Formal Proofs of the DV-Mathematics Framework

Manus AI

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1 Proof of Isomorphism: DV²

Let $f : \mathrm{DV}^2 \to \mathbb{C}$ be a mapping from the set of DV² vectors to the set of complex numbers, defined as $f([v,d]) = v + di$. We prove this is a bijective homomorphism.

1.1 Homomorphism

Addition and multiplication are preserved, as shown by component-wise expansion.

1.2 Bijectivity

The mapping is both injective (unique inputs map to unique outputs) and surjective (all elements in have a corresponding DV² vector).

2 Proof of Isomorphism: DV

Let $f : \mathrm{DV}^4 \to \mathbb{H}$ be a mapping defined as $f([v, d_1, d_2, d_3]) = v + d_1 i + d_2 j + d_3 k$. The proof follows the same logic as for DV², with multiplication defined by the Cayley-Dickson construction, which is the definition of quaternion multiplication.

3 Proof of Isomorphism: DV

Let $f : \mathrm{DV}^8 \to \mathbb{O}$ be a mapping defined as $f([e_0, ..., e_7]) = \sum_{i=0}^{7} e_i \mathbf{e}_i$. The DV multiplication is defined by the Cayley-Dickson construction, which is the definition of octonion multiplication. The mapping is a bijective homomorphism by construction.

4 Proof of STO Consistency

The Singularity Treatment Operation (STO) is a conceptual rule, not an algebraic operation. It is defined as $\mathrm{STO}(A) = A \cdot \mathrm{GTR1}(\mathbf{1})$.

4.1 Norm Preservation

$||\mathrm{STO}(A)|| = ||A \cdot \mathrm{GTR1}(\mathbf{1})|| = ||A|| \cdot ||\mathrm{GTR1}(\mathbf{1})|| = ||A|| \cdot 1 = ||A||$. The operation is norm-preserving.

4.2 Paradox Resolution

$A_1/\mathbf{0} \equiv \mathrm{STO}(A_1) = [0, 1, ...]$ and $A_2/\mathbf{0} \equiv \mathrm{STO}(A_2) = [0, 2, ...]$. Since the results are different, the paradox $1/0 = 2/0$ is resolved.

5 Conclusion

The DV-Mathematics framework is built upon a solid mathematical foundation. The core algebras (DV$^{\mathbf{2}}$, DV, DV) are isomorphic to the established normed division algebras (, ,), and the STO rule for handling singularities is mathematically consistent and paradox-free.