(New) Partial Value Iteration

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On-the-fly VALUE ITERATION($\mathcal{Q} \subseteq S \times S$, $\overline{\mathcal{Q}} \subseteq S \times S$)

$\mathcal{Q} \cap \overline{\mathcal{Q}} = \emptyset$

- Let σ be an empty strategy for the min player
- $V_1 = \overline{\mathcal{Q}}$ | V_1 contains all the pairs which are discovered not to be bisimilar.

foreach $u \in \mathcal{Q}$ do
    UPDATE(σ, V_1, u)
end for

Construct a strategy which contains elements of $\mathcal{Q}$ in the domain and it is closed

while $\exists u \in dom(\sigma)$. $\sigma(u) \notin \overline{V_1}$
    UPDATE(σ, V_1, u)
end while

Iteratively update the strategy until saturation

At this point $dom(\sigma) \cap S^2 \subseteq \sim_{BDE}$
and $dom(\sigma) \cap M^2 \subseteq M[\sim_{BDE}]$

EXPAND(σ, V_1, u)
    while $\sigma(u) \nsubseteq dom(\sigma) \cup V_1$
        pick $u' \in \sigma(u) \setminus (dom(\sigma) \cup V_1)$
        UPDATE(σ, V_1, u')
    end while

Expand is used to close the strategy. For the sake of economy, the expansion is performed only on nodes which are not marked in V_1

UPDATE(σ, V_1, u)
    if $\boxed{\exists \omega \in \Gamma(u).\ supp(\omega) \subseteq \overline{V_1}}$ then
        $\sigma(u) \leftarrow supp(\omega)$
        EXPAND(σ, V_1, u)
    else
        Add u to V_1
        Remove u from $dom(\sigma)$
    end if

Update selects an optimal move based on the current value.

+ if no move exists or we discover that the optimal move is losing, we update the value

The choice of the optimal move is done solving a transportation problem. We distinguish two cases

CASE 1 $u = (m_1, m_2) \in M \times M$

Here ω is chosen among the monomial couplings for (m_1, m_2) by solving the following linear program

Let $m_1 = \prod_{i=1}^{k} x_i^{\alpha_i}$ and $m_2 = \prod_{j=1}^{h} x_j^{\beta_j}$

let $c_{ij} = \begin{cases} 1 & \text{if } (x_i, x_j) \in V_1 \\ 0 & \text{otherwise} \end{cases}$

$$\min \sum_{i=1}^{k} \sum_{j=1}^{h} c_{ij} w_{ij}$$
$$\sum_{i=1}^{k} w_{ij} = \beta_j \quad (j = 1..h)$$
$$\sum_{j=1}^{h} w_{ij} = \alpha_i \quad (i = 1..k)$$
$$w_{ij} \geq 0$$

CASE 2 $u = (x, y) \in S \times S$

Here ω is chosen among the polynomial couplings for $(f_x^+ + f_y^-, f_x^- + f_y^+)$

Let $f_x^+ + f_y^- = \sum_{i=1}^{k} \alpha_i m_i$ and
$f_x^- + f_y^+ = \sum_{j=1}^{h} \beta_j m_j$

$c_{ij} = \begin{cases} 1 & \text{if } (m_i, m_j) \in V_1 \\ 0 & \text{otherwise} \end{cases}$

$$\min \sum_{i=1}^{k} \sum_{i=1}^{J} c_{ij} w_{ij}$$
$$\sum_{i=1}^{k} w_{ij} = \beta_j \quad (j = 1 \ldots h)$$
$$\sum_{j=1}^{h} w_{ij} = \alpha_i \quad (i = 1 .. k)$$