System Dynaics for 8-Body System in 3 Dimensions with Newtonian Point-Masses

Polaris via Python Code March 24, 2024

 $0 \quad 0 \quad 0$ $0 \quad 0 \quad 0$ 0 0 0 0 0 0 0 $0 \quad 0 \quad 0$ $0 \quad 0 \quad -\frac{Gm_1}{r_3^3} - \frac{Gm_2}{r_3^3} - \frac{Gm_3}{r_3^3} - \frac{Gm_4}{r_3^3} - \frac{Gm_6}{r_3^3} - \frac{Gm_7}{r_3^3} - \frac{Gm_8}{r_3^3}$ $0 \ 0 \ 0$ 0 0 0 $egin{array}{cccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \end{array}$ $0 \quad 0 \quad 0$ 0 0 0 $0 \quad 0 \quad 0$ $0 \quad 0 \quad 0$

First Order ODEs $\frac{\mathrm{d}t}{\mathrm{d}t}\dot{x_1} = -\frac{Gm_2}{r_{1,2}^3}(x_1 - x_2) - \frac{Gm_3}{r_{1,3}^3}(x_1 - x_3) - \frac{Gm_4}{r_{1,4}^3}(x_1 - x_4) - \frac{Gm_5}{r_{1,5}^3}(x_1 - x_5) - \frac{Gm_6}{r_{1,6}^3}(x_1 - x_6) - \frac{Gm_7}{r_{1,7}^3}(x_1 - x_7) - \frac{Gm_8}{r_{1,8}^3}(x_1 - x_8)$

$\frac{\mathrm{d}}{\mathrm{d}t}y_1 = \dot{y_1}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{y_1} = -\frac{Gm_2}{r_{1,2}^3}(x_1 - x_2) - \frac{Gm_3}{r_{1,3}^3}(x_1 - x_3) - \frac{Gm_4}{r_{1,4}^3}(x_1 - x_4) - \frac{Gm_5}{r_{1,5}^3}(x_1 - x_5) - \frac{Gm_6}{r_{1,6}^3}(x_1 - x_6) - \frac{Gm_7}{r_{1,7}^3}(x_1 - x_7) - \frac{Gm_8}{r_{1,8}^3}(x_1 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}z_{1} = \dot{z}_{1}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{z}_{1} = -\frac{Gm_{2}}{r_{1,2}^{3}}(x_{1} - x_{2}) - \frac{Gm_{3}}{r_{1,3}^{3}}(x_{1} - x_{3}) - \frac{Gm_{4}}{r_{1,4}^{3}}(x_{1} - x_{4}) - \frac{Gm_{5}}{r_{1,5}^{3}}(x_{1} - x_{5}) - \frac{Gm_{6}}{r_{1,6}^{3}}(x_{1} - x_{6}) - \frac{Gm_{7}}{r_{1,7}^{3}}(x_{1} - x_{7}) - \frac{Gm_{8}}{r_{1,8}^{3}}(x_{1} - x_{8})$ $\frac{\mathrm{d}}{\mathrm{d}t}x_2 = \dot{x_2}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{x_2} = -\frac{Gm_1}{r_{2,1}^3}(x_2 - x_1) - \frac{Gm_3}{r_{2,3}^3}(x_2 - x_3) - \frac{Gm_4}{r_{2,4}^3}(x_2 - x_4) - \frac{Gm_5}{r_{2,5}^3}(x_2 - x_5) - \frac{Gm_6}{r_{2,6}^3}(x_2 - x_6) - \frac{Gm_7}{r_{2,7}^3}(x_2 - x_7) - \frac{Gm_8}{r_{2,8}^3}(x_2 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}y_2 = \dot{y}_2$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{y}_2 = -\frac{Gm_1}{r_{2,1}^3}(x_2 - x_1) - \frac{Gm_3}{r_{2,3}^3}(x_2 - x_3) - \frac{Gm_4}{r_{2,4}^3}(x_2 - x_4) - \frac{Gm_5}{r_{2,5}^3}(x_2 - x_5) - \frac{Gm_6}{r_{2,6}^3}(x_2 - x_6) - \frac{Gm_7}{r_{2,7}^3}(x_2 - x_7) - \frac{Gm_8}{r_{2,8}^3}(x_2 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}z_2 = \dot{z}_2$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{z}_2 = -\frac{Gm_1}{r_{2,1}^3}(x_2 - x_1) - \frac{Gm_3}{r_{2,3}^3}(x_2 - x_3) - \frac{Gm_4}{r_{2,4}^3}(x_2 - x_4) - \frac{Gm_5}{r_{2,5}^3}(x_2 - x_5) - \frac{Gm_6}{r_{2,6}^3}(x_2 - x_6) - \frac{Gm_7}{r_{2,7}^3}(x_2 - x_7) - \frac{Gm_8}{r_{2,8}^3}(x_2 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}x_3 = \dot{x}_3$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{x}_3 = -\frac{Gm_1}{r_{3,1}^3}(x_3 - x_1) - \frac{Gm_2}{r_{3,2}^3}(x_3 - x_2) - \frac{Gm_4}{r_{3,4}^3}(x_3 - x_4) - \frac{Gm_5}{r_{3,5}^3}(x_3 - x_5) - \frac{Gm_6}{r_{3,6}^3}(x_3 - x_6) - \frac{Gm_7}{r_{3,7}^3}(x_3 - x_7) - \frac{Gm_8}{r_{3,8}^3}(x_3 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}y_3 = \dot{y}_3$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{y}_3 = -\frac{Gm_1}{r_{3,1}^3}(x_3 - x_1) - \frac{Gm_2}{r_{3,2}^3}(x_3 - x_2) - \frac{Gm_4}{r_{3,4}^3}(x_3 - x_4) - \frac{Gm_5}{r_{3,5}^3}(x_3 - x_5) - \frac{Gm_6}{r_{3,6}^3}(x_3 - x_6) - \frac{Gm_7}{r_{3,7}^3}(x_3 - x_7) - \frac{Gm_8}{r_{3,8}^3}(x_3 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}z_3 = \dot{z}_3$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{z}_3 = -\frac{Gm_1}{r_{3,1}^3}(x_3 - x_1) - \frac{Gm_2}{r_{3,2}^3}(x_3 - x_2) - \frac{Gm_4}{r_{3,4}^3}(x_3 - x_4) - \frac{Gm_5}{r_{3,5}^3}(x_3 - x_5) - \frac{Gm_6}{r_{3,6}^3}(x_3 - x_6) - \frac{Gm_7}{r_{3,7}^3}(x_3 - x_7) - \frac{Gm_8}{r_{3,8}^3}(x_3 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}x_4 = \dot{x_4}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{x_4} = -\frac{Gm_1}{r_{4,1}^3}(x_4 - x_1) - \frac{Gm_2}{r_{4,2}^3}(x_4 - x_2) - \frac{Gm_3}{r_{4,3}^3}(x_4 - x_3) - \frac{Gm_5}{r_{4,5}^3}(x_4 - x_5) - \frac{Gm_6}{r_{4,6}^3}(x_4 - x_6) - \frac{Gm_7}{r_{4,7}^3}(x_4 - x_7) - \frac{Gm_8}{r_{4,8}^3}(x_4 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}y_4 = \dot{y}_4$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{y}_4 = -\frac{Gm_1}{r_{4,1}^3}(x_4 - x_1) - \frac{Gm_2}{r_{4,2}^3}(x_4 - x_2) - \frac{Gm_3}{r_{4,3}^3}(x_4 - x_3) - \frac{Gm_5}{r_{4,5}^3}(x_4 - x_5) - \frac{Gm_6}{r_{4,6}^3}(x_4 - x_6) - \frac{Gm_7}{r_{4,7}^3}(x_4 - x_7) - \frac{Gm_8}{r_{4,8}^3}(x_4 - x_8)$ $\frac{\mathrm{d}t^{24}}{\mathrm{d}t}\dot{z}_{4} = -\frac{Gm_{1}}{r_{4,1}^{3}}(x_{4} - x_{1}) - \frac{Gm_{2}}{r_{4,2}^{3}}(x_{4} - x_{2}) - \frac{Gm_{3}}{r_{4,3}^{3}}(x_{4} - x_{3}) - \frac{Gm_{5}}{r_{4,5}^{3}}(x_{4} - x_{5}) - \frac{Gm_{6}}{r_{4,6}^{3}}(x_{4} - x_{6}) - \frac{Gm_{7}}{r_{4,7}^{3}}(x_{4} - x_{7}) - \frac{Gm_{8}}{r_{4,8}^{3}}(x_{4} - x_{8})$ $\frac{\mathrm{d}}{\mathrm{d}t}x_5 = \dot{x_5}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{x_5} = -\frac{Gm_1}{r_{5,1}^3}(x_5 - x_1) - \frac{Gm_2}{r_{5,2}^3}(x_5 - x_2) - \frac{Gm_3}{r_{5,3}^3}(x_5 - x_3) - \frac{Gm_4}{r_{5,4}^3}(x_5 - x_4) - \frac{Gm_6}{r_{5,6}^3}(x_5 - x_6) - \frac{Gm_7}{r_{5,7}^3}(x_5 - x_7) - \frac{Gm_8}{r_{5,8}^3}(x_5 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}y_5 = \dot{y}_5$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{y}_5 = -\frac{Gm_1}{r_{5,1}^3}(x_5 - x_1) - \frac{Gm_2}{r_{5,2}^3}(x_5 - x_2) - \frac{Gm_3}{r_{5,3}^3}(x_5 - x_3) - \frac{Gm_4}{r_{5,4}^3}(x_5 - x_4) - \frac{Gm_6}{r_{5,6}^3}(x_5 - x_6) - \frac{Gm_7}{r_{5,7}^3}(x_5 - x_7) - \frac{Gm_8}{r_{5,8}^3}(x_5 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}z_5 = \dot{z}_5$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{z}_5 = -\frac{Gm_1}{r_{5,1}^3}(x_5 - x_1) - \frac{Gm_2}{r_{5,2}^3}(x_5 - x_2) - \frac{Gm_3}{r_{5,3}^3}(x_5 - x_3) - \frac{Gm_4}{r_{5,4}^3}(x_5 - x_4) - \frac{Gm_6}{r_{5,6}^3}(x_5 - x_6) - \frac{Gm_7}{r_{5,7}^3}(x_5 - x_7) - \frac{Gm_8}{r_{5,8}^3}(x_5 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}x_6 = \dot{x}_6$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{x}_6 = -\frac{Gm_1}{r_{6,1}^3}(x_6 - x_1) - \frac{Gm_2}{r_{6,2}^3}(x_6 - x_2) - \frac{Gm_3}{r_{6,3}^3}(x_6 - x_3) - \frac{Gm_4}{r_{6,4}^3}(x_6 - x_4) - \frac{Gm_5}{r_{6,5}^3}(x_6 - x_5) - \frac{Gm_7}{r_{6,7}^3}(x_6 - x_7) - \frac{Gm_8}{r_{6,8}^3}(x_6 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}y_6 = \dot{y_6}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{y_6} = -\frac{Gm_1}{r_{6,1}^3}(x_6 - x_1) - \frac{Gm_2}{r_{6,2}^3}(x_6 - x_2) - \frac{Gm_3}{r_{6,3}^3}(x_6 - x_3) - \frac{Gm_4}{r_{6,4}^3}(x_6 - x_4) - \frac{Gm_5}{r_{6,5}^3}(x_6 - x_5) - \frac{Gm_7}{r_{6,7}^3}(x_6 - x_7) - \frac{Gm_8}{r_{6,8}^3}(x_6 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}z_{6} = \dot{z}_{6}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{z}_{6} = -\frac{Gm_{1}}{r_{6,1}^{3}}(x_{6} - x_{1}) - \frac{Gm_{2}}{r_{6,2}^{3}}(x_{6} - x_{2}) - \frac{Gm_{3}}{r_{6,3}^{3}}(x_{6} - x_{3}) - \frac{Gm_{4}}{r_{6,4}^{3}}(x_{6} - x_{4}) - \frac{Gm_{5}}{r_{6,5}^{3}}(x_{6} - x_{5}) - \frac{Gm_{7}}{r_{6,7}^{3}}(x_{6} - x_{7}) - \frac{Gm_{8}}{r_{6,8}^{3}}(x_{6} - x_{8})$ $\frac{\mathrm{d}}{\mathrm{d}t}x_7 = \dot{x_7}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{x_7} = -\frac{Gm_1}{r_{7,1}^3}(x_7 - x_1) - \frac{Gm_2}{r_{7,2}^3}(x_7 - x_2) - \frac{Gm_3}{r_{7,3}^3}(x_7 - x_3) - \frac{Gm_4}{r_{7,4}^3}(x_7 - x_4) - \frac{Gm_5}{r_{7,5}^3}(x_7 - x_5) - \frac{Gm_6}{r_{7,6}^3}(x_7 - x_6) - \frac{Gm_8}{r_{7,8}^3}(x_7 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}y_7 = \dot{y_7}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{y_7} = -\frac{Gm_1}{r_{7,1}^3}(x_7 - x_1) - \frac{Gm_2}{r_{7,2}^3}(x_7 - x_2) - \frac{Gm_3}{r_{7,3}^3}(x_7 - x_3) - \frac{Gm_4}{r_{7,4}^3}(x_7 - x_4) - \frac{Gm_5}{r_{7,5}^3}(x_7 - x_5) - \frac{Gm_6}{r_{7,6}^3}(x_7 - x_6) - \frac{Gm_8}{r_{7,8}^3}(x_7 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}z_7 = \dot{z}_7$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{z}_7 = -\frac{Gm_1}{r_{7,1}^3}(x_7 - x_1) - \frac{Gm_2}{r_{7,2}^3}(x_7 - x_2) - \frac{Gm_3}{r_{7,3}^3}(x_7 - x_3) - \frac{Gm_4}{r_{7,4}^3}(x_7 - x_4) - \frac{Gm_5}{r_{7,5}^3}(x_7 - x_5) - \frac{Gm_6}{r_{7,6}^3}(x_7 - x_6) - \frac{Gm_8}{r_{7,8}^3}(x_7 - x_8)$ $\frac{\mathrm{d}}{\mathrm{d}t}x_8 = \dot{x_8}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{x_8} = -\frac{Gm_1}{r_{8,1}^3}(x_8 - x_1) - \frac{Gm_2}{r_{8,2}^3}(x_8 - x_2) - \frac{Gm_3}{r_{8,3}^3}(x_8 - x_3) - \frac{Gm_4}{r_{8,4}^3}(x_8 - x_4) - \frac{Gm_5}{r_{8,5}^3}(x_8 - x_5) - \frac{Gm_6}{r_{8,6}^3}(x_8 - x_6) - \frac{Gm_7}{r_{8,7}^3}(x_8 - x_7)$ $\frac{\mathrm{d}}{\mathrm{d}t}y_8 = \dot{y_8}$ $\frac{\mathrm{d}}{\mathrm{d}t}\dot{y_8} = -\frac{Gm_1}{r_{8,1}^3}(x_8 - x_1) - \frac{Gm_2}{r_{8,2}^3}(x_8 - x_2) - \frac{Gm_3}{r_{8,3}^3}(x_8 - x_3) - \frac{Gm_4}{r_{8,4}^3}(x_8 - x_4) - \frac{Gm_5}{r_{8,5}^3}(x_8 - x_5) - \frac{Gm_6}{r_{8,6}^3}(x_8 - x_6) - \frac{Gm_7}{r_{8,7}^3}(x_8 - x_7)$ $\frac{\mathrm{d}\,t}{\mathrm{d}\,t}\dot{z}_{8} = -\frac{Gm_{1}}{r_{8,1}^{3}}(x_{8} - x_{1}) - \frac{Gm_{2}}{r_{8,2}^{3}}(x_{8} - x_{2}) - \frac{Gm_{3}}{r_{8,3}^{3}}(x_{8} - x_{3}) - \frac{Gm_{4}}{r_{8,4}^{3}}(x_{8} - x_{4}) - \frac{Gm_{5}}{r_{8,5}^{3}}(x_{8} - x_{5}) - \frac{Gm_{6}}{r_{8,6}^{3}}(x_{8} - x_{6}) - \frac{Gm_{7}}{r_{8,7}^{3}}(x_{8} - x_{7})$



Vector Form

- $\mathbf{r}_{1}'' = -\frac{Gm_{2}\left(\mathbf{r}_{1} \mathbf{r}_{2}\right)}{\left|\mathbf{r}_{1} \mathbf{r}_{2}\right|^{3}} \frac{Gm_{3}\left(\mathbf{r}_{1} \mathbf{r}_{3}\right)}{\left|\mathbf{r}_{1} \mathbf{r}_{3}\right|^{3}} \frac{Gm_{4}\left(\mathbf{r}_{1} \mathbf{r}_{4}\right)}{\left|\mathbf{r}_{1} \mathbf{r}_{4}\right|^{3}} \frac{Gm_{5}\left(\mathbf{r}_{1} \mathbf{r}_{5}\right)}{\left|\mathbf{r}_{1} \mathbf{r}_{5}\right|^{3}} \frac{Gm_{6}\left(\mathbf{r}_{1} \mathbf{r}_{6}\right)}{\left|\mathbf{r}_{1} \mathbf{r}_{6}\right|^{3}} \frac{Gm_{7}\left(\mathbf{r}_{1} \mathbf{r}_{7}\right)}{\left|\mathbf{r}_{1} \mathbf{r}_{7}\right|^{3}} \frac{Gm_{8}\left(\mathbf{r}_{1} \mathbf{r}_{8}\right)}{\left|\mathbf{r}_{1} \mathbf{r}_{8}\right|^{3}}$
- $\mathbf{r}_{2}^{\prime\prime} = -\frac{Gm_{1}\left(\mathbf{r}_{2} \mathbf{r}_{1}\right)}{\left|\mathbf{r}_{2} \mathbf{r}_{1}\right|^{3}} \frac{Gm_{3}\left(\mathbf{r}_{2} \mathbf{r}_{3}\right)}{\left|\mathbf{r}_{2} \mathbf{r}_{3}\right|^{3}} \frac{Gm_{4}\left(\mathbf{r}_{2} \mathbf{r}_{4}\right)}{\left|\mathbf{r}_{2} \mathbf{r}_{4}\right|^{3}} \frac{Gm_{5}\left(\mathbf{r}_{2} \mathbf{r}_{5}\right)}{\left|\mathbf{r}_{2} \mathbf{r}_{5}\right|^{3}} \frac{Gm_{6}\left(\mathbf{r}_{2} \mathbf{r}_{6}\right)}{\left|\mathbf{r}_{2} \mathbf{r}_{6}\right|^{3}} \frac{Gm_{7}\left(\mathbf{r}_{2} \mathbf{r}_{7}\right)}{\left|\mathbf{r}_{2} \mathbf{r}_{7}\right|^{3}} \frac{Gm_{8}\left(\mathbf{r}_{2} \mathbf{r}_{8}\right)}{\left|\mathbf{r}_{2} \mathbf{r}_{8}\right|^{3}}$
