

Sec. 1.1

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- a) This is not a proposition;
- b) This is not a proposition;
- c) This is a proposition that is false;
- d) This is not a proposition; its truth value depends on the value of x .
- e) This is a proposition that is false.
- f) This is not a proposition; its truth value depends on the value of n .

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- d) false
- e) false.

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- e) $(p \wedge q) \rightarrow r$

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- a) If you get promoted, then you wash the boss's car.
- b) If the winds are from the south, then there will be a spring thaw.
- c) If you bought the computer less than a year ago, then the warranty is good.
- d) If Willy cheats, then he gets caught.
- e) If you access the website, then you must pay a subscription fee.
- f) If you know the right people, then you will be elected.
- g) If Carol is on a boat, then she gets seasick.

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b)

converse(逆命题): Whenever I go to the beach, it is a sunny summer day.

inverse(否命题): Whenever it is not a sunny day, I do not go to the beach.

contrapositive(逆否命题): Whenever I do not go to the beach, it is not a sunny summer day.

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f)

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

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This cannot be a proposition, because it cannot have a truth value. if it were true, then it would be truly asserting that it is false, a contradiction; on the other hand if it were false, then its assertion that it is false must be false, so that it would be true—again a contradiction.

Sec. 1.2

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To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service. Express your answer in terms of w: "You can use the wireless network in the airport," d: "You pay the daily fee," and s: "You are a subscriber to the service."

$$w \rightarrow (d \vee s)$$

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Are these system specifications consistent? "Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."

p : the system software is being upgraded

q: users can access the file system

r: users can save new files

$$p \rightarrow \neg q$$

$$q \rightarrow r$$

$$\neg r \rightarrow \neg p$$

Note that we can make all the conclusion true by making p false and r true.

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When planning a party you want to know whom to invite. Among the people you would like to invite are three touchy friends. You know that if Jasmine attends, she will become unhappy if Samir is there, Samir will attend only if Kanti will be there, and Kanti will not attend unless Jasmine also does. Which combinations of these three friends can you invite

so as not to make someone unhappy?

I use j , s , and k for the propositions that Jasmine, Samir, and Kanti attend, respectively. And I get the following statements:

$$j \rightarrow \neg s$$

$$s \rightarrow k$$

$$k \rightarrow j$$

Since I want all the statements to be true, they can be combined into a formula.

$$(j \rightarrow \neg s) \wedge (s \rightarrow k) \wedge (k \rightarrow j)$$

$$= (\neg j \vee \neg s) \wedge (\neg s \vee k) \wedge (\neg k \vee j)$$

$$= (\neg s + \neg j * k) * (\neg k + j)$$

$$= \neg s * \neg k + \neg s * j$$

So, the only combinations of friends that make everybody happy are Jasmine and Kanti, or Jasmine alone (or no one!).

Sec 1.3

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p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

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Use De Morgan's laws to find the negation of each of the following statements.

a) Kwame will take a job in industry or go to graduate school.

Kwame will not take a job in industry and will not go to graduate school.

b) Yoshiko knows Java and calculus.

Yoshiko does not know Java or does not know calculus.

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$$b) [(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

$$= \neg[(\neg p \vee q) \wedge (\neg q \vee r)] \vee (\neg p \vee r)$$

$$= (p \wedge \neg q) \vee (q \wedge \neg r) \vee \neg p \vee r$$

$$= [p \wedge \neg q \wedge (r \vee \neg r)] \vee [q \wedge \neg r \wedge (p \vee \neg p)] \vee \neg p \vee r$$

$$= (p \wedge \neg r) \vee \neg p \vee r$$

$$= \neg(\neg p \vee r) \vee (\neg p \vee r)$$

$$= \text{True}$$

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Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

$$\begin{aligned} \text{原式} &= (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee q \vee r \\ &= (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg r \wedge r) \vee (p \wedge \neg r \wedge \neg r) \vee q \vee r \\ &= (\neg q \wedge r) \vee q \vee r \\ &= \neg(q \vee r) \vee q \vee r \\ &= \text{True} \end{aligned}$$

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Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

$$\begin{aligned} (p \wedge q) \rightarrow r &= \neg p \vee \neg q \vee r \\ (p \rightarrow r) \wedge (q \rightarrow r) &= (\neg p \vee r) \wedge (\neg q \vee r) \\ &= (\neg p \wedge \neg q) \vee (r \wedge \neg q) \vee (\neg p \wedge r) \vee r \\ &= \neg(p \vee q) \vee r \end{aligned}$$

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Find a compound proposition involving the propositional variables p , q , and r that is true when p and q are true and r is false, but is false otherwise. [Hint: Use a conjunction of each propositional variable or its negation.]

$$p \wedge q \wedge \neg r$$

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Find a compound proposition logically equivalent to $p \rightarrow q$ using only the logical operator \downarrow

The proposition p NOR q is true when both p and q are false, and it is false otherwise. The operator \downarrow is called Peirce arrow.

$$p \rightarrow q$$

$$= \neg p \vee q$$

$$= (\neg p \downarrow q) \downarrow (\neg p \downarrow q)$$

$$= ((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q)$$