Sec. 10.1

1. Draw graph models, stating the type of graph (from Table 1) used, to represent airline routes where every day there are four flights from Boston to Newark, two flights from Newark to Boston, three flights from Newark to Miami, two flights from Miami to Newark, one flight from Newark to Detroit, two flights from Detroit to Newark, three flights from Newark to Washington, two flights from Washington to Newark, and one flight from Washington to Miami, with

a) an edge between vertices representing cities that have a flight between them (in either direction).

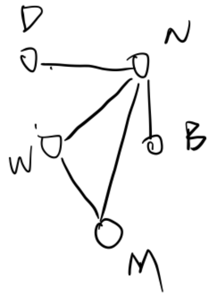
b) an edge between vertices representing cities for each flight that operates between them (in either direction).

c) an edge between vertices representing cities for each flight that operates between them (in either direction), plus a loop for a special sightseeing trip that takes off and lands in Miami.

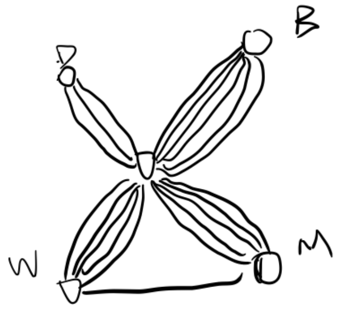
d) an edge from a vertex representing a city where a flight starts to the vertex representing the city where it ends.

e) an edge for each flight from a vertex representing a city where the flight begins to the vertex representing the city where the flight ends.

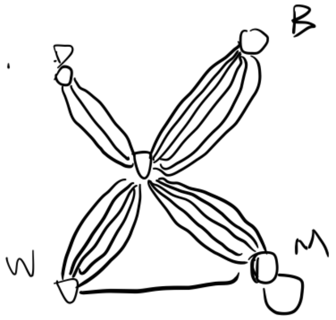
a)



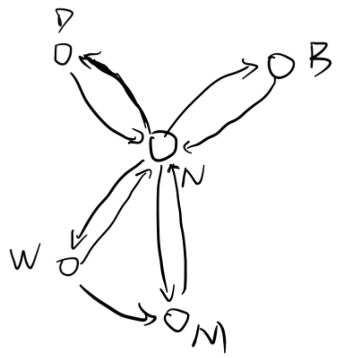
b)



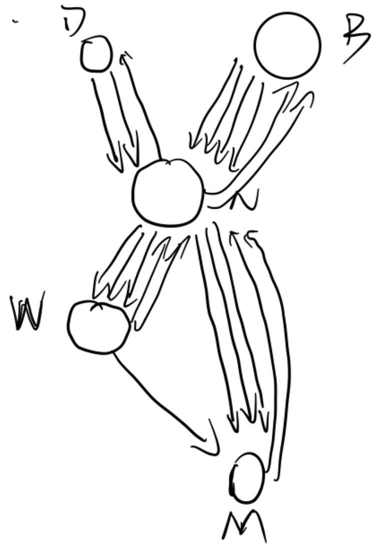
c).



d).

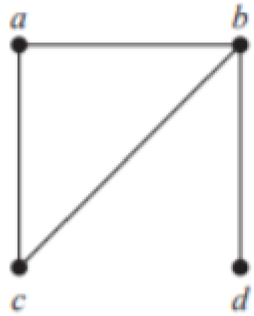


e).



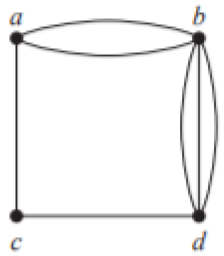
For Exercises 3–9, determine whether the graph shown has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph in Table 1 this graph is.

3.



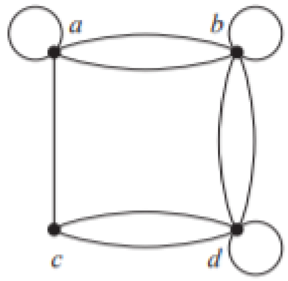
undirected， no multiple edges, no loops

simple graph

4. 

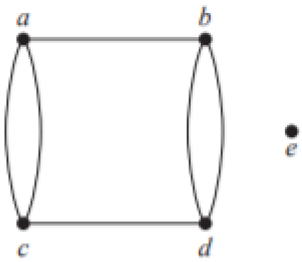
undirected， multiple edges, no loops

multigraph

5. 