- $\mathbf{r}_{3}'' = -\frac{Gm_{1}\left(\mathbf{r}_{3} \mathbf{r}_{1}\right)}{\left|\mathbf{r}_{3} \mathbf{r}_{1}\right|^{3}} \frac{Gm_{2}\left(\mathbf{r}_{3} \mathbf{r}_{2}\right)}{\left|\mathbf{r}_{3} \mathbf{r}_{2}\right|^{3}} \frac{Gm_{4}\left(\mathbf{r}_{3} \mathbf{r}_{4}\right)}{\left|\mathbf{r}_{3} \mathbf{r}_{4}\right|^{3}} \frac{Gm_{5}\left(\mathbf{r}_{3} \mathbf{r}_{5}\right)}{\left|\mathbf{r}_{3} \mathbf{r}_{5}\right|^{3}} \frac{Gm_{6}\left(\mathbf{r}_{3} \mathbf{r}_{6}\right)}{\left|\mathbf{r}_{3} \mathbf{r}_{6}\right|^{3}} \frac{Gm_{7}\left(\mathbf{r}_{3} \mathbf{r}_{7}\right)}{\left|\mathbf{r}_{3} \mathbf{r}_{7}\right|^{3}} \frac{Gm_{8}\left(\mathbf{r}_{3} \mathbf{r}_{8}\right)}{\left|\mathbf{r}_{3} \mathbf{r}_{8}\right|^{3}}$

 $\mathbf{r}_{5}'' = -\frac{Gm_{1}\left(\mathbf{r}_{5} - \mathbf{r}_{1}\right)}{\left|\mathbf{r}_{5} - \mathbf{r}_{1}\right|^{3}} - \frac{Gm_{2}\left(\mathbf{r}_{5} - \mathbf{r}_{2}\right)}{\left|\mathbf{r}_{5} - \mathbf{r}_{2}\right|^{3}} - \frac{Gm_{3}\left(\mathbf{r}_{5} - \mathbf{r}_{3}\right)}{\left|\mathbf{r}_{5} - \mathbf{r}_{3}\right|^{3}} - \frac{Gm_{4}\left(\mathbf{r}_{5} - \mathbf{r}_{4}\right)}{\left|\mathbf{r}_{5} - \mathbf{r}_{4}\right|^{3}} - \frac{Gm_{6}\left(\mathbf{r}_{5} - \mathbf{r}_{6}\right)}{\left|\mathbf{r}_{5} - \mathbf{r}_{6}\right|^{3}} - \frac{Gm_{7}\left(\mathbf{r}_{5} - \mathbf{r}_{7}\right)}{\left|\mathbf{r}_{5} - \mathbf{r}_{7}\right|^{3}} - \frac{Gm_{8}\left(\mathbf{r}_{5} - \mathbf{r}_{8}\right)}{\left|\mathbf{r}_{5} - \mathbf{r}_{8}\right|^{3}}$

- $\mathbf{r}_{4}'' = -\frac{Gm_{1}\left(\mathbf{r}_{4} \mathbf{r}_{1}\right)}{\left|\mathbf{r}_{4} \mathbf{r}_{1}\right|^{3}} \frac{Gm_{2}\left(\mathbf{r}_{4} \mathbf{r}_{2}\right)}{\left|\mathbf{r}_{4} \mathbf{r}_{2}\right|^{3}} \frac{Gm_{3}\left(\mathbf{r}_{4} \mathbf{r}_{3}\right)}{\left|\mathbf{r}_{4} \mathbf{r}_{3}\right|^{3}} \frac{Gm_{5}\left(\mathbf{r}_{4} \mathbf{r}_{5}\right)}{\left|\mathbf{r}_{4} \mathbf{r}_{5}\right|^{3}} \frac{Gm_{6}\left(\mathbf{r}_{4} \mathbf{r}_{6}\right)}{\left|\mathbf{r}_{4} \mathbf{r}_{6}\right|^{3}} \frac{Gm_{7}\left(\mathbf{r}_{4} \mathbf{r}_{7}\right)}{\left|\mathbf{r}_{4} \mathbf{r}_{7}\right|^{3}} \frac{Gm_{8}\left(\mathbf{r}_{4} \mathbf{r}_{8}\right)}{\left|\mathbf{r}_{4} \mathbf{r}_{8}\right|^{3}}$
