Sec. 1.4

6. Let N(x) be the statement “x has visited North Dakota,” where the domain consists of the students in your school. Express each of these quantifications in English.

c) ¬∃xN(x)

There doesn't exist a student in my school who has visited North Dakota.

d) ∃x¬N(x)

There exists a student in my school who has not visited North Dakota.

e) ¬∀xN(x)

Not all students in the school have visited North Dakota.

f) ∀x¬N(x)

All students in the school have not visited North Dakota.

9. Let P(x) be the statement “x can speak Russian” and let Q(x) be the statement “x knows the computer language C++.” Express each of these sentences in terms of P(x) , Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

b) There is a student at your school who can speak Russian but who doesn’t know C++.

$\exists x(P(X) \wedge \neg Q(X))$

d) No student at your school can speak Russian or knows C++.

$\forall x \neg(P(X) \wedge \neg Q(X))$

20.Suppose that the domain of the propositional function P(x) consists of−5,−3,−1, 1, 3, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

e) ∃x(¬P(x))∧∀x((x < 0) → P(x))

$(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge \forall x((x \geq 0) \vee P(X))$

$(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge P(-5) \wedge P(-3) \wedge P(-1)$

24. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

b) Somebody in your class has seen a foreign movie.

Let P(x): x has seen a foreign movie.

Q(x): x is in my class.

1: $\exists x P(x)$

2: $\exists x(P(x) \wedge Q(x))$

d) All students in your class can solve quadratic equations.

Let R(x): x can solve quadratic equations.

1: $\forall x R(x)$

2: $\forall x(Q(x) \rightarrow R(x))$

40. Express each of these system specifications using predicates, quantifiers, and logical connectives.

a) When there is less than 30 megabytes free on the hard disk, a warning message is sent to all users.

Let F(x) be “There is less than x megabytes free on the hard disk.” with the domain of discourse being positive numbers

Let W(x) be “User x is sent a warning message.”

$F(30) \rightarrow \forall x W(x)$

b) No directories in the file system can be opened and no files can be closed when system errors have been detected.

Let P(x) be “Directory x can be opened.” with the domain of the directory in the file system.

Let Q(x) be "File x can be closed" with the domain of the file in the file system.

Let R(x) be “The number of system errors have been detected” with the domain of positive number.

$\forall x R(x) \rightarrow ((\forall x \neg P(x)) \wedge (\forall x \neg Q(x)))$

c) The file system cannot be backed up if there is a user currently logged on.

Let B be the proposition “The file system can be backed up.”

Let L(x) be “User x is currently logged on.”

$(\exists x L(x)) \rightarrow \neg B$

d) Video on demand can be delivered when there are at least 8 megabytes of memory available and the connection speed is at least 56 kilobits per second.

Let P(x) be the proposition "“There are at least x megabytes of memory available" with the domain of positive numbers.

Let Q(x) be the proposition "The connection speed is at least x kilobits per second" with the domain of positive numbers.

Let R be the proposition “Video on demand can be delivered.”

$R \rightarrow (P(8) \wedge Q(56))$

44. Determine whether ∀x(P(x)↔ Q(x)) and ∀x P(x) ↔ ∀x Q(x) are logically equivalent. Justify your answer.

they are not logically equivalent.

Since $F \leftrightarrow F$ is True, we can construct a proposition like P(x) is odd number and Q(x) is even number with the domain of positive numbers.

Then the proposition ∀x(P(x)↔Q(x)) is false while ∀x P(x) ↔ ∀x Q(x) is true.

49. Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.

a) ∀x(P(x)→ A) ≡ ∃xP(x) → A

- If A is true, both sides are true.
- If A is false.
  - If P(x) is false for all x, P(x)->A and ∃xP(x) → A are both true. So both sides are true.
  - if P(x) is true for all x, P(x)->A and ∃xP(x) → A are both false. So both sides are false.
  - if P(x) can be true or false, ∃xP(x) is true and ∃xP(x) → A is false, and for the left side, ∀x(P(x)→ A) is false.

