Sec. 9.1

7.Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where (x, y) $\in$ R if and only if

a) $x \neq y$.

symmetric

c) $x = y + 1 \; or \; x = y - 1$.

symmetric

h)$x \geq y^2$.

antisymmetric， transitive

26. Let R be the relation R = {(a, b) | a<b} on the set of integers. Find

a)R^{-1}

$\{(b,a)|(a,b) \in R\} = \{(b,a)|a < b\} = \{(a,b)|a > b\}$

b)$\overline{R}$

$R = \{(a,b)|a \geq b\}$

32. Let R be the relation {(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)}, and let S be the relation {(2, 1), (3, 1), (3, 2), (4, 2)}. Find S ∘ R.

$$R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$S \circ R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

The answer is $\{(1,1),(1,2),(2,1),(2,2)\}$.

47. How many relations are there on a set with n elements that are

a) symmetric?

$2^n * 2^{\frac{n^2-n}{2}} = 2^{\frac{n^2+n}{2}}$

b) antisymmetric?

$2^n * 3^{\frac{n^2-n}{2}}$

c) asymmetric?

$3^{\frac{n^2-n}{2}}$

d) irreflexive?

2^{n^2-n}

e) reflexive and symmetric?

$2^{\frac{n^2-n}{2}}$

f) neither reflexive nor irreflexive?

$2^{n^2} - 2^{n^2-n+1}$

51. Show that the relation R on a set A is symmetric if and only if $R = R^{-1}$, where R^{-1} is the inverse relation.

if R is symmetric and $(a, b) \in R$, then $(b, a) \in R$, so $(a, b) \in R^{-1}$. Hence, $R \subseteqq R^{-1}$

Similarly, $R^{-1} \subseteqq R$. So $R = R^{-1}$.

Conversely, if $R = R^{-1}$ and $(a, b) \in R$, then $(a, b) \in R^{-1}$, so $(b, a) \in R$.

Thus R is symmetric.

Sec. 9.3

13. Let R be the relation represented by the matrix $M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Find the matrix representing

a) R^{-1}.

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

b) $\overline{R}$.

$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

c) R^2.

$$M_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

14. Let R1 and R2 be relations on a set A represented by the matrices $M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Find the matrices that represent

a) R1 ∪ R2.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

b) R1 ∩ R2.

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

c) R2 ◦ R1.

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

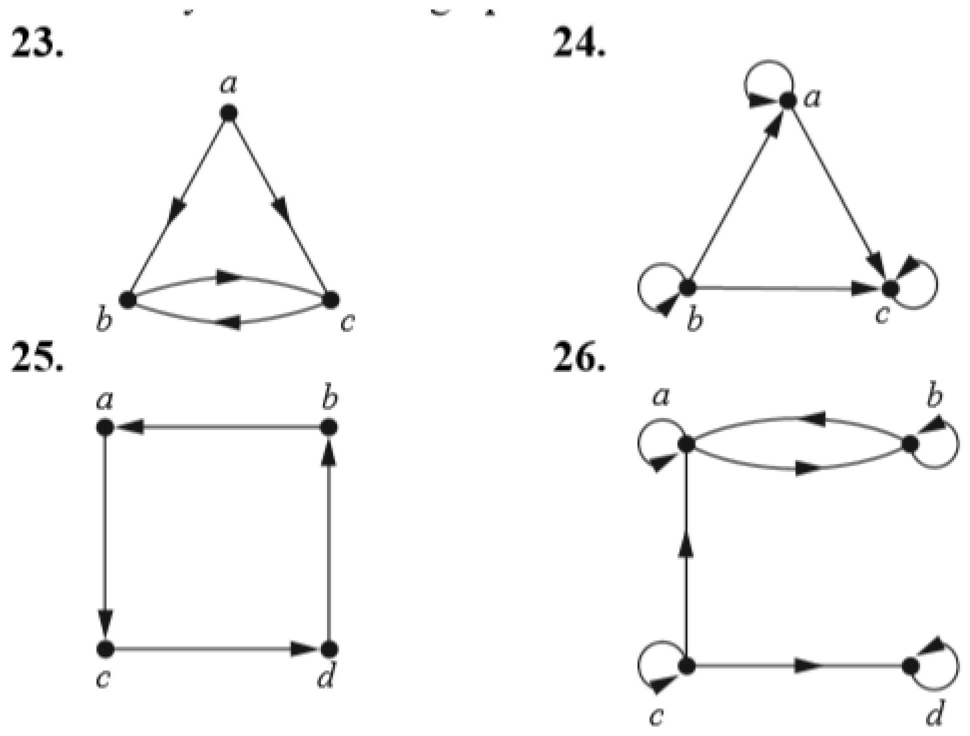
d) R1 ◦ R1.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

e) R1 ⊕ R2.

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

31. Determine whether the relations represented by the directed graphs shown in Exercises 23–25 are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.



23.

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

irreflexive

24

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

reflexive antisymmetric transitive

25

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

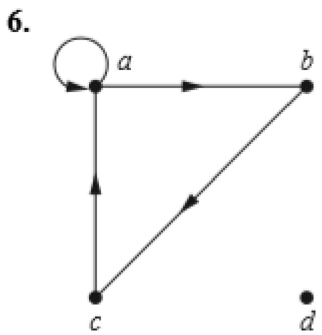
irreflexive, antisymmetric,

Sec. 9.4

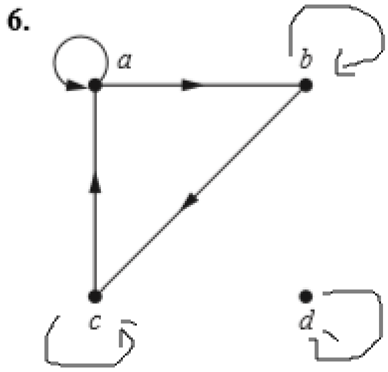
2. Let R be the relation $\{(a,b)|a \neq b\}$ on the set of integers. What is the reflexive closure of R?

$Z \times Z$

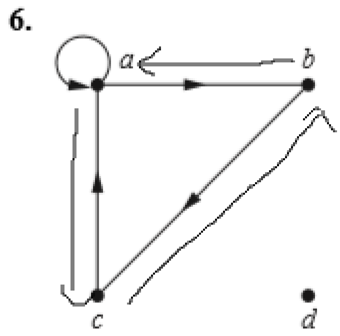
6. draw the directed graph of the reflexive closure of the relations with the directed graph shown. Find the directed graphs of the symmetric closures of the relations. Find the directed graph of the smallest relation that is both reflexive and symmetric that contains each of the relations.



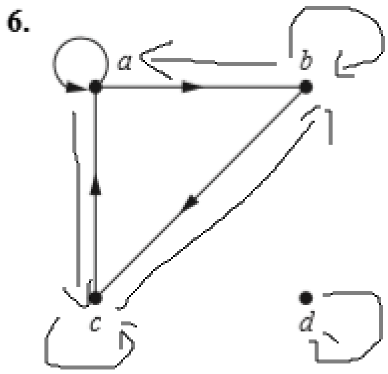
reflexive closure:



symmetric closures:



both reflexive and symmetric :



20. Let R be the relation that contains the pair (a,b) if a and b are cities such that there is a direct non-stop airline flight from a to b. When is (a,b) in

R^2

it is possible to fly from a to b with a scheduled stop (and possibly a plane change) in some intermediate city.

R^3

it is possible to fly from a to b with two scheduled stops in some intermediate city.

R^*

it is possible to fly from a to b.

28. Use Warshall's algorithm to find the transitive closures of the relations .

a) {(a,c),(b,d),(c,a),(d,b),(e,d)}

$$R = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} W_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} W_3 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$W_4 = W_5 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

29. Find the smallest relation containing the relation {(1,2),(1,4),(3,3),(4,1)} that is

a) reflexive and transitive.

transive: {(1,1),(1,2),(1,4),(3,3),(4,1),(4,2),(4,4)}

answer: {(1,1),(1,2),(1,4),(2,2),(3,3),(4,1),(4,2),(4,4)}

b) symmetric and transitive.

transive: {(1,1),(1,2),(1,4),(3,3),(4,1),(4,2),(4,4)}

answer: {(1,1),(1,2),(1,4),(2,1),(2,4),(3,3),(4,1),(4,2),(4,4)}

c) reflexive, symmetric, and transitive.

answer: {(1,1),(1,2),(1,4),(2,1),(2,2),(2,4),(3,3),(4,1),(4,2),(4,4)}