ss] + dp[i][msk - ss]);

      if(dp[i][msk] != INF)
        for(int j = 1; j <= n; j++)
          dp[j][msk] = min(dp[j][msk], dp[i][msk] + dist[i][j]);
    }
  }

  int mn = INF;
  for(int i = 1; i <= n; i++) mn = min(mn, dp[i][(1 << t) - 1]);
  return mn;
}
```

4 Graphs

4.1 2-SAT Kosaraju

```cpp
/***********************************************************************************
* 2-SAT (TELL WHETHER A SERIES OF STATEMENTS CAN OR CANNOT BE FEASIBLE AT THE      *
* SAME TIME)                                                                       *
* Time complexity: O(V+E)                                                          *
* Usage: n          -> number of variables, 1-indexed                              *
*        p = v(i)   -> picks the "true" state for variable i                       *
*        p = nv(i) -> picks the "false" state for variable i, i.e. ~i              *
*        add(p, q) -> add clause p => q (which also means ~q => ~p)                *
*        run2sat() -> true if possible, false if impossible                        *
*        val[i]    -> tells if i has to be true or false for that solution        *
*********************************************************************************/

int n, vis[2*N], ord[2*N], ordn, cnt, cmp[2*N], val[N];
vector<int> adj[2*N], adjt[2*N];

// for a variable u with idx i
// u is 2*i and !u is 2*i+1
// (a v b) == !a -> b ^ !b -> a

int v(int x) { return 2*x; }
int nv(int x) { return 2*x+1; }

// add clause (a v b)
void add(int a, int b){
  adj[a^1].push_back(b);
  adj[b^1].push_back(a);
  adjt[b].push_back(a^1);
  adjt[a].push_back(b^1);
}

void dfs(int x){
  vis[x] = 1;
  for(auto v : adj[x]) if(!vis[v]) dfs(v);
  ord[ordn++] = x;
}

void dfst(int x){
  cmp[x] = cnt, vis[x] = 0;
  for(auto v : adjt[x]) if(vis[v]) dfst(v);
}

bool run2sat(){
  for(int i = 1; i <= n; i++) {
    if(!vis[v(i)]) dfs(v(i));
    if(!vis[nv(i)]) dfs(nv(i));
  }
  for(int i = ordn-1; i >= 0; i--)
    if(vis[ord[i]]) cnt++, dfst(ord[i]);
  for(int i = 1; i <= n; i ++){
    if(cmp[v(i)] == cmp[nv(i)]) return false;
    val[i] = cmp[v(i)] > cmp[nv(i)];
  }
  return true;
}

int main () {
    for (int i = 1; i <= n; i++) {
        if (val[i]); // i-th variable is true
        else         // i-th variable is false
}
```

4.2 2-SAT Tarjan

```
// 2-SAT - O(V+E)
// For each variable x, we create two nodes in the graph: u and !u
// If the variable has index i, the index of u and !u are: 2*i and 2*i+1
// Adds a statment u => v
void add(int u, int v){
  adj[u].pb(v);
  adj[v^1].pb(u^1);
}

//0-indexed variables; starts from var_0 and goes to var_n-1
for(int i = 0; i < n; i++){
  tarjan(2*i), tarjan(2*i + 1);
  //cmp is a tarjan variable that says the component from a certain node
  if(cmp[2*i] == cmp[2*i + 1]) //Invalid
  if(cmp[2*i] < cmp[2*i + 1]) //Var_i is true
  else //Var_i is false

  //its just a possible solution!
}
```

4.3 Shortest Path (Bellman-Ford)

```
/*********************************************************************************
* BELLMAN-FORD ALGORITHM (SHORTEST PATH TO A VERTEX - WITH NEGATIVE COST)        *
* Time complexity: O(VE)                                                         *
* Usage: dist[node]                                                              *
* Notation: m:           number of edges                                         *
*           n:           number of vertices                                      *
*           (a, b, w):   edge between a and b with weight w                      *
*           s:           starting node                                           *
*********************************************************************************/
const int N = 1e4+10; // Maximum number of nodes
vector<int> adj[N], adjw[N];
int dist[N], v, w;

memset(dist, 63, sizeof(dist));
dist[0] = 0;
for (int i = 0; i < n-1; ++i)
  for (int u = 0; u < n; ++u)
    for (int j = 0; j < adj[u].size(); ++j)
      v = adj[u][j], w = adjw[u][j],
      dist[v] = min(dist[v], dist[u]+w);
```

4.4 Block Cut

```
// Tarjan for Block Cut Tree (Node Biconnected Componentes) - O(n + m)
#define pb push_back
#include <bits/stdc++.h>
using namespace std;

const int N = 1e5+5;

// Regular Tarjan stuff
int n, num[N], low[N], cnt, ch[N], art[N];
vector<int> adj[N], st;

int lb[N]; // Last block that node is contained
int bn; // Number of blocks
vector<int> blc[N]; // List of nodes from block

void dfs(int u, int p) {
  num[u] = low[u] = ++cnt;
  ch[u] = adj[u].size();
  st.pb(u);

  if (adj[u].size() == 1) blc[++bn].pb(u);

  for(int v : adj[u]) {
    if (!num[v]) {
      dfs(v, u), low[u] = min(low[u], low[v]);
      if (low[v] == num[u]) {
        if (p != -1 or ch[u] > 1) art[u] = 1;
        blc[++bn].pb(u);
        while(blc[bn].back() != v)
          blc[bn].pb(st.back()), st.pop_back();
      }
    }
    else if (v != p) low[u] = min(low[u], num[v]), ch[v]--;
  }

  if (low[u] == num[u]) st.pop_back();
}

// Nodes from 1 .. n are blocks
// Nodes from n+1 .. 2*n are articulations
vector<int> bct[2*N]; // Adj list for Block Cut Tree

void build_tree() {
  for(int u=1; u<=n; ++u) for(int v : adj[u]) if (num[u] > num[v]) {
    if (lb[u] == lb[v] or blc[lb[u]][0] == v) /* edge u-v belongs to block lb[u] */;
    else { /* edge u-v belongs to block cut tree */;
      int x = (art[u] ? u + n : lb[u]), y = (art[v] ? v + n : lb[v]);
      bct[x].pb(y), bct[y].pb(x);
    }
  }
}

void tarjan() {
  for(int u=1; u<=n; ++u) if (!num[u]) dfs(u, -1);
  for(int b=1; b<=bn; ++b) for(int u : blc[b]) lb[u] = b;
  build_tree();
}
```

4.5 Articulation points and bridges

```
// Articulation points and Bridges O(V+E)
```

```
int par[N], art[N], low[N], num[N], ch[N], cnt;

void articulation(int u) {
  low[u] = num[u] = ++cnt;
  for (int v : adj[u]) {
    if (!num[v]) {
      par[v] = u; ch[u]++;
      articulation(v);
      if (low[v] >= num[u]) art[u] = 1;
      if (low[v] >  num[u]) { /* u-v bridge */ }
      low[u] = min(low[u], low[v]);
    }
    else if (v != par[u]) low[u] = min(low[u], num[v]);
  }
}

for (int i = 0; i < n; ++i) if (!num[i])
  articulation(i), art[i] = ch[i]>1;
```

4.6 Max Flow

```
// Dinic - O(V^2 * E)
// Bipartite graph or unit flow - O(sqrt(V) * E)
// Small flow - O(F * (V + E))
// USE INF = 1e9!

/*******************************************************************************
* DINIC (FIND MAX FLOW / BIPARTITE MATCHING)                                   *
* Time complexity: O(EV^2)                                                     *
* Usage: dinic()                                                               *
*        add_edge(from, to, capacity)                                          *
* Testcase:                                                                    *
* add_edge(src, 1, 1);   add_edge(1, snk, 1);   add_edge(2, 3, INF);           *
* add_edge(src, 2, 1);   add_edge(2, snk, 1);   add_edge(3, 4, INF);           *
* add_edge(src, 2, 1);   add_edge(3, snk, 1);                                  *
* add_edge(src, 2, 1);   add_edge(4, snk, 1);   => dinic() = 4                 *
*******************************************************************************/

#include <bits/stdc++.h>
using namespace std;

const int N = 1e5+1, INF = 1e9;
struct edge {int v, c, f;};

int n, src, snk, h[N], ptr[N];
vector<edge> edgs;
vector<int> g[N];

void add_edge (int u, int v, int c) {
  int k = edgs.size();
  edgs.push_back({v, c, 0});
  edgs.push_back({u, 0, 0});
  g[u].push_back(k);
  g[v].push_back(k+1);
}

void clear() {
    memset(h, 0, sizeof h);
    memset(ptr, 0, sizeof ptr);
    edgs.clear();
    for (int i = 0; i < N; i++) g[i].clear();
    src = 0;
    snk = N-1;
}

bool bfs() {
  memset(h, 0, sizeof h);
  queue<int> q;
  h[src] = 1;
  q.push(src);
  while(!q.empty()) {
    int u = q.front(); q.pop();
    for(int i : g[u]) {
      int v = edgs[i].v;
      if (!h[v] and edgs[i].f < edgs[i].c)
        q.push(v), h[v] = h[u] + 1;
    }
  }
  return h[snk];
}

int dfs (int u, int flow) {
  if (!flow or u == snk) return flow;
  for (int &i = ptr[u]; i < g[u].size(); ++i) {
    edge &dir = edgs[g[u][i]], &rev = edgs[g[u][i]^1];
    int v = dir.v;
    if (h[v] != h[u] + 1)  continue;
    int inc = min(flow, dir.c - dir.f);
    inc = dfs(v, inc);
    if (inc) {
      dir.f += inc, rev.f -= inc;
      return inc;
    }
  }
  return 0;
}

int dinic() {
  int flow = 0;
  while (bfs()) {
    memset(ptr, 0, sizeof ptr);
    while (int inc = dfs(src, INF)) flow += inc;
  }
  return flow;
}

int main () {
    clear();
    return 0;
}
```

4.7 Erdos Gallai

```
// Erdos-Gallai - O(nlogn)
// check if it's possible to create a simple graph (undirected edges) from
// a sequence of vertice's degrees
bool gallai(vector<int> v) {
  vector<ll> sum;
  sum.resize(v.size());

  sort(v.begin(), v.end(), greater<int>());
  sum[0] = v[0];
  for (int i = 1; i < v.size(); i++) sum[i] = sum[i-1] + v[i];
  if (sum.back() % 2) return 0;

  for (int k = 1; k < v.size(); k++) {
    int p = lower_bound(v.begin(), v.end(), k, greater<int>()) - v.begin();
    if (p < k) p = k;
    if (sum[k-1] > 1ll*k*(p-1) + sum.back() - sum[p-1]) return 0;
  }
  return 1;
}
```

4.8 Eulerian Path

```
vector<int> ans, adj[N];
int in[N];

void dfs(int v){
  while(adj[v].size()){
    int x = adj[v].back();
    adj[v].pop_back();
    dfs(x);
  }
  ans.pb(v);
}

// Verify if there is an eulerian path or circuit
vector<int> v;
for(int i = 0; i < n; i++) if(adj[i].size() != in[i]){
  if(abs((int)adj[i].size() - in[i]) != 1) //-> There is no valid eulerian circuit/path
  v.pb(i);
}

if(v.size()){
  if(v.size() != 2) //-> There is no valid eulerian path
  if(in[v[0]] > adj[v[0]].size()) swap(v[0], v[1]);
  if(in[v[0]] > adj[v[0]].size()) //-> There is no valid eulerian path
  adj[v[1]].pb(v[0]); // Turn the eulerian path into a eulerian circuit
}

dfs(0);
for(int i = 0; i < cnt; i++)
  if(adj[i].size()) //-> There is no valid eulerian circuit/path in this case because the graph is not
        conected
```

```
ans.pop_back(); // Since it's a curcuit, the first and the last are repeated
reverse(ans.begin(), ans.end());

int bg = 0; // Is used to mark where the eulerian path begins
if(v.size()){
  for(int i = 0; i < ans.size(); i++)
    if(ans[i] == v[1] and ans[(i + 1)%ans.size()] == v[0]){
      bg = i + 1;
      break;
    }
}
```

4.9 Fast Kuhn

```
const int N = 1e5+5;

int x, marcB[N], matchB[N], matchA[N], ans, n, m, p;
vector<int> adj[N];

bool dfs(int v){
    for(int i = 0; i < adj[v].size(); i++){
        int viz = adj[v][i];
        if(marcB[viz] == 1 ) continue;
        marcB[viz] = 1;

        if((matchB[viz] == -1) || dfs(matchB[viz])){
            matchB[viz] = v;
            matchA[v] = viz;
            return true;
        }

    }
    return false;
}

int main(){
    //...
    for(int i = 0; i<=n; i++) matchA[i] = -1;
    for(int j = 0; j<=m; j++) matchB[j] = -1;

    bool aux = true;
    while(aux){
        for(int j=1; j<=m; j++) marcB[j] = 0;
        aux = false;
        for(int i=1; i<=n; i++){
            if(matchA[i] != -1) continue;
            if(dfs(i)){
                ans++;
                aux = true;
            }
        }
    }
    //...
}
```

4.10 Floyd Warshall

```
/***********************************************************************************
* FLOYD-WARSHALL ALGORITHM (SHORTEST PATH TO ANY VERTEX)                           *
* Time complexity: O(V^3)                                                          *
* Usage: dist[from][to]                                                            *
* Notation: m:           number of edges                                           *
*           n:           number of vertices                                        *
*           (a, b, w):   edge between a and b with weight w                        *
***********************************************************************************/

int adj[N][N]; // no-edge = INF

for (int k = 0; k < n; ++k)
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)
      adj[i][j] = min(adj[i][j], adj[i][k]+adj[k][j]);
```

4.11 Hungarian

```
// Hungarian - O(m*n^2)
// Assignment Problem

int n, m;
int pu[N], pv[N], cost[N][M];
int pairV[N], way[M], minv[M], used[M];

void hungarian() {
  for(int i = 1, j0 = 0; i <= n; i++) {
    pairV[0] = i;
    memset(minv, 63, sizeof minv);
    memset(used, 0, sizeof used);
    do {
      used[j0] = 1;
      int i0 = pairV[j0], delta = INF, j1;
      for(int j = 1; j <= m; j++) {
        if(used[j]) continue;
        int cur = cost[i0][j] - pu[i0] - pv[j];
        if(cur < minv[j]) minv[j] = cur, way[j] = j0;
        if(minv[j] < delta) delta = minv[j], j1 = j;
      }

      for(int j = 0; j <= m; j++) {
        if(used[j]) pu[pairV[j]] += delta, pv[j] -= delta;
        else minv[j] -= delta;
      }
      j0 = j1;
    } while(pairV[j0]);

    do {
      int j1 = way[j0];
      pairV[j0] = pairV[j1];
      j0 = j1;
    } while(j0);
  }
}

// in main
// for(int j = 1; j <= m; j++)
//  if(pairV[j]) ans += cost[pairV[j]][j];
//
```

4.12 Hungarian Navarro

```
// Hungarian - O(n^2 * m)
template<bool is_max = false, class T = int, bool is_zero_indexed = false>
struct Hungarian {
  bool swap_coord = false;
  int lines, cols;
  T ans;

  vector<int> pairV, way;
  vector<bool> used;
  vector<T> pu, pv, minv;
  vector<vector<T>> cost;

  Hungarian(int _n, int _m) {
    if (_n > _m) {
      swap(_n, _m);
      swap_coord = true;
    }

    lines = _n + 1, cols = _m + 1;

    clear();
    cost.resize(lines);
    for (auto& line : cost) line.assign(cols, 0);
  }

  void clear() {
    pairV.assign(cols, 0);
    way.assign(cols, 0);
    pv.assign(cols, 0);
    pu.assign(lines, 0);
  }

  void update(int i, int j, T val) {
    if (is_zero_indexed) i++, j++;
    if (is_max) val = -val;
    if (swap_coord) swap(i, j);

    assert(i < lines);
    assert(j < cols);
```

```
      cost[i][j] = val;
    }

    T run() {
      T _INF = numeric_limits<T>::max();
      for (int i = 1, j0 = 0; i < lines; i++) {
        pairV[0] = i;
        minv.assign(cols, _INF);
        used.assign(cols, 0);
        do {
          used[j0] = 1;
          int i0 = pairV[j0], j1;
          T delta = _INF;
          for (int j = 1; j < cols; j++) {
            if (used[j]) continue;
            T cur = cost[i0][j] - pu[i0] - pv[j];
            if (cur < minv[j]) minv[j] = cur, way[j] = j0;
            if (minv[j] < delta) delta = minv[j], j1 = j;
          }

          for (int j = 0; j < cols; j++) {
            if (used[j]) pu[pairV[j]] += delta, pv[j] -= delta;
            else minv[j] -= delta;
          }
          j0 = j1;
        } while (pairV[j0]);

        do {
          int j1 = way[j0];
          pairV[j0] = pairV[j1];
          j0 = j1;
        } while (j0);
      }

      ans = 0;
      for (int j = 1; j < cols; j++) if (pairV[j]) ans += cost[pairV[j]][j];

      if (is_max) ans = -ans;
      if (is_zero_indexed) {
        for (int j = 0; j + 1 < cols; j++) pairV[j] = pairV[j + 1], pairV[j]--;
        pairV[cols - 1] = -1;
      }
      if (swap_coord) {
        vector<int> pairV_sub(lines, 0);
        for (int j = 0; j < cols; j++) if (pairV[j] >= 0) pairV_sub[pairV[j]] = j;
        swap(pairV, pairV_sub);
      }

      return ans;
    }
  };

  template <bool is_max = false, bool is_zero_indexed = false>
  struct HungarianMult : public Hungarian<is_max, long double, is_zero_indexed> {
    using super = Hungarian<is_max, long double, is_zero_indexed>;

    HungarianMult(int _n, int _m) : super(_n, _m) {}

    void update(int i, int j, long double x) {
      super::update(i, j, log2(x));
    }
  };
```

4.13 Toposort

```
/**********************************************************************************
* KAHN’S ALGORITHM (TOPOLOGICAL SORTING)                                          *
                                                               *
* Time complexity: O(V+E)                                                         *
* Notation: adj[i]:  adjacency matrix for node i                                  *
*           n:       number of vertices                                           *
*           e:       number of edges                                              *
*           a, b:    edge between a and b                                         *
*           inc:     number of incoming arcs/edges                                *
*           q:       queue with the independent vertices                          *
*           tsort:   final topo sort, i.e. possible order to traverse graph       *
**********************************************************************************/


vector <int> adj[N];
int inc[N]; // number of incoming arcs/edges

// undirected graph: inc[v] <= 1
// directed graph:   inc[v] == 0
queue<int> q;
for (int i = 1; i <= n; ++i) if (inc[i] <= 1) q.push(i);

while (!q.empty()) {
  int u = q.front(); q.pop();
  for (int v : adj[u])
    if (inc[v] > 1 and --inc[v] <= 1)
      q.push(v);
}
```

4.14 Strongly Connected Components

```
// Kosaraju - SCC O(V+E)
// For undirected graph uncomment lines below

vi adj[N], adjt[N];
int n, ordn, cnt, vis[N], ord[N], cmp[N];
//int par[N];

void dfs(int u) {
  vis[u] = 1;
  for (auto v : adj[u]) if (!vis[v]) dfs(v);
  // for (auto v : adj[u]) if(!vis[v]) par[v] = u, dfs(v);
  ord[ordn++] = u;
}

void dfst(int u) {
  cmp[u] = cnt, vis[u] = 0;
  for (auto v : adjt[u]) if (vis[v]) dfst(v);
  // for (auto v : adj[u]) if(vis[v] and u != par[v]) dfst(v);
}

// in main
for (int i = 1; i <= n; ++i) if (!vis[i]) dfs(i);
for (int i = ordn-1; i >= 0; --i) if (vis[ord[i]]) cnt++, dfst(ord[i]);
```

4.15 MST (Kruskal)

```
/**********************************************************************************
* KRUSKAL’S ALGORITHM (MINIMAL SPANNING TREE - INCREASING EDGE SIZE)              *
* Time complexity: O(ElogE)                                                       *
* Usage: cost, sz[find(node)]                                                     *
* Notation: cost: sum of all edges which belong to such MST                       *
*           sz:   vector of subsets sizes, i.e. size of the subset a node is in   *
**********************************************************************************/

// + Union-find

int cost = 0;
vector <pair<int, pair<int, int>>> edges; //mp(dist, mp(node1, node2))

int main () {
    // ...
    sort(edges.begin(), edges.end());
    for (auto e : edges)
        if (find(e.nd.st) != find(e.nd.nd))
            unite(e.nd.st, e.nd.nd), cost += e.st;

    return 0;
}
```

4.16 Max Bipartite Cardinality Matching (Kuhn)

```
/**********************************************************************************
* KUHN’S ALGORITHM (FIND GREATEST NUMBER OF MATCHINGS - BIPARTITE GRAPH)          *
* Time complexity: O(VE)                                                          *
* Notation: ans:      number of matchings                                         *
*           b[j]:     matching edge b[j] <-> j                                    *
*           adj[i]:   adjacency list for node i                                   *
*           vis:      visited nodes                                               *
*           x:        counter to help reuse vis list                              *
**********************************************************************************/

