Sec. 1.6

12. Show that the argument form with premises (p∧t)→ (r ∨s), q → (u∧t), u → p, and ¬s and co；nclusion q → r is valid by first using Exercise 11 and then using rules of inference from Table 1.

(p∧t)→ (r ∨s) => $\neg p \vee \neg t \vee r \vee s$

q → (u∧t) => $\neg q \vee (u \wedge t)$

u → p => $\neg u \vee p$

q, ¬s

q, $\neg q \vee (u \wedge t)$ => $u \wedge t$

$u \wedge t$, $\neg u \vee p$ => p

14. For each of these arguments, explain which rules of inference are used for each step.

d) "There is someone in this class who has been to France. Everyone who goes to France visits the Louvre. Therefore, someone in this class has visited the Louvre."

Let c(x) be "x is in this class,"

Let f(x) be "x has been to France,"

Let l(x) be "x has visited the Louvre."

	Step	Reason
1	$\exists x(c(x) \wedge f(x))$	Premise
2	$\forall x(f(x) \rightarrow l(x))$	Premise
3	$c(y) \wedge f(y)$	Existential instantiation using (1)
4	c(y)	Simplification using (3)
5	f(y)	Simplification using (3)
6	$f(y) \rightarrow l(y)$	Universal instantiation using (2)
7	l(y)	Modus ponens using (5) and (6)
8	$c(y) \wedge l(y)$	Conjunction using (4) and (7)
9	$\exists x(c(x) \wedge l(x))$	Existential generalization using (8)

18. What is wrong with this argument? Let $S(x,y)$ be "x is shorter than y." Given the premise $\exists s S(s,Max)$, it follows that $S(Max, Max)$. Then by existential generalization it follows that $\exists x S(x,x)$, so that someone is shorter than himself.

We know that some s exists that makes S(s,Max) true, but we cannot conclude that Max is one such s.

24. Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x)\vee Q(x))$ is true then $\forall xP(x)\vee\forall xQ(x)$ is true.

1. $\forall x(P(x)\vee Q(x))$ Premise

2. $P(c)\vee Q(c)$ Universal instantiation from (1)

3. $P(c)$ Simplification from (2)

4. $\forall xP(x)$ Universal generalization from (3)

5. $Q(c)$ Simplification from (2)

6. $\forall xQ(x)$ Universal generalization from (5)

7. $\forall x(P(x)\vee\forall xQ(x))$ Conjunction from (4) and (6)

Steps 3 and 5 are incorrect. conjunction cannot use the simplification.

29. Use rules of inference to show that if $\forall x(P(x)\vee Q(x))$, $\forall x(\neg Q(x)\vee S(x))$, $\forall x(R(x)\rightarrow\neg S(x))$, and $\exists x\neg P(x)$ are true, then $\exists x\neg R(x)$ is true.

$\exists x\neg P(x) \Rightarrow \neg P(m)$

$\forall x(P(x)\vee Q(x)) \Rightarrow P(m)\vee Q(m)$

$\forall x(\neg Q(x)\vee S(x)) \Rightarrow \neg Q(m)\vee S(m)$

$\forall x(R(x)\rightarrow\neg S(x)) \Rightarrow R(m)\rightarrow\neg S(m) \Rightarrow \neg R(m)\vee\neg S(m)$

$\neg P(m), P(m)\vee Q(m) \Rightarrow Q(m)$

$\neg Q(m)\vee S(m),\quad Q(m) \Rightarrow S(m)$

$S(m), \neg R(m)\vee\neg S(m) \Rightarrow R(m)$

$R(m) \Rightarrow \exists x\neg R(x)$

34. The Logic Problem, taken from WFF' N PROOF, The Game of Logic, has these two assumptions: 1. "Logic is difficult or not many students like logic." 2. "If mathematics is easy, then logic is not difficult." By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions:

a) That mathematics is not easy, if many students like logic.

Let d for "logic is difficult";

Let s for "many students like logic"

Let e for “mathematics is easy.”

the assumptions are $d \vee \neg s$ and $e \rightarrow \neg d$.

the conclusion is $s \rightarrow \neg e$

$d \vee \neg s$ => $s \rightarrow d$

$e \rightarrow \neg d$ => $d \rightarrow \neg e$

we can conclude $s \rightarrow \neg e$

Sec 1.7

7. Use a direct proof to show that every odd integer is the difference of two squares.

Let n be an odd integer, and we can write n = 2k+1.

Then we know that $(k+1)^2 - k^2 = 2k + 1$

So every odd integer is the difference of two squares

8. Prove that if n is a perfect square, then n+2 is not a perfect square.

Let's assume n+2 is a perfect square.

Let $n = k^2$ (k is a nonnegative integer)

Then n+2 can be $(k+1)^2$

$(k+1)^2 - k^2 = 2k + 1 \geq 3 > n + 2 - n = 2$

So, the assumption is wrong, n+2 cannot be a perfect square.