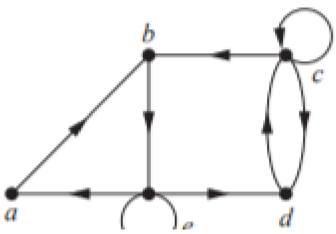
undirected， multiple edges, has loops

pseudograph

6. 

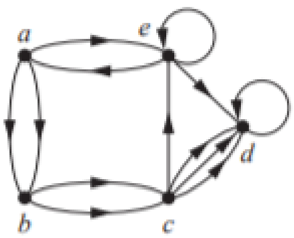
undirected， multiple edges, no loops

multigraph

7. 

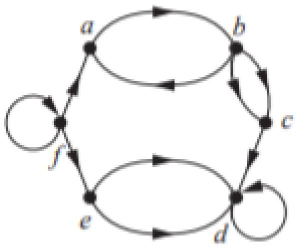
directed,

directed graph

8. 

directed, multiple edges

directed multigraph

9. 

b->c directed, multiple edges

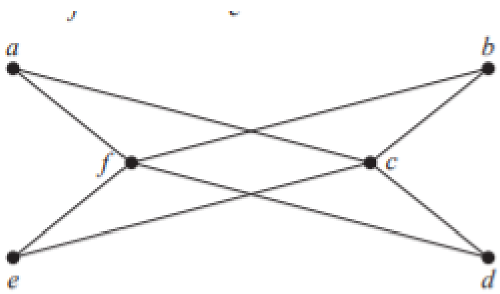
directed multigraph

Sec. 10.2

5. Can a simple graph exist with 15 vertices each of degree five?

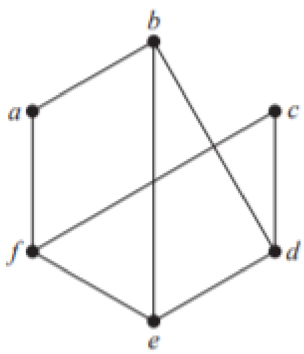
No, degree must be even number.

24.In Exercises 21–25 determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.



it's bipartite.

$V_1 = f, c$, $V_2 = a, b, e, d$

25. 

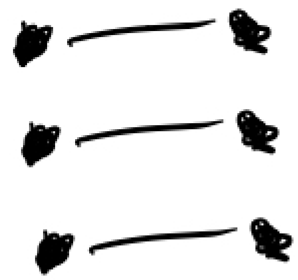
Not bipartite.

42. Determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence.

b) 6, 5, 4, 3, 2, 1

Since this sequence has odd-degree vertices , no graph exists with these degrees.

f) 1, 1, 1, 1, 1, 1



h) 5, 5, 4, 3, 2, 1

Since this sequence has odd-degree vertices , no graph exists with these degrees.

53. For which values of n are these graphs regular?

a) K_n

$n \geq 1$

b) C_n

$n \geq 3$

c) W_n

n = 3

d) Q_n

$n \geq 1$

60. If G is a simple graph with 15 edges and $\overline{G}$ has 13 edges, how many vertices does G have?

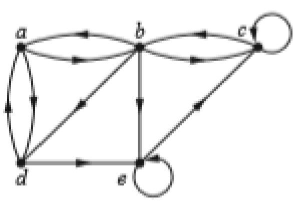
$G + \overline{G} = 28$

$n(n-1)/2 = 28$ -> $n = 8$

Sec 10.3

8. Represent the graph in Exercise 4 with an adjacency matrix.

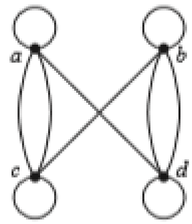
4.



$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

15. Represent the given graph using an adjacency matrix.

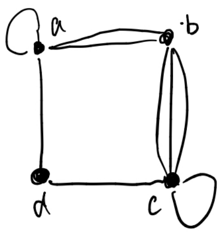
15.



$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

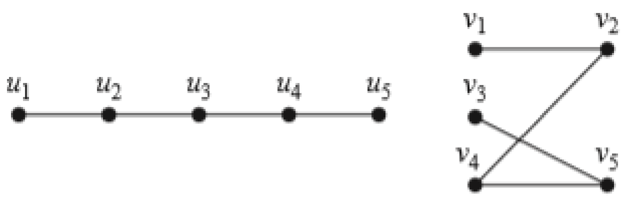
17. draw an undirected graph represented by the given adjacency matrix.