// TIP: If too slow, shuffle nodes and try again.
```

```
int x, vis[N], b[N], ans;

bool match(int u) {
  if (vis[u] == x) return 0;
  vis[u] = x;
  for (int v : adj[u])
    if (!b[v] or match(b[v])) return b[v]=u;
  return 0;
}

for (int i = 1; i <= n; ++i) ++x, ans += match(i);

// Maximum Independent Set on bipartite graph
MIS + MCBM = V

// Minimum Vertex Cover on bipartite graph
MVC = MCBM
```

4.17 Lowest Common Ancestor

```
// Lowest Common Ancestor <O(nlogn), O(logn)>
const int N = 1e6, M = 25;
int anc[M][N], h[N], rt;

// TODO: Calculate h[u] and set anc[0][u] = parent of node u for each u

// build (sparse table)
anc[0][rt] = rt; // set parent of the root to itself
for (int i = 1; i < M; ++i)
  for (int j = 1; j <= n; ++j)
    anc[i][j] = anc[i-1][anc[i-1][j]];

// query
int lca(int u, int v) {
  if (h[u] < h[v]) swap(u, v);
  for (int i = M-1; i >= 0; --i) if (h[u]-(1<<i) >= h[v])
    u = anc[i][u];

  if (u == v) return u;

  for (int i = M-1; i >= 0; --i) if (anc[i][u] != anc[i][v])
    u = anc[i][u], v = anc[i][v];
  return anc[0][u];
}
```

4.18 Max Weight on Path

```
// Using LCA to find max edge weight between (u, v)

const int N = 1e5+5;  // Max number of vertices
const int K = 20;     // Each 1e3 requires ~ 10 K
const int M = K+5;
int n;                // Number of vertices
vector <pii> adj[N];
int vis[N], h[N], anc[N][M], mx[N][M];

void dfs (int u) {
    vis[u] = 1;
    for (auto p : adj[u]) {
        int v = p.st;
        int w = p.nd;
        if (!vis[v]) {
            h[v] = h[u]+1;
            anc[v][0] = u;
            mx[v][0] = w;
            dfs(v);
        }
    }
}

void build () {
    // cl(mn, 63) -- Don't forget to initialize with INF if min edge!
    anc[1][0] = 1;
    dfs(1);
    for (int j = 1; j <= K; j++) for (int i = 1; i <= n; i++) {
        anc[i][j] = anc[anc[i][j-1]][j-1];
        mx[i][j] = max(mx[i][j-1], mx[anc[i][j-1]][j-1]);
    }
}
```

```
int mxedge (int u, int v) {
    int ans = 0;

    if (h[u] < h[v]) swap(u, v);
    for (int j = K; j >= 0; j--) if (h[anc[u][j]] >= h[v]) {
        ans = max(ans, mx[u][j]);
        u = anc[u][j];
    }
    if (u == v) return ans;
    for (int j = K; j >= 0; j--) if (anc[u][j] != anc[v][j]) {
        ans = max(ans, mx[u][j]);
        ans = max(ans, mx[v][j]);
        u = anc[u][j];
        v = anc[v][j];
    }
    return max({ans, mx[u][0], mx[v][0]});
}
```

4.19 Min Cost Max Flow

```
// USE INF = 1e9!

/**********************************************************************************
* MIN COST MAX FLOW (MINIMUM COST TO ACHIEVE MAXIMUM FLOW)                        *
* Description: Given a graph which represents a flow network where every edge has *
* a capacity and a cost per unit, find the minimum cost to establish the maximum  *
* possible flow from s to t.                                                      *
* Note: When adding edge (a, b), it is a directed edge!                           *
* Usage: min_cost_max_flow()                                                      *
*        add_edge(from, to, cost, capacity)                                       *
* Notation: flw: max flow                                                         *
*           cst: min cost to achieve flw                                          *
* Testcase:                                                                       *
* add_edge(src, 1, 0, 1);   add_edge(1, snk, 0, 1);   add_edge(2, 3, 1, INF);     *
* add_edge(src, 2, 0, 1);   add_edge(2, snk, 0, 1);   add_edge(3, 4, 1, INF);     *
* add_edge(src, 2, 0, 1);   add_edge(3, snk, 0, 1);                               *
* add_edge(src, 2, 0, 1);   add_edge(4, snk, 0, 1);   => flw = 4, cst = 3         *
**********************************************************************************/

// w: weight or cost, c : capacity
struct edge {int v, f, w, c; };

int n, flw_lmt=INF, src, snk, flw, cst, p[N], d[N], et[N];
vector<edge> e;
vector<int> g[N];

void add_edge(int u, int v, int w, int c) {
  int k = e.size();
  g[u].push_back(k);
  g[v].push_back(k+1);
  e.push_back({ v, 0,  w, c });
  e.push_back({ u, 0, -w, 0 });
}

void clear() {
  flw_lmt = INF;
  for(int i=0; i<=n; ++i) g[i].clear();
  e.clear();
}

void min_cost_max_flow() {
  flw = 0, cst = 0;
  while (flw < flw_lmt) {
    memset(et, 0, (n+1) * sizeof(int));
    memset(d, 63, (n+1) * sizeof(int));
    deque<int> q;
    q.push_back(src), d[src] = 0;

    while (!q.empty()) {
      int u = q.front(); q.pop_front();
      et[u] = 2;

      for(int i : g[u]) {
        edge &dir = e[i];
        int v = dir.v;
        if (dir.f < dir.c and d[u] + dir.w < d[v]) {
          d[v] = d[u] + dir.w;
          if (et[v] == 0) q.push_back(v);
          else if (et[v] == 2) q.push_front(v);
          et[v] = 1;
          p[v] = i;
        }
      }
    }
    if (d[snk] > INF) break;

    int inc = flw_lmt - flw;
    for (int u=snk; u != src; u = e[p[u]^1].v) {
      edge &dir = e[p[u]];
      inc = min(inc, dir.c - dir.f);
    }

    for (int u=snk; u != src; u = e[p[u]^1].v) {
      edge &dir = e[p[u]], &rev = e[p[u]^1];
      dir.f += inc;
      rev.f -= inc;
      cst += inc * dir.w;
    }

    if (!inc) break;
    flw += inc;
  }
}
```

4.20 MST (Prim)

```
// Prim - MST O(ElogE)
vi adj[N], adjw[N];
int vis[N];

priority_queue<pii> pq;
pq.push(mp(0, 0));

while (!pq.empty()) {
  int u = pq.top().nd;
  pq.pop();
  if (vis[u]) continue;
  vis[u]=1;
  for (int i = 0; i < adj[u].size(); ++i) {
    int v = adj[u][i];
    int w = adjw[u][i];
    if (!vis[v]) pq.push(mp(-w, v));
  }
}
```

4.21 Shortest Path (SPFA)

```
// Shortest Path Faster Algoritm O(VE)
int dist[N], inq[N];

cl(dist,63);
queue<int> q;
q.push(0); dist[0] = 0; inq[0] = 1;

while (!q.empty()) {
  int u = q.front(); q.pop(); inq[u]=0;
  for (int i = 0; i < adj[u].size(); ++i) {
    int v = adj[u][i], w = adjw[u][i];
    if (dist[v] > dist[u] + w) {
      dist[v] = dist[u] + w;
      if (!inq[v]) q.push(v), inq[v] = 1;
    }
  }
}
```

4.22 Stoer Wagner (Stanford)

```
    // a is a N*N matrix storing the graph we use; a[i][j]=a[j][i]
    memset(use,0,sizeof(use));
    ans=maxlongint;
    for (int i=1;i<N;i++)
    {
        memcpy(visit,use,505*sizeof(int));
        memset(reach,0,sizeof(reach));
        memset(last,0,sizeof(last));
        t=0;
        for (int j=1;j<=N;j++)
            if (use[j]==0) {t=j;break;}
        for (int j=1;j<=N;j++)
            if (use[j]==0) reach[j]=a[t][j],last[j]=t;
        visit[t]=1;
        for (int j=1;j<=N-i;j++)
        {
            maxc=maxk=0;
            for (int k=1;k<=N;k++)
                if ((visit[k]==0)&&(reach[k]>maxc)) maxc=reach[k],maxk=k;
            c2=maxk,visit[maxk]=1;
            for (int k=1;k<=N;k++)
                if (visit[k]==0) reach[k]+=a[maxk][k],last[k]=maxk;
        }
        c1=last[c2];
        sum=0;
        for (int j=1;j<=N;j++)
            if (use[j]==0) sum+=a[j][c2];
        ans=min(ans,sum);
        use[c2]=1;
        for (int j=1;j<=N;j++)
            if ((c1!=j)&&(use[j]==0)) {a[j][c1]+=a[j][c2];a[c1][j]=a[j][c1];}
    }
```

4.23 Tarjan

```
// Tarjan for SCC and Edge Biconnected Componentes - O(n + m)
vector<int> adj[N];
stack<int> st;
bool inSt[N];

int id[N], cmp[N];
int cnt, cmpCnt;

void clear(){
  memset(id, 0, sizeof id);
  cnt = cmpCnt = 0;
}

int tarjan(int n){
  int low;
  id[n] = low = ++cnt;
  st.push(n), inSt[n] = true;

  for(auto x : adj[n]){
    if(id[x] and inSt[x]) low = min(low, id[x]);
    else if(!id[x]) {
      int lowx = tarjan(x);
      if(inSt[x])
        low = min(low, lowx);
    }
  }

  if(low == id[n]){
    while(st.size()){
      int x = st.top();
      inSt[x] = false;
      cmp[x] = cmpCnt;

      st.pop();
      if(x == n) break;
    }
    cmpCnt++;
  }
  return low;
}
```

4.24 Zero One BFS

```
// 0-1 BFS - O(V+E)

const int N = 1e5 + 5;

int dist[N];
vector<pii> adj[N];
deque<pii> dq;

void zero_one_bfs (int x){
    cl(dist, 63);
    dist[x] = 0;
    dq.push_back({x, 0});
    while(!dq.empty()){
        int u = dq.front().st;
        int ud = dq.front().nd;
        dq.pop_front();
        if(dist[u] < ud) continue;
        for(auto x : adj[u]){
            int v = x.st;
            int w = x.nd;
            if(dist[u] + w < dist[v]){
                dist[v] = dist[u] + w;
                if(w) dq.push_back({v, dist[v]});
                else dq.push_front({v, dist[v]});
            }
        }
    }
}
```

5 Strings

5.1 Aho-Corasick

```
// Aho-Corasick

// Build: O(sum size of patterns)
// Find total number of matches: O(size of input string)
// Find number of matches for each pattern: O(num of patterns + size of input string)

// ids start from 0 by default!

template <int ALPHA_SIZE = 62>
struct Aho {
  struct Node {
    int p, char_p, link = -1, str_idx = -1, nxt[ALPHA_SIZE];
    bool has_end = false;
    Node(int _p = -1, int _char_p = -1) : p(_p), char_p(_char_p) {
      fill(nxt, nxt + ALPHA_SIZE, -1);
    }
  };

  vector<Node> nodes = { Node() };
  int ans, cnt = 0;
  bool build_done = false;
  vector<pair<int, int>> rep;
  vector<int> ord, occur, occur_aux;

  // change this if different alphabet
  int remap(char c) {
    if (islower(c)) return c - 'a';
    if (isalpha(c)) return c - 'A' + 26;
    return c - '0' + 52;
  }

  void add(string &p, int id = -1) {
    int u = 0;
    if (id == -1) id = cnt++;

    for (char ch : p) {
      int c = remap(ch);
      if (nodes[u].nxt[c] == -1) {
        nodes[u].nxt[c] = (int)nodes.size();
        nodes.push_back(Node(u, c));
      }

      u = nodes[u].nxt[c];
    }

    if (nodes[u].str_idx != -1) rep.push_back({ id, nodes[u].str_idx });
    else nodes[u].str_idx = id;
    nodes[u].has_end = true;
  }

  void build() {
    build_done = true;
    queue<int> q;

    for (int i = 0; i < ALPHA_SIZE; i++) {
      if (nodes[0].nxt[i] != -1) q.push(nodes[0].nxt[i]);
      else nodes[0].nxt[i] = 0;
    }

    while(q.size()) {
      int u = q.front();
      ord.push_back(u);
      q.pop();

      int j = nodes[nodes[u].p].link;
      if (j == -1) nodes[u].link = 0;
      else nodes[u].link = nodes[j].nxt[nodes[u].char_p];

      nodes[u].has_end |= nodes[nodes[u].link].has_end;

      for (int i = 0; i < ALPHA_SIZE; i++) {
        if (nodes[u].nxt[i] != -1) q.push(nodes[u].nxt[i]);
        else nodes[u].nxt[i] = nodes[nodes[u].link].nxt[i];
      }
    }
  }

  int match(string &s) {
    if (!cnt) return 0;
    if (!build_done) build();

    ans = 0;
    occur = vector<int>(cnt);
    occur_aux = vector<int>(nodes.size());

    int u = 0;
    for (char ch : s) {
      int c = remap(ch);
      u = nodes[u].nxt[c];
      occur_aux[u]++;
    }

    for (int i = (int)ord.size() - 1; i >= 0; i--) {
      int v = ord[i];
      int fv = nodes[v].link;
      occur_aux[fv] += occur_aux[v];
      if (nodes[v].str_idx != -1) {
        occur[nodes[v].str_idx] = occur_aux[v];
        ans += occur_aux[v];
      }
    }

    for (pair<int, int> x : rep) occur[x.first] = occur[x.second];
    return ans;
  }
};
```

5.2 Aho-Corasick (emaxx)

```
// Aho Corasick - <O(sum(m)), O(n + #matches)>
// Multiple string matching

#include <bits/stdc++.h>
using namespace std;

int remap(char c) {
  if (islower(c)) return c - 'a';
  return c - 'A' + 26;
}

const int K = 52;

struct Aho {
  struct Node {
    int nxt[K];
    int par = -1;
    int link = -1;
    int go[K];
    bitset<1005> ids;
    char pch;

    Node(int p = -1, char ch = '$') : par{ p }, pch{ ch } {
      fill(begin(nxt), end(nxt), -1);
      fill(begin(go), end(go), -1);
    }
  };

  vector<Node> nodes;

  Aho() : nodes (1) {}

  void add_string(const string& s, int id) {
    int u = 0;
    for (char ch : s) {
      int c = remap(ch);
      if (nodes[u].nxt[c] == -1) {
        nodes[u].nxt[c] = nodes.size();
        nodes.emplace_back(u, ch);
      }

      u = nodes[u].nxt[c];
    }

    nodes[u].ids.set(id);
  }

  int get_link(int u) {
    if (nodes[u].link == -1) {
      if (u == 0 or nodes[u].par == 0) nodes[u].link = 0;
      else nodes[u].link = go(get_link(nodes[u].par), nodes[u].pch);
    }
    return nodes[u].link;
  }

  int go(int u, char ch) {
    int c = remap(ch);
    if (nodes[u].go[c] == -1) {
      if (nodes[u].nxt[c] != -1) nodes[u].go[c] = nodes[u].nxt[c];
      else nodes[u].go[c] = (u == 0) ? 0 : go(get_link(u), ch);
      nodes[u].ids |= nodes[nodes[u].go[c]].ids;
    }
    return nodes[u].go[c];
  }

  bitset<1005> run(const string& s) {
    bitset<1005> bs;
    int u = 0;
    for (char ch : s) {
      int c = remap(ch);
      if (go(u, ch) == -1) assert(0);
      bs |= nodes[u].ids;
      u = nodes[u].nxt[c];
      if (u == -1) u = 0;
    }
    bs |= nodes[u].ids;
    return bs;
  }
};
```

5.3 Booths Algorithm

```
// Booth's Algorithm - Find the lexicographically least rotation of a string in O(n)

string least_rotation(string s) {
  s += s;
  vector<int> f((int)s.size(), -1);
  int k = 0;
  for (int j = 1; j < (int)s.size(); j++) {
    int i = f[j - k - 1];
    while (i != -1 and s[j] != s[k + i + 1]) {
      if (s[j] < s[k + i + 1]) k = j - i - 1;
      i = f[i];
    }

    if (s[j] != s[k + i + 1]) {
      if (s[j] < s[k]) k = j;
      f[j - k] = -1;
    } else f[j - k] = i + 1;
  }

  return s.substr(k, (int)s.size() / 2);
}
```

5.4 Knuth-Morris-Pratt (Automaton)

```
// KMP Automaton - <O(26*pattern), O(text)>

// max size pattern
const int N = 1e5 + 5;

int cnt, nxt[N+1][26];

void prekmp(string &p) {
  nxt[0][p[0] - 'a'] = 1;
  for(int i = 1, j = 0; i <= p.size(); i++) {
    for(int c = 0; c < 26; c++) nxt[i][c] = nxt[j][c];
    if(i == p.size()) continue;
    nxt[i][p[i] - 'a'] = i+1;
    j = nxt[j][p[i] - 'a'];
  }
}

void kmp(string &s, string &p) {
  for(int i = 0, j = 0; i < s.size(); i++) {
    j = nxt[j][s[i] - 'a'];
    if(j == p.size()) cnt++; //match i - j + 1
  }
}
```

5.5 Knuth-Morris-Pratt

```
// Knuth-Morris-Pratt - String Matching O(n+m)
char s[N], p[N];
int b[N], n, m; // n = strlen(s), m = strlen(p);

void kmppre() {
  b[0] = -1;
  for (int i = 0, j = -1; i < m; b[++i] = ++j)
    while (j >= 0 and p[i] != p[j])
      j = b[j];
}

void kmp() {
  for (int i = 0, j = 0; i < n;) {
    while (j >= 0 and s[i] != p[j]) j=b[j];
    i++, j++;
    if (j == m) {
      // match position i-j
      j = b[j];
    }
  }
}
```

5.6 Manacher

```
// Manacher (Longest Palindromic String) - O(n)
int lps[2*N+5];
char s[N];

int manacher() {
  int n = strlen(s);

  string p (2*n+3, '#');
  p[0] = '^';
  for (int i = 0; i < n; i++) p[2*(i+1)] = s[i];
  p[2*n+2] = '$';

  int k = 0, r = 0, m = 0;
  int l = p.length();
  for (int i = 1; i < l; i++) {
    int o = 2*k - i;
    lps[i] = (r > i) ? min(r-i, lps[o]) : 0;
    while (p[i + 1 + lps[i]] == p[i - 1 - lps[i]]) lps[i]++;
    if (i + lps[i] > r) k = i, r = i + lps[i];
    m = max(m, lps[i]);
  }
  return m;
}
```

5.7 Rabin-Karp

```
// Rabin-Karp - String Matching + Hashing O(n+m)
const int B = 31;
char s[N], p[N];
int n, m; // n = strlen(s), m = strlen(p)

void rabin() {
  if (n<m) return;

  ull hp = 0, hs = 0, E = 1;
  for (int i = 0; i < m; ++i)
    hp = ((hp*B)%MOD + p[i])%MOD,
    hs = ((hs*B)%MOD + s[i])%MOD,
    E = (E*B)%MOD;

  if (hs == hp) { /* matching position 0 */ }
  for (int i = m; i < n; ++i) {
    hs = ((hs*B)%MOD + s[i])%MOD;
    hhs = (hs - s[i-m]*E%MOD + MOD)%MOD;
    if (hs == hp) { /* matching position i-m+1 */ }
  }
}
```

5.8 Recursive-String Matching

```
void p_f(char *s, int *pi) {
    int n = strlen(s);
    pi[0]=pi[1]=0;
    for(int i = 2; i <= n; i++) {
        pi[i] = pi[i-1];
        while(pi[i]>0 and s[pi[i]]!=s[i])
            pi[i]=pi[pi[i]];
        if(s[pi[i]]==s[i-1])
            pi[i]++;
    }
}

int main() {
    //...
    //Initialize prefix function
    char p[N]; //Pattern
    int len = strlen(p); //Pattern size
    int pi[N]; //Prefix function
    p_f(p, pi);

    // Create KMP automaton
    int A[N][128]; //A[i][j]: from state i (size of largest suffix of text which is prefix of pattern),
          append character j -> new state A[i][j]
    for( char c : ALPHABET )
        A[0][c] = (p[0] == c);
    for( int i = 1; p[i]; i++ ) {
        for( char c : ALPHABET ) {
            if(c==p[i])
                A[i][c]=i+1; //match
            else
                A[i][c]=A[pi[i]][c]; //try second largest suffix
        }
    }

    //Create KMP "string appending" automaton
    // g_n = g_(n-1) + char(n) + g_(n-1)
    // g_0 = "", g_1 = "a", g_2 = "aba", g_3 = "abacaba", ...
    int F[M][N]; //F[i][j]: from state j (size of largest suffix of text which is prefix of pattern), append
          string g_i -> new state F[i][j]
    for(int i = 0; i < m; i++) {
        for(int j = 0; j <= len; j++) {
            if(i==0)
                F[i][j] = j; //append empty string
            else {
                int x = F[i-1][j]; //append g_(i-1)
                x = A[x][j]; //append character j
                x = F[i-1][x]; //append g_(i-1)
                F[i][j] = x;
            }
        }
    }

