Scientific Report on DV-Mathematics (Dimensions-Vectors)

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November 27, 2025

Abstract

As Ivano Franco Malaspina, I present DV-Mathematics, or Dimensions Vector Space (DV-Space), born from a thought experiment to grasp singularities better, crafted in collaboration with an AI. This hypothetical algebraic space rethinks singularities like division by zero. It builds on real numbers by adding an orthogonal depth dimension, where rotations shift information at tricky points without any loss. DV stands out by holding onto norms and details firmly, dodging issues in setups like the Riemann sphere (which packs singularities into infinity, creating endless values) or wheel algebras (which allow zero-division but drop basics like associativity). Our AI partnership tested how these rotations keep things finite, paving the way for steadier calculations in areas like numerical methods and physics models. This report lays out the space, proves its algebra, weighs it against kin structures, and spotlights uses, showing DV as a sleek extension that tackles arithmetic edges geometrically.

1 Introduction

DV-Mathematics, or Dimensions Vector Space (DV-Space), offers a fresh take on handling singularities like division by zero in a hypothetical algebraic framework. It expands the real numbers with an orthogonal depth dimension, shifting information at problematic points through geometric rotation—preserving everything without loss. What sets DV apart is its commitment to maintaining norms and data integrity, avoiding the pitfalls of methods like the Riemann sphere, which introduces infinity, or wheel algebras, which compromise key properties such as associativity—all while keeping things straightforward and contradiction-free. My journey with DV started from a childhood obsession with black holes, those mysterious cosmic hearts where reality seems to collapse. This sparked a thought experiment: Could a singularity mark not an abrupt end, but the dawn of deeper complexity? Through rotation in an extra dimension, DV gives these "hidden cores" a tangible structure, keeping values finite and intact. Building on this, I delved into arithmetic's blind spots, like how dividing by zero stalls calculations or spawns infinities. Teaming up with an AI helped simulate and hone the rotation idea, essentially "storing" data sideways. Not only does this sidestep paradoxes—such as equating 1 and 2 from $1/0 = 2/0$—but it also seamlessly integrates real numbers for full compatibility. The sections ahead outline the space's basics, core operations including depth rotation, algebraic proofs, comparisons, and potential uses. With its Euclidean metrics and robust structure (forming a field except at zero), DV promises a stable, finite toolkit that could bolster models in computing and theory, even hinting at ways to frame black hole singularities as shifts into concealed realms.

2 Definition and Foundations of the DV-Space

DV-Space unfolds as a two-dimensional vector space over the reals $\mathbb{R}$, naturally stretching the one-dimensional real line. Each element, a dimensions vector, appears as $[v, d]$: v handles the familiar value, linking back to everyday numbers, while d acts as the depth layer, a perpendicular spot for rerouted info. This setup neatly folds in reals—any x becomes $[x, 0]$—so standard ops roll on smoothly until singularities pop up. The basics align with vector space rules: commutative, associative, distributive. Adding $[v_1, d_1] + [v_2, d_2] =$

$[v_1 + v_2, d_1 + d_2]$, with $[0, 0]$ as zero. Scaling $k \cdot [v, d] = [kv, kd]$. The inverse? $[-v, -d]$. To confirm, DV satisfies all eight vector space axioms over $\mathbb{R}$:

Addition is associative, commutative, has identity $[0, 0]$ and inverses. Scalar multiplication is associative, distributive over vector addition and scalar addition, with 1 as scalar identity. These hold component-wise, as in $\mathbb{R}^2$. Unlike complex numbers' quick leap to multiplication, DV stays vector-focused at first, primed for singularity tweaks with room to grow.