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$



InExercises34–37 determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

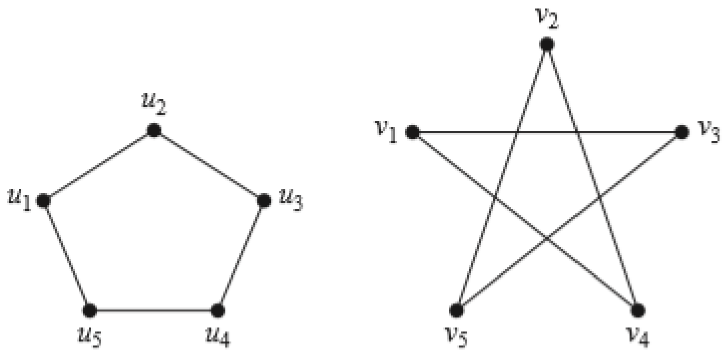
34.



f(u1) = v1, f(u2) = v2, f(u3) = v4, f(u4) = v5, and f(u5) = v3.

isomorphic

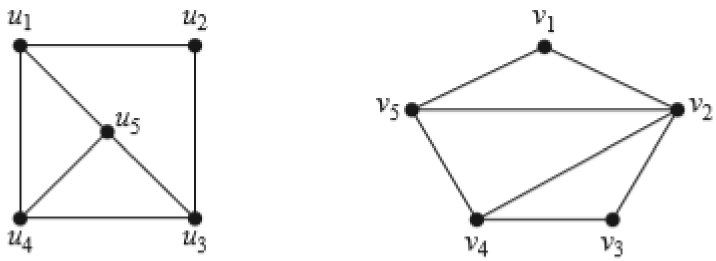
35.



f(u1) = v1, f(u2) = v4, f(u3) = v2, f(u4) = v5, and f(u5) = v3.

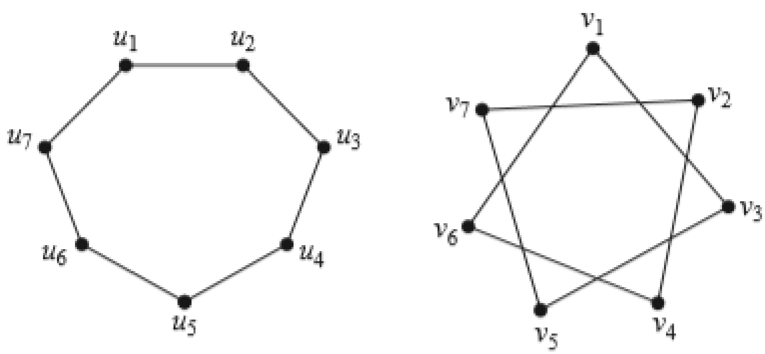
isomorphic

36.



These graphs are not isomorphic. The second has a vertex of degree 4, whereas the first does not.

37.

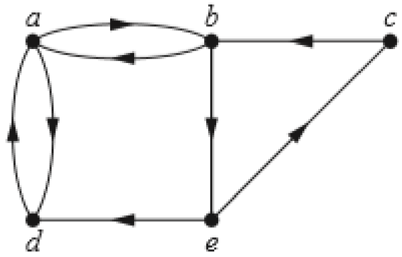


f(u1) = v1, f(u2) = v3, f(u3) = v5, f(u4) = v7, f(u5) = v2, f(u6) = v4, f(u7) = v6.

isomorphic

Sec 10.4

27. Find the number of paths from a to e in the directed graph in Exercise 2 of length.



e) 6.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} 12 & 3 & 1 & 2 & 5 \\ 4 & 8 & 3 & 10 & 2 \\ 7 & 2 & 1 & 2 & 3 \\ 1 & 5 & 2 & 7 & 1 \\ 6 & 4 & 1 & 4 & 3 \end{bmatrix}$$

The answer is 5.