Thus in all cases, the 2 propositions have the same truth value.

Sec. 1.5

6. Let C(x,y) mean that student x is enrolled in class y, where the domain for x consists of all students in your school and the domain for y consists of all classes being given at your school. Express each of these statements by a simple English sentence.

e) ∃x∃y∀z((x $\neq$ y)∧(C(x,z) → C(y,z)))

There exist two distinct people, the second of whom is enrolled in every course that the first is enrolled in.

f) ∃x∃y∀z((x $\neq$ y)∧(C(x,z) ↔ C(y,z)))

There exist two distinct people enrolled in exactly the same courses.

12. Let I(x)be the statement “x has an Internet connection” and C(x,y) be the statement “x and y have chatted over the Internet,” where the domain for the variables x and y consists of all students in your class. Use quantifiers to express each of these statements.

d) No one in the class has chatted with Bob.

$\forall x \neg C(x, Bob)$

h) Exactly one student in your class has an Internet connection.

$\exists x \forall y((x = y) \leftrightarrow I(y))$

k) Someone in your class has an Internet connection but has not chatted with anyone else in your class.

$\exists x(I(x) \land \forall y((x \neq y) \to \neg C(x, y)))$

n) There are at least two students in your class who have not chatted with the same person in your class.

$\exists x \exists y((x \neq y) \land \forall z \neg(C(x,z) \land C(y,z)))$

14. Use quantifiers and predicates with more than one variable to express these statements.

c) Some student in this class has visited Alaska but has not visited Hawaii.

Let P(x,y) mean that person x has visited state y.

$\exists x(P(x, Alaska) \land \neg P(x, Hawaii))$

d) All students in this class have learned at least one programming language.

Let L(x,y) mean that person x has learned programming language y.

$\forall x \exists y L(x,y)$

e) There is a student in this class who has taken every course offered by one of the departments in this school.

Let P(x,y) mean that student x take course y.

Let Q(x,y) mean that course y is offered by department z.

$\exists x \exists z \forall y Q(y,z) \rightarrow P(x,y)z$

f) Some student in this class grew up in the same town as exactly one other student in this class.

Let P(x,y) mean that student x and y grew up in the same town.

$\exists x \exists y(x \neq y) \land P(x,y) \land \forall z(P(x,z) \rightarrow (x = y \lor x = z))$

24. Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

a) ∃x∀y(x+y = y)

There is a number that when added to every number does not change its value.

d) ∀x∀y((x $\neq$ 0)∧(y $\neq$ 0) ↔ (xy $\neq$ 0))

The product of two numbers is nonzero if and only if both factors are nonzero.

32. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

d) ∀y∃x∃z(T(x,y,z)∨Q(x,y))

∃y∀x∀z(¬T(x,y,z)∧¬Q(x,y))

34. Find a common domain for the variables x, y, and z for which the statement∀x∀y((x $\neq$ y)→∀z((z = x)∨ (z = y))) is true and another domain for which it is false.

True : A company has two departments, x,y,z means that a department.

False: A company has three departments, x,y,z means that a department.

38. Express the negations of these propositions using quantifiers, and in English.

b) There is a student in this class who has never seen a computer.

Let P(x) be “student x has seen a computer.”

$\forall x P(x)$

Every student in this class has seen a computer.

d) There is a student in this class who has been in at least one room of every building on campus.

Let P(z,y) be “Room z is in building y,”

Let Q(x,z) be “Student x has been in room z.”

the original statement is $\exists x \exists z \forall y (P(z, y) \wedge Q(x, z))$

the negation of the statement is $\forall x \forall z \exists y (\neg P(z, y) \vee \neg Q(x, z))$

There is a building such that for every room in that building, every student in this class has not been in that room.

42. Use quantifiers to express the distributive laws of multiplication over addition for real numbers.

x,y,z has the domain of the real number

$\forall x \forall y \forall z (x(y + z) = xy + xz)$