Sec 3.1

2.Determine which characteristics of an algorithm described in the text (after Algorithm 1) the following procedures have and which they lack.

a) procedure double(n: positive integer)

```
while n>0
    n := 2n
```

This procedure is not finite.

b) procedure divide(n: positive integer)

```
while n ≥ 0
    m := 1/n
    n := n−1
```

when n=0, the step m:= 1/n can not be performed.

c) procedure sum(n: positive integer)

```
sum:= 0
while i<10
    sum:=sum+i
```

The value i is never set. This procedure lacks definiteness.

d) procedure choose(a,b: integers)

```
x :=either a or b
```

This procedure lacks definiteness, since the statement does not tell whether x is to be set equal to a or to b.

4. Describe an algorithm that takes as input a list of n integers and produces as output the largest difference obtained by subtracting an integer in the list from the one following it.

assume a is the list of integers.

```
output := -inf
for i:= 1 to n-1
    if output < a[i+1] - a[i]
        output := a[i+1] - a[i]
return output
```

Sec 3.2

8. Find the least integer n such that f(x)is O(x^n) for each of these functions.

c) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$

$f(x) = 1 + \frac{x^2}{x^4+1} \leq 2$,

n = 0

26. Give a big-O estimate for each of these functions. For the function g in your estimate f(x) is O(g(x)), use a simple function g of smallest order.

a) $(n^3 + n^2 logn)(logn + 1) + (17logn + 19)(n^3 + 2)$

$O(n^3 * logn)$

54. Show that $x^5y^3 + x^4y^4 + x^3y^5$ is $\Omega(x^3y^3)$.

Each term of the given expression is greater than x^3y^3.

$x^5y^3 + x^4y^4 + x^3y^5 \geq x^3y^3$ when k=1

56. Show that $\lceil xy \rceil$ is $\Omega(xy)$.

It's apparently that $\lceil xy \rceil \geq xy$, Thus $\lceil xy \rceil$ is $\Omega(xy)$, taking C = 1 and k1 = k2 = 0.

Challenge Problem: Find explicit constants ε,C such that $\varepsilon\sqrt{n} \leq \frac{1\cdot3\cdot5\cdots(2n+1)}{2\cdot4\cdot6\cdots2n} \leq C\sqrt{n}$,for all positive integers.

Sec 3.3

7. Suppose that an element is known to be among the first four elements in a list of 32 elements. Would a linear search or a binary search locate this element more rapidly?

linear search.

10.

a) Show that this algorithm determines the number of 1 bits in the bit string S:

```
procedure bit count(S: bit string)
count := 0
while S != 0
    count :=count+1
    S :=S ∧(S -1)
return count {count is the number of 1s in S}
```

Here S −1 is the bit string obtained by changing the rightmost 1 bit of S to a 0 and all the 0 bits to the right of this to 1s. [Recall that S ∧(S −1) is the bitwise AND of S and S −1.]

S ∧(S −1) turns the rightmost 1s in S to 0s, so in this way, we can count the 1s in S.

b) How many bitwise AND operations are needed to find the number of 1 bits in a string S using the algorithm in part (a)?

Obviously the number of bitwise AND operations is equal to the final value of count.

Sec 5.1

46. Prove that a set with n elements has n(n−1)(n−2)/6 subsets containing exactly three elements whenever n is an integer greater than or equal to 3.

- When n = 3, only itself can be the subset containing exactly 3 elements. 3*2*1/6 = 1.
- Assume a set with k elements has k(k−1)(k−2)/6 subsets containing exactly three elements.

  Now we need to prove that a set with k+1 elements has k(k−1)(k+1)/6 subsets containing exactly three elements.

  We can divide the k+1 elements into 2 sets containing k and 1 elements, corresponding. So, the subsets have 3 elements can be k(k-1)(k-2)/6 + (k-1)*k/2 = k(k−1)(k+1)/6.

In Exercises 47 and 48 we consider the problem of placing towers along a straight road, so that every building on the road receives cellular service. Assume that a building receives cellular service if it is within one mile of a tower.

47. Devise a greedy algorithm that uses the minimum number of towers possible to provide cell service to d buildings located at positions $x_1, x_2, \ldots, x_d$ from the start of the road. [Hint: At each step, go as far as possible along the road before adding a tower so as not to leave any buildings without coverage.]

assume x_k is the nearest building without coverage. build a tower at $x_k + 1$, loop this step until all the buildings are coverd.

48. Use mathematical induction to prove that the algorithm you devised in Exercise 47 produces an optimal solution, that is, that it uses the fewest towers possible to provide cellular service to all buildings.

Let $s_1 < s_2 < \cdots < s_k$ be an optimal locations of the towers (i.e., so as to minimize k), and let $t_1 < t_2 < \cdots < t_l$ be the locations produced by the algorithm from Exercise 47.

- we must have$s_1 \leq x_1 + 1 = t_1$. If $s1 \neq t1$, then we can move the first tower in the optimal solution to position t1 without losing cell service for any building. Therefore we can assume that $s_1 = t_1$.
- assume $s_i = t_i$ for i from 1 to m. then x_j is smallest location of a building out of range of the tower at s_m. thus $x_j > s_i + 1$. In order to serve that building there must be a tower s_{i+1} such that $s_{i+1} \leq x_j + 1 = t_{i+1}$. Like the step before, when i=0, this is the first case.

74. Suppose that we want to prove that $\frac{1}{2} * \frac{3}{4} * \ldots * \frac{2n-1}{2n} < \frac{1}{\sqrt{3n}}$ for all positive integers n.

a) Show that if we try to prove this inequality using mathematical induction, the basis step works, but the inductive step fails.

- for n = 1 the statement $1/2 < 1/\sqrt{3}$ is true.

- The inductive step would require proving that $\frac{1}{\sqrt{3n}} * \frac{2n+1}{2n+2} < \frac{1}{\sqrt{3n+3}}$

  Squaring both sides and clearing fractions, we see that this is equivalent to $4n^2 + 4n + 1 < 4n^2 + 4n$, which of course is not true.

b) Show that mathematical induction can be used to prove the stronger inequality $\frac{1}{2} * \frac{3}{4} * ... * \frac{2n-1}{2n} < \frac{1}{\sqrt{3n+1}}$ for all integers greater than 1, which, together with a verification for the case where n = 1, establishes the weaker inequality we originally tried to prove using mathematical induction.

- for n = 2 , the statement $3/8 < 1/\sqrt{7}$ is true.
- The inductive step this time requires proving that $\frac{1}{\sqrt{3n+1}} * \frac{2n+1}{2n+2} < \frac{1}{\sqrt{3n+4}}$.

  After manipulations, the statement is equivalent to $12n^3 + 28n^2 + 19n + 4 < 12n^3 + 28n^2 + 20n + 4$, which is true.