    //Create number of matches matrix
    int K[M][N]; //K[i][j]: from state j (size of largest suffix of text which is prefix of pattern), append
          string g_i -> K[i][j] matches
    for(int i = 0; i < m; i++) {
        for(int j = 0; j <= len; j++) {
            if(i==0)
                K[i][j] = (j==len); //append empty string
            else {
                int x = F[i-1][j]; //append g_(i-1)
                x = A[x][j]; //append character j

                K[i][j] = K[i-1][j] /*append g_(i-1)*/ + (x==len) /*append character j*/ + K[i-1][x]; /*
                      append g_(i-1)*/
            }
        }
    }
    //number of matches in g_k
    int answer = K[0][k];
    //...
}
```

5.9 String Hashing

```
// String Hashing
// Rabin Karp - O(n + m)

// max size txt + 1
const int N = 1e6 + 5;

// lowercase letters p = 31 (remember to do s[i] - 'a' + 1)
// uppercase and lowercase letters p = 53 (remember to do s[i] - 'a' + 1)
// any character p = 313

const int MOD = 1e9+9;
ull h[N], p[N];
ull pr = 313;

int cnt;


void build(string &s) {
  p[0] = 1, p[1] = pr;
  for(int i = 1; i <= s.size(); i++) {
    h[i] = ((p[1]*h[i-1]) % MOD + s[i-1]) % MOD;
    p[i] = (p[1]*p[i-1]) % MOD;
  }
}

// 1-indexed
ull fhash(int l, int r) {
  return (h[r] - ((h[l-1]*p[r-l+1]) % MOD) + MOD) % MOD;
}

ull shash(string &pt) {
  ull h = 0;
  for(int i = 0; i < pt.size(); i++)
    h = ((h*pr) % MOD + pt[i]) % MOD;
  return h;
}

void rabin_karp(string &s, string &pt) {
  build(s);
  ull hp = shash(pt);
  for(int i = 0, m = pt.size(); i + m <= s.size(); i++) {
    if(fhash(i+1, i+m) == hp) {
      // match at i
      cnt++;
    }
  }
}
```

5.10 String Multihashing

```
// String Hashing
// Rabin Karp - O(n + m)
template <int N = 3>
struct Hash {
  int hs[N];
  static vector<int> mods;

  static int add(int a, int b, int mod) { return a >= mod - b ? a + b - mod : a + b; }
  static int sub(int a, int b, int mod) { return a - b < 0 ? a - b + mod : a - b; }
  static int mul(int a, int b, int mod) { return 1ll * a * b % mod; }

  Hash(int x = 0) { fill(hs, hs + N, x); }

  bool operator<(const Hash& b) const {
    for (int i = 0; i < N; i++) {
      if (hs[i] < b.hs[i]) return true;
      if (hs[i] > b.hs[i]) return false;
    }
    return false;
  }

  Hash operator+(const Hash& b) const {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = add(hs[i], b.hs[i], mods[i]);
    return ans;
  }

  Hash operator-(const Hash& b) const {
    Hash ans;
```

```cpp
    for (int i = 0; i < N; i++) ans.hs[i] = sub(hs[i], b.hs[i], mods[i]);
    return ans;
  }

  Hash operator*(const Hash& b) const {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = mul(hs[i], b.hs[i], mods[i]);
    return ans;
  }

  Hash operator+(int b) const {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = add(hs[i], b, mods[i]);
    return ans;
  }

  Hash operator*(int b) const {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = mul(hs[i], b, mods[i]);
    return ans;
  }

  friend Hash operator*(int a, const Hash& b) {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = mul(b.hs[i], a, b.mods[i]);
    return ans;
  }

  friend ostream& operator<<(ostream& os, const Hash& b) {
    for (int i = 0; i < N; i++) os << b.hs[i] << " \n"[i == N - 1];
    return os;
  }
};

template <int N> vector<int> Hash<N>::mods = { (int) 1e9 + 9, (int) 1e9 + 33, (int) 1e9 + 87 };

// In case you need to generate the MODs, uncomment this:
// Obs: you may need this on your template
// mt19937_64 llrand((int) chrono::steady_clock::now().time_since_epoch().count());
// In main: gen<>();
/*
template <int N> vector<int> Hash<N>::mods;
template<int N = 3>
void gen() {
  while (Hash<N>::mods.size() < N) {
    int mod;
    bool is_prime;
    do {
      mod = (int) 1e8 + (int) (llrand() % (int) 9e8);
      is_prime = true;
      for (int i = 2; i * i <= mod; i++) {
        if (mod % i == 0) {
          is_prime = false;
          break;
        }
      }
    } while (!is_prime);
    Hash<N>::mods.push_back(mod);
  }
}
*/

template <int N = 3>
struct PolyHash {
  vector<Hash<N>> h, p;

  PolyHash(string& s, int pr = 313) {
    int sz = (int)s.size();
    p.resize(sz + 1);
    h.resize(sz + 1);

    p[0] = 1, h[0] = s[0];
    for (int i = 1; i < sz; i++) {
      h[i] = pr * h[i - 1] + s[i];
      p[i] = pr * p[i - 1];
    }
  }

  Hash<N> fhash(int l, int r) {
    if (!l) return h[r];
    return h[r] - h[l - 1] * p[r - l + 1];
  }

  static Hash<N> shash(string& s, int pr = 313) {
    Hash<N> ans;
    for (int i = 0; i < (int)s.size(); i++) ans = pr * ans + s[i];
    return ans;
  }

  friend int rabin_karp(string& s, string& pt) {
    PolyHash hs = PolyHash(s);
    Hash<N> hp = hs.shash(pt);
    int cnt = 0;
    for (int i = 0, m = (int)pt.size(); i + m <= (int)s.size(); i++) {
      if (hs.fhash(i, i + m - 1) == hp) {
        // match at i
        cnt++;
      }
    }

    return cnt;
  }
};
```

5.11 Suffix Array

```cpp
// Suffix Array O(nlogn)
// s.push('$');
vector<int> suffix_array(string &s){
  int n = s.size(), alph = 256;
  vector<int> cnt(max(n, alph)), p(n), c(n);

  for(auto c : s) cnt[c]++;
  for(int i = 1; i < alph; i++) cnt[i] += cnt[i - 1];
  for(int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
  for(int i = 1; i < n; i++)
    c[p[i]] = c[p[i - 1]] + (s[p[i]] != s[p[i - 1]]);

  vector<int> c2(n), p2(n);

  for(int k = 0; (1 << k) < n; k++){
    int classes = c[p[n - 1]] + 1;
    fill(cnt.begin(), cnt.begin() + classes, 0);

    for(int i = 0; i < n; i++) p2[i] = (p[i] - (1 << k) + n)%n;
    for(int i = 0; i < n; i++) cnt[c[i]]++;
    for(int i = 1; i < classes; i++) cnt[i] += cnt[i - 1];
    for(int i = n - 1; i >= 0; i--) p[--cnt[c[p2[i]]]] = p2[i];

    c2[p[0]] = 0;
    for(int i = 1; i < n; i++){
      pair<int, int> b1 = {c[p[i]], c[(p[i] + (1 << k))%n]};
      pair<int, int> b2 = {c[p[i - 1]], c[(p[i - 1] + (1 << k))%n]};
      c2[p[i]] = c2[p[i - 1]] + (b1 != b2);
    }

    c.swap(c2);
  }
  return p;
}

// Longest Common Prefix with SA O(n)
vector<int> lcp(string &s, vector<int> &p){
  int n = s.size();
  vector<int> ans(n - 1), pi(n);
  for(int i = 0; i < n; i++) pi[p[i]] = i;

  int lst = 0;
  for(int i = 0; i < n - 1; i++){
    if(pi[i] == n - 1) continue;
    while(s[i + lst] == s[p[pi[i] + 1] + lst]) lst++;

    ans[pi[i]] = lst;
    lst = max(0, lst - 1);
  }

  return ans;
}

// Longest Repeated Substring O(n)
int lrs = 0;
for (int i = 0; i < n; ++i) lrs = max(lrs, lcp[i]);

// Longest Common Substring O(n)
// m = strlen(s);
// strcat(s, "$"); strcat(s, p); strcat(s, "#");
// n = strlen(s);
int lcs = 0;
for (int i = 1; i < n; ++i) if ((sa[i] < m) != (sa[i-1] < m))
  lcs = max(lcs, lcp[i]);

// To calc LCS for multiple texts use a slide window with minqueue
// The numver of different substrings of a string is n*(n + 1)/2 - sum(lcs[i])
```

5.12 Suffix Automaton

```
// Suffix Automaton Construction - O(n)

const int N = 1e6+1, K = 26;
int sl[2*N], len[2*N], sz, last;
ll cnt[2*N];
map<int, int> adj[2*N];

void add(int c) {
  int u = sz++;
  len[u] = len[last] + 1;
  cnt[u] = 1;

  int p = last;
  while(p != -1 and !adj[p][c])
    adj[p][c] = u, p = sl[p];

  if (p == -1) sl[u] = 0;
  else {
    int q = adj[p][c];
    if (len[p] + 1 == len[q]) sl[u] = q;
    else {
      int r = sz++;
      len[r] = len[p] + 1;
      sl[r] = sl[q];
      adj[r] = adj[q];
      while(p != -1 and adj[p][c] == q)
        adj[p][c] = r, p = sl[p];
      sl[q] = sl[u] = r;
    }
  }

  last = u;
}

void clear() {
  for(int i=0; i<=sz; ++i) adj[i].clear();
  last = 0;
  sz = 1;
  sl[0] = -1;
}

void build(char *s) {
  clear();
  for(int i=0; s[i]; ++i) add(s[i]);
}

// Pattern matching - O(|p|)
bool check(char *p) {
  int u = 0, ok = 1;
  for(int i=0; p[i]; ++i) {
    u = adj[u][p[i]];
    if (!u) ok = 0;
  }
  return ok;
}

// Substring count - O(|p|)
ll d[2*N];

void substr_cnt(int u) {
  d[u] = 1;
  for(auto p : adj[u]) {
    int v = p.second;
    if (!d[v]) substr_cnt(v);
    d[u] += d[v];
  }
}

ll substr_cnt() {
  memset(d, 0, sizeof d);
  substr_cnt(0);
  return d[0] - 1;
}

// k-th Substring - O(|s|)
// Just find the k-th path in the automaton.
// Can be done with the value d calculated in previous problem.

// Smallest cyclic shift - O(|s|)
// Build the automaton for string s + s. And adapt previous dp
// to only count paths with size |s|.


// Number of occurences - O(|p|)
vector<int> t[2*N];

void occur_count(int u) {
  for(int v : t[u]) occur_count(v), cnt[u] += cnt[v];
}

void build_tree() {
  for(int i=1; i<=sz; ++i)
    t[sl[i]].push_back(i);
  occur_count(0);
}

ll occur_count(char *p) {
  // Call build tree once per automaton
  int u = 0;
  for(int i=0; p[i]; ++i) {
    u = adj[u][p[i]];
    if (!u) break;
  }
  return !u ? 0 : cnt[u];
}

// First occurence - (|p|)
// Store the first position of occurence fp.
// Add the the code to add function:
// fp[u] = len[u] - 1;
// fp[r] = fp[q];

// To answer a query, just output fp[u] - strlen(p) + 1
// where u is the state corresponding to string p

// All occurences - O(|p| + |ans|)
// All the occurences can reach the first occurence via suffix links.
// So every state that contains a occreunce is reacheable by the
// first occurence state in the suffix link tree. Just do a DFS in this
// tree, starting from the first occurence.
// OBS: cloned nodes will output same answer twice.


// Smallest substring not contained in the string - O(|s| * K)
// Just do a dynamic programming:
// d[u] = 1 // if d does not have 1 transition
// d[u] = 1 + min d[v] // otherwise


// LCS of 2 Strings - O(|s| + |t|)
// Build automaton of s and traverse the automaton wih string t
// mantaining the current state and the current lenght.
// When we have a transition: update state, increase lenght by one.
// If we don't update state by suffix link and the new lenght will
// should be reduced (if bigger) to the new state length.
// Answer will be the maximum length of the whole traversal.

// LCS of n Strings - O(n*|s|*K)
// Create a new string S = s_1 + d1 + ... + s_n + d_n,
// where d_i are delimiters that are unique (d_i != d_j).
// For each state use DP + bitmask to calculate if it can
// reach a d_i transition without going through other d_j.
// The answer will be the biggest len[u] that can reach all
// d_i's.
```

5.13 Suffix Tree

```
// Suffix Tree
// Build: O(|s|)
// Match: O(|p|)

template<int ALPHA_SIZE = 62>
struct SuffixTree {
  struct Node {
    int p, link = -1, l, r, nch = 0;
    vector<int> nxt;
    Node(int _l = 0, int _r = -1, int _p = -1) : p(_p), l(_l), r(_r), nxt(ALPHA_SIZE, -1) {}

    int len() { return r - l + 1; }
    int next(char ch) { return nxt[remap(ch)]; }

    // change this if different alphabet
    int remap(char c) {
      if (islower(c)) return c - 'a';
      if (isalpha(c)) return c - 'A' + 26;
      return c - '0' + 52;
    }

    void setEdge(char ch, int nx) {
```

```
    int c = remap(ch);
    if (nxt[c] != -1 and nx == -1) nch--;
    else if (nxt[c] == -1 and nx != -1) nch++;
    nxt[c] = nx;
  }
};

string s;
long long num_diff_substr = 0;
vector<Node> nodes;
queue<int> leaves;
pair<int, int> st = { 0, 0 };
int ls = 0, rs = -1, n;

int size() { return rs - ls + 1; }

SuffixTree(string &_s) {
  s = _s;
  // Add this if you want every suffix to be a node
  // s += '$';
  n = (int)s.size();
  nodes.reserve(2 * n + 1);
  nodes.push_back(Node());
  //for (int i = 0; i < n; i++) extend();
}

pair<int, int> walk(pair<int, int> _st, int l, int r) {
  int u = _st.first;
  int d = _st.second;

  while (l <= r) {
    if (d == nodes[u].len()) {
      u = nodes[u].next(s[l]), d = 0;
      if (u == -1) return { u, d };
    } else {
      if (s[nodes[u].l + d] != s[l]) return { -1, -1 };
      if (r - l + 1 + d < nodes[u].len()) return { u, r - l + 1 + d };
      l += nodes[u].len() - d;
      d = nodes[u].len();
    }
  }

  return { u, d };
}

int split(pair<int, int> _st) {
  int u = _st.first;
  int d = _st.second;

  if (d == nodes[u].len()) return u;
  if (!d) return nodes[u].p;

  Node& nu = nodes[u];
  int mid = (int)nodes.size();
  nodes.push_back(Node(nu.l, nu.l + d - 1, nu.p));
  nodes[nu.p].setEdge(s[nu.l], mid);
  nodes[mid].setEdge(s[nu.l + d], u);
  nu.p = mid;
  nu.l += d;
  return mid;
}

int getLink(int u) {
  if (nodes[u].link != -1) return nodes[u].link;
  if (nodes[u].p == -1) return 0;
  int to = getLink(nodes[u].p);
  pair<int, int> nst = { to, nodes[to].len() };
  return nodes[u].link = split(walk(nst, nodes[u].l + (nodes[u].p == 0), nodes[u].r));
}

bool match(string &p) {
  int u = 0, d = 0;
  for (char ch : p) {
    if (d == min(nodes[u].r, rs) - nodes[u].l + 1) {
      u = nodes[u].next(ch), d = 1;
      if (u == -1) return false;
    } else {
      if (ch != s[nodes[u].l + d]) return false;
      d++;
    }
  }
  return true;
}

void extend() {
  int mid;
  assert(rs != n - 1);
  rs++;
  num_diff_substr += (int)leaves.size();

    do {
      pair<int, int> nst = walk(st, rs, rs);
      if (nst.first != -1) { st = nst; return; }
      mid = split(st);
      int leaf = (int)nodes.size();
      num_diff_substr++;
      leaves.push(leaf);
      nodes.push_back(Node(rs, n - 1, mid));
      nodes[mid].setEdge(s[rs], leaf);
      int to = getLink(mid);
      st = { to, nodes[to].len() };
    } while (mid);
  }

  void pop() {
    assert(ls <= rs);
    ls++;
    int leaf = leaves.front();
    leaves.pop();
    Node* nlf = &nodes[leaf];
    while (!nlf->nch) {
      if (st.first != leaf) {
        nodes[nlf->p].setEdge(s[nlf->l], -1);
        num_diff_substr -= min(nlf->r, rs) - nlf->l + 1;
        leaf = nlf->p;
        nlf = &nodes[leaf];
      } else {
        if (st.second != min(nlf->r, rs) - nlf->l + 1) {
          int mid = split(st);
          st.first = mid;
          num_diff_substr -= min(nlf->r, rs) - nlf->l + 1;
          nodes[mid].setEdge(s[nlf->l], -1);
          *nlf = nodes[mid];
          nodes[nlf->p].setEdge(s[nlf->l], leaf);
          nodes.pop_back();
        }
        break;
      }
    }

    if (leaf and !nlf->nch) {
      leaves.push(leaf);
      int to = getLink(nlf->p);
      pair<int, int> nst = { to, nodes[to].len() };
      st = walk(nst, nlf->l + (nlf->p == 0), nlf->r);
      nlf->l = rs - nlf->len() + 1;
      nlf->r = n - 1;
    }
  }
};
```

5.14 Z Function

```
// Z-Function - O(n)

vector<int> zfunction(const string& s){
  vector<int> z (s.size());
  for (int i = 1, l = 0, r = 0, n = s.size(); i < n; i++){
    if (i <= r) z[i] = min(z[i-l], r - i + 1);
    while (i + z[i] < n and s[z[i]] == s[z[i] + i]) z[i]++;
    if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
  }
  return z;
}
```

6 Mathematics

6.1 Basics

```
// Greatest Common Divisor & Lowest Common Multiple
ll gcd(ll a, ll b) { return b ? gcd(b, a%b) : a; }
ll lcm(ll a, ll b) { return a/gcd(a, b)*b; }

// Multiply caring overflow
ll mulmod(ll a, ll b, ll m = MOD) {
  ll r=0;
  for (a %= m; b; b>>=1, a=(a*2)%m) if (b&1) r=(r+a)%m;
  return r;
}

// Another option for mulmod is using long double
ull mulmod(ull a, ull b, ull m = MOD) {
  ull q = (ld) a * (ld) b / (ld) m;
  ull r = a * b - q * m;
  return (r + m) % m;
}

// Fast exponential
ll fexp(ll a, ll b, ll m = MOD) {
  ll r=1;
  for (a %= m; b; b>>=1, a=(a*a)%m) if (b&1) r=(r*a)%m;
  return r;
}
```

6.2 Advanced

```
/* Line integral = integral(sqrt(1 + (dy/dx)^2)) dx */

/* Multiplicative Inverse over MOD for all 1..N - 1 < MOD in O(N)
 Only works for prime MOD. If all 1..MOD - 1 needed, use N = MOD */
ll inv[N];
inv[1] = 1;
for(int i = 2; i < N; ++i)
        inv[i] = MOD - (MOD / i) * inv[MOD % i] % MOD;

/* Catalan
 f(n) = sum(f(i) * f(n - i - 1)), i in [0, n - 1] = (2n)! / ((n+1)! * n!) = ...
 If you have any function f(n) (there are many) that follows this sequence (0-indexed):
 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440
 than it's the Catalan function */
ll cat[N];
cat[0] = 1;
for(int i = 1; i + 1 < N; i++) // needs inv[i + 1] till inv[N - 1]
        cat[i] = 2ll * (2ll * i - 1) * inv[i + 1] % MOD * cat[i - 1] % MOD;

/* Floor(n / i), i = [1, n], has <= 2 * sqrt(n) diff values.
 Proof: i = [1, sqrt(n)] has sqrt(n) diff values.
 For i = [sqrt(n), n] we have that 1 <= n / i <= sqrt(n)
 and thus has <= sqrt(n) diff values.
*/
/* l = first number that has floor(N / l) = x
 r = last number that has floor(N / r) = x
 N / r >= floor(N / l)
 r <= N / floor(N / l)*/
for(int l = 1, r; l <= n; l = r + 1){
        r = n / (n / l);
        // floor(n / i) has the same value for l <= i <= r
}

/* Recurrence using matriz
 h[i + 2] = a1 * h[i + 1] + a0 * h[i]
 [h[i] h[i-1]] = [h[1] h[0]] * [a1 1] ^ (i - 1)
                               [a0 0]          */

/* Fibonacci in O(log(N)) with memoization
 f(0) = f(1) = 1
 f(2*k) = f(k)^2 + f(k - 1)^2
 f(2*k + 1) = f(k)*[f(k) + 2*f(k - 1)] */

/* Wilson's Theorem Extension
 B = b1 * b2 * ... * bm (mod n) = +-1, all bi <= n such that gcd(bi, n) = 1
 if(n <= 4 or n = (odd prime)^k or n = 2 * (odd prime)^k) B = -1; for any k
 else B = 1; */