- $\mathbf{r}_{6}'' = -\frac{Gm_{1}\left(\mathbf{r}_{6} \mathbf{r}_{1}\right)}{\left|\mathbf{r}_{6} \mathbf{r}_{1}\right|^{3}} \frac{Gm_{2}\left(\mathbf{r}_{6} \mathbf{r}_{2}\right)}{\left|\mathbf{r}_{6} \mathbf{r}_{2}\right|^{3}} \frac{Gm_{3}\left(\mathbf{r}_{6} \mathbf{r}_{3}\right)}{\left|\mathbf{r}_{6} \mathbf{r}_{3}\right|^{3}} \frac{Gm_{4}\left(\mathbf{r}_{6} \mathbf{r}_{4}\right)}{\left|\mathbf{r}_{6} \mathbf{r}_{5}\right|^{3}} \frac{Gm_{5}\left(\mathbf{r}_{6} \mathbf{r}_{5}\right)}{\left|\mathbf{r}_{6} \mathbf{r}_{7}\right|^{3}} \frac{Gm_{7}\left(\mathbf{r}_{6} \mathbf{r}_{7}\right)}{\left|\mathbf{r}_{6} \mathbf{r}_{7}\right|^{3}} \frac{Gm_{8}\left(\mathbf{r}_{6} \mathbf{r}_{8}\right)}{\left|\mathbf{r}_{6} \mathbf{r}_{8}\right|^{3}}$
- $\mathbf{r}_{7}^{\prime\prime} = -\frac{Gm_{1}\left(\mathbf{r}_{7} \mathbf{r}_{1}\right)}{\left|\mathbf{r}_{7} \mathbf{r}_{1}\right|^{3}} \frac{Gm_{2}\left(\mathbf{r}_{7} \mathbf{r}_{2}\right)}{\left|\mathbf{r}_{7} \mathbf{r}_{2}\right|^{3}} \frac{Gm_{3}\left(\mathbf{r}_{7} \mathbf{r}_{3}\right)}{\left|\mathbf{r}_{7} \mathbf{r}_{3}\right|^{3}} \frac{Gm_{4}\left(\mathbf{r}_{7} \mathbf{r}_{4}\right)}{\left|\mathbf{r}_{7} \mathbf{r}_{4}\right|^{3}} \frac{Gm_{5}\left(\mathbf{r}_{7} \mathbf{r}_{5}\right)}{\left|\mathbf{r}_{7} \mathbf{r}_{5}\right|^{3}} \frac{Gm_{6}\left(\mathbf{r}_{7} \mathbf{r}_{6}\right)}{\left|\mathbf{r}_{7} \mathbf{r}_{6}\right|^{3}} \frac{Gm_{8}\left(\mathbf{r}_{7} \mathbf{r}_{8}\right)}{\left|\mathbf{r}_{7} \mathbf{r}_{8}\right|^{3}}$ $\mathbf{r}_{8}'' = -\frac{Gm_{1}\left(\mathbf{r}_{8} - \mathbf{r}_{1}\right)}{\left|\mathbf{r}_{8} - \mathbf{r}_{1}\right|^{3}} - \frac{Gm_{2}\left(\mathbf{r}_{8} - \mathbf{r}_{2}\right)}{\left|\mathbf{r}_{8} - \mathbf{r}_{2}\right|^{3}} - \frac{Gm_{3}\left(\mathbf{r}_{8} - \mathbf{r}_{3}\right)}{\left|\mathbf{r}_{8} - \mathbf{r}_{3}\right|^{3}} - \frac{Gm_{4}\left(\mathbf{r}_{8} - \mathbf{r}_{4}\right)}{\left|\mathbf{r}_{8} - \mathbf{r}_{4}\right|^{3}} - \frac{Gm_{5}\left(\mathbf{r}_{8} - \mathbf{r}_{5}\right)}{\left|\mathbf{r}_{8} - \mathbf{r}_{5}\right|^{3}} - \frac{Gm_{6}\left(\mathbf{r}_{8} - \mathbf{r}_{6}\right)}{\left|\mathbf{r}_{8} - \mathbf{r}_{6}\right|^{3}} - \frac{Gm_{7}\left(\mathbf{r}_{8} - \mathbf{r}_{7}\right)}{\left|\mathbf{r}_{8} - \mathbf{r}_{7}\right|^{3}}$