Exercise 7: Strain

06.12.2021 - 10.12.2021

Question 1 . . . .

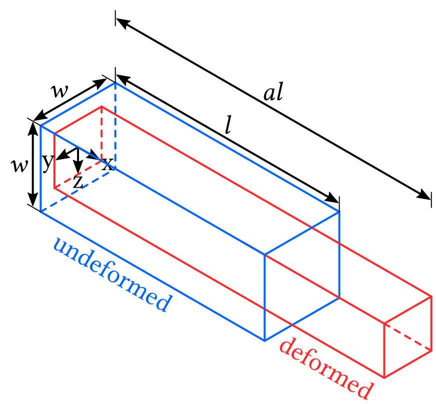
Consider the following displacement field,

$$\mathbf{u}(x, y, z) = k \begin{bmatrix} 2x + y^2 \\ x^2 - 3y^2 \\ 0 \end{bmatrix},$$

where k is a nonzero constant. Calculate the strain tensor ε!

Question 2 . . . .

A solid bar with dimensions $l \times w \times w$ (see below) is stretched along its length to a final length al. The volume of the bar does not change during deformation. Calculate the displacement vector $\mathbf{u}$ and the strain tensor ε!



Question 3 . . . .

(Saint-Venant's compatibility conditions)

The strain tensor ε has six distinct components. However, these six components are computed from only three components of the displacement vector $\mathbf{u}$. Thus, if we want to solve for the components of $\mathbf{u}$ *given* the component of ε, we have six equations for three unknowns. For this system of equations to have a solution, some of the strain components must be related. Show that they are by considering their second derivatives! For example, differentiate ε_{xx} twice with respect to y, ε_{yy} twice with respect to x and ε_{xy} with respect to x and y, and compare!

Question 4 . . . .

In the first question, you computed the strain tensor ε for displacement field

$$\mathbf{u}(x, y, z) = k \begin{bmatrix} 2x + y^2 \\ x^2 - 3y^2 \\ 0 \end{bmatrix}.$$

Now show that ε fulfills the compatibility conditions!