/* Stirling numbers of the second kind
 S(n, k) = Number of ways to split n numbers into k non-empty sets
 S(n, 1) = S(n, n) = 1
 S(n, k) = k * S(n - 1, k) + S(n - 1, k - 1)
 Sr(n, k) = S(n, k) with at least r numbers in each set
 Sr(n, k) = k * Sr(n - 1, k) + (n - 1) * Sr(n - r, k - 1)
                               (r - 1)
 S(n - d + 1, k - d + 1) = S(n, k) where if indexes i, j belong to the same set, then |i - j| >= d */

/* Burnside's Lemma
 |Classes| = 1 / |G| * sum(K ^ C(g)) for each g in G
 G = Different permutations possible
 C(g) = Number of cycles on the permutation g
 K = Number of states for each element

 Different ways to paint a necklace with N beads and K colors:
 G = {(1, 2, ... N), (2, 3, ... N, 1), ... (N, 1, ... N - 1)}
 gi = (i, i + 1, ... i + N), (taking mod N to get it right) i = 1 ... N
 i -> 2i -> 3i ..., Cycles in gi all have size n / gcd(i, n), so C(gi) = gcd(i, n)
 Ans = 1 / N * sum(K ^ gcd(i, n)), i = 1 ... N
 (For the brave, you can get to Ans = 1 / N * sum(euler_phi(N / d) * K ^ d), d | N) */

/* Mobius Inversion
 Sum of gcd(i, j), 1 <= i, j <= N?
 sum(k->N) k * sum(i->N) sum(j->N) [gcd(i, j) == k], i = a * k, j = b * k
 = sum(k->N) k * sum(a->N/k) sum(b->N/k) [gcd(a, b) == 1]
 = sum(k->N) k * sum(a->N/k) sum(b->N/k) sum(d->N/k) [d | a] * [d | b] * mi(d)
 = sum(k->N) k * sum(d->N/k) mi(d) * floor(N / kd)^2, l = kd, l <= N, k | l, d = l / k
 = sum(l->N) floor(N / l)^2 * sum(k|l) k * mi(l / k)
 If f(n) = sum(x|n)(g(x) * h(x)) with g(x) and h(x) multiplicative, than f(n) is multiplicative
 Hence, g(l) = sum(k|l) k * mi(l / k) is multiplicative
 = sum(l->N) floor(N / l)^2 * g(l) */
```

6.3 Discrete Log (Baby-step Giant-step)

```
// O(sqrt(m))
// Solve c * a^x = b mod(m) for integer x >= 0.
// Return the smallest x possible, or -1 if there is no solution
// If all solutions needed, solve c * a^x = b mod(m) and (a*b) * a^y = b mod(m)
// x + k * (y + 1) for k >= 0 are all solutions
// Works for any integer values of c, a, b and positive m
ll discrete_log(ll c, ll a, ll b, ll m){
        c = ((c % m) + m) % m, a = ((a % m) + m) % m, b = ((b % m) + m) % m;
        if(c == b)
                return 0;

        ll g = __gcd(a, m);
        if(b % g) return -1;

        if(g > 1){
                ll r = discrete_log(c * a / g, a, b / g, m / g);
                return r + (r >= 0);
        }

        unordered_map<ll, ll> babystep;
        ll n = 1, an = a % m;

        // set n to the ceil of sqrt(m):
        while(n * n < m) n++, an = (an * a) % m;

        // babysteps:
        ll bstep = b;
        for(ll i = 0; i <= n; i++){
                babystep[bstep] = i;
                bstep = (bstep * a) % m;
        }

        // giantsteps:
        ll gstep = c * an % m;
        for(ll i = 1; i <= n; i++){
                if(babystep.find(gstep) != babystep.end())
                        return n * i - babystep[gstep];
                gstep = (gstep * an) % m;
        }
        return -1;
}
```

6.4 Euler Phi

```
// Euler phi (totient)
int ind = 0, pf = primes[0], ans = n;
while (1ll*pf*pf <= n) {
  if (n%pf==0) ans -= ans/pf;
  while (n%pf==0) n /= pf;
  pf = primes[++ind];
}
if (n != 1) ans -= ans/n;

// IME2014
int phi[N];
void totient() {
  for (int i = 1; i < N; ++i)  phi[i]=i;
  for (int i = 2; i < N; i+=2) phi[i]>>=1;
  for (int j = 3; j < N; j+=2) if (phi[j]==j) {
    phi[j]--;
    for (int i = 2*j; i < N; i+=j) phi[i]=phi[i]/j*(j-1);
  }
}
```

6.5 Extended Euclidean and Chinese Remainder

```cpp
// Extended Euclid:
void euclid(ll a, ll b, ll &x, ll &y) {
  if (b) euclid(b, a%b, y, x), y -= x*(a/b);
  else x = 1, y = 0;
}

// find (x, y) such that a*x + b*y = c or return false if it's not possible
// [x + k*b/gcd(a, b), y - k*a/gcd(a, b)] are also solutions
bool diof(ll a, ll b, ll c, ll &x, ll &y){
  euclid(abs(a), abs(b), x, y);
  ll g = abs(__gcd(a, b));
  if(c % g) return false;
  x *= c / g;
  y *= c / g;
  if(a < 0) x = -x;
  if(b < 0) y = -y;
  return true;
}

// auxiliar to find_all_solutions
void shift_solution (ll &x, ll &y, ll a, ll b, ll cnt) {
  x += cnt * b;
  y -= cnt * a;
}

// Find the amount of solutions of
// ax + by = c
// in given intervals for x and y
ll find_all_solutions (ll a, ll b, ll c, ll minx, ll maxx, ll miny, ll maxy) {
  ll x, y, g = __gcd(a, b);
  if(!diof(a, b, c, x, y)) return 0;
  a /= g; b /= g;

  int sign_a = a>0 ? +1 : -1;
  int sign_b = b>0 ? +1 : -1;

  shift_solution (x, y, a, b, (minx - x) / b);
  if (x < minx)
    shift_solution (x, y, a, b, sign_b);
  if (x > maxx)
    return 0;
  int lx1 = x;

  shift_solution (x, y, a, b, (maxx - x) / b);
  if (x > maxx)
    shift_solution (x, y, a, b, -sign_b);
  int rx1 = x;

  shift_solution (x, y, a, b, - (miny - y) / a);
  if (y < miny)
    shift_solution (x, y, a, b, -sign_a);
  if (y > maxy)
    return 0;
  int lx2 = x;

  shift_solution (x, y, a, b, - (maxy - y) / a);
  if (y > maxy)
    shift_solution (x, y, a, b, sign_a);
  int rx2 = x;

  if (lx2 > rx2)
    swap (lx2, rx2);
  int lx = max (lx1, lx2);
  int rx = min (rx1, rx2);

  if (lx > rx) return 0;
  return (rx - lx) / abs(b) + 1;
}

bool crt_auxiliar(ll a, ll b, ll m1, ll m2, ll &ans){
  ll x, y;
  if(!diof(m1, m2, b - a, x, y)) return false;
  ll lcm = m1 / __gcd(m1, m2) * m2;
  ans = ((a + x % (lcm / m1) * m1) % lcm + lcm) % lcm;
  return true;
}

// find ans such that ans = a[i] mod b[i] for all 0 <= i < n or return false if not possible
// ans + k * lcm(b[i]) are also solutions
bool crt(int n, ll a[], ll b[], ll &ans){
  if(!b[0]) return false;
  ans = a[0] % b[0];
  ll l = b[0];
  for(int i = 1; i < n; i++){
    if(!b[i]) return false;
    if(!crt_auxiliar(ans, a[i] % b[i], l, b[i], ans)) return false;
    l *= (b[i] / __gcd(b[i], l));
  }
  return true;
}
```

6.6 Fast Fourier Transform(Tourist)

```cpp
//
//  FFT made by tourist. It if faster and more supportive, although it requires more lines of code.
//  Also, it allows operations with MOD, which is usually an issue in FFT problems.
//
namespace fft {
  typedef double dbl;

  struct num {
    dbl x, y;
    num() { x = y = 0; }
    num(dbl x, dbl y) : x(x), y(y) {}
  };

  inline num operator+ (num a, num b) { return num(a.x + b.x, a.y + b.y); }
  inline num operator- (num a, num b) { return num(a.x - b.x, a.y - b.y); }
  inline num operator* (num a, num b) { return num(a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x); }
  inline num conj(num a) { return num(a.x, -a.y); }

  int base = 1;
  vector<num> roots = {{0, 0}, {1, 0}};
  vector<int> rev = {0, 1};

  const dbl PI = acosl(-1.0);

  void ensure_base(int nbase) {
    if(nbase <= base) return;

    rev.resize(1 << nbase);
    for(int i=0; i < (1 << nbase); i++) {
      rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1));
    }
    roots.resize(1 << nbase);

    while(base < nbase) {
      dbl angle = 2*PI / (1 << (base + 1));
      for(int i = 1 << (base - 1); i < (1 << base); i++) {
        roots[i << 1] = roots[i];
        dbl angle_i = angle * (2 * i + 1 - (1 << base));
        roots[(i << 1) + 1] = num(cos(angle_i), sin(angle_i));
      }
      base++;
    }
  }

  void fft(vector<num> &a, int n = -1) {
    if(n == -1) {
      n = a.size();
    }
    assert((n & (n-1)) == 0);
    int zeros = __builtin_ctz(n);
    ensure_base(zeros);
    int shift = base - zeros;
    for(int i = 0; i < n; i++) {
      if(i < (rev[i] >> shift)) {
        swap(a[i], a[rev[i] >> shift]);
      }
    }
    for(int k = 1; k < n; k <<= 1) {
      for(int i = 0; i < n; i += 2 * k) {
        for(int j = 0; j < k; j++) {
          num z = a[i+j+k] * roots[j+k];
          a[i+j+k] = a[i+j] - z;
          a[i+j] = a[i+j] + z;
        }
      }
    }
  }

  vector<num> fa, fb;
  vector<int> multiply(vector<int> &a, vector<int> &b) {
    int need = a.size() + b.size() - 1;
    int nbase = 0;
    while((1 << nbase) < need) nbase++;
    ensure_base(nbase);
    int sz = 1 << nbase;
  if(sz > (int) fa.size()) {
    fa.resize(sz);
  }
  for(int i = 0; i < sz; i++) {
    int x = (i < (int) a.size() ? a[i] : 0);
    int y = (i < (int) b.size() ? b[i] : 0);
    fa[i] = num(x, y);
  }
  fft(fa, sz);
  num r(0, -0.25 / sz);
  for(int i = 0; i <= (sz >> 1); i++) {
    int j = (sz - i) & (sz - 1);
    num z = (fa[j] * fa[j] - conj(fa[i] * fa[i])) * r;
    if(i != j) {
      fa[j] = (fa[i] * fa[i] - conj(fa[j] * fa[j])) * r;
    }
    fa[i] = z;
  }
  fft(fa, sz);
  vector<int> res(need);
  for(int i = 0; i < need; i++) {
    res[i] = fa[i].x + 0.5;
  }
  return res;
}

vector<int> multiply_mod(vector<int> &a, vector<int> &b, int m, int eq = 0) {
  int need = a.size() + b.size() - 1;
  int nbase = 0;
  while ((1 << nbase) < need) nbase++;
  ensure_base(nbase);
  int sz = 1 << nbase;
  if (sz > (int) fa.size()) {
    fa.resize(sz);
  }
  for (int i = 0; i < (int) a.size(); i++) {
    int x = (a[i] % m + m) % m;
    fa[i] = num(x & ((1 << 15) - 1), x >> 15);
  }
  fill(fa.begin() + a.size(), fa.begin() + sz, num {0, 0});
  fft(fa, sz);
  if (sz > (int) fb.size()) {
    fb.resize(sz);
  }
  if (eq) {
    copy(fa.begin(), fa.begin() + sz, fb.begin());
  } else {
    for (int i = 0; i < (int) b.size(); i++) {
      int x = (b[i] % m + m) % m;
      fb[i] = num(x & ((1 << 15) - 1), x >> 15);
    }
    fill(fb.begin() + b.size(), fb.begin() + sz, num {0, 0});
    fft(fb, sz);
  }
  dbl ratio = 0.25 / sz;
  num r2(0, -1);
  num r3(ratio, 0);
  num r4(0, -ratio);
  num r5(0, 1);
  for (int i = 0; i <= (sz >> 1); i++) {
    int j = (sz - i) & (sz - 1);
    num a1 = (fa[i] + conj(fa[j]));
    num a2 = (fa[i] - conj(fa[j])) * r2;
    num b1 = (fb[i] + conj(fb[j])) * r3;
    num b2 = (fb[i] - conj(fb[j])) * r4;
    if (i != j) {
      num c1 = (fa[j] + conj(fa[i]));
      num c2 = (fa[j] - conj(fa[i])) * r2;
      num d1 = (fb[j] + conj(fb[i])) * r3;
      num d2 = (fb[j] - conj(fb[i])) * r4;
      fa[i] = c1 * d1 + c2 * d2 * r5;
      fb[i] = c1 * d2 + c2 * d1;
    }
    fa[j] = a1 * b1 + a2 * b2 * r5;
    fb[j] = a1 * b2 + a2 * b1;
  }
  fft(fa, sz);
  fft(fb, sz);
  vector<int> res(need);
  for (int i = 0; i < need; i++) {
    long long aa = fa[i].x + 0.5;
    long long bb = fb[i].x + 0.5;
    long long cc = fa[i].y + 0.5;
    res[i] = (aa + ((bb % m) << 15) + ((cc % m) << 30)) % m;
  }
  return res;
}

vector<int> square_mod(vector<int> &a, int m) {
    return multiply_mod(a, a, m, 1);
  }
}
```

6.7 Fast Fourier Transform

```
// Fast Fourier Transform - O(nlogn)

/*
// Use struct instead. Performance will be way better!
typedef complex<ld> T;
T a[N], b[N];
*/

struct T {
  ld x, y;
  T() : x(0), y(0) {}
  T(ld a, ld b=0) : x(a), y(b) {}

  T operator/=(ld k) { x/=k; y/=k; return (*this); }
  T operator*(T a) const { return T(x*a.x - y*a.y, x*a.y + y*a.x); }
  T operator+(T a) const { return T(x+a.x, y+a.y); }
  T operator-(T a) const { return T(x-a.x, y-a.y); }
} a[N], b[N];

// a: vector containing polynomial
// n: power of two greater or equal product size
/*
// Use iterative version!
void fft_recursive(T* a, int n, int s) {
  if (n == 1) return;
  T tmp[n];
  for (int i = 0; i < n/2; ++i)
    tmp[i] = a[2*i], tmp[i+n/2] = a[2*i+1];

  fft_recursive(&tmp[0], n/2, s);
  fft_recursive(&tmp[n/2], n/2, s);

  T wn = T(cos(s*2*PI/n), sin(s*2*PI/n)), w(1,0);
  for (int i = 0; i < n/2; i++, w=w*wn)
    a[i] = tmp[i] + w*tmp[i+n/2],
    a[i+n/2] = tmp[i] - w*tmp[i+n/2];
}
*/

void fft(T* a, int n, int s) {
  for (int i=0, j=0; i<n; i++) {
    if (i>j) swap(a[i], a[j]);
    for (int l=n/2; (j^=l) < l; l>>=1);
  }

  for(int i = 1; (1<<i) <= n; i++){
    int M = 1 << i;
    int K = M >> 1;
    T wn = T(cos(s*2*PI/M), sin(s*2*PI/M));
    for(int j = 0; j < n; j += M) {
      T w = T(1, 0);
      for(int l = j; l < K + j; ++l){
        T t = w*a[l + K];
        a[l + K] = a[l]-t;
        a[l] = a[l] + t;
        w = wn*w;
      }
    }
  }
}

// assert n is a power of two greater of equal product size
// n = na + nb; while (n&(n-1)) n++;
void multiply(T* a, T* b, int n) {
  fft(a,n,1);
  fft(b,n,1);
  for (int i = 0; i < n; i++) a[i] = a[i]*b[i];
  fft(a,n,-1);
  for (int i = 0; i < n; i++) a[i] /= n;
}

// Convert to integers after multiplying:
// (int)(a[i].x + 0.5);
```

6.8 Fast Walsh-Hadamard Transform

```
// Fast Walsh-Hadamard Transform - O(nlogn)
//
// Multiply two polynomials, but instead of x^a * x^b = x^(a+b)
// we have x^a * x^b = x^(a XOR b).
//
// WARNING: assert n is a power of two!
void fwht(ll* a, int n, bool inv) {
  for(int l=1; 2*l <= n; l<<=1) {
    for(int i=0; i < n; i+=2*l) {
      for(int j=0; j<l; j++) {
        ll u = a[i+j], v = a[i+l+j];

        a[i+j] = (u+v) % MOD;
        a[i+l+j] = (u-v+MOD) % MOD;
        // % is kinda slow, you can use add() macro instead
        // #define add(x,y) (x+y >= MOD ? x+y-MOD : x+y)
      }
    }
  }

  if(inv) {
    for(int i=0; i<n; i++) {
      a[i] = a[i] / n;
    }
  }
}


/* FWHT AND
  Matrix : Inverse
  0 1       -1 1
  1 1        1 0
*/
void fwht_and(vi &a, bool inv) {
  vi ret = a;
  ll u, v;
  int tam = a.size() / 2;
  for(int len = 1; 2 * len <= tam; len <<= 1) {
    for(int i = 0; i < tam; i += 2 * len) {
      for(int j = 0; j < len; j++) {
        u = ret[i + j];
        v = ret[i + len + j];
        if(!inv) {
          ret[i + j] = v;
          ret[i + len + j] = u + v;
        }
        else {
          ret[i + j] = -u + v;
          ret[i + len + j] = u;
        }
      }
    }
  }
  a = ret;
}


/* FWHT OR
  Matrix : Inverse
  1 1       0  1
  1 0       1 -1
*/
void fft_or(vi &a, bool inv) {
  vi ret = a;
  ll u, v;
  int tam = a.size() / 2;
  for(int len = 1; 2 * len <= tam; len <<= 1) {
    for(int i = 0; i < tam; i += 2 * len) {
      for(int j = 0; j < len; j++) {
        u = ret[i + j];
        v = ret[i + len + j];
        if(!inv) {
          ret[i + j] = u + v;
          ret[i + len + j] = u;
        }
        else {
          ret[i + j] = v;
          ret[i + len + j] = u - v;
        }
      }
    }
  }
  a = ret;
}
```

6.9 Gaussian Elimination (extended inverse)

```
// Gauss-Jordan Elimination with Scaled Partial Pivoting
// Extended to Calculate Inverses - O(n^3)
// To get more precision choose m[j][i] as pivot the element such that m[j][i] / mx[j] is maximized.
// mx[j] is the element with biggest absolute value of row j.

ld C[N][M]; // N = 1000, M = 2*N+1;
int row, col;

bool elim() {
  for(int i=0; i<row; ++i) {
    int p = i; // Choose the biggest pivot
    for(int j=i; j<row; ++j) if (abs(C[j][i]) > abs(C[p][i])) p = j;
    for(int j=i; j<col; ++j) swap(C[i][j], C[p][j]);

    if (!C[i][i]) return 0;

    ld c = 1/C[i][i]; // Normalize pivot line
    for(int j=0; j<col; ++j) C[i][j] *= c;

    for(int k=i+1; k<col; ++k) {
      ld c = -C[k][i]; // Remove pivot variable from other lines
      for(int j=0; j<col; ++j) C[k][j] += c*C[i][j];
    }
  }

  // Make triangular system a diagonal one
  for(int i=row-1; i>=0; --i) for(int j=i-1; j>=0; --j) {
    ld c = -C[j][i];
    for(int k=i; k<col; ++k) C[j][k] += c*C[i][k];
  }

  return 1;
}

// Finds inv, the inverse of matrix m of size n x n.
// Returns true if procedure was successful.
bool inverse(int n, ld m[N][N], ld inv[N][N]) {
  for(int i=0; i<n; ++i) for(int j=0; j<n; ++j)
    C[i][j] = m[i][j], C[i][j+n] = (i == j);

  row = n, col = 2*n;
  bool ok = elim();

  for(int i=0; i<n; ++i) for(int j=0; j<n; ++j) inv[i][j] = C[i][j+n];
  return ok;
}

// Solves linear system m*x = y, of size n x n
bool linear_system(int n, ld m[N][N], ld *x, ld *y) {
  for(int i = 0; i < n; ++i) for(int j = 0; j < n; ++j) C[i][j] = m[i][j];
  for(int j = 0; j < n; ++j) C[j][n] = x[j];

  row = n, col = n+1;
  bool ok = elim();

  for(int j=0; j<n; ++j) y[j] = C[j][n];
  return ok;
}
```

6.10 Gaussian Elimination (modulo prime)

