

**Exercise 6: Stress invariants****29.11.2021 - 03.12.2021****Question 1** .....

Analyse the plane stress

$$\sigma = \begin{pmatrix} 3 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 2 \end{pmatrix}$$

- Find the rotation angle  $\phi_{\sigma, \max}$  at which the diagonal stress entries are maximal.
- Which values take the principal or main stresses  $\sigma_1$  and  $\sigma_2$ ? You can use the formulas derived in the lecture.
- As well find the rotation angle  $\phi_{\tau, \max}$  at which the shear stress is maximal and compute the value for the maximal shear stress  $\tau_{\max}$ .
- In the lecture it was shown that not only the principal stresses can characterize a stress state but also the stress invariants. Compute the stress invariants  $I_1$  and  $I_2$ .
- The dimension of a stress as well as the dimension of the principal stresses is force per area. What are the dimensions of the two stress invariants  $I_1$  and  $I_2$ .

**Question 2** .....

The following stress tensor characterises a special stress state

$$\sigma = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\sqrt{2} \end{pmatrix}$$

- Compute the angle  $\phi_{\sigma, \max}$  at which the normal stresses takes its maximal value.
- Use the general rotation matrix and the computed angle  $\phi_{\sigma, \max}$  to rotate the stress state in the coordinate system of maximal normal stress. What are the values for the principal stresses?
- What is the special name for the stress state found in (b)?
- In the lecture we have derived two rotation angles to rotate the stress from maximal diagonal elements into the coordinate system where the shear stress is maximal. Take one of the two angles and rotate the stress computed in (b) by the rotation matrix to find the stress state with maximal shear stress.

**Question 3** .....

Now we have a more general three dimensional stress state given by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

- Compute the three principal stresses which are the eigenvalues of the stress tensor.
- What are the values of the three invariants  $I_1$ ,  $I_2$  and  $I_3$  of the given stress state?
- Compute the hydrostatic stress  $\sigma_h$ .
- Compute the deviatoric stress  $s_{ij}$  which is also called stress deviator.
- Which values take the invariants  $J_1$ ,  $J_2$  and  $J_3$  of the stress deviator.
- What is the value of the von Mises stress?
- What is special about  $J_2$  and why is the von Mises stress derived from  $J_2$ ?