

Mathematical concepts for computer science

Relations

- Relationships between elements of sets are represented using the structure called a relation, which is just a **subset of the Cartesian product** of the sets.
- Sets of ordered pairs are called **binary relations**.

DEFINITION

- Let A and B be sets. A **binary relation from A to B** is a **subset of $A \times B$** .

Relations

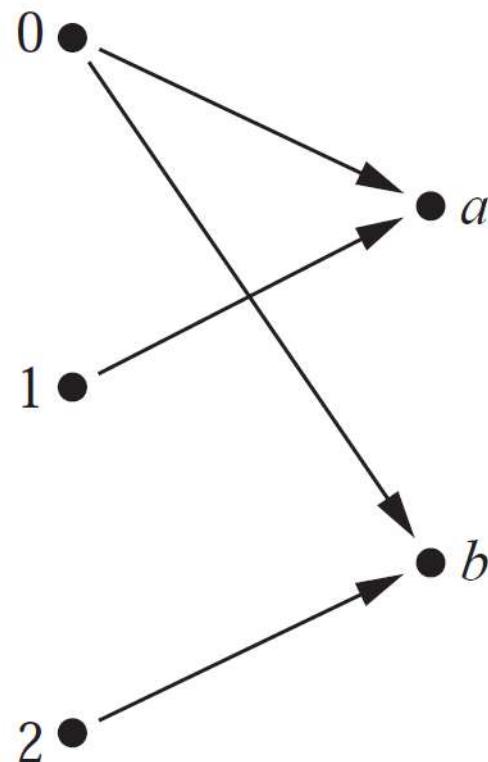
- In other words, a binary relation from A to B is a set R of ordered pairs where the **first element** of each ordered pair comes **from A** and the **second element** comes **from B**.
- $a R b$ to denote that $(a, b) \in R$ and $a \not R b$ to denote that $(a, b) / \in R$. Moreover, when (a, b) belongs to R , **a** is said to be **related to b by R**.

Relations

- Let **A** be the **set of students** in your school, and let **B** be the **set of courses**. Let **R** be the **relation** that consists of those pairs **(a, b)**, where a is a student enrolled in course b.
- For instance, if Vishnu and Arjun are enrolled in **CSC3101**, the pairs **(Vishnu, CSC3101)** and **(Arjun, CSC3101)** belong to **R**.
- However, if Vivek is not enrolled in **CSC3101**, then the pair **(Vivek, CSC3101)** is not in **R**.

Relations

- Let $A = \{0, 1, 2\}$ and $B = \{a, b\}$. Then $\{(0, a), (0, b), (1, a), (2, b)\}$ is a relation from A to B.



R	a	b
0	×	×
1	×	
2		×

Relations on a Set

- A **relation on a set A** is a **relation from A to A**.
- In other words, a **relation on a set A** is a **subset of $A \times A$** .
- Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b) \mid a \text{ divides } b\}$?

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$$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)\}.$$

Relations on a Set

- Consider these relations on the set of integers:
 - $R_1 = \{(a, b) \mid a \leq b\},$
 - $R_2 = \{(a, b) \mid a > b\},$
 - $R_3 = \{(a, b) \mid a = b \text{ or } a = -b\},$
 - $R_4 = \{(a, b) \mid a = b\},$
 - $R_5 = \{(a, b) \mid a = b + 1\},$
 - $R_6 = \{(a, b) \mid a + b \leq 3\}.$

Which of these relations contain each of the pairs
(1, 1), (1, 2), (2, 1), (1, -1), and (2, 2)?

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 - $R_5 = \{(a, b) \mid a = b + 1\}$,
 - $R_6 = \{(a, b) \mid a + b \leq 3\}$.
- $(1, 1)$ is in R_1 , R_3 , R_4 , and R_6 ;
- $(1, 2)$ is in R_1 and R_6 ;
- $(2, 1)$ is in R_2 , R_5 , R_6 ;
- $(1, -1)$ is in R_2 , R_3 , and R_6 ;
- $(2, 2)$ is in R_1 , R_3 , and R_4 .

