Calculo Vectorial Taller #1 Corte #2 Andrés Felipe Bernal Urrea. 7003748

① Multiplicadores de Lagrange, valores maximos y minimos.

ⓐ $f(x,y,z) = x^2 + y^2 + z^2$; $x^4 + y^4 + z^4 = 1$

$g(x,y,z) = x^4 + y^4 + z^4 - 1$ $\qquad x^4 + y^4 + z^4 - 1 = 0$

$\frac{\partial}{\partial x}\left((x^2+y^2+z^2) = \lambda(x^4+y^4+z^4-1)\right) = 2x = \lambda 4x^3 \quad x = \lambda 2x^3 \quad \lambda = \frac{1}{2x^2}$

$\frac{\partial}{\partial y}\left((x^2+y^2+z^2) = \lambda(x^4+y^4+z^4-1)\right) = 2y = \lambda 4y^3 \quad y = \lambda 2y^3 \quad \lambda = \frac{1}{2y^2}$

$\frac{\partial}{\partial z}\left((x^2+y^2+z^2) = \lambda(x^4+y^4+z^4-1)\right) = 2z = \lambda 4z^3 \quad z = \lambda 2z^3 \quad \lambda = \frac{1}{2z^2}$

$f\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = \boxed{\frac{3}{\sqrt{3}}}\ \boxed{M}$

$x^4 + y^4 + z^4 = 1$

$3x^4 = 1$

$x^4 = \frac{1}{3}$

$x = \sqrt[4]{\frac{4}{3}}$

$x = \pm\frac{1}{\sqrt{3}}$

$\frac{1}{2y^2} = \frac{1}{2x^2} = \frac{1}{2z^2}$

$\frac{2}{2y^2} = \frac{1}{x^2} = \frac{2}{2z^2}$

$\frac{1}{y^2} = \frac{1}{x^2} = \frac{1}{z^2}$

$\boxed{x^2 = y^2 = z^2}$

(b) $f(x,y,z) = yz + xy$; $(xy = 1$; $y^2 + z^2 = 1)$

$$\frac{\partial}{\partial x}\left((yz+xy) = \lambda(xy-1) + \mu(y^2+z^2-1)\right) = y = \lambda y \quad \boxed{1=\lambda}$$

$$\frac{\partial}{\partial y}\left((yz+xy) = \lambda(xy-1) + \mu(y^2+z^2-1)\right) = z + \cancel{x} = \cancel{\lambda x} + \mu 2y \rightarrow \underbrace{z = \mu 2y}$$

$$\frac{\partial}{\partial z}\left((yz+xy) + \lambda(xy-1) + \mu(y^2+z^2-1)\right) = y = \lambda(0) + \mu 2z \rightarrow \underbrace{y = \mu 2z}$$

$\underbrace{xy - 1 = 0}$, $\underbrace{y^2 + z^2 - 1 = 0}$

$x = \frac{2}{\sqrt{2}}$

$y = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$

$y = \pm\sqrt{\frac{2}{2}}$

$z = \pm\sqrt{\frac{2}{2}}$

$z^2 + z^2 = 1$

$2z^2 = 1$

$y = z = \sqrt{\frac{1}{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$xy = 1$

$y = \frac{1}{x}$

$x = \frac{1}{y}$

$z = \frac{\mu 2}{x}$

$x = \frac{\mu 2}{z}$

$z = \frac{1}{2}(2)$

$f(\sqrt{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) = \frac{1}{2} + \frac{2}{2} = \boxed{\frac{3}{2}}$ M

$-y = -z = \boxed{-\frac{\sqrt{2}}{2}}$

$\mu^2 = \frac{1}{4} = \boxed{\pm\frac{1}{2} = \mu}$

$\frac{1}{x} = \mu 2z$

$z = y$

$f(-\sqrt{2}, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}) = -\frac{1}{2} + \frac{2}{2} = \boxed{\frac{1}{2}}$

$\mu 2 = \frac{z}{\mu 2 z}$

$x = \frac{1}{\mu 2 z}$

$f(-\sqrt{2}, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) = -\frac{1}{2} - \frac{2}{2} = \boxed{-\frac{3}{2}}$ m

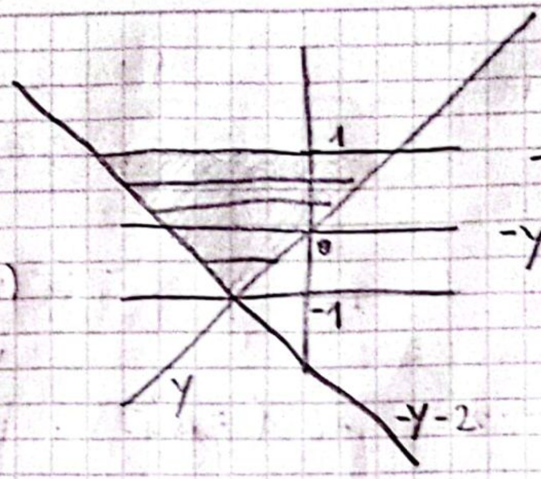
$\mu 2 = \frac{1}{\mu 2}$

$\frac{\mu 2}{z} = \frac{1}{\mu 2 z}$

$f(-\sqrt{2}, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}) = \frac{1}{2} - \frac{2}{2} = \boxed{\frac{-1}{2}}$

② Evalúe la integral doble

$$\iint_D y^2\,dA,\quad D=\{(x,y)\mid -1\le y\le 1,\ -y-2\le x\le y$$



$-1\le y\le 1$

$-y-2\le x\le y$

$$\int_{-1}^{1}\int_{-y-2}^{y} y^2\,dx\,dy=\int_{-1}^{1} xy^2\Big|_{-y-2}^{y}\,dy$$

$(y)y^2-(-y-2)y^2$

$y^3-(-y^3-2y^2)=2y^3+2y^2$

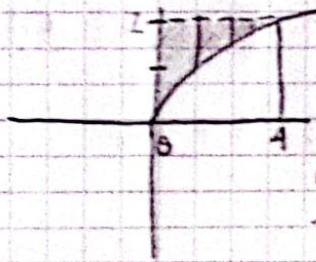
$$\int_{-1}^{1} 2y^3+2y^2\,dy=2\frac{y^4}{4}+2\frac{y^3}{3}\Big|_{-1}^{1}$$

$$\frac{2(1)^4}{4}+\frac{2(1)^3}{3}-\left(\frac{2(-1)^4}{4}+\frac{2(-1)^3}{3}\right)$$

$$\cancel{\frac{1}{2}}+\frac{2}{3}-\cancel{\frac{1}{2}}+\frac{2}{3}=\boxed{\frac{4}{3}}$$

③ Bosqueje la región de integración y cambie el orden de integración:

$$\int_0^4\int_{\sqrt{x}}^{2}\frac{1}{y^3+1}\,dy\,dx \qquad \int_0^2\int_0^{y^2}\frac{1}{y^3+1}\,dx\,dy$$

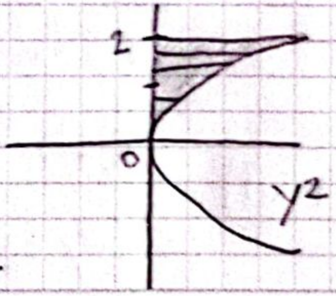


$\sqrt{x}=y$

$x=y^2$

$0<x<4$

$\sqrt{x}<y<2$



$0\le x\le y^2$

$0\le y\le 2$