3 The Depth Rotation as Core Operation

Depth rotation TR anchors DV-Mathematics, dodging arithmetic snags with a clean twist. It's a linear map: a 90-degree counterclockwise spin in the v-d plane, $\mathrm{TR}([v, d]) = [-d, v]$. It kicks in around the pure depth $[0, 1]$ or near zero-division. Take $\mathrm{TR}([5, 0]) = [0, 5]$: value slides into depth, safe and sound. Or $\mathrm{TR}([3, 4]) = [-4, 3]$, swapping with a sign flip. Linearity shines through: $\mathrm{TR}(k \cdot X + Y) = k \cdot \mathrm{TR}(X) + \mathrm{TR}(Y)$. Proofs flow from the def: scalar and add checks match up. Apply it repeatedly, and a cycle of 4 emerges, proving it's reversible: $\mathrm{TR}^2([v, d]) = [-v, -d]$ (180 degrees), $\mathrm{TR}^3 = [d, -v]$, TR^4 back to start. This loop marks it as unitary, handy for cycles or loops in systems. For singularities, DV treats zero-division as a TR cue, skipping old contradictions and outpacing simpler fixes that breed logic knots.

4 Metric and Norm Preservation in the DV-Space

To give DV-Space geometric or physical flair, it gets the Euclidean inner product: $\langle [v_1, d_1], [v_2, d_2] \rangle = v_1 v_2 + d_1 d_2$. Bilinear, symmetric, positive-definite—it ortho-separates bases $[1, 0]$ and $[0, 1]$. The norm follows: $\|[v, d]\| = \sqrt{v^2 + d^2}$, measuring "length" or "energy." Rotation preserves norms, key to it all. Check: $\|\mathrm{TR}([v, d])\|^2 = (-d)^2 + v^2 = d^2 + v^2 = \|[v, d]\|^2$. Thus, invariance holds, like energy hold in physics spins. Unlike hyperbolic twists in other singularity handles, DV sticks to Euclidean ease, smoothing calcs.

5 Extended Algebra and Multiplication Operation

Pushing DV to full algebra over $\mathbb{R}$, bilinear multiplication joins in, meshing with rotation to form a unity ring: $[v_1, d_1] \cdot [v_2, d_2] = [v_1 v_2 - d_1 d_2, v_1 d_2 + d_1 v_2]$. Linear per arg, distributive over adds. Unity's $[1, 0]$. Multiply by $[0, 1]$? Gets TR exactly: $[0, 1] \cdot [v, d] = [-d, v]$. Echoes complex mult, highlighting isomorphism but layered for singularities. Lets it scale to DV matrices, broadening linear algebra reach.

6 Proofs of the Algebra Properties

Multiplication's algebra holds via tight proofs:

1. Associativity: For $A = [v_1, d_1]$, $B = [v_2, d_2]$, $C = [v_3, d_3]$, left and right $(A \cdot B) \cdot C$ vs. $A \cdot (B \cdot C)$ expand to matching terms, allowing bracket-free chains.

2. Distributivity over Addition: $A \cdot (B + C)$ splits evenly to $A \cdot B + A \cdot C$, thanks to per-arg linearity.

3. Commutativity: $[v_1, d_1] \cdot [v_2, d_2] = [v_2, d_2] \cdot [v_1, d_1]$. Proof: Left: $[v_1 v_2 - d_1 d_2, v_1 d_2 + d_1 v_2]$; right: $[v_2 v_1 - d_2 d_1, v_2 d_1 + d_2 v_1]$—symmetric since multiplication in $\mathbb{R}$ commutes.

4. Norm Preservation in Multiplication: $\|A \cdot B\| = \|A\| \cdot \|B\|$. Squared: expands, cross terms cancel, yielding $(v_1^2 + d_1^2)(v_2^2 + d_2^2)$. Makes DV a normed division algebra (except at zero), akin to reals, complexes, quaternions—fuel for conservation-driven apps.

6.1 Comparisons to Similar Structures and Added Value of DV

DV shares traits with math extensions tackling zero-division but shines via minimalism and norm hold. Compare to the Riemann sphere: it adds infinity to complexes, sending zero-div to ∞. DV keeps finite, rotating info orthogonally—big plus for stability-needy algos dodging infinities. To wheel algebras: they ring-extend for zero-div via absorbing nulls but ditch associativity or distributivity. DV keeps them, weaving singularities via rotation for field-like extensions (sans zero). Dual numbers bring nilpotent ϵ ($\epsilon^2 = 0$) for infinitesimal diffs, not singularities per se—DV's rotation stores orthogonally sans nilpotency, wider for stability. On complex numbers ($\mathbb{C}$): DV mirrors structurally ($[v, d] \leftrightarrow v + di$, TR like i-mult), but adds singularity interp missing in $\mathbb{C}$ (zero-div to infinity). DV favors finite norms, ideal for sims shunning infinities, retaining $\mathbb{C}$ perks. DV's edge: norm hold sans infinity or property drops—blends vector simplicity with geometric edge fixes for robust computes, skipping others' complexity.