Properties of Relations

- A relation R on a set A is called **reflexive** if $(a, a) \in R$ for every element $a \in A$.
- Consider the following relations on $\{1, 2, 3, 4\}$:
- $R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$,
- $R_2 = \{(1, 1), (1, 2), (2, 1)\}$,
- $R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$,
- $R_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$,
- $R_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$,
- $R_6 = \{(3, 4)\}$.

Which of these relations are reflexive?

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R3 and R5 are reflexive

Properties of Relations

- Is the “divides” relation on the set of positive integers reflexive?

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- **Because $a \mid a$ whenever a is a positive integer, the “divides” relation is reflexive.**
- Is the “divides” relation on the set of all integers reflexive?

Properties of Relations

- Is the “divides” relation on the set of positive integers reflexive?
- **Because $a \mid a$ whenever a is a positive integer, the “divides” relation is reflexive.**
- Is the “divides” relation on the set of all integers reflexive?
- **not reflexive because by definition 0 does not divide 0.**

Properties of Relations

- A relation R on a set A is called **symmetric** if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.
- A relation R on a set A such that for all $a, b \in A$, if $(a, b) \in R$ but $(b, a) \notin R$, or vice versa is called **anti-symmetric**.

Properties of Relations

- Consider the following relations on $\{1, 2, 3, 4\}$:
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- $R5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$,
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Which of these relations are **symmetric**?

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R2 and R3 are symmetric

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Properties of Relations

- Consider these relations on the set of integers:
 - $R1 = \{(a, b) \mid a \leq b\},$
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 - $R3 = \{(a, b) \mid a = b \text{ or } a = -b\},$
 - $R4 = \{(a, b) \mid a = b\},$
 - $R5 = \{(a, b) \mid a = b + 1\},$
 - $R6 = \{(a, b) \mid a + b \leq 3\}.$

Which of the relations are symmetric and which are anti-symmetric?

Properties of Relations

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 - $R2 = \{(a, b) \mid a > b\}$,
 - **R3** = $\{(a, b) \mid a = b \text{ or } a = -b\}$, $\Rightarrow a = b \text{ or } a = -b$, then $b = a$ or $b = -a$.
 - **R4** = $\{(a, b) \mid a = b\}$, $\Rightarrow a = b$ implies that $b = a$.
 - $R5 = \{(a, b) \mid a = b + 1\}$,
 - **R6** = $\{(a, b) \mid a + b \leq 3\}$. $\Rightarrow a + b \leq 3$ implies that $b + a \leq 3$.

The relations **R3, R4, and R6** are **symmetric**.

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 - $R4 = \{(a, b) \mid a = b\},$
 - $R5 = \{(a, b) \mid a = b + 1\},$
 - $R6 = \{(a, b) \mid a + b \leq 3\}.$

The relations **R1, R2, and R5** are **antisymmetric**.

Properties of Relations

- A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, **then $(a, c) \in R$** , for all $a, b, c \in A$.

Properties of Relations

- Consider the following relations on $\{1, 2, 3, 4\}$:
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Which of these relations are **transitive**?

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R4, R5, and R6 are **transitive**

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Which of the relations are **transitive**?

Properties of Relations

- Consider these relations on the set of integers:
 - $R1 = \{(a, b) \mid a \leq b\}$, $\Rightarrow a \leq b$ and $b \leq c$ imply that $a \leq c$.
 - $R2 = \{(a, b) \mid a > b\}$, $\Rightarrow a > b$ and $b > c$ imply that $a > c$.
 - $R3 = \{(a, b) \mid a = b \text{ or } a = -b\}$, $\Rightarrow a = \pm b$ and $b = \pm c \Rightarrow a = \pm c$
 - $R4 = \{(a, b) \mid a = b\}$,
 - $R5 = \{(a, b) \mid a = b + 1\}$,
 - $R6 = \{(a, b) \mid a + b \leq 3\}$.

Relations **R1, R2, R3, and R4** are **transitive**

Properties of Relations

- Is the “divides” relation on the set of positive integers transitive?

Properties of Relations

- Is the “divides” relation on the set of positive integers transitive?
- Suppose that a divides b and b divides c . Then there are positive integers k and l such that $b = ak$ and $c = bl$. Hence, $c = a(kl)$, so a divides c .

It follows that this relation is transitive.

Representing Relations Using Matrices

- A relation between finite sets can be represented using a **zero–one matrix**.
- Suppose that R is a relation from $A = \{a_1, a_2, \dots, a_m\}$ to $B = \{b_1, b_2, \dots, b_n\}$. The relation R can be represented by the matrix $\mathbf{MR} = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R, \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

Representing Relations Using Matrices

- Suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Let R be the relation from A to B containing (a, b) if $a \in A, b \in B$, and $a > b$. What is the matrix representing R if $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$, and $b_1 = 1$ and $b_2 = 2$?

Representing Relations Using Matrices

- Suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Let R be the relation from A to B containing (a, b) if $a \in A$, $b \in B$, and $a > b$. What is the matrix representing R if $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$, and $b_1 = 1$ and $b_2 = 2$?
- $R = \{(2, 1), (3, 1), (3, 2)\}$

$$M_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Representing Relations Using Matrices

- Suppose that the relation R on a set is represented by the matrix. Is R reflexive, symmetric, and/or anti-symmetric?

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

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- Suppose that the relation R on a set is represented by the matrix. Is R reflexive, symmetric, and/or anti-symmetric?

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- Because all the **diagonal elements** of this matrix are equal to **1**, R is **reflexive**. Moreover, because \mathbf{M}_R is **symmetric**, it follows that R is **symmetric**.

Representing Relations Using Matrices

- Suppose that the relations R_1 and R_2 on a set A are represented by the matrices. What are the matrices representing $R_1 \cup R_2$ and $R_1 \cap R_2$?

$$\mathbf{M}_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

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- Suppose that the relations R_1 and R_2 on a set A are represented by the matrices. What are the matrices representing $R_1 \cup R_2$ and $R_1 \cap R_2$?

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$$\mathbf{M}_{R_1 \cup R_2} = \mathbf{M}_{R_1} \vee \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \mathbf{M}_{R_1 \cap R_2} = \mathbf{M}_{R_1} \wedge \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Representing Relations Using Matrices

- Let the zero-one matrices for S^oR , R , and S be M ($S^oR = [t_{ij}]$, $M(R) = [r_{ij}]$, and $M(S) = [s_{ij}]$, respectively (these matrices have sizes $m \times p$, $m \times n$, and $n \times p$, respectively)).
- The ordered pair (a_i, c_j) belongs to S^oR if and only if there is an element b_k such that (a_i, b_k) belongs to R and (b_k, c_j) belongs to S .
- It follows that $t_{ij} = 1$ if and only if $r_{ik} = s_{kj} = 1$ for some k .

Representing Relations Using Matrices

- Find the matrix representing the relations $S \circ R$, where the matrices representing R and S are

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{M}_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{S \circ R} = \mathbf{M}_R \odot \mathbf{M}_S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Representing Relations Using Matrices

- Find the matrix representing the relation R^2 , where the matrix representing R is

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Representing Relations Using Matrices

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$$\mathbf{M}_{R^2} = \mathbf{M}_R^{[2]} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Closures of Relations

- let R be a relation on a set A . R may or may not have some property P , such as reflexivity, symmetry, or transitivity. If there is a relation S with property P **containing R** such that S is a subset of every relation with property P **containing R** , then S is called the **closure** of R with respect to P .

Closures of Relations

- The relation $R = \{(1, 1), (1, 2), (2, 1), (3, 2)\}$ on the set $A = \{1, 2, 3\}$ is not reflexive.
- How can we produce a reflexive relation containing R that is as small as possible?
- This can be done by adding **(2, 2) and (3, 3)** to R , because these are the only pairs of the form (a, a) that are not in R .
- Clearly, this new relation contains R . Furthermore, **any reflexive relation that contains R must also contain (2, 2) and (3, 3)**. Because this relation contains R , is reflexive, and is contained within every reflexive relation that contains R , it is called the **reflexive closure of R** .

Closures of Relations

- What is the reflexive closure of the relation $R = \{(a, b) \mid a < b\}$ on the set of integers?

$$R \cup \Delta = \{(a, b) \mid a < b\} \cup \{(a, a) \mid a \in \mathbf{Z}\} = \{(a, b) \mid a \leq b\}$$

Closures of Relations

- The relation $\{(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 2)\}$ on $\{1, 2, 3\}$ is not symmetric.
- How can we produce a symmetric relation that is as small as possible and contains R?

Closures of Relations

- The relation $\{(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 2)\}$ on $\{1, 2, 3\}$ is not symmetric.
- How can we produce a symmetric relation that is as small as possible and contains R ?
- To do this, we need only add $(2, 1)$ and $(1, 3)$, because these are the only pairs of the form (b, a) with $(a, b) \in R$ that are not in R .

Symmetric Closures of Relations

- The relation $\{(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 2)\}$ on $\{1, 2, 3\}$ is not symmetric.
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- To do this, we need only add $(2, 1)$ and $(1, 3)$, because these are the only pairs of the form (b, a) with $(a, b) \in R$ that are not in R .
- The symmetric closure of a relation R can be constructed by **adding all ordered pairs of the form (b, a)** , where (a, b) is in the relation, that are not already present in R .

Symmetric Closures of Relations

- The symmetric closure of a relation can be constructed by taking the union of a relation with its inverse, that is, $R \cup R^{-1}$ is the symmetric closure of R , where

$$R^{-1} = \{(b, a) \mid (a, b) \in R\}.$$

Symmetric Closures of Relations

- What is the symmetric closure of the relation $R = \{(a, b) \mid a > b\}$ on the set of positive integers?

Symmetric Closures of Relations

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$$R \cup R^{-1} = \{(a, b) \mid a > b\} \cup \{(b, a) \mid a > b\} = \{(a, b) \mid a \neq b\}$$

Transitive Closures

Let R be a relation on a set A . The *connectivity relation* R^* consists of the pairs (a, b) such that there is a path of length at least one from a to b in R .

The transitive closure of a relation R equals the connectivity relation R^* .

Let \mathbf{M}_R be the zero–one matrix of the relation R on a set with n elements. Then the zero–one matrix of the transitive closure R^* is

$$\mathbf{M}_{R^*} = \mathbf{M}_R \vee \mathbf{M}_R^{[2]} \vee \mathbf{M}_R^{[3]} \vee \dots \vee \mathbf{M}_R^{[n]}.$$

Transitive Closures

Find the zero–one matrix of the transitive closure of the relation R where

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{M}_{R^*} = \mathbf{M}_R \vee \mathbf{M}_R^{[2]} \vee \mathbf{M}_R^{[3]}$$

Transitive Closures

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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Transitive Closures

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$$\mathbf{M}_{R^*} = \mathbf{M}_R \vee \mathbf{M}_R^{[2]} \vee \mathbf{M}_R^{[3]}$$

Transitive Closures

ALGORITHM 1 A Procedure for Computing the Transitive Closure.

procedure *transitive closure* (\mathbf{M}_R : zero–one $n \times n$ matrix)

$\mathbf{A} := \mathbf{M}_R$

$\mathbf{B} := \mathbf{A}$

for $i := 2$ **to** n

$\mathbf{A} := \mathbf{A} \odot \mathbf{M}_R$

$\mathbf{B} := \mathbf{B} \vee \mathbf{A}$

return \mathbf{B} { \mathbf{B} is the zero–one matrix for R^* }

Transitive Closures

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return \mathbf{B} { \mathbf{B} is the zero–one matrix for R^* }

Reference

- **Rosen, Kenneth H., and Kamala Krithivasan.** *Discrete mathematics and its applications: with combinatorics and graph theory*. Tata McGraw-Hill Education, 2016.