34. Is this reasoning for finding the solutions of the equation $\sqrt{2x^2 - 1} = x$ correct? (1)$\sqrt{2x^2 - 1} = x$ is given; (2)$2x^2 - 1 = x^2$, obtained by squaring both sides of (1); (3) $x^2 - 1 = 0$, obtained by subtracting x^2 from both sides of (2); (4) $(x-1)(x+1) = 0$, obtained by factoring the left-hand side of $x^2 - 1$; (5) x = 1 or x =−1, which follows because ab= 0 implies that a = 0 or b = 0.

(1) -> (2) Note that x must lager than zero.

Sec 1.8

22. Use forward reasoning to show that if x is a nonzero real number, then $x^2 + 1/x^2 \geq 2$. [Hint: Start with the inequality $(x - 1/x)^2 \geq 0$ which holds for all nonzero real numbers x.]

$(x - 1/x)^2 \geq 0$

$x^2 - 2 + (1/x)^2 \geq 0$

$x^2 + 1/x^2 \geq 2$

24 The quadratic mean of two real numbers x and y equals $\sqrt{(x^2+y^2)/2}$. By computing the arithmetic and quadratic means of different pairs of positive real numbers, formulate a conjecture about their relative sizes and prove your conjecture.

conjecture: $\frac{x+y}{2} \leq \sqrt{\frac{x^2+y^2}{2}}$

$x^2+y^2 \geq 2xy$

$2x^2+2y^2 \geq (x+y)^2$

$\frac{(x+y)^2}{4} \leq \frac{x^2+y^2}{2}$

$\frac{x+y}{2} \leq \sqrt{\frac{x^2+y^2}{2}}$

so,we can prove the conjecture.

Sec 2.1

11. Determine whether each of these statements is true or false

a) x ∈{ x} T

b) {x}⊆{ x} T

c) {x}∈{ x} F

d) {x}∈{{ x}} T

e) ∅⊆{ x} T

f) ∅∈{ x} F

18. Find two sets A and B such that A ∈ B and A ⊆ B.

A = ∅

B = {∅}

22. Can you conclude that A = B if A and B are two sets with the same power set?

Yes, The union of all the sets in the power set of a set X must be exactly X.

24. Determine whether each of these sets is the power set of a set, where a and b are distinct elements.

a) ∅ F

b) {∅,{a}} T {a}

c) {∅,{a},{∅,a}} F

d) {∅,{a},{b},{a,b}} T {a,b}

32. Let A ={a,b,c}, B ={x,y}, and C ={ 0,1}. Find

a) A×B ×C.

{(a,x,0),(a,x,1),(a,y,0),(a,y,1),(b,x,0),(b,x,1),(b,y,0),(b,y,1),(c,x,0),(c,x,1),(c,y,0),(c,y,1)}

c) C ×A×B

{(0,a,x),(0,a,y),(0,b,x),(0,b,y),(0,c,x),(0,c,y),(1,a,x),(1,a,y),(1,b,x),(1,b,y),(1,c,x),(1,c,y)}

Sec 2.2

17. Show that if A, B, and C are sets, then $\overline{A \cap B \cap C}$= $\bar{A} \cup \bar{B} \cup \bar{C}$

a) by showing each side is a subset of the other side.

$$\begin{aligned} x &\in \overline{A \cap B \cap C} \\ x &\notin A \cap B \cap C \\ x &\notin A \vee x \notin B \vee x \notin C \\ x &\in \bar{A} \vee x \in \bar{B} \vee x \in \bar{C} \\ x &\in \bar{A} \cup \bar{B} \cup \bar{C} \end{aligned} \tag{1}$$

b) using a membership table.

A	B	C	$\overline{A \cap B \cap C}$	$\bar{A} \cup \bar{B} \cup \bar{C}$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

48. Let A_i ={...,−2,−1,0,1,...,i}. Find

a)$\bigcup_{i=1}^{n} A_i$

A_n

b)$\bigcap_{i=1}^{n} A_i$

A_1

57. Show how bitwise operations on bit strings can be used to find these combinations of A ={a,b,c,d,e}, B ={b,c,d,g,p,t,v}, C ={c,e,i,o,u,x,y,z}, and D ={d,e,h,i, n,o,t,u,x,y}.

c) (A∪D)∩(B ∪C)

$A \cup B \cup C \cup D = \{a, b, c, d, e, g, h, i, n, o, p, t, u, v, x, y, z\}$

A = 1 1111 0000 0000 0000

B = 0 1110 1000 0110 1000

C = 0 0101 0010 1001 0111

D = 0 0011 0111 1011 0110

$A \cup D$ = 1 1111 0111 1011 0110

$B \cup C$ = 0 1111 1010 1111 1111

$(A \cup D) \cap (B \cup C)$ = 0 1111 0010 1011 0110 = {b,c,d,e,i,o,t,u,x,y}