Kinematics

The transport theorem states that the inertial derivative of a vector u is

$$\frac{\mathrm{d}^f \vec{u}}{\mathrm{d}t} = \frac{\mathrm{d}^r \vec{u}}{\mathrm{d}t} + \vec{\omega}^{rf} \times \vec{u}$$

Where $\frac{\mathrm{d}^f}{\mathrm{d}t}$ denotes the derivative in the coordinates of the fixed frame f, and $\frac{\mathrm{d}^r}{\mathrm{d}t}$ denotes derivative in the coordinates of the rotating frame r, and $\vec{\omega}^{rf}$ denotes the angular velocity of r in f.

For a position vector $\vec{r}$, we will find the velocity $\dot{\vec{r}}$ and acceleration $\ddot{\vec{r}}$. Note that dot notation implies inertial.

Diagrams

The circular restricted three body problem (CR3BP) is a special case of the three body problem. In the CR3BP (much like in Keplerian 2-body dynamics), we neglect the mass of satellite S, while treating the larger celestial body B_1 and smaller celestial body B_2 as point masses. Crucially, these bodies must orbit one another in circular orbits. In other words, they both orbit about their inertially stationary barycenter c *at constant velocity and distance*.

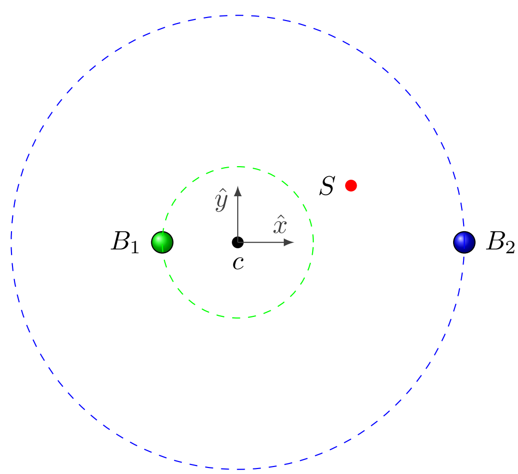

Figure 1: Geometry. S is the satellite, B are the celestial bodies. $\hat{z}$ implied out of page

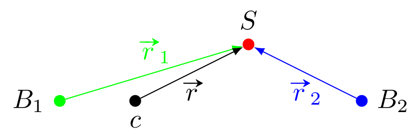

Figure 2: Vectors. Note d_1 refers to the distance from c to B_1 and d_2 refers to the distance from c to B_2. Predictably, $d_{12} = d_1 + d_2$ is the distance between the bodies

Because $\hat{x}$ points from c to B_2, which is not inertially stationary, the xyz frame is rotating. Specifically, it is rotating positively about z. Because the celestial bodies are in a circular orbit, their rates of rotation about c are constant. This means that the xyz frame rotates at a constant rate of $\vec{\omega} = \Omega\hat{z}$