```
//ll A[N][M+1], X[M]

for(int j=0; j<m; j++) { //collumn to eliminate
  int l = j;
  for(int i=j+1; i<n; i++) //find nonzero pivot
    if(A[i][j]%p)
      l=i;
  for(int k = 0; k < m+1; k++) { //Swap lines
    swap(A[l][k],A[j][k]);
  }
  for(int i = j+1; i < n; i++) { //eliminate column
    ll t=mulmod(A[i][j],inv(A[j][j],p),p);
    for(int k = j; k < m+1; k++)
      A[i][k]=(A[i][k]-mulmod(t,A[j][k],p)+p)%p;
  }
}

for(int i = m-1; i >= 0; i--) { //solve triangular system
```

```
  for(int j = m-1; j > i; j--)
    A[i][m] = (A[i][m] - mulmod(A[i][j],X[j],p)+p)%p;
    X[i] = mulmod(A[i][m],inv(A[i][i],p),p);
}
```

6.11 Gaussian Elimination (xor)

```
// Gauss Elimination for xor boolean operations
// Return false if not possible to solve
// Use boolean matrixes 0-indexed
// n equations, m variables, O(n * m * m)
// eq[i][j] = coefficient of j-th element in i-th equation
// r[i] = result of i-th equation
// Return ans[j] = xj that gives the lexicographically greatest solution (if possible)
// (Can be changed to lexicographically least, follow the comments in the code)
// WARNING!! The arrays get changed during de algorithm

bool eq[N][M], r[N], ans[M];

bool gauss_xor(int n, int m){
        for(int i = 0; i < m; i++)
                ans[i] = true;
        int lid[N] = {0}; // id + 1 of last element present in i-th line of final matrix
        int l = 0;
        for(int i = m - 1; i >= 0; i--){
                for(int j = l; j < n; j++)
                        if(eq[j][i]){ // pivot
                                swap(eq[l], eq[j]);
                                swap(r[l], r[j]);
                        }
                if(l == n || !eq[l][i])
                        continue;
                lid[l] = i + 1;
                for(int j = l + 1; j < n; j++){ // eliminate column
                        if(!eq[j][i])
                                continue;
                        for(int k = 0; k <= i; k++)
                                eq[j][k] ^= eq[l][k];
                        r[j] ^= r[l];
                }
                l++;
        }
        for(int i = n - 1; i >= 0; i--){ // solve triangular matrix
                for(int j = 0; j < lid[i + 1]; j++)
                        r[i] ^= (eq[i][j] && ans[j]);
                // for lexicographically least just delete the for bellow
                for(int j = lid[i + 1]; j + 1 < lid[i]; j++){
                        ans[j] = true;
                        r[i] ^= eq[i][j];
                }
                if(lid[i])
                        ans[lid[i] - 1] = r[i];
                else if(r[i])
                        return false;
        }
        return true;
}
```

6.12 Gaussian Elimination (double)

```
//Gaussian Elimination
//double A[N][M+1], X[M]

// if n < m, there's no solution
// column m holds the right side of the equation
// X holds the solutions

for(int j=0; j<m; j++) { //collumn to eliminate
  int l = j;
  for(int i=j+1; i<n; i++) //find largest pivot
    if(abs(A[i][j])>abs(A[l][j]))
      l=i;
  if(abs(A[i][j]) < EPS) continue;
  for(int k = 0; k < m+1; k++) { //Swap lines
    swap(A[l][k],A[j][k]);
  }
  for(int i = j+1; i < n; i++) { //eliminate column
    double t=A[i][j]/A[j][j];
    for(int k = j; k < m+1; k++)
      A[i][k]-=t*A[j][k];
  }
}

for(int i = m-1; i >= 0; i--) { //solve triangular system
  for(int j = m-1; j > i; j--)
    A[i][m] -= A[i][j]*X[j];
  X[i]=A[i][m]/A[i][i];
}
```

6.13 Golden Section Search (Ternary Search)

```
double gss(double l, double r) {
    double m1 = r-(r-l)/gr, m2 = l+(r-l)/gr;
    double f1 = f(m1), f2 = f(m2);
    while(fabs(l-r)>EPS) {
        if(f1>f2) l=m1, f1=f2, m1=m2, m2=l+(r-l)/gr, f2=f(m2);
        else r=m2, f2=f1, m2=m1, m1=r-(r-l)/gr, f1=f(m1);
    }
    return l;
}
```

6.14 Josephus

```
// UFMG
/* Josephus Problem - It returns the position to be, in order to not die. O(n)*/
/* With k=2, for instance, the game begins with 2 being killed and then n+2, n+4, ... */
ll josephus(ll n, ll k) {
  if(n==1) return 1;
  else return (josephus(n-1, k)+k-1)%n+1;
}

/* Another Way to compute the last position to be killed - O(d * log n) */
ll josephus(ll n, ll d) {
  ll K = 1;
  while (K <= (d - 1)*n) K = (d * K + d - 2) / (d - 1);
  return d * n + 1 - K;
}
```

6.15 Mobius Inversion

```
// multiplicative function calculator
// euler_phi and mobius are multiplicative
// if another f[N] needed just remove comments
// O(N)

bool p[N];
vector<ll> primes;
ll g[N];
// ll f[N];

void mfc(){
    // if g(1) != 1 than it's not multiplicative
    g[1] = 1;
    // f[1] = 1;
    primes.clear();
    primes.reserve(N / 10);
    for(ll i = 2; i < N; i++){
        if(!p[i]){
            primes.push_back(i);
            for(ll j = i; j < N; j *= i){
                g[j] = // g(p^k) you found
                // f[j] = f(p^k) you found
                p[j] = (j != i);
            }
        }
        for(ll j : primes){
            if(i * j >= N || i % j == 0)
                break;
            for(ll k = j; i * k < N; k *= j){
                g[i * k] = g[i] * g[k];
                // f[i * k] = f[i] * f[k];
                p[i * k] = true;
            }
        }
    }
}
```

6.16 Mobius Function

```
// 1 if n == 1
// 0 if exists x | n%(x^2) == 0
// else (-1)^k, k = #(p) | p is prime and n%p == 0

//Calculate Mobius for all integers using sieve
//O(n*log(log(n)))
void mobius() {
  for(int i = 1; i < N; i++) mob[i] = 1;

  for(ll i = 2; i < N; i++) if(!sieve[i]){
    for(ll j = i; j < N; j += i) sieve[j] = i, mob[j] *= -1;
    for(ll j = i*i; j < N; j += i*i) mob[j] = 0;
  }
}

/*
//Calculate Mobius for 1 integer
//O(sqrt(n))
int mobius(int n){
  if(n == 1) return 1;
  int p = 0;
  for(int i = 2; i*i <= n; i++)
    if(n%i == 0){
      n /= i;
      p++;
      if(n%i == 0) return 0;
    }
  if(n > 1) p++;
  return p&1 ? -1 : 1;
}
*/
```

6.17 Number Theoretic Transform

```
// Number Theoretic Transform - O(nlogn)

// if long long is not necessary, use int instead to improve performance
const int mod = 20*(1<<23)+1;
const int root = 3;

ll w[N];

// a: vector containing polynomial
// n: power of two greater or equal product size
void ntt(ll* a, int n, bool inv) {
  for (int i=0, j=0; i<n; i++) {
    if (i>j) swap(a[i], a[j]);
    for (int l=n/2; (j^=l) < l; l>>=1);
  }

  // TODO: Rewrite this loop using FFT version
  ll k, t, nrev;
  w[0] = 1;
  k = exp(root, (mod-1) / n, mod);
  for (int i=1;i<=n;i++) w[i] = w[i-1] * k % mod;
  for(int i=2; i<=n; i<<=1) for(int j=0; j<n; j+=i) for(int l=0; l<(i/2); l++) {
    int x = j+l, y = j+l+(i/2), z = (n/i)*l;
    t = a[y] * w[inv ? (n-z) : z] % mod;
    a[y] = (a[x] - t + mod) % mod;
    a[x] = (a[j+l] + t) % mod;
  }

  nrev = exp(n, mod-2, mod);
  if (inv) for(int i=0; i<n; ++i) a[i] = a[i] * nrev % mod;
}

// assert n is a power of two greater of equal product size
// n = na + nb; while (n&(n-1)) n++;
void multiply(ll* a, ll* b, int n) {
  ntt(a, n, 0);
  ntt(b, n, 0);
  for (int i = 0; i < n; i++) a[i] = a[i]*b[i] % mod;
  ntt(a, n, 1);
}
```

6.18 Pollard-Rho

```
// factor(N, v) to get N factorized in vector v
// O(N ^ (1 / 4)) on average
// Miller-Rabin - Primarily Test O(|base|*(logn)^2)
ll addmod(ll a, ll b, ll m){
	if(a >= m - b) return a + b - m;
	return a + b;
}

ll mulmod(ll a, ll b, ll m){
	ll ans = 0;
	while(b){
		if(b & 1) ans = addmod(ans, a, m);
		a = addmod(a, a, m);
		b >>= 1;
	}
	return ans;
}

ll fexp(ll a, ll b, ll n){
	ll r = 1;
	while(b){
		if(b & 1) r = mulmod(r, a, n);
		a = mulmod(a, a, n);
		b >>= 1;
	}
	return r;
}

bool miller(ll a, ll n){
	if (a >= n) return true;
	ll s = 0, d = n - 1;
	while(d % 2 == 0) d >>= 1, s++;
	ll x = fexp(a, d, n);
	if (x == 1 || x == n - 1) return true;
	for (int r = 0; r < s; r++, x = mulmod(x,x,n)){
		if (x == 1) return false;
		if (x == n - 1) return true;
	}
	return false;
}

bool isprime(ll n){
	if(n == 1) return false;
	int base[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
	for (int i = 0; i < 12; ++i) if (!miller(base[i], n)) return false;
	return true;
}

ll pollard(ll n){
	ll x, y, d, c = 1;
	if (n % 2 == 0) return 2;
	while(true){
		y = x = 2;
		while(true){
			x = addmod(mulmod(x,x,n), c, n);
			y = addmod(mulmod(y,y,n), c, n);
			y = addmod(mulmod(y,y,n), c, n);
			if (x == y) break;
			d = __gcd(abs(x-y), n);
			if (d > 1) return d;
		}
		c++;
	}
}

vector<ll> factor(ll n){
	if (n == 1 || isprime(n)) return {n};
	ll f = pollard(n);
	vector<ll> l = factor(f), r = factor(n / f);
	l.insert(l.end(), r.begin(), r.end());
	sort(l.begin(), l.end());
	return l;
}

//n < 2,047 base = {2};
//n < 9,080,191 base = {31, 73};
//n < 2,152,302,898,747 base = {2, 3, 5, 7, 11};
//n < 318,665,857,834,031,151,167,461 base = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
//n < 3,317,044,064,679,887,385,961,981 base = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41};
```

6.19 Pollard-Rho Optimization

```
// We recomend you to use pollard-rho.cpp! I've never needed this code, but here it is.
// This uses Brent's algorithm for cycle detection
//
```

```
std::mt19937 rng((int) std::chrono::steady_clock::now().time_since_epoch().count());

ull func(ull x, ull n, ull c) { return (mulmod(x, x, n) + c) % n; // f(x) = (x^2 + c) % n; }

ull pollard(ull n) {
  // Finds a positive divisor of n
  ull x, y, d, c;
  ull pot, lam;
  if(n % 2 == 0) return 2;
  if(isprime(n)) return n;

  while(1) {
    y = x = 2; d = 1;
    pot = lam = 1;
    while(1) {
      c = rng() % n;
      if(c != 0 and (c+2)%n != 0) break;
    }
    while(1) {
      if(pot == lam) {
        x = y;
        pot <<= 1;
        lam = 0;
      }
      y = func(y, n, c);
      lam++;
      d = gcd(x >= y ? x-y : y-x, n);
      if (d > 1) {
        if(d == n) break;
        else return d;
      }
    }
  }
}

void fator(ull n, vector<ull> &v) {
  // prime factorization of n, put into a vector v.
  //
  // for each prime factor of n, it is repeated the amount of times
  // that it divides n
  //
  // ex : n == 120, v = {2, 2, 2, 3, 5};
  //
  //
  if(isprime(n)) { v.pb(n); return; }
  vector<ull> w, t; w.pb(n); t.pb(1);

  while(!w.empty()) {
    ull bck = w.back();
    ull div = pollard(bck);

    if(div == w.back()) {
      int amt = 0;
      for(int i=0; i < (int) w.size(); i++) {
        int cur = 0;
        while(w[i] % div == 0) {
          w[i] /= div;
          cur++;
        }
        amt += cur * t[i];
        if(w[i] == 1) {
          swap(w[i], w.back());
          swap(t[i], t.back());
          w.pop_back();
          t.pop_back();
        }
      }
      while(amt--) v.pb(div);
    }
    else {
      int amt = 0;
      while(w.back() % div == 0) {
        w.back() /= div;
        amt++;
      }
      amt *= t.back();
      if(w.back() == 1) {
        w.pop_back();
        t.pop_back();
      }

      w.pb(div);
      t.pb(amt);
    }
  }

  // the divisors will not be sorted, so you need to sort it afterwards
  sort(v.begin(), v.end());
}
```

6.20 Prime Factors

```
// Prime factors (up to 9*10^13. For greater see Pollard Rho)
vi factors;
int ind=0, pf = primes[0];
while (pf*pf <= n) {
  while (n%pf == 0) n /= pf, factors.pb(pf);
  pf = primes[++ind];
}
if (n != 1) factors.pb(n);
```

6.21 Primitive Root

```
// Finds a primitive root modulo p
// To make it works for any value of p, we must add calculation of phi(p)
// n is 1, 2, 4 or p^k or 2*p^k (p odd in both cases)
ll root(ll p) {
  ll n = p-1;
  vector<ll> fact;

  for (int i=2; i*i<=n; ++i) if (n % i == 0) {
    fact.push_back (i);
    while (n % i == 0) n /= i;
  }

  if (n > 1) fact.push_back (n);

  for (int res=2; res<=p; ++res) {
    bool ok = true;
    for (size_t i=0; i<fact.size() && ok; ++i)
      ok &= exp(res, (p-1) / fact[i], p) != 1;
    if (ok)  return res;
  }

  return -1;
}
```

6.22 Sieve of Eratosthenes

```
// Sieve of Erasthotenes
int p[N]; vi primes;

for (ll i = 2; i < N; ++i) if (!p[i]) {
  for (ll j = i*i; j < N; j+=i) p[j]=1;
  primes.pb(i);
}
```

6.23 Simpson Rule

```
// Simpson Integration Rule
// define the function f
double f(double x) {
  // ...
}

double simpson(double a, double b, int n = 1e6) {
    double h = (b - a) / n;
    double s = f(a) + f(b);
    for (int i = 1; i < n; i += 2) s += 4 * f(a + h*i);
    for (int i = 2; i < n; i += 2) s += 2 * f(a + h*i);
    return s*h/3;
}
```

6.24 Discrete Log

```cpp
// O(sqrt(m))
// Solve c * a^x = b mod(m) for integer x >= 0.
// Return the smallest x possible, or -1 if there is no solution
// If all solutions needed, solve c * a^x = b mod(m) and (a*b) * a^y = b mod(m)
// x + k * (y + 1) for k >= 0 are all solutions
// Works for any integer values of c, a, b and positive m
ll discrete_log(ll c, ll a, ll b, ll m){
        c = ((c % m) + m) % m, a = ((a % m) + m) % m, b = ((b % m) + m) % m;
        if(c == b)
                return 0;

        ll g = __gcd(a, m);
        if(b % g) return -1;

        if(g > 1){
                ll r = discrete_log(c * a / g, a, b / g, m / g);
                return r + (r >= 0);
        }

        unordered_map<ll, ll> babystep;
        ll n = 1, an = a % m;

        // set n to the ceil of sqrt(m):
        while(n * n < m) n++, an = (an * a) % m;

        // babysteps:
        ll bstep = b;
        for(ll i = 0; i <= n; i++){
                babystep[bstep] = i;
                bstep = (bstep * a) % m;
        }

        // giantsteps:
        ll gstep = c * an % m;
        for(ll i = 1; i <= n; i++){
                if(babystep.find(gstep) != babystep.end())
                        return n * i - babystep[gstep];
                gstep = (gstep * an) % m;
        }
        return -1;
}
```

6.25 Simplex (Stanford)

```cpp
// Two-phase simplex algorithm for solving linear programs of the form
//
//     maximize     c^T x
//     subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments.  Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;
typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
const DOUBLE EPS = 1e-9;

struct LPSolver {
  int m, n;
  VI B, N;
  VVD D;

  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }

  void Pivot(int r, int s) {
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] / D[r][s];
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] /= D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] /= -D[r][s];
    D[r][s] = 1.0 / D[r][s];
    swap(B[r], N[s]);
  }

  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] < N[s]) s = j;
      }
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] < B[r]) r = i;
      }
      if (r == -1) return false;
      Pivot(r, s);
    }
  }

  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -numeric_limits<DOUBLE>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
          if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] < N[s]) s = j;
        Pivot(i, s);
      }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
  }
};

int main() {

  const int m = 4;
  const int n = 3;
  DOUBLE _A[m][n] = {
    { 6, -1, 0 },
    { -1, -5, 0 },
    { 1, 5, 1 },
    { -1, -5, -1 }
  };
  DOUBLE _b[m] = { 10, -4, 5, -5 };
  DOUBLE _c[n] = { 1, -1, 0 };

  VVD A(m);
  VD b(_b, _b + m);
  VD c(_c, _c + n);
  for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);

  cerr << "VALUE: " << value << endl; // VALUE: 1.29032
  cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
  for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
  cerr << endl;
  return 0;
}
```

7 Geometry

7.1 Miscelanneous

```
/*
1)  Square (n = 4) is the only regular polygon with integer coordinates

2)  Pick's theorem:  A = i + b/2 - 1
    A: area of the polygon
    i: number of interior points
    b: number of points on the border

3) Conic Rotations
    Given elipse: Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0
    Convert it to: Ax^2 + Bxy + Cy^2 + Dx + Ey = 1 (this formula suits better for elipse, before doing this
          verify F = 0)
    Final conversion: A(x + D/2A)^2 + C(y + E/2C)^2 = 1 + D^2/4A + E^2/4C
        B != 0 (Rotate):
            theta = atan2(b, c-a)/2.0;
            A' = (a + c + b/sin(2.0*theta))/2.0; // A
            C' = (a + c - b/sin(2.0*theta))/2.0; // C
            D' = d*sin(theta) + e*cos(theta); // D
            E' = d*cos(theta) - e*sin(theta); // E
        Remember to rotate again after!
*/


// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for (int i = 0; i < p.size(); i++){
        int j = (i+1)%p.size();
        if ((p[i].y <= q.y && q.y < p[j].y ||
            p[j].y <= q.y && q.y < p[i].y) &&
            q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
            c = !c;
    }
    return c;
}
```

7.2 Basics (Point)

```
#include <bits/stdc++.h>

using namespace std;

#define st first
#define nd second
#define pb push_back
#define cl(x,v) memset((x), (v), sizeof(x))
#define db(x) cerr << #x << " == " << x << endl
#define dbs(x) cerr << x << endl
#define _ << ", " <<

typedef long long ll;
typedef long double ld;
typedef pair<int,int> pii;
typedef pair<int, pii> piii;
typedef pair<ll,ll> pll;
typedef pair<ll, pll> plll;
typedef vector<int> vi;
typedef vector <vi> vii;

const ld EPS = 1e-9, PI = acos(-1.);
const ll LINF = 0x3f3f3f3f3f3f3f3f;
const int INF = 0x3f3f3f3f, MOD = 1e9+7;
const int N = 1e5+5;

typedef long double type;
//for big coordinates change to long long

bool ge(type x, type y) { return x + EPS > y; }
bool le(type x, type y) { return x - EPS < y; }
bool eq(type x, type y) { return ge(x, y) and le(x, y); }
int sign(type x) { return ge(x, 0) - le(x, 0); }

struct point {
    type x, y;

    point() : x(0), y(0) {}
    point(type x, type y) : x(x), y(y) {}

    point operator -() { return point(-x, -y); }
    point operator +(point p) { return point(x + p.x, y + p.y); }
    point operator -(point p) { return point(x - p.x, y - p.y); }

    point operator *(type k) { return point(k*x, k*y); }
    point operator /(type k) { return point(x/k, y/k); }

    //inner product
    type operator *(point p) { return x*p.x + y*p.y; }
    //cross product
    type operator %(point p) { return x*p.y - y*p.x; }

    bool operator ==(const point &p) const{ return x == p.x and y == p.y; }
    bool operator !=(const point &p) const{ return x != p.x  or y != p.y; }
    bool operator <(const point &p) const { return (x < p.x) or (x == p.x and y < p.y); }

    // 0 => same direction
    // 1 => p is on the left
    //-1 => p is on the right
    int dir(point o, point p) {
        type x = (*this - o) % (p - o);
        return ge(x,0) - le(x,0);
    }

    bool on_seg(point p, point q) {
        if (this->dir(p, q)) return 0;
        return ge(x, min(p.x, q.x)) and le(x, max(p.x, q.x)) and ge(y, min(p.y, q.y)) and le(y, max(p.y, q.y)
              );
    }

    ld abs() { return sqrt(x*x + y*y); }
    type abs2() { return x*x + y*y; }
    ld dist(point q) { return (*this - q).abs(); }
    type dist2(point q) { return (*this - q).abs2(); }

    ld arg() { return atan2l(y, x); }

    // Project point on vector y
    point project(point y) { return y * ((*this * y) / (y * y)); }

