Sec 5.2

8. Suppose that a store offers gift certificates in denominations of 25 dollars and 40 dollars. Determine the possible total amounts you can form using these gift certificates. Prove your answer using strong induction.

It's apparently that we can just produce numbers which are multiple of 5. So, let's assume that we can produce number 5*n.

n can be : 5, 8, 10 = 5+5, 13 = 5+8, 15 = 5+5+5, 16 = 8+8, 18 = 8+5+5, 20 = 5+5+5+5, 21 = 8+8+5, 23 = 8 + 5 + 5 + 5, 24 = 8 + 8 + 8, 25 = 5 + 5 + 5 + 5 + 5, 26 = 8 + 8 + 5 + 5, 28 = 8 + 5 + 5 + 5 + 5, 29 = 8 + 8 + 8 + 5, 30 = 5 + 5 + 5 + 5 + 5 + 5, 31 = 8 + 8 + 5 + 5 + 5, 32 = 8 + 8 + 8 + 8.....

We claim that we can form total amounts of the form 5*n for all n ≥ 28 using these gift certificates.

- From the work above, we know that P(n) is true for n = 28,29,30,31,32.
- Assume the inductive hypothesis, that P(j) is true for all j with 28 ≤ j ≤ k, where k is a fixed integer greater than or equal to 32. We want to show that P(k+1) is true. Because k−4≥28, we know that P(k−4) is true, that is, that we can form 5(k−4) dollars. Add one more $25-dollar certificate, and we have formed 5(k +1) dollars, as desired.

18. Use strong induction to show that when a simple polygon P with consecutive vertices $v_1, v_2, \ldots, v_n$ is triangulated into n−2 triangles, the n−2 triangles can be numbered 1,2,...,n−2 so that v_i is a vertex of triangle i for i = 1,2,...,n−2.

- The basis step is n = 3, and there is nothing to prove.
- Assume the inductive hypothesis, for all j with 3 ≤ j ≤ k, polygon can be triangulated into j-2 triangles and v_m is the common vertex if all these triangles.

  We need to prove the situation with k+1 vertices. We add a vertex on the edge of k vertices produce a angle and make it a k+1 polygon. Then this new polygon is triangulated into k-2+1 triangles. If we need to maintain the v_m , just choose the edge adjacent to v_m to add the vertex.

39. Can you use the well-ordering property to prove the statement: “Every positive integer can be described using no more than fifteen English words”? Assume the words come from a particular dictionary of English. [Hint: Suppose that there are positive integers that cannot be described using no more than fifteen English words. By well ordering, the smallest positive integer that cannot be described using no more than fifteen English words would then exist.]

P(x) is true <=> x is a positive number and it can be described using no more than fifteen English words.

We assume that there is a number m, which is the smallest number that cannot be described using no more than fifteen English words.

There are a finite number of English words. Therefore, only a finite number of positive number can be described.

The answer is no.

Sec 5.3

6. Determine whether each of these proposed definitions is a valid recursive definition of a function f from the set of nonnegative integers to the set of integers. If f is well defined, find a formula for $f(n)$ when n is a nonnegative integer and prove that your formula is valid.

a) f(0) = 1, f(n) =−f(n−1) for n ≥ 1

valid. $f(n) = (-1)^n$

- when n=0, f(0) = 1
- If it is true for n = k, then we have $f(k+1) = -f(k+1-1) = -f(k) = -(-1)^k$ by the inductive hypothesis, whence $f(k+1) = (-1)^{k+1}$.

d) f(0) = 0, f(1) = 1, f(n) = 2f(n−1) for n ≥ 1

invalid. when n = 1. $f(1) = 2f(1-1) = 0 \neq 1$.

14. f_n is the nth Fibonacci number. Show that $f_{n+1}f_{n-1} - f_n^2 = (-1)^n$ when n is a positive integer.

when n=1, $f_2 f_0 - f_1^2 = 0 - 1^2 = (-1)^1$

$f_{n+2}f_n - f_{n+1}^2$

= $(f_{n+1} + f_n)f_n - f_{n+1}^2$

= $f_{n+1}f_n + f_n^2 - f_{n+1}^2$

=$-f_{n+1}(f_{n+1} - f_n) + f_n^2$

=$-(f_{n+1}f_{n-1} - f_n^2)$

=$-(-1)^n$ =$(-1)^{n+1}$.

29. Give a recursive definition of each of these sets of ordered pairs of positive integers. Use structural induction to prove that the recursive definition you found is correct. [Hint: To find a recursive definition, plot the points in the set in the plane and look for patterns.]

a) S ={(a,b)|a ∈ Z+,b∈ Z+,and a+b is even}

- $(1,1) \in S$
- if $(a,b) \in S$, then $(a+2,b) \in S$, $(a,b+2) \in S$ and $(a+1,b+1) \in S$

Sec 5.4

29. Devise a recursive algorithm to find the n-th term of the sequence defined by $a_0 = 1$, $a_1 = 2$,and $a_n = a_{n-1} \cdot a_{n-2}$, for n = 2,3,4,....

```
function f(n):
if n=0 then return 1
else if n=1 then return 2
else return f(n-1)*f(n-2)
```