6.2 Inverse Elements and Division in the DV-Space

For non-zero norm $B = [v_2, d_2]$, inverse is conjugate over squared norm: $B^{-1} = [v_2, -d_2]/(v_2^2 + d_2^2)$. Proof: $B \cdot B^{-1} = [1, 0]$. Division: $A/B = A \cdot B^{-1}$. Ex: $[5, 0]/[0, 1] = [0, -5]$, norm 5 intact. For $[0, 0]$, fallback to TR(A)—diff results like $[0, 1]$ for $1/0$, $[0, 2]$ for $2/0$ dodge paradoxes. This ensures DV (except $[0, 0]$) forms a field: It has additive/ multiplicative identities ($[0, 0]$ and $[1, 0]$), inverses (as proven), and satisfies associativity, commutativity, distributivity (from section 5). The exception at zero aligns with structures like $\mathbb{C}$, maintaining consistency without full ring division there.

6.3 Validity, Consistency, and Comparisons to Established Structures

DV's validity flows from axiom fulfillment and no contradictions. Classic zero-div paradox ($1/0 = 2/0 \Rightarrow 1 = 2$) gets rotated away sans assumption breaks. To solidify, consider the isomorphism to $\mathbb{C}$: Define $\phi([v, d]) = v + di$. Then:

Addition: $\phi([v_1, d_1] + [v_2, d_2]) = (v_1 + v_2) + (d_1 + d_2)i = \phi([v_1, d_1]) + \phi([v_2, d_2])$. Multiplication: $\phi([v_1, d_1] \cdot [v_2, d_2]) = (v_1 v_2 - d_1 d_2) + (v_1 d_2 + d_1 v_2)i = (v_1 + d_1 i)(v_2 + d_2 i) = \phi([v_1, d_1]) \cdot \phi([v_2, d_2])$. Rotation: $\phi(\text{TR}([v, d])) = \phi([-d, v]) = -d + vi = i(v + di) = i \cdot \phi([v, d])$. Since $\mathbb{C}$ is a field, DV inherits algebraic solidity (except at zero). Sims in code confirm: $[3, 2] \cdot [1, 4] = [-5, 14]$, norm $\sqrt{13} \cdot \sqrt{17} \approx 14.866$, no glitches. Building on earlier comparisons, DV sidesteps associativity losses seen in wheels while keeping norms finite unlike the Riemann sphere. As noted, DV integrates both seamlessly unlike dual numbers. Thus, a valid minimalist stretch offering fresh edge views sans theory clashes. DV codes easy in Python, C++, MATLAB via class for vectors, ops, rotation, mult—quick prototypes. Hits comp sci (stable algos, matrix inverts, error handles), signal proc (phase shifts, filters, compression), theoretical physics (singularity models in gravity, cosmology, quantum fields). Hypotheticals: ML optimizers rotating gradients at singularities for stability (PyTorch loss funcs), data compression shifting/rebuilding orthogonally (SciPy FFT near-zero freqs), quantum sims viewing divergences as dim shifts (QuTiP Lindblad eqs at singularities), med imaging stable reconstructs (Radon trans zero-densities). Economics (Pandas finance sims at limits), biology (BioPython compartment models zero-rates) could gain, but need more theo-empirical checks for real relevance.

7 Conclusion

Wrapping up, DV-Mathematics crafts a seamless way to navigate arithmetic singularities via geometric rotation in an extended space, guarding norms and props sans infinities. As Ivano Franco Malaspina, this grew from my childhood black hole awe, with AI collab unveiling singularities as complexity portals over voids. Proofs and comparisons affirm DV's edges. Ahead: higher dims, DV matrices, tool tests. Enriches math theory, eyes practical steadiness across fields. Open questions include integrating DV with non-Euclidean geometries or quantum mechanics for wider singularity frames.

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