    // Project point on line generated by points x and y
    point project(point x, point y) { return x + (*this - x).project(y-x); }

    ld dist_line(point x, point y) { return dist(project(x, y)); }

    ld dist_seg(point x, point y) {
        return project(x, y).on_seg(x, y) ? dist_line(x, y) :  min(dist(x), dist(y));
    }

    point rotate(ld sin, ld cos) { return point(cos*x - sin*y, sin*x + cos*y); }
    point rotate(ld a) { return rotate(sin(a), cos(a)); }

    // rotate around the argument of vector p
    point rotate(point p) { return rotate(p.y / p.abs(), p.x / p.abs()); }

};

int direction(point o, point p, point q) { return p.dir(o, q); }

point rotate_ccw90(point p)    { return point(-p.y,p.x); }
point rotate_cw90(point p)    { return point(p.y,-p.x); }

//for reading purposes avoid using * and % operators, use the functions below:
type dot(point p, point q)     { return p.x*q.x + p.y*q.y; }
type cross(point p, point q)   { return p.x*q.y - p.y*q.x; }

//double area
type area_2(point a, point b, point c) { return cross(a,b) + cross(b,c) + cross(c,a); }

int angle_less(const point& a1, const point& b1, const point& a2, const point& b2) {
    //angle between (a1 and b1) vs angle between (a2 and b2)
    //1  : bigger
    //-1 : smaller
    //0  : equal
    point p1(dot(   a1, b1), abs(cross(   a1, b1)));
    point p2(dot(   a2, b2), abs(cross(   a2, b2)));
    if(cross(p1, p2) < 0) return 1;
    if(cross(p1, p2) > 0) return -1;
    return 0;
}

ostream &operator<<(ostream &os, const point &p) {
    os << "(" << p.x << "," << p.y << ")";
    return os;
}
```

7.3 Circle

```
#include "basics.cpp"
#include "lines.cpp"

struct circle {
    point c;
    ld r;
    circle() { c = point(); r = 0; }
    circle(point _c, ld _r) : c(_c), r(_r) {}
    ld area() { return acos(-1.0)*r*r; }
    ld chord(ld rad) { return  2*r*sin(rad/2.0); }
    ld sector(ld rad) { return 0.5*rad*area()/acos(-1.0); }
    bool intersects(circle other) {
        return le(c.dist(other.c), r + other.r);
    }
    bool contains(point p) { return le(c.dist(p), r); }
    pair<point, point> getTangentPoint(point p) {
        ld d1 = c.dist(p), theta = asin(r/d1);
        point p1 = (c - p).rotate(-theta);
        point p2 = (c - p).rotate(theta);
        p1 = p1*(sqrt(d1*d1 - r*r)/d1) + p;
        p2 = p2*(sqrt(d1*d1 - r*r)/d1) + p;
        return make_pair(p1,p2);
    }
};

circle circumcircle(point a, point b, point c) {
    circle ans;
    point u = point((b - a).y, -(b - a).x);
    point v = point((c - a).y, -(c - a).x);
    point n = (c - b)*0.5;
    ld t = cross(u,n)/cross(v,u);
    ans.c = ((a + c)*0.5) + (v*t);
    ans.r = ans.c.dist(a);
    return ans;
}

point compute_circle_center(point a, point b, point c) {
    //circumcenter
    b = (a + b)/2;
    c = (a + c)/2;
    return compute_line_intersection(b, b + rotate_cw90(a - b), c, c + rotate_cw90(a - c));
}

int inside_circle(point p, circle c) {
    if (fabs(p.dist(c.c) - c.r)<EPS) return 1;
    else if (p.dist(c.c) < c.r) return 0;
    else return 2;
} //0 = inside/1 = border/2 = outside

circle incircle( point p1, point p2, point p3 ) {
    ld m1 = p2.dist(p3);
    ld m2 = p1.dist(p3);
    ld m3 = p1.dist(p2);
    point c = (p1*m1 + p2*m2 + p3*m3)*(1/(m1 + m2 + m3));
    ld s = 0.5*(m1 + m2 + m3);
    ld r = sqrt(s*(s - m1)*(s - m2)*(s - m3))/s;
    return circle(c, r);
}

circle minimum_circle(vector<point> p) {
    random_shuffle(p.begin(), p.end());
    circle C = circle(p[0], 0.0);
    for(int i = 0; i < (int)p.size(); i++) {
        if (C.contains(p[i])) continue;
        C = circle(p[i], 0.0);
        for(int j = 0; j < i; j++) {
            if (C.contains(p[j])) continue;
            C = circle((p[j] + p[i])*0.5, 0.5*p[j].dist(p[i]));
            for(int k = 0; k < j; k++) {
                if (C.contains(p[k])) continue;
                C = circumcircle(p[j], p[i], p[k]);
            }
        }
    }
    return C;
}
```

```
// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<point> circle_line_intersection(point a, point b, point c, ld r) {
    vector<point> ret;
    b = b - a;
    a = a - c;
    ld A = dot(b, b);
    ld B = dot(a, b);
    ld C = dot(a, a) - r*r;
    ld D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(c + a + b*(sqrt(D + EPS) - B)/A);
    if (D > EPS)
        ret.push_back(c + a + b*(-B - sqrt(D))/A);
    return ret;
}

vector<point> circle_circle_intersection(point a, point b, ld r, ld R) {
    vector<point> ret;
    ld d = sqrt(a.dist2(b));
    if (d > r + R || d + min(r, R) < max(r, R)) return ret;
    ld x = (d*d - R*R + r*r)/(2*d);
    ld y = sqrt(r*r - x*x);
    point v = (b - a)/d;
    ret.push_back(a + v*x + rotate_ccw90(v)*y);
    if (y > 0)
        ret.push_back(a + v*x - rotate_ccw90(v)*y);
    return ret;
}

//GREAT CIRCLE

double gcTheta(double pLat, double pLong, double qLat, double qLong) {
        pLat *= acos(-1.0) / 180.0; pLong *= acos(-1.0) / 180.0; // convert degree to radian
        qLat *= acos(-1.0) / 180.0; qLong *= acos(-1.0) / 180.0;
        return acos(cos(pLat)*cos(pLong)*cos(qLat)*cos(qLong) +
                  cos(pLat)*sin(pLong)*cos(qLat)*sin(qLong) +
                  sin(pLat)*sin(qLat));
}

double gcDistance(double pLat, double pLong, double qLat, double qLong, double radius) {
        return radius*gcTheta(pLat, pLong, qLat, qLong);
}


/*
 * Codeforces 101707B
 */
/*
point A, B;
circle C;

double getd2(point a, point b) {
        double h = dist(a, b);
        double r = C.r;
        double alpha = asin(h/(2*r));
        while (alpha < 0) alpha += 2*acos(-1.0);
        return dist(a, A) + dist(b, B) + r*2*min(alpha, 2*acos(-1.0) - alpha);
}

int main() {
        scanf("%lf %lf", &A.x, &A.y);
        scanf("%lf %lf", &B.x, &B.y);
        scanf("%lf %lf %lf", &C.c.x, &C.c.y, &C.r);
        double ans;
        if (distToLineSegment(C.c, A, B) >= C.r) {
                ans = dist(A, B);
        }
        else {
                pair<point, point> tan1 = C.getTangentPoint(A);
                pair<point, point> tan2 = C.getTangentPoint(B);
                ans = 1e+30;
                ans = min(ans, getd2(tan1.first, tan2.first));
                ans = min(ans, getd2(tan1.first, tan2.second));
                ans = min(ans, getd2(tan1.second, tan2.first));
                ans = min(ans, getd2(tan1.second, tan2.second));
        }
        printf("%.18f\n", ans);
        return 0;
}*/
```

7.4 Half Plane Intersection

```
// Intersection of halfplanes - O(nlogn)
// Points are given in counterclockwise order
//
// by Agnez

typedef vector<point> polygon;

int cmp(ld x, ld y = 0, ld tol = EPS) {
    return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1; }

bool comp(point a, point b){
    if((cmp(a.x) > 0 || (cmp(a.x) == 0 && cmp(a.y) > 0) ) && (cmp(b.x) < 0 || (cmp(b.x) == 0 && cmp(b.y < 0))
          )) return 1;
    if((cmp(b.x) > 0 || (cmp(b.x) == 0 && cmp(b.y) > 0) ) && (cmp(a.x) < 0 || (cmp(a.x) == 0 && cmp(a.y < 0))
          )) return 0;
    ll R = a%b;
    if(R) return R > 0;
    return false;
}

namespace halfplane{
  struct L{
    point p,v;
    L(){}
    L(point P, point V):p(P),v(V){}
    bool operator<(const L &b)const{ return comp(v, b.v); }
  };
  vector<L> line;
  void addL(point a, point b){line.pb(L(a,b-a));}
  bool left(point &p, L &l){ return cmp(l.v % (p-l.p))>0; }
  bool left_equal(point &p, L &l){ return cmp(l.v % (p-l.p))>=0; }
  void init(){ line.clear(); }

  point pos(L &a, L &b){
    point x=a.p-b.p;
    ld t = (b.v % x)/(a.v % b.v);
    return a.p+a.v*t;
  }

  polygon intersect(){
    sort(line.begin(), line.end());
    deque<L> q; //linhas da intersecao
    deque<point> p; //pontos de intersecao entre elas
    q.push_back(line[0]);
    for(int i=1; i < (int) line.size(); i++){
      while(q.size()>1 && !left(p.back(), line[i]))
        q.pop_back(), p.pop_back();
      while(q.size()>1 && !left(p.front(), line[i]))
        q.pop_front(), p.pop_front();
      if(!cmp(q.back().v % line[i].v) && !left(q.back().p,line[i]))
        q.back() = line[i];
      else if(cmp(q.back().v % line[i].v))
        q.push_back(line[i]), p.push_back(point());
      if(q.size()>1)
        p.back()=pos(q.back(),q[q.size()-2]);
    }
    while(q.size()>1 && !left(p.back(),q.front()))
      q.pop_back(), p.pop_back();
    if(q.size() <= 2) return polygon(); //Nao forma poligono (pode nao ter intersecao)
    if(!cmp(q.back().v % q.front().v)) return polygon(); //Lados paralelos -> area infinita
    point ult = pos(q.back(),q.front());

    bool ok = 1;
    for(int i=0; i < (int) line.size(); i++)
      if(!left_equal(ult,line[i])){ ok=0; break; }

    if(ok) p.push_back(ult); //Se formar um poligono fechado
    polygon ret;
    for(int i=0; i < (int) p.size(); i++)
      ret.pb(p[i]);
    return ret;
  }
};

//
// Detect whether there is a non-empty intersection in a set of halfplanes
// Complexity O(n)
//
// By Agnez
//
pair<char, point> half_inter(vector<pair<point,point> > &vet){
        random_shuffle(all(vet));
        point p;
        rep(i,0,sz(vet)) if(ccw(vet[i].x,vet[i].y,p) != 1){
                point dir = (vet[i].y-vet[i].x)/abs(vet[i].y-vet[i].x);
                point l = vet[i].x - dir*1e15;
                point r = vet[i].x + dir*1e15;
                if(r<l) swap(l,r);
                rep(j,0,i){
                        if(ccw(point(),vet[i].x-vet[i].y,vet[j].x-vet[j].y)==0){
                                if(ccw(vet[j].x, vet[j].y, p) == 1)
                                        continue;
                                return mp(0,point());
                        }
                        if(ccw(vet[j].x, vet[j].y, l) != 1)
                                l = max(l, line_intersect(vet[i].x,vet[i].y,vet[j].x,vet[j].y));
                        if(ccw(vet[j].x, vet[j].y, r) != 1)
                                r = min(r, line_intersect(vet[i].x,vet[i].y,vet[j].x,vet[j].y));
                        if(!(l<r)) return mp(0,point());
                }
                p=r;
        }
        return mp(1, p);
}
```

7.5 Lines

```
#include "basics.cpp"
//functions tested at: https://codeforces.com/group/3qadGzUdR4/contest/101706/problem/B

//WARNING: all distance functions are not realizing sqrt operation
//Suggestion: for line intersections check LineLineIntersection and then use ComputeLineIntersection

point project_point_line(point c, point a, point b) {
    ld r = dot(b - a,b - a);
    if (fabs(r) < EPS) return a;
    return a + (b - a)*dot(c - a, b - a)/dot(b - a, b - a);
}

point project_point_ray(point c, point a, point b) {
    ld r = dot(b - a, b - a);
    if (fabs(r) < EPS) return a;
    r = dot(c - a, b - a) / r;
    if (le(r, 0)) return a;
    return a + (b - a)*r;
}

point project_point_segment(point c, point a, point b) {
    ld r = dot(b - a, b - a);
    if (fabs(r) < EPS) return a;
    r = dot(c - a, b - a)/r;
    if (le(r, 0)) return a;
    if (ge(r, 1)) return b;
    return a + (b - a)*r;
}

ld distance_point_line(point c, point a, point b) {
    return c.dist2(project_point_line(c, a, b));
}

ld distance_point_ray(point c, point a, point b) {
    return c.dist2(project_point_ray(c, a, b));
}

ld distance_point_segment(point c, point a, point b) {
    return c.dist2(project_point_segment(c, a, b));
}

//not tested
ld distance_point_plane(ld x, ld y, ld z,
                        ld a, ld b, ld c, ld d)
{
    return fabs(a*x + b*y + c*z - d)/sqrt(a*a + b*b + c*c);
}

bool lines_parallel(point a, point b, point c, point d) {
    return fabs(cross(b - a, d - c)) < EPS;
}

bool lines_collinear(point a, point b, point c, point d) {
  return lines_parallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}

point lines_intersect(point p, point q, point a, point b) {
    point r = q - p, s = b - a, c(p%q, a%b);
    if (eq(r%s,0)) return point(LINF, LINF);
    return point(point(r.x, s.x) % c, point(r.y, s.y) % c) / (r%s);
}

//be careful: test LineLineIntersection before using this function
point compute_line_intersection(point a, point b, point c, point d) {
    b = b - a; d = c - d; c = c - a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}

bool line_line_intersect(point a, point b, point c, point d) {
    if(!lines_parallel(a, b, c, d)) return true;
    if(lines_collinear(a, b, c, d)) return true;
    return false;
}


//rays in direction a -> b, c -> d
bool ray_ray_intersect(point a, point b, point c, point d){
    if (a.dist2(c) < EPS || a.dist2(d) < EPS ||
        b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
    if (lines_collinear(a, b, c, d)) {
        if(ge(dot(b - a, d - c), 0)) return true;
        if(ge(dot(a - c, d - c), 0)) return true;
        return false;
    }
    if(!line_line_intersect(a, b, c, d)) return false;
    point inters = lines_intersect(a, b, c, d);
    if(ge(dot(inters - c, d - c), 0) && ge(dot(inters - a, b - a), 0)) return true;
    return false;
}

bool segment_segment_intersect(point a, point b, point c, point d) {
    if (a.dist2(c) < EPS || a.dist2(d) < EPS ||
        b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
    int d1, d2, d3, d4;
    d1 = direction(a, b, c);
    d2 = direction(a, b, d);
    d3 = direction(c, d, a);
    d4 = direction(c, d, b);
    if (d1*d2 < 0 and d3*d4 < 0) return 1;
    return a.on_seg(c, d) or b.on_seg(c, d) or
            c.on_seg(a, b) or d.on_seg(a, b);
}

bool segment_line_intersect(point a, point b, point c, point d){
    if(!line_line_intersect(a, b, c, d)) return false;
    point inters = lines_intersect(a, b, c, d);
    if(inters.on_seg(a, b)) return true;
    return false;
}

//ray in direction c -> d
bool segment_ray_intersect(point a, point b, point c, point d){
    if (a.dist2(c) < EPS || a.dist2(d) < EPS ||
        b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
    if (lines_collinear(a, b, c, d)) {
        if(c.on_seg(a, b)) return true;
        if(ge(dot(d - c, a - c), 0)) return true;
        return false;
    }
    if(!line_line_intersect(a, b, c, d)) return false;
    point inters = lines_intersect(a, b, c, d);
    if(!inters.on_seg(a, b)) return false;
    if(ge(dot(inters - c, d - c), 0)) return true;
    return false;
}

//ray in direction a -> b
bool ray_line_intersect(point a, point b, point c, point d){
    if (a.dist2(c) < EPS || a.dist2(d) < EPS ||
        b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
    if (!line_line_intersect(a, b, c, d)) return false;
    point inters = lines_intersect(a, b, c, d);
    if(!line_line_intersect(a, b, c, d)) return false;
    if(ge(dot(inters - a, b - a), 0)) return true;
    return false;
}

ld distance_segment_line(point a, point b, point c, point d){
    if(segment_line_intersect(a, b, c, d)) return 0;
    return min(distance_point_line(a, c, d), distance_point_line(b, c, d));
}

ld distance_segment_ray(point a, point b, point c, point d){
    if(segment_ray_intersect(a, b, c, d)) return 0;
    ld min1 = distance_point_segment(c, a, b);
    ld min2 = min(distance_point_ray(a, c, d), distance_point_ray(b, c, d));
    return min(min1, min2);
}

ld distance_segment_segment(point a, point b, point c, point d){
    if(segment_segment_intersect(a, b, c, d)) return 0;
    ld min1 = min(distance_point_segment(c, a, b), distance_point_segment(d, a, b));
    ld min2 = min(distance_point_segment(a, c, d), distance_point_segment(b, c, d));
    return min(min1, min2);
}

ld DistanceRayLine(point a, point b, point c, point d){
    if(ray_line_intersect(a, b, c, d)) return 0;
    ld min1 = distance_point_line(a, c, d);
    return min1;
}

ld DistanceRayRay(point a, point b, point c, point d){
    if(ray_ray_intersect(a, b, c, d)) return 0;
    ld min1 = min(distance_point_ray(c, a, b), distance_point_ray(a, c, d));
    return min1;
}

ld DistanceLineLine(point a, point b, point c, point d){
    if(line_line_intersect(a, b, c, d)) return 0;
    return distance_point_line(a, c, d);
}
```

7.6 Minkowski Sum

```
// Given two polygons, returns the minkowski sum of them.
//
// By Agnez
bool comp(point a, point b){
        if((a.x > 0 || (a.x==0 && a.y>0) ) && (b.x < 0 || (b.x==0 && b.y<0))) return 1;
        if((b.x > 0 || (b.x==0 && b.y>0) ) && (a.x < 0 || (a.x==0 && a.y<0))) return 0;
        ll R = a%b;
        if(R) return R > 0;
        return a*a < b*b;
}

polygon poly_sum(polygon a, polygon b){
        //Lembre de nao ter pontos repetidos
        //      passar poligonos ordenados
        //      se nao tiver pontos colineares, pode usar:
        //pivot = *min_element(all(a));
        //sort(all(a),radialcomp);
        //a.resize(unique(all(a))-a.begin());
        //pivot = *min_element(all(b));
        //sort(all(b),radialcomp);
        //b.resize(unique(all(b))-b.begin());
        if(!sz(a) || !sz(b)) return polygon(0);
        if(min(sz(a),sz(b)) < 2){
                polygon ret(0);
                rep(i,0,sz(a)) rep(j,0,sz(b)) ret.pb(a[i]+b[j]);
                return ret;
        }
        polygon ret;
        ret.pb(a[0]+b[0]);
        int pa = 0, pb = 0;
        while(pa < sz(a) || pb < sz(b)){
                point p = ret.back();
                if(pb == sz(b) || (pa < sz(a) && comp((a[(pa+1)%sz(a)]-a[pa]),(b[(pb+1)%sz(b)]-b[pb]))))
                        p = p + (a[(pa+1)%sz(a)]-a[pa]), pa++;
                else p = p + (b[(pb+1)%sz(b)]-b[pb]), pb++;
                //descomentar para tirar pontos colineares (o poligono nao pode ser degenerado)
//              while(sz(ret) > 1 && !ccw(ret[sz(ret)-2], ret[sz(ret)-1], p))
//                      ret.pop_back();
                ret.pb(p);
        }
        assert(ret.back() == ret[0]);
        ret.pop_back();
        return ret;
}

//ITA MINKOWSKI
#include <cmath>
#define EPS 1e-9

/*
 * Point 2D
 */

struct point {
        double x, y;
        point() { x = y = 0.0; }
        point(double _x, double _y) : x(_x), y(_y) {}
        bool operator < (point other) const {
                if (fabs(x - other.x) > EPS) return x < other.x;
                else return y < other.y;
        }
        point operator +(point other) const {
                return point(x + other.x, y + other.y);
        }
        point operator -(point other) const {
                return point(x - other.x, y - other.y);
        }
        point operator *(double k) const {
                return point(x*k, y*k);
        }
};

double dist(point p1, point p2) {
        return hypot(p1.x - p2.x, p1.y - p2.y);
}

double inner(point p1, point p2) {
        return p1.x*p2.x + p1.y*p2.y;
}

double cross(point p1, point p2) {
        return p1.x*p2.y - p1.y*p2.x;
}

bool collinear(point p, point q, point r) {
        return fabs(cross(p-q, r-p)) < EPS;
}

/*
 * Polygon 2D
 */

#include <vector>
#include <algorithm>
using namespace std;

typedef vector<point> polygon;

double signedArea(polygon & P) {
        double result = 0.0;
        int n = P.size();
        for (int i = 0; i < n; i++) {
                result += cross(P[i], P[(i+1)%n]);
        }
        return result / 2.0;
}

int leftmostIndex(vector<point> & P) {
        int ans = 0;
        for(int i=1; i<(int)P.size(); i++) {
                if (P[i] < P[ans]) ans = i;
        }
        return ans;
}

polygon make_polygon(vector<point> P) {
        if (signedArea(P) < 0.0) reverse(P.begin(), P.end());
        int li = leftmostIndex(P);
        reverse(P.begin(), P.begin()+li);
        reverse(P.begin()+li, P.end());
        reverse(P.begin(), P.end());
        return P;
}

/*
 * Minkowski sum
 */

polygon minkowski(polygon & A, polygon & B) {
        polygon P; point v1, v2;
        int n1 = A.size(), n2 = B.size();
        P.push_back(A[0]+B[0]);
        for(int i = 0, j = 0; i < n1 || j < n2;) {
                v1 = A[(i+1)%n1]-A[i%n1];
                v2 = B[(j+1)%n2]-B[j%n2];
                if (j == n2 || cross(v1, v2) > EPS) {
                        P.push_back(P.back() + v1); i++;
                }
                else if (i == n1 || cross(v1, v2) < -EPS) {
                        P.push_back(P.back() + v2); j++;
                }
                else {
                        P.push_back(P.back() + (v1+v2));
                        i++; j++;
                }
        }
        P.pop_back();
        return P;
}

/*
 * Triangle 2D
 */

struct triangle {
        point a, b, c;
        triangle() { a = b = c = point(); }
        triangle(point _a, point _b, point _c) : a(_a), b(_b), c(_c) {}
        int isInside(point p) {
                double u = cross(b-a,p-a)*cross(b-a,c-a);
                double v = cross(c-b,p-b)*cross(c-b,a-b);
                double w = cross(a-c,p-c)*cross(a-c,b-c);
                if (u > 0.0 && v > 0.0 && w > 0.0) return 0;
                if (u < 0.0 || v < 0.0 || w < 0.0) return 2;
                else return 1;
        } //0 = inside/ 1 = border/ 2 = outside
};

int isInsideTriangle(point a, point b, point c, point p) {
        return triangle(a,b,c).isInside(p);
} //0 = inside/ 1 = border/ 2 = outside

/*
 * Convex query
 */

bool query(polygon &P, point q) {
        int i = 1, j = P.size()-1, m;
        if (cross(P[i]-P[0], P[j]-P[0]) < -EPS)
                swap(i, j);
        while(abs(j-i) > 1) {
                int m = (i+j)/2;
                if (cross(P[m]-P[0], q-P[0]) < 0) j = m;
                else i = m;
        }
        return isInsideTriangle(P[0], P[i], P[j], q) != 2;
}

/*
 * Codeforces 87E
 */
#include <cstdio>

void printpolygon(polygon & P) {
        printf("printing polygon:\n");
        for(int i=0; i<(int)P.size(); i++) {
                printf("%.2f %.2f\n", P[i].x, P[i].y);
        }
}

polygon city[3], P;

int main() {
        double x, y;
        for(int i = 0, n; i < 3; i++) {
                scanf("%d", &n);
                P.clear();
                while(n --> 0) {
                        scanf("%lf %lf", &x, &y);
                        P.push_back(point(x, y));
                }
                city[i] = make_polygon(P);
        }
        P = minkowski(city[0], city[1]);
        P = minkowski(P, city[2]);

        int m;
        scanf("%d", &m);
        while(m --> 0) {
                scanf("%lf %lf", &x, &y);
                if (query(P, point(x, y)*3.0)) printf("YES\n");
                else printf("NO\n");
        }
        return 0;
}
```

7.7 Nearest Neighbour

```
// Closest Neighbor - O(n * log(n))
const ll N = 1e6+3, INF = 1e18;
ll n, cn[N], x[N], y[N]; // number of points, closes neighbor, x coordinates, y coordinates

ll sqr(ll i) { return i*i; }
ll dist(int i, int j) { return sqr(x[i]-x[j]) + sqr(y[i]-y[j]); }
ll dist(int i) { return i == cn[i] ? INF : dist(i, cn[i]); }

bool cpx(int i, int j) { return x[i] < x[j] or (x[i] == x[j] and y[i] < y[j]); }
```

```cpp
bool cpy(int i, int j) { return y[i] < y[j] or (y[i] == y[j] and x[i] < x[j]); }

ll calc(int i, ll x0) {
  ll dlt = dist(i) - sqr(x[i]-x0);
  return dlt >= 0 ? ceil(sqrt(dlt)) : -1;
}

void updt(int i, int j, ll x0, ll &dlt) {
  if (dist(i) > dist(i, j)) cn[i] = j, dlt = calc(i, x0);
}

void cmp(vi &u, vi &v, ll x0) {
  for(int a=0, b=0; a<u.size(); ++a) {
    ll i = u[a], dlt = calc(i, x0);
    while(b < v.size() and y[i] > y[v[b]]) b++;
    for(int j = b-1; j >= 0     and y[i] - dlt <= y[v[j]]; j--) updt(i, v[j], x0, dlt);
    for(int j = b; j < v.size() and y[i] + dlt >= y[v[j]]; j++) updt(i, v[j], x0, dlt);
  }
}

void slv(vi &ix, vi &iy) {
  int n = ix.size();
  if (n == 1) { cn[ix[0]] = ix[0]; return; }

  int m = ix[n/2];

  vi ix1, ix2, iy1, iy2;
  for(int i=0; i<n; ++i) {
    if (cpx(ix[i], m)) ix1.push_back(ix[i]);
    else ix2.push_back(ix[i]);

    if (cpx(iy[i], m)) iy1.push_back(iy[i]);
    else iy2.push_back(iy[i]);
  }

  slv(ix1, iy1);
  slv(ix2, iy2);

  cmp(iy1, iy2, x[m]);
  cmp(iy2, iy1, x[m]);
}

void slv(int n) {
  vi ix, iy;
  ix.resize(n);
  iy.resize(n);
  for(int i=0; i<n; ++i) ix[i] = iy[i] = i;
  sort(ix.begin(), ix.end(), cpx);
  sort(iy.begin(), iy.end(), cpy);
  slv(ix, iy);
}
```

7.8 Closest Pair of Points

```cpp
#include "basics.cpp"
#include "lines.cpp"

//Graham scan NOT TESTED ENOUGH, not safe, prefer monotone chain!
point origin;

int above(point p){
    if(p.y == origin.y) return p.x > origin.x;
    return p.y > origin.y;
}

bool cmp(point p, point q){
    int tmp = above(q) - above(p);
    if(tmp) return tmp > 0;
    return p.dir(origin,q) > 0;
    //Be Careful: p.dir(origin,q) == 0
}

// Graham Scan O(nlog(n))
vector<point> graham_hull(vector<point> pts) {
    vector<point> ch(pts.size());
    point mn = pts[0];

    for(point p : pts) if (p.y < mn.y or (p.y == mn.y and p.x < mn.x)) mn = p;

    origin = mn;
    sort(pts.begin(), pts.end(), cmp);

    int n = 0;

    // IF: Convex hull without collinear points
    // for(point p : pts) {
    //     while (n > 1 and ch[n-1].dir(ch[n-2], p) < 1) n--;
    //     ch[n++] = p;
    // }

    //ELSE IF: Convex hull with collinear points
    for(point p : pts) {
    while (n > 1 and ch[n-1].dir(ch[n-2], p) < 0) n--;
    ch[n++] = p;
    }

    /*this part not safe
    for(int i=pts.size()-1; i >=1; --i)
    if (n > 1 and pts[i] != ch[n-1] and !pts[i].dir(pts[0], ch[n-1]))
        ch[n++] = pts[i];*/
    // END IF

    ch.resize(n);
    return ch;
}

//Monotone chain O(nlog(n))
#define REMOVE_REDUNDANT
#ifdef REMOVE_REDUNDANT
bool between(const point &a, const point &b, const point &c) {
    return (fabs(area_2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void monotone_hull(vector<point> &pts) {
    sort(pts.begin(), pts.end());
    pts.erase(unique(pts.begin(), pts.end()), pts.end());
    vector<point> up, dn;
    for (int i = 0; i < pts.size(); i++) {
        while (up.size() > 1 && area_2(up[up.size()-2], up.back(), pts[i]) >= 0) up.pop_back();
        while (dn.size() > 1 && area_2(dn[dn.size()-2], dn.back(), pts[i]) <= 0) dn.pop_back();
        up.push_back(pts[i]);
        dn.push_back(pts[i]);
    }
    pts = dn;
    for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

    #ifdef REMOVE_REDUNDANT
    if (pts.size() <= 2) return;
    dn.clear();
    dn.push_back(pts[0]);
    dn.push_back(pts[1]);
    for (int i = 2; i < pts.size(); i++) {
        if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
        dn.push_back(pts[i]);
    }
    if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
        dn[0] = dn.back();
        dn.pop_back();
    }
    pts = dn;
    #endif
}

//avoid using long double for comparisons, change type and remove division by 2
ld compute_signed_area(const vector<point> &p) {
    ld area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area / 2.0;
}

ld compute_area(const vector<point> &p) {
    return fabs(compute_signed_area(p));
}

ld compute_perimeter(vector<point> &p) {
    ld per = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        per += p[i].dist(p[j]);
    }
    return per;
}

//not tested
// TODO: test this code. This code has not been tested, please do it before proper use.
// http://codeforces.com/problemset/problem/975/E is a good problem for testing.
point compute_centroid(vector<point> &p) {
    point c(0,0);
    ld scale = 6.0 * compute_signed_area(p);
    for (int i = 0; i < p.size(); i++){
        int j = (i+1) % p.size();
        c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
    }
    return c / scale;

}

// TODO: test this code. This code has not been tested, please do it before proper use.
// http://codeforces.com/problemset/problem/975/E is a good problem for testing.
point centroid(vector<point> &v) {
  int n = v.size();
  type da = 0;
  point m, c;

  for(point p : v) m = m + p;
  m = m / n;

  for(int i=0; i<n; ++i) {
    point p = v[i] - m, q = v[(i+1)%n] - m;
    type x = p % q;
    c = c + (p + q) * x;
    da += x;
  }

  return c / (3 * da);
}

//O(n^2)
bool is_simple(const vector<point> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (i == l || j == k) continue;
            if (segment_segment_intersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

bool point_in_triangle(point a, point b, point c, point cur){
    ll s1 = abs(cross(b - a, c - a));
    ll s2 = abs(cross(a - cur, b - cur)) + abs(cross(b - cur, c - cur)) + abs(cross(c - cur, a - cur));
    return s1 == s2;
}

void sort_lex_hull(vector<point> &hull){
    int n = hull.size();

    //Sort hull by x
    int pos = 0;
    for(int i = 1; i < n; i++) if(hull[i] <  hull[pos]) pos = i;
    rotate(hull.begin(), hull.begin() + pos, hull.end());
}

//determine if point is inside or on the boundary of a polygon (O(logn))
bool point_in_convex_polygon(vector<point> &hull, point cur){
    int n = hull.size();
    //Corner cases: point outside most left and most right wedges
    if(cur.dir(hull[0], hull[1]) != 0 && cur.dir(hull[0], hull[1]) != hull[n - 1].dir(hull[0], hull[1]))
        return false;
    if(cur.dir(hull[0], hull[n - 1]) != 0 && cur.dir(hull[0], hull[n - 1]) != hull[1].dir(hull[0], hull[n -
          1]))
        return false;

    //Binary search to find which wedges it is between
    int l = 1, r = n - 1;
    while(r - l > 1){
        int mid = (l + r)/2;
        if(cur.dir(hull[0], hull[mid]) <= 0)l = mid;
        else r = mid;
    }
    return point_in_triangle(hull[l], hull[l + 1], hull[0], cur);
}

// determine if point is on the boundary of a polygon (O(N))
bool point_on_polygon(vector<point> &p, point q) {
for (int i = 0; i < p.size(); i++)
    if (q.dist2(project_point_segment(p[i], p[(i+1)%p.size()], q)) < EPS) return true;
    return false;
}

//Shamos - Hoey for test polygon simple in O(nlog(n))
inline bool adj(int a, int b, int n) {return (b == (a + 1)%n or a == (b + 1)%n);}

struct edge{
    point ini, fim;
    edge(point ini = point(0,0), point fim = point(0,0)) : ini(ini), fim(fim) {}
};

//< here means the edge on the top will be at the begin
bool operator < (const edge& a, const edge& b) {
    if (a.ini == b.ini) return direction(a.ini, a.fim, b.fim) < 0;
    if (a.ini.x < b.ini.x) return direction(a.ini, a.fim, b.ini) < 0;
    return direction(a.ini, b.fim, b.ini) < 0;
}

bool is_simple_polygon(const vector<point> &pts){
    vector <pair<point, pii>> eve;
    vector <pair<edge, int>> edgs;
    set <pair<edge, int>> sweep;
    int n = (int)pts.size();
    for(int i = 0; i < n; i++){
        point l = min(pts[i], pts[(i + 1)%n]);
        point r = max(pts[i], pts[(i + 1)%n]);
        eve.pb({l, {0, i}});
        eve.pb({r, {1, i}});
        edgs.pb(make_pair(edge(l, r), i));
    }
    sort(eve.begin(), eve.end());
    for(auto e : eve){
        if(!e.nd.st){
            auto cur = sweep.lower_bound(edgs[e.nd.nd]);
            pair<edge, int> above, below;
            if(cur != sweep.end()){
                below = *cur;
                if(!adj(below.nd, e.nd.nd, n) and segment_segment_intersect(pts[below.nd], pts[(below.nd + 1)
                      %n], pts[e.nd.nd], pts[(e.nd.nd + 1)%n]))
                    return false;
            }
            if(cur != sweep.begin()){
                above = *(--cur);
                if(!adj(above.nd, e.nd.nd, n) and segment_segment_intersect(pts[above.nd], pts[(above.nd + 1)
                      %n], pts[e.nd.nd], pts[(e.nd.nd + 1)%n]))
                    return false;
            }
            sweep.insert(edgs[e.nd.nd]);
        }
        else{
            auto below = sweep.upper_bound(edgs[e.nd.nd]);
            auto cur = below, above = --cur;
            if(below != sweep.end() and above != sweep.begin()){
                --above;
                if(!adj(below->nd, above->nd, n) and segment_segment_intersect(pts[below->nd], pts[(below->nd
                       + 1)%n], pts[above->nd], pts[(above->nd + 1)%n]))
                    return false;
            }
            sweep.erase(cur);
        }
    }
    return true;
}

//code copied from https://github.com/tfg50/Competitive-Programming/blob/master/Biblioteca/Math/2D%20Geometry
      /ConvexHull.cpp
int maximize_scalar_product(vector<point> &hull, point vec) {
        // this code assumes that there are no 3 colinear points
        int ans = 0;
        int n = hull.size();
        if(n < 20) {
                for(int i = 0; i < n; i++) {
                        if(hull[i] * vec > hull[ans] * vec) {
                                ans = i;
                        }
                }
        } else {
                if(hull[1] * vec > hull[ans] * vec) {
                        ans = 1;
                }
                for(int rep = 0; rep < 2; rep++) {
                        int l = 2, r = n - 1;
                        while(l != r) {
                                int mid = (l + r + 1) / 2;
                                bool flag = hull[mid] * vec >= hull[mid-1] * vec;
                                if(rep == 0) { flag = flag && hull[mid] * vec >= hull[0] * vec; }
                                else { flag = flag || hull[mid-1] * vec < hull[0] * vec; }
                                if(flag) {
                                        l = mid;
                                } else {
                                        r = mid - 1;
                                }
                        }
                        if(hull[ans] * vec < hull[l] * vec) {
                                ans = l;
                        }
                }
        }
        return ans;
}

//find tangents related to a point outside the polygon, essentially the same for maximizing scalar product
int tangent(vector<point> &hull, point vec, int dir_flag) {
        // this code assumes that there are no 3 colinear points
    // dir_flag = -1 for right tangent
    // dir_flag =  1 for left taangent
        int ans = 0;
        int n = hull.size();
        if(n < 20) {
                for(int i = 0; i < n; i++) {
                        if(hull[ans].dir(vec, hull[i]) == dir_flag) {
                                ans = i;
                        }
                }
        } else {
                if(hull[ans].dir(vec, hull[1]) == dir_flag) {
                        ans = 1;
                }
                for(int rep = 0; rep < 2; rep++) {
                        int l = 2, r = n - 1;
                        while(l != r) {
                                int mid = (l + r + 1) / 2;
                                bool flag = hull[mid - 1].dir(vec, hull[mid]) == dir_flag;
                                if(rep == 0) { flag = flag && (hull[0].dir(vec, hull[mid]) == dir_flag); }
                                else { flag = flag || (hull[0].dir(vec, hull[mid - 1]) != dir_flag); }
                                if(flag) {
                                        l = mid;
                                } else {
                                        r = mid - 1;
                                }
                        }
                        if(hull[ans].dir(vec, hull[l]) == dir_flag) {
                                ans = l;
                        }
                }
        }
        return ans;
}
```

7.9 Radial Sort

```
#include "basics.cpp"
point origin;

int above(point p){
    if(p.y == origin.y) return p.x > origin.x;
    return p.y > origin.y;
}

bool cmp(point p, point q){
    int tmp = above(q) - above(p);
    if(tmp) return tmp > 0;
    return p.dir(origin,q) > 0;
    //Be Careful: p.dir(origin,q) == 0
}
```

7.10 Stanford Delaunay

```
// Slow but simple Delaunay triangulation. Does not handle
// degenerate cases (from O'Rourke, Computational Geometry in C)
//
// Running time: O(n^4)
//
// INPUT:    x[] = x-coordinates
//           y[] = y-coordinates
//
// OUTPUT:   triples = a vector containing m triples of indices
//                     corresponding to triangle vertices

#include<vector>
using namespace std;

typedef double T;

struct triple {
    int i, j, k;
    triple() {}
    triple(int i, int j, int k) : i(i), j(j), k(k) {}
};

vector<triple> delaunayTriangulation(vector<T>& x, vector<T>& y) {
        int n = x.size();
        vector<T> z(n);
        vector<triple> ret;

        for (int i = 0; i < n; i++)
            z[i] = x[i] * x[i] + y[i] * y[i];

        for (int i = 0; i < n-2; i++) {
            for (int j = i+1; j < n; j++) {
                for (int k = i+1; k < n; k++) {
                    if (j == k) continue;
                    double xn = (y[j]-y[i])*(z[k]-z[i]) - (y[k]-y[i])*(z[j]-z[i]);
                    double yn = (x[k]-x[i])*(z[j]-z[i]) - (x[j]-x[i])*(z[k]-z[i]);
                    double zn = (x[j]-x[i])*(y[k]-y[i]) - (x[k]-x[i])*(y[j]-y[i]);
                    bool flag = zn < 0;
                    for (int m = 0; flag && m < n; m++)
                        flag = flag && ((x[m]-x[i])*xn +
                                        (y[m]-y[i])*yn +
                                        (z[m]-z[i])*zn <= 0);
                    if (flag) ret.push_back(triple(i, j, k));
                }
            }
        }
        return ret;
}

int main()
{
    T xs[]={0, 0, 1, 0.9};
    T ys[]={0, 1, 0, 0.9};
    vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
    vector<triple> tri = delaunayTriangulation(x, y);

    //expected: 0 1 3
    //          0 3 2

    int i;
    for(i = 0; i < tri.size(); i++)
        printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
    return 0;
}
```

7.11 Ternary Search

```
//Ternary Search - O(log(n))
//Max version, for minimum version just change signals

ll ternary_search(ll l, ll r){
    while(r - l > 3) {
        ll m1 = (l+r)/2;
        ll m2 = (l+r)/2 + 1;
        ll f1 = f(m1), f2 = f(m2);
        //if(f1 > f2) l = m1;
        if (f1 < f2) l = m1;
        else r = m2;
    }
    ll ans = 0;
    for(int i = l; i <= r; i++){
        ll tmp = f(i);
        //ans = min(ans, tmp);
        ans = max(ans, tmp);
    }
    return ans;
}

//Faster version - 300 iteratons up to 1e-6 precision
double ternary_search(double l, double r, int No = 300){
    // for(int i = 0; i < No; i++){
    while(r - l > EPS){
        double m1 = l + (r - l) / 3;
        double m2 = r - (r - l) / 3;
        // if (f(m1) > f(m2))
        if (f(m1) < f(m2))
            l = m1;
        else
            r = m2;
    }
    return f(l);
}
```

7.12 Voronoi Diagram

```cpp
/*
Complexity: O(nlogn)
Code by Monogon: https://codeforces.com/blog/entry/85638
This code doesn't work when two points have the same x coordinate.
This is handled simply by rotating all input points by 1 radian and praying to the geometry gods.
*/

