Parcial Final Ecuaciones Diferenciales

Andrés Felipe Bernal Urrea 2003748

Desarrollo

① $F(a) = \begin{cases} 1, & 0 \leq a \leq 1 \quad \mathcal{U}(t) \\ -1, & 1 \leq a \leq 2 \quad -\mathcal{U}(t-1) \end{cases}$

$\cancel{1} + 1\,\mathcal{U}(t) + \cancel{(-1)} + (-1)(-2\mathