Note: There are five sections in this questions paper, one question from each section may be attempted.

I

1.

a) By mathematical induction prove that (a) $1 + 3 + 5 + \cdots \ldots + (2n - 1) = n^2$ (b) Sum of "n" natural numbers is $n * \frac{n+1}{2}$

(6 Marks)

b) A patient takes a lab test and the result comes back positive. It is known that the test returns a correct positive result in only 99% of the cases and a correct negative result in only 95% of the cases. Furthermore, only 0.03 of the entire population has this disease. What is the probability that this patient has cancer?

(4 Marks)

OR

2.

a) A desk lamp produced by luminar company was found to be defective (D). There are three factories (A.B.C) where such desk lamps are manufactured. The quality control manager has the information tabulated below. If he randomly selects a lamp and if it is defective, what is the probability that the lamp was manufactured by A,B or C? (7 Marks)

Factory	% of Production	Probability of Defective lamp
A	0.3	0.021
B	0.3	0.015
C	0.4	0.010

b) In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both cold and hot drinks? (3 Marks)

II

3. Find the shortest spanning tree using Kruskal's Algorithm and Prims algorithm. Show each step of the algorithms. Also, write algorithms for these two methods. (10 Marks).
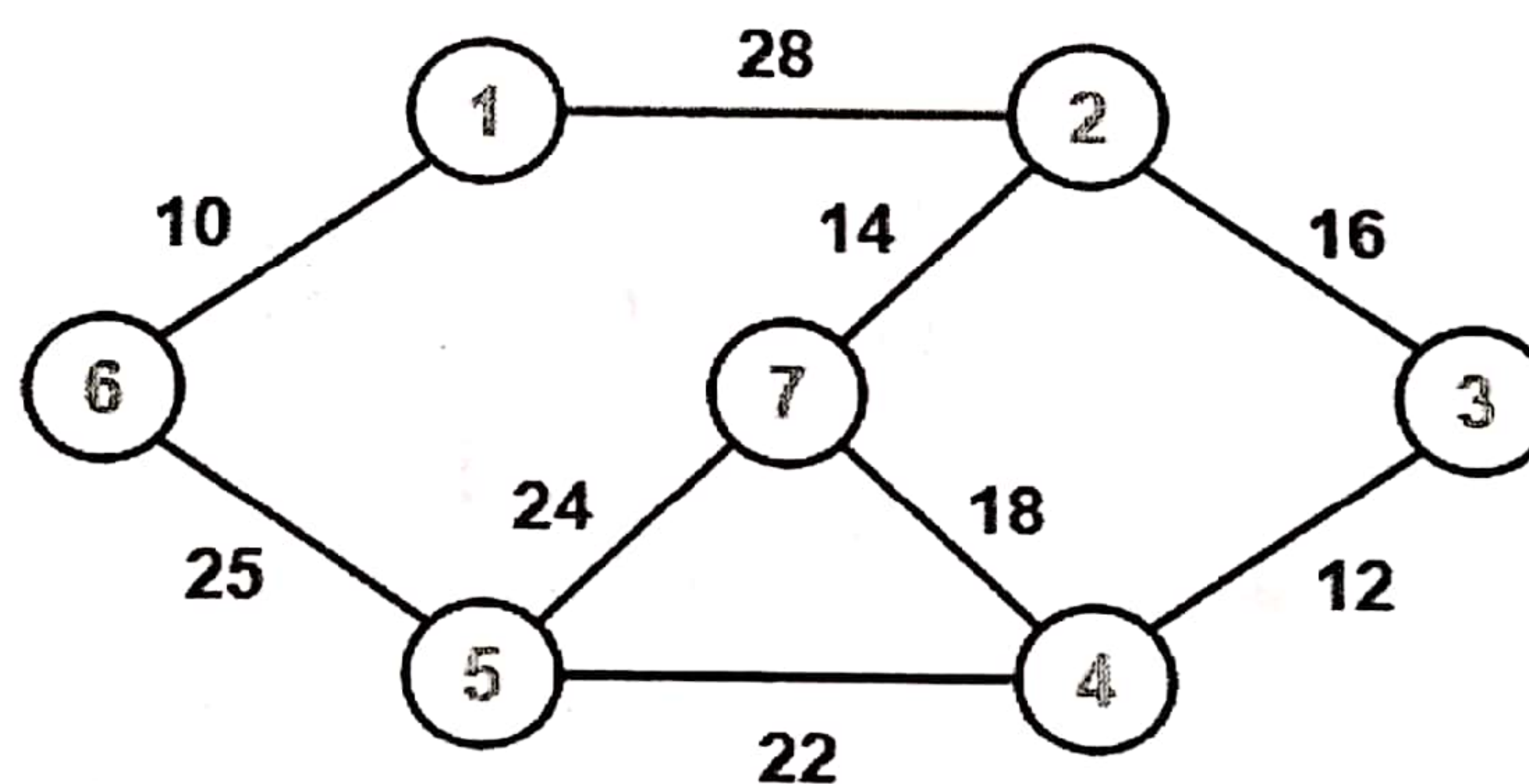

OR

4.

a) Make an adjacency matrix and adjacency list from the above graph (in question 3). What are the properties of the adjacency matrix? (4 Marks)

b) Show the tree (of the above graph in question 3) after breadth first and depth fist search. (6 Marks)

III

5.

a) A jet airliner, flying due east at 500 mph in still air, encounters a 70-mph tailwind blowing in the direction 60° north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they? (4 Marks)

b) If v = 3i - 4j is a velocity vector, express v as a product of its speed times a unit vector in the direction of motion. (3 Marks)

c) Consider a 50-N weight suspended by two wires as shown in the following figure (see the figure on next page). If the magnitude of vector F1 is 35 N, find angle a and the magnitude of vector F2. (3 Marks)

OR

6.
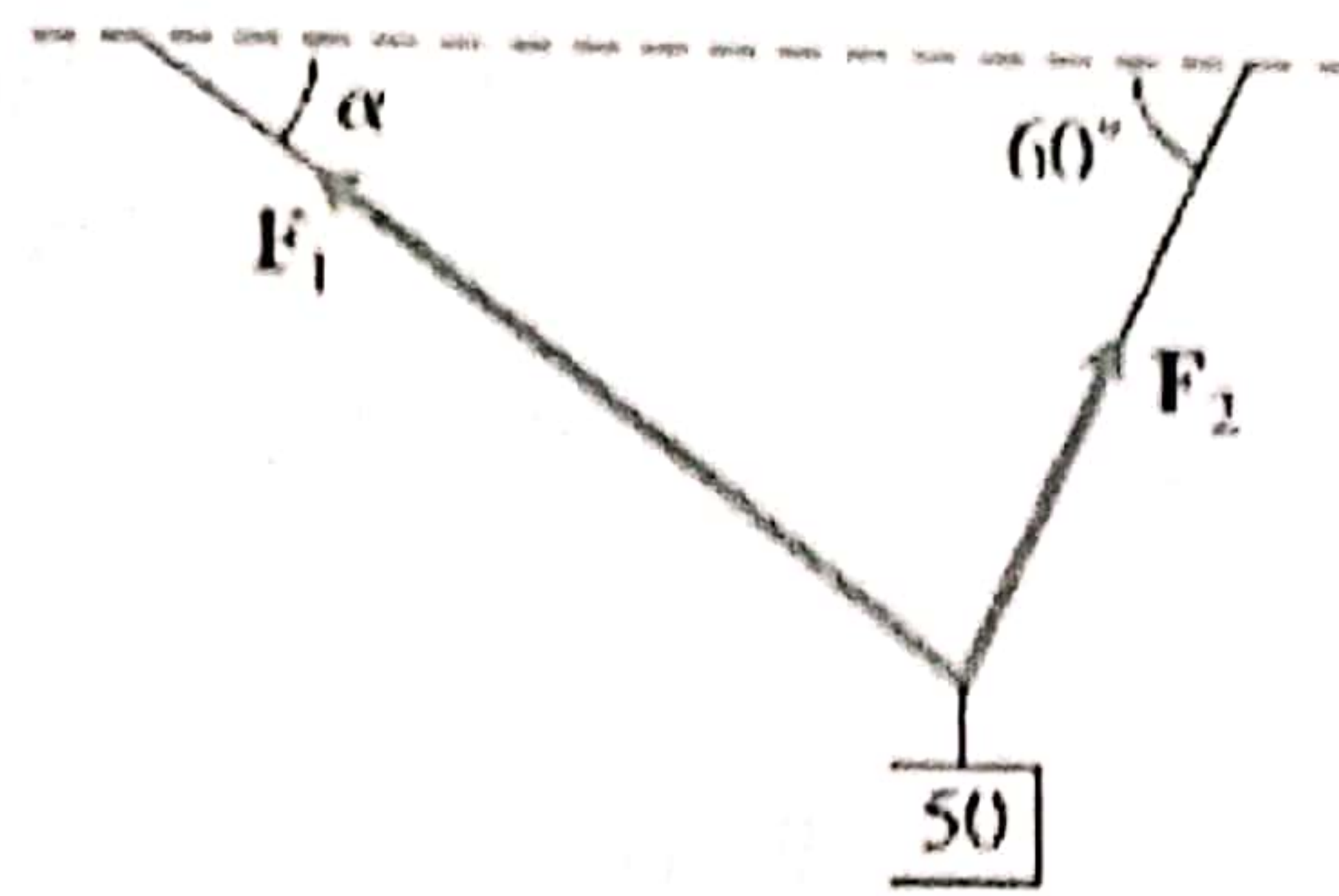
a) Find the vector projection of u = 6i + 3j + 2k onto v = i - 2j – 2k and the scalar component of u in the direction of v. (3 Marks)

b) Find the volume of the box (parallelepiped) determined by u = i + 2j – k, v = - 2i + 3k, and w = 7j – 4k. (3 Marks)

c) Find the distance from the point S(1, 1, 5) to the line
   L: x = 1 + t, y = 3 – t, z = 2t . (2 Marks)

d) Find the angle between the planes 3x - 6y - 2z = 15 and 2x + y - 2z = 5. (2 Marks)

IV

7.

a) Let $f(x, y) = x^2 + 3xy + 4y + 2.$ Evaluate the gradient (vector) of the function at (4,-5). Find its magnitude and direction. (4 Marks)

b) The plane x=2 intersect the parabola $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to parabola at (2,2,5). (3 Marks)

c) Find the derivative of $w = xy$ with respect to t along the path $x = \cos t, y = \sin t$. What is the derivative's value at $t = \frac{\pi}{2}$? (3 Marks)

OR

8.

a) Let $f(x, y) = x^2 + 3xy^2 + 4yx + 2.$ Evaluate the Hessian matrix of the function at (3,4). What are the properties of this matrix? (5 Marks)

b) Find the partial derivative of the function w with respect to r and s, where $w = x^2 + y^2$ and $x = r - s$ and $y = r + s$. (5 Marks)

V

9.

a) Consider a linear system of three equations $x + 2y - z = 2, 2x - 3y + z = -1, 5x - y - 2z = -3$. Find x, y and z. What is the determinant of the coefficient matrix? (5 Marks)

b) Consider the coefficient matrix in the above example (in question. 9a). Perform a LDU decomposition of the same. (5 Marks)

OR

10. Consider the following dataset. What is the probability of playing tennis when the attributes take the values <Sunny, Mild, Normal, Weak>? Assume that the attributes are independent. (10 Marks)

Day	Outlook	Temperature	Humidity	Wind	Play Tennis
Day1	Sunny	Hot	High	Weak	No
Day2	Sunny	Hot	High	Strong	No
Day3	Overcast	Hot	High	Weak	Yes
Day4	Rain	Mild	High	Weak	Yes
Day5	Rain	Cool	Normal	Weak	Yes
Day6	Rain	Cool	Normal	Strong	No
Day7	Overcast	Cool	Normal	Strong	Yes
Day8	Sunny	Mild	High	Weak	No
Day9	Sunny	Cool	Normal	Weak	Yes
Day10	Rain	Mild	Normal	Weak	Yes
Day11	Sunny	Mild	Normal	Strong	Yes
Day12	Overcast	Mild	High	Strong	Yes
Day13	Overcast	Hot	Normal	Weak	Yes
Day14	Rain	Mild	High	Strong	No