

Sec 8.5

7. There are 2504 computer science students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Linux, and 345 have taken a course in C. Further, 876 have taken courses in both Java and Linux, 231 have taken courses in both Linux and C, and 290 have taken courses in both Java and C. If 189 of these students have taken courses in Linux, Java, and C, how many of these 2504 students have not taken a course in any of these three programming languages?

$$2504 - 1876 - 999 - 345 + 876 + 231 + 290 - 189 = 492$$

10. Find the number of positive integers not exceeding 100 that are not divisible by 5 or by 7.

$$100 - 20 - \lfloor 100/7 \rfloor + \lfloor 100/35 \rfloor = 68$$

18. How many terms are there in the formula for the number of elements in the union of 10 sets given by the principle of inclusion-exclusion?

$$C(10, 1) + C(10, 2) + \dots + C(10, 10) = 2^{10} - C(10, 0) = 1023$$

Sec 8.6

6. An integer is called square free if it is not divisible by the square of a positive integer greater than 1. Find the number of square free positive integers less than 100.

the square of a positive integer less than 100 : 4, 9, 16, 25, 36, 49, 64, 81

But we can find the number can be divisible by 16, 64 and 81 must can be divisible by 4 and 9 too.

$$99 - (99/4) - (99/9) - (99/25) - (99/49) + (99/36) = 61.$$

11. In how many ways can seven different jobs be assigned to four different employees so that each employee is assigned at least one job and the most difficult job is assigned to the best employee?

let x_i be the employee.

$$x_1 + x_2 + x_3 + x_4 = 6, x_1 \geq 0, x_2, x_3, x_4 \geq 1$$

When $x_1 = 0$:

$$3^6 - C(3, 1)(3 - 1)^6 + C(3, 2)(3 - 2)^6 = 729 - 3 * 64 + 3 = 540$$

When $x_1 \geq 1$:

$$4^6 - C(4, 1)3^6 + C(4, 2)2^6 - C(4, 3) = 1560$$

The answer is 2100.

16. A group of n students is assigned seats for each of two classes in the same classroom. How many ways can these seats be assigned if no student is assigned the same seat for both classes?

There are $n!$ ways to make the first assignment. Then the next seating must be a derangement with respect to this numbering, so there are D_n second seatings possible. Therefore the answer is $n!D_n$.

$$D_n = n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \right]$$