28. Show that every connected graph with n vertices has at least n−1 edges.

We show this by induction on n. For n = 1 there is nothing to prove. Now assume the inductive hypothesis, and let G be a connected graph with n +1 vertices and fewer than n edges, where n ≥ 1. Since the sum of the degrees of the vertices of G is equal to 2 times the number of edges, we know that the sum of the degrees is less than 2n, which is less than 2(n +1). Therefore some vertex has degree less than 2. Since G is connected, this vertex is not isolated, so it must have degree 1. Remove this vertex and its edge. Clearly the result is still connected, and it has n vertices and fewer than n−1 edges, contradicting the inductive hypothesis. Therefore the statement holds for G, and the proof is complete.

29. Let G = (V,E) be a simple graph. Let R be the relation on V consisting of pairs of vertices (u,v) such that there is a path from u to v or such that u = v. Show that R is an equivalence relation.

It's apparently that R is reflexive.

Since it's a simple graph , When there is a path from v to u, namely, there is a path from u to v. R is symmetric.

Assume that there is a path from u to v and from v to w. Putting these two paths together gives a path from u to w. Hence, R is transitive.

62. Use Theorem2 to show that the graph G1 in Figure 2 is connected whereas the graph G2 in that figure is not connected.

THEOREM 2: Let G be a graph with adjacency matrix A with respect to the ordering $v_1, v_2, ..., v_n$ of the vertices of the graph (with directed or undirected edges, with multiple edges and loops allowed). The number of different paths of length r from v_i to v_j, where r is a positive integer, equals the $(i, j)th$ entry of A^r.

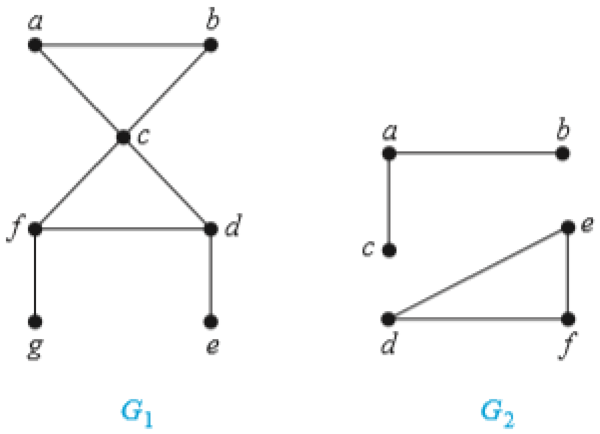


FIGURE 2 **The Graphs G_1 and G_2.**

- $$G_1 = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad G_1^3 = \begin{bmatrix} 2 & 3 & 5 & 2 & 1 & 2 & 1 \\ 3 & 2 & 5 & 2 & 1 & 2 & 1 \\ 5 & 5 & 4 & 6 & 1 & 6 & 1 \\ 2 & 2 & 6 & 2 & 3 & 5 & 1 \\ 1 & 1 & 1 & 3 & 0 & 1 & 1 \\ 2 & 2 & 6 & 5 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 3 & 0 \end{bmatrix}$$

  Already every off-diagonal entry in G_1^3 is nonzero, so we know that there is a path of length 3 between every pair of distinct vertices in this graph. Therefore the graph G is connected.

- $$G_2 = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \quad G_2^2 = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & 2 \end{bmatrix} \quad G_2^3 = \begin{bmatrix} 0 & 2 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 3 & 3 \\ 0 & 0 & 0 & 3 & 2 & 3 \\ 0 & 0 & 0 & 3 & 3 & 2 \end{bmatrix}$$

$$G_2^4 = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 5 & 5 \\ 0 & 0 & 0 & 5 & 6 & 5 \\ 0 & 0 & 0 & 5 & 5 & 6 \end{bmatrix} \quad G_2^5 = \begin{bmatrix} 0 & 4 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 11 & 11 \\ 0 & 0 & 0 & 11 & 10 & 11 \\ 0 & 0 & 0 & 11 & 11 & 10 \end{bmatrix}$$

$$\sum_{i=1}^5 G_2^i = \begin{bmatrix} 6 & 7 & 7 & 0 & 0 & 0 \\ 7 & 3 & 3 & 0 & 0 & 0 \\ 7 & 3 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 21 & 21 \\ 0 & 0 & 0 & 21 & 20 & 21 \\ 0 & 0 & 0 & 21 & 21 & 20 \end{bmatrix}$$

Thus the fact that there was a 0 as an off-diagonal entry in the sum told us that the graph was not connected.