TransERR: Translation-based Knowledge Graph Embedding via Efficient Relation Rotation

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Abstract content

Appendix

1. Proof of Propositions

Proposition 1. *TransERR can infer the symmetry relation pattern.* If $(e_1, r, e_2) \in \mathcal{T}, (e_2, r, e_1) \in \mathcal{T}$, we have

$$\left.\begin{aligned} \mathbf{e_1} \otimes \mathbf{r}^{\triangleleft \mathbf{H}} = \mathbf{e_2} \otimes \mathbf{r}^{\triangleleft \mathbf{T}} \\ \mathbf{e_2} \otimes \mathbf{r}^{\triangleleft \mathbf{H}} = \mathbf{e_1} \otimes \mathbf{r}^{\triangleleft \mathbf{T}} \end{aligned}\right\} \Rightarrow \mathbf{r}^{\triangleleft \mathbf{H}} = -\mathbf{r}^{\triangleleft \mathbf{T}}. \tag{1}$$

Proposition 2. *TransERR can infer the antisymmetry relation pattern.* If $(e_1, r, e_2) \in \mathcal{T}, (e_2, r, e_1) \notin \mathcal{T}$, we have

$$\left.\begin{aligned} \mathbf{e_1} \otimes \mathbf{r}^{\triangleleft \mathbf{H}} = \mathbf{e_2} \otimes \mathbf{r}^{\triangleleft \mathbf{T}} \\ \mathbf{e_2} \otimes \mathbf{r}^{\triangleleft \mathbf{H}} \neq \mathbf{e_1} \otimes \mathbf{r}^{\triangleleft \mathbf{T}} \end{aligned}\right\} \Rightarrow \mathbf{r}^{\triangleleft \mathbf{H}} \neq -\mathbf{r}^{\triangleleft \mathbf{T}}. \tag{2}$$

Proposition 3. *TransERR can infer the inversion relation pattern.* If $(e_1, r_1, e_2) \in \mathcal{T}, (e_2, r_2, e_1) \in \mathcal{T}$, we have

$$\begin{gathered} \left.\begin{aligned} \mathbf{e_1} \otimes \mathbf{r_1}^{\triangleleft \mathbf{H}} + \mathbf{r_1} = \mathbf{e_2} \otimes \mathbf{r_1}^{\triangleleft \mathbf{T}} \\ \mathbf{e_2} \otimes \mathbf{r_2}^{\triangleleft \mathbf{H}} + \mathbf{r_2} = \mathbf{e_1} \otimes \mathbf{r_2}^{\triangleleft \mathbf{T}} \end{aligned}\right\} \Rightarrow \\ (\mathbf{r_1}^{\triangleleft \mathbf{H}} \otimes \mathbf{r_2}^{\triangleleft \mathbf{H}} = \mathbf{r_1}^{\triangleleft \mathbf{T}} \otimes \mathbf{r_2}^{\triangleleft \mathbf{T}}) \wedge (\mathbf{r_1} = -\mathbf{r_2}). \end{gathered} \tag{3}$$

Proposition 4. *TransERR can infer the composition relation pattern.* If $(e_1, r_1, e_2) \in \mathcal{T}, (e_2, r_2, e_3) \in \mathcal{T}, (e_1, r_3, e_3) \in \mathcal{T}$, we have

$$\begin{gathered} \left.\begin{aligned} \mathbf{e_1} \otimes \mathbf{r_1}^{\triangleleft \mathbf{H}} + \mathbf{r_1} = \mathbf{e_2} \otimes \mathbf{r_1}^{\triangleleft \mathbf{T}} \\ \mathbf{e_2} \otimes \mathbf{r_2}^{\triangleleft \mathbf{H}} + \mathbf{r_2} = \mathbf{e_3} \otimes \mathbf{r_2}^{\triangleleft \mathbf{T}} \\ \mathbf{e_1} \otimes \mathbf{r_3}^{\triangleleft \mathbf{H}} + \mathbf{r_3} = \mathbf{e_3} \otimes \mathbf{r_3}^{\triangleleft \mathbf{T}} \end{aligned}\right\} \Rightarrow \\ (\mathbf{r_3}^{\triangleleft \mathbf{H}} = \mathbf{r_1}^{\triangleleft \mathbf{H}} \otimes \mathbf{r_2}^{\triangleleft \mathbf{H}}) \wedge (\mathbf{r_3}^{\triangleleft \mathbf{T}} = \mathbf{r_1}^{\triangleleft \mathbf{T}} \otimes \mathbf{r_2}^{\triangleleft \mathbf{T}}) \\ \wedge (\mathbf{r_3} = \mathbf{r_1} \otimes \mathbf{r_2}^{\triangleleft \mathbf{H}} + \mathbf{r_2} \otimes \mathbf{r_1}^{\triangleleft \mathbf{T}}). \end{gathered} \tag{4}$$

Proposition 5. *TransERR can infer the subrelation pattern.* Consider $\forall e_1, e_2 \in \mathcal{E}$ and $\forall r_1, r_2 \in \mathcal{R} : (e_1, r_1, e_2) \rightarrow (e_1, r_2, e_2)$ as a subrelation relation pattern, and we add an constraint as $\frac{\mathbf{r_2}^{\triangleleft \mathbf{H}}}{\mathbf{r_1}^{\triangleleft \mathbf{H}}} = \frac{\mathbf{r_2}^{\triangleleft \mathbf{T}}}{\mathbf{r_1}^{\triangleleft \mathbf{T}}} = \frac{\mathbf{r_2}}{\mathbf{r_1}} = \mu$, where μ is a learnable parameter and $|\mu| < 1$. Hence, when the constraints are satisfied, we have

$$f_{r1}(\mathbf{h}, \mathbf{t}) < f_{r2}(\mathbf{h}, \mathbf{t}). \tag{5}$$

Specifically, the model focus more on the subrelation r_2. Hence, TransERR can infer the subrelation pattern when the above constraints are satisfied.