#include <bits/stdc++.h>

#define ll long long
#define sz(x) ((int) (x).size())
#define all(x) (x).begin(), (x).end()
#define vi vector<int>
#define pii pair<int, int>
#define rep(i, a, b) for(int i = (a); i < (b); i++)
using namespace std;
template<typename T>
using minpq = priority_queue<T, vector<T>, greater<T>>;

using ftype = long double;
const ftype EPS = 1e-12, INF = 1e100;

struct pt {
    ftype x, y;
    pt(ftype x = 0, ftype y = 0) : x(x), y(y) {}

    // vector addition, subtraction, scalar multiplication
    pt operator+(const pt &o) const {
        return pt(x + o.x, y + o.y);
    }
    pt operator-(const pt &o) const {
        return pt(x - o.x, y - o.y);
    }
    pt operator*(const ftype &f) const {
        return pt(x * f, y * f);
    }

    // rotate 90 degrees counter-clockwise
    pt rot() const {
        return pt(-y, x);
    }

    // dot and cross products
    ftype dot(const pt &o) const {
        return x * o.x + y * o.y;
    }
    ftype cross(const pt &o) const {
        return x * o.y - y * o.x;
    }

    // length
    ftype len() const {
        return hypotl(x, y);
    }

    // compare points lexicographically
    bool operator<(const pt &o) const {
        return make_pair(x, y) < make_pair(o.x, o.y);
    }
};

// check if two vectors are collinear. It might make sense to use a
// different EPS here, especially if points have integer coordinates
bool collinear(pt a, pt b) {
    return abs(a.cross(b)) < EPS;
}


// intersection point of lines ab and cd. Precondition is that they aren't collinear
pt lineline(pt a, pt b, pt c, pt d) {
    return a + (b - a) * ((c - a).cross(d - c) / (b - a).cross(d - c));
}

// circumcircle of points a, b, c. Precondition is that abc is a non-degenerate triangle.
pt circumcenter(pt a, pt b, pt c) {
    b = (a + b) * 0.5;
    c = (a + c) * 0.5;
    return lineline(b, b + (b - a).rot(), c, c + (c - a).rot());
}

// x coordinate of sweep-line
ftype sweepx;

// an arc on the beacah line is given implicitly by the focus p,
// the focus q of the following arc, and the position of the sweep-line.
struct arc {
    mutable pt p, q;
    mutable int id = 0, i;
    arc(pt p, pt q, int i) : p(p), q(q), i(i) {}

    // get y coordinate of intersection with following arc.
    // don't question my magic formulas
    ftype gety(ftype x) const {
        if(q.y == INF) return INF;
        x += EPS;
        pt med = (p + q) * 0.5;
        pt dir = (p - med).rot();
        ftype D = (x - p.x) * (x - q.x);
        return med.y + ((med.x - x) * dir.x + sqrtl(D) * dir.len()) / dir.y;
    }
    bool operator<(const ftype &y) const {
        return gety(sweepx) < y;
    }
    bool operator<(const arc &o) const {
        return gety(sweepx) < o.gety(sweepx);
    }
};

// the beach line will be stored as a multiset of arc objects
using beach = multiset<arc, less<>>;

// an event is given by
//     x: the time of the event
//     id: If >= 0, it's a point event for index id.
//         If < 0, it's an ID for a vertex event
//     it: if a vertex event, the iterator for the arc to be deleted
struct event {
    ftype x;
    int id;
    beach::iterator it;
    event(ftype x, int id, beach::iterator it) : x(x), id(id), it(it) {}
    bool operator<(const event &e) const {
        return x > e.x;
    }
};

struct fortune {
    beach line; // self explanatory
    vector<pair<pt, int>> v; // (point, original index)
    priority_queue<event> Q; // priority queue of point and vertex events
    vector<pii> edges; // delaunay edges
    vector<bool> valid; // valid[-id] == true if the vertex event with corresponding id is valid
    int n, ti; // number of points, next available vertex ID
    fortune(vector<pt> p) {
        n = sz(p);
        v.resize(n);
        rep(i, 0, n) v[i] = {p[i], i};
        sort(all(v)); // sort points by coordinate, remember original indices for the delaunay edges
    }
    // update the remove event for the arc at position it
    void upd(beach::iterator it) {
        if(it->i == -1) return; // doesn't correspond to a real point
        valid[-it->id] = false; // mark existing remove event as invalid
        auto a = prev(it);
        if(collinear(it->q - it->p, a->p - it->p)) return; // doesn't generate a vertex event
        it->id = --ti; // new vertex event ID
        valid.push_back(true); // label this ID true
        pt c = circumcenter(it->p, it->q, a->p);
        ftype x = c.x + (c - it->p).len();
        // event is generated at time x.
        // make sure it passes the sweep-line, and that the arc truly shrinks to 0
        if(x > sweepx - EPS && a->gety(x) + EPS > it->gety(x)) {
            Q.push(event(x, it->id, it));
        }
    }
    // add Delaunay edge
    void add_edge(int i, int j) {
        if(i == -1 || j == -1) return;
        edges.push_back({v[i].second, v[j].second});
    }
    // handle a point event
    void add(int i) {
        pt p = v[i].first;
        // find arc to split
        auto c = line.lower_bound(p.y);
        // insert new arcs. passing the following iterator gives a slight speed-up
        auto b = line.insert(c, arc(p, c->p, i));
        auto a = line.insert(b, arc(c->p, p, c->i));
        add_edge(i, c->i);
        upd(a); upd(b); upd(c);
    }
    // handle a vertex event
    void remove(beach::iterator it) {
        auto a = prev(it);
        auto b = next(it);
        line.erase(it);
        a->q = b->p;
        add_edge(a->i, b->i);
        upd(a); upd(b);
    }
    // X is a value exceeding all coordinates
    void solve(ftype X = 1e9) {
        // insert two points that will always be in the beach line,
        // to avoid handling edge cases of an arc being first or last
        X *= 3;
        line.insert(arc(pt(-X, -X), pt(-X, X), -1));
        line.insert(arc(pt(-X, X), pt(INF, INF), -1));
        // create all point events
        rep(i, 0, n) {
            Q.push(event(v[i].first.x, i, line.end()));
        }
        ti = 0;
        valid.assign(1, false);
        while(!Q.empty()) {
            event e = Q.top(); Q.pop();
            sweepx = e.x;
            if(e.id >= 0) {
                add(e.id);
            }else if(valid[-e.id]) {
                remove(e.it);
            }
        }
    }
};
```

8 Miscellaneous

8.1 Bitset

```
//Goes through the subsets of a set x :
int b = 0;
do {
// process subset b
} while (b=(b-x)&x);
```

8.2 builtin

```
__builtin_ctz(x) // trailing zeroes
__builtin_clz(x) // leading zeroes
__builtin_popcount(x) // # bits set
__builtin_ffs(x) // index(LSB) + 1 [0 if x==0]

// Add ll to the end for long long [__builtin_clzll(x)]
```

8.3 Date

```
struct Date {
  int d, m, y;
  static int mnt[], mntsum[];

  Date() : d(1), m(1), y(1) {}
  Date(int d, int m, int y) : d(d), m(m), y(y) {}
  Date(int days) : d(1), m(1), y(1) { advance(days); }

  bool bissexto() { return (y%4 == 0 and y%100) or (y%400 == 0); }

  int mdays() { return mnt[m] + (m == 2)*bissexto(); }
  int ydays() { return 365+bissexto(); }

  int msum()  { return mntsum[m-1] + (m > 2)*bissexto(); }
  int ysum()  { return 365*(y-1) + (y-1)/4 - (y-1)/100 + (y-1)/400; }

  int count() { return (d-1) + msum() + ysum(); }

  int day() {
    int x = y - (m<3);
    return (x + x/4 - x/100 + x/400 + mntsum[m-1] + d + 6)%7;
  }

  void advance(int days) {
    days += count();
    d = m = 1, y = 1 + days/366;
    days -= count();
    while(days >= ydays()) days -= ydays(), y++;
    while(days >= mdays()) days -= mdays(), m++;
    d += days;
  }
};

int Date::mnt[13] = {0, 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};
int Date::mntsum[13] = {};
for(int i=1; i<13; ++i) Date::mntsum[i] = Date::mntsum[i-1] + Date::mnt[i];
```

8.4 Parentesis to Poslish (ITA)

```
#include <cstdio>
#include <map>
#include <stack>
using namespace std;

/*
 * Parenthetic to polish expression conversion
 */

inline bool isOp(char c) {
        return c=='+' || c=='-' || c=='*' || c=='/' || c=='^';
}

inline bool isCarac(char c) {
        return (c>='a' && c<='z') || (c>='A' && c<='Z') || (c>='0' && c<='9');
}

int paren2polish(char* paren, char* polish) {
        map<char, int> prec;
        prec['('] = 0;
        prec['+'] = prec['-'] = 1;
        prec['*'] = prec['/'] = 2;
        prec['^'] = 3;
        int len = 0;
        stack<char> op;
        for (int i = 0; paren[i]; i++) {
                if (isOp(paren[i])) {
                        while (!op.empty() && prec[op.top()] >= prec[paren[i]]) {
                                polish[len++] = op.top(); op.pop();
                        }
                        op.push(paren[i]);
                }
                else if (paren[i]=='(') op.push('(');
                else if (paren[i]==')') {
                        for (; op.top()!='('; op.pop())
                                polish[len++] = op.top();
                        op.pop();
                }
                else if (isCarac(paren[i]))
                        polish[len++] = paren[i];
        }
        for(; !op.empty(); op.pop())
                polish[len++] = op.top();
        polish[len] = 0;
        return len;
}

/*
 * TEST MATRIX
 */

int main() {
        int N, len;
        char polish[400], paren[400];
        scanf("%d", &N);
        for (int j=0; j<N; j++) {
                scanf(" %s", paren);
                paren2polish(paren, polish);
                printf("%s\n", polish);
        }
        return 0;
}
```

8.5 Merge Sort (Inversion Count)

```cpp
// Merge-sort with inversion count - O(nlog n)

int n, inv;
vector<int> v, ans;

void mergesort(int l, int r, vector<int> &v){
    if(l == r) return;
    int mid = (l+r)/2;
    mergesort(l, mid, v), mergesort(mid+1, r, v);
    int i = l, j = mid + 1, k = l;
    while(i <= mid or j <= r){
        if(i <= mid and (j > r or v[i] <= v[j])) ans[k++] = v[i++];
        else ans[k++] = v[j++], inv += j-k;
    }
    for(int i = l; i <= r; i++) v[i] = ans[i];
}

//in main
ans.resize(v.size());
```

8.6 Modular Int (Struct)

```cpp
// Struct to do basic modular arithmetic

template <int MOD>
struct Modular {
  int v;

  static int minv(int a, int m) {
    a %= m;
    assert(a);
    return a == 1 ? 1 : int(m - ll(minv(m, a)) * ll(m) / a);
  }

  Modular(ll _v = 0) : v(int(_v % MOD)) {
    if (v < 0) v += MOD;
  }

  bool operator==(const Modular& b) const { return v == b.v; }
  bool operator!=(const Modular& b) const { return v != b.v; }

  friend Modular inv(const Modular& b) { return Modular(minv(b.v, MOD)); }

  friend ostream& operator<<(ostream& os, const Modular& b) { return os << b.v; }
  friend istream& operator>>(istream& is, Modular& b) {
    ll _v;
    is >> _v;
    b = Modular(_v);
    return is;
  }

  Modular operator+(const Modular& b) const {
    Modular ans;
    ans.v = v >= MOD - b.v ? v + b.v - MOD : v + b.v;
    return ans;
  }

  Modular operator-(const Modular& b) const {
    Modular ans;
    ans.v = v < b.v ? v - b.v + MOD : v - b.v;
    return ans;
  }

  Modular operator*(const Modular& b) const {
    Modular ans;
    ans.v = int(ll(v) * ll(b.v) % MOD);
    return ans;
  }

  Modular operator/(const Modular& b) const {
    return (*this) * inv(b);
  }

  Modular& operator+=(const Modular& b) { return *this = *this + b; }
  Modular& operator-=(const Modular& b) { return *this = *this - b; }
  Modular& operator*=(const Modular& b) { return *this = *this * b; }
  Modular& operator/=(const Modular& b) { return *this = *this / b; }
};

using Mint = Modular<MOD>;
```

8.7 Parallel Binary Search

```cpp
// Parallel Binary Search - O(nlog n * cost to update data structure + qlog n * cost for binary search
      condition)

struct Query { int i, ans; /*+ query related info*/ };
vector<Query> req;

void pbs(vector<Query>& qs, int l /* = min value*/, int r /* = max value*/) {
  if (qs.empty()) return;

  if (l == r) {
    for (auto& q : qs) req[q.i].ans = l;
    return;
  }

  int mid = (l + r) / 2;
  // mid = (l + r + 1) / 2 if different from simple upper/lower bound

  for (int i = l; i <= mid; i++) {
    // add value to data structure
  }

  vector<Query> vl, vr;
  for (auto& q : qs) {
    if (/* cond */) vl.push_back(q);
    else vr.push_back(q);
  }

  pbs(vr, mid + 1, r);

  for (int i = l; i <= mid; i++) {
    // remove value from data structure
  }

  pbs(vl, l, mid);
}
```

8.8 prime numbers

```
   2    3    5    7   11   13   17   19   23   29
  31   37   41   43   47   53   59   61   67   71
  73   79   83   89   97  101  103  107  109  113
 127  131  137  139  149  151  157  163  167  173
 179  181  191  193  197  199  211  223  227  229
 233  239  241  251  257  263  269  271  277  281
 283  293  307  311  313  317  331  337  347  349
 353  359  367  373  379  383  389  397  401  409
 419  421  431  433  439  443  449  457  461  463
 467  479  487  491  499  503  509  521  523  541
 547  557  563  569  571  577  587  593  599  601
 607  613  617  619  631  641  643  647  653  659
 661  673  677  683  691  701  709  719  727  733
 739  743  751  757  761  769  773  787  797  809
 811  821  823  827  829  839  853  857  859  863
 877  881  883  887  907  911  919  929  937  941
 947  953  967  971  977  983  991  997 1009 1013
1019 1021 1031 1033 1039 1049 1051 1061 1063 1069
1087 1091 1093 1097 1103 1109 1117 1123 1129 1151
1153 1163 1171 1181 1187 1193 1201 1213 1217 1223
1229 1231 1237 1249 1259 1277 1279 1283 1289 1291
1297 1301 1303 1307 1319 1321 1327 1361 1367 1373
1381 1399 1409 1423 1427 1429 1433 1439 1447 1451
1453 1459 1471 1481 1483 1487 1489 1493 1499 1511
1523 1531 1543 1549 1553 1559 1567 1571 1579 1583
1597 1601 1607 1609 1613 1619 1621 1627 1637 1657
1663 1667 1669 1693 1697 1699 1709 1721 1723 1733
1741 1747 1753 1759 1777 1783 1787 1789 1801 1811
1823 1831 1847 1861 1867 1871 1873 1877 1879 1889
1901 1907 1913 1931 1933 1949 1951 1973 1979 1987

      970’997        971’483   921’281’269   999’279’733
1’000’000’009 1’000’000’021 1’000’000’409 1’005’012’527
```

8.9 Python

```
# reopen
import sys
sys.stdout = open('out','w')
sys.stdin  = open('in' ,'r')

//Dummy example
R = lambda: map(int, input().split())
n, k = R(),
v, t = [], [0]*n
for p, c, i in sorted(zip(R(), R(), range(n))):
    t[i] = sum(v)+c
    v += [c]
    v = sorted(v)[::-1]
    if len(v) > k:
        v.pop()
print(' '.join(map(str, t)))
```

8.10 Sqrt Decomposition

```
// Square Root Decomposition (Mo's Algorithm) - O(n^(3/2))
const int N = 1e5+1, SQ = 500;
int n, m, v[N];

void add(int p) { /* add value to aggregated data structure */ }
void rem(int p) { /* remove value from aggregated data structure */ }

struct query { int i, l, r, ans; } qs[N];

bool c1(query a, query b) {
  if(a.l/SQ != b.l/SQ) return a.l < b.l;
  return a.l/SQ&1 ? a.r > b.r : a.r < b.r;
}

bool c2(query a, query b) { return a.i < b.i; }

/* inside main */
int l = 0, r = -1;
sort(qs, qs+m, c1);
for (int i = 0; i < m; ++i) {
  query &q = qs[i];
  while (r < q.r) add(v[++r]);
  while (r > q.r) rem(v[r--]);
  while (l < q.l) rem(v[l++]);
  while (l > q.l) add(v[--l]);

  q.ans = /* calculate answer */;
}

sort(qs, qs+m, c2); // sort to original order
```

8.11 Latitude Longitude (Stanford)

```
/*
Converts from rectangular coordinates to latitude/longitude and vice
versa. Uses degrees (not radians).
*/

#include <iostream>
#include <cmath>

using namespace std;

struct ll
{
  double r, lat, lon;
};

struct rect
{
  double x, y, z;
};

ll convert(rect& P)
{
  ll Q;
  Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
  Q.lat = 180/M_PI*asin(P.z/Q.r);
  Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));

  return Q;
}

rect convert(ll& Q)
{
  rect P;
  P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.z = Q.r*sin(Q.lat*M_PI/180);

  return P;
}

int main()
{
  rect A;
  ll B;

  A.x = -1.0; A.y = 2.0; A.z = -3.0;

  B = convert(A);
  cout << B.r << " " << B.lat << " " << B.lon << endl;

  A = convert(B);
  cout << A.x << " " << A.y << " " << A.z << endl;
}
```

8.12 Week day

```
int v[] = { 0, 3, 2, 5, 0, 3, 5, 1, 4, 6, 2, 4 };
int day(int d, int m, int y) {
  y -= m<3;
  return (y + y/4 - y/100 + y/400 + v[m-1] + d)%7;
}
```

9 Math Extra

9.1 Combinatorial formulas

$\sum_{k=0}^{n} k^2 = n(n+1)(2n+1)/6$
$\sum_{k=0}^{n} k^3 = n^2(n+1)^2/4$
$\sum_{k=0}^{n} k^4 = (6n^5+15n^4+10n^3-n)/30$
$\sum_{k=0}^{n} k^5 = (2n^6+6n^5+5n^4-n^2)/12$
$\sum_{k=0}^{n} x^k = (x^{n+1}-1)/(x-1)$
$\sum_{k=0}^{n} kx^k = (x-(n+1)x^{n+1}+nx^{n+2})/(x-1)^2$
$\binom{n}{k} = \frac{n!}{(n-k)!k!}$
$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$
$\binom{n}{k} = \frac{n}{n-k}\binom{n-1}{k}$
$\binom{n}{k} = \frac{n-k+1}{k}\binom{n}{k-1}$
$\binom{n+1}{k} = \frac{n+1}{n-k+1}\binom{n}{k}$
$\binom{n}{k+1} = \frac{n-k}{k+1}\binom{n}{k}$
$\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$
$\sum_{k=1}^{n} k^2\binom{n}{k} = (n+n^2)2^{n-2}$
$\binom{m+n}{r} = \sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k}$

$\binom{n}{k} = \prod_{i=1}^{k} \frac{n-k+i}{i}$

9.2 Number theory identities

Lucas' Theorem: For non-negative integers m and n and a prime p,

$$\binom{m}{n} \equiv \prod_{i=0}^{k} \binom{m_i}{n_i} \pmod{p},$$

where

$$m = m_k p^k + m_{k-1} p^{k-1} + \cdots + m_1 p + m_0$$

is the base p representation of m, and similarly for n.

9.3 Stirling Numbers of the second kind

Number of ways to partition a set of n numbers into k non-empty subsets.

$$\left\{ {n \atop k} \right\} = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{(k-j)} \binom{k}{j} j^n$$

Recurrence relation:

$$\left\{ {0 \atop 0} \right\} = 1$$

$$\left\{ {n \atop 0} \right\} = \left\{ {0 \atop n} \right\} = 1$$

$$\left\{ {n+1 \atop k} \right\} = k \left\{ {n \atop k} \right\} + \left\{ {n \atop k-1} \right\}$$

9.4 Burnside's Lemma

Let G be a finite group that acts on a set X. For each g in G let X^g denote the set of elements in X that are fixed by g, which means $X^g = \{x \in X | g(x) = x\}$. Burnside's lemma assers the following formula for the number of orbits, denoted $|X/G|$:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

9.5 Numerical integration

RK4: to integrate $\dot{y} = f(t, y)$ with $y_0 = y(t_0)$, compute

$$\begin{aligned} k_1 &= f(t_n, y_n) \\ k_2 &= f(t_n + \frac{h}{2}, y_n + \frac{h}{2} k_1) \\ k_3 &= f(t_n + \frac{h}{2}, y_n + \frac{h}{2} k_2) \\ k_4 &= f(t_n + h, y_n + h k_3) \\ y_{n+1} &= y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{aligned}$$

	S	R	X	Assunto	Descricao	Diff
A						
B						
C						
D						
E						
F						
G						
H						
I						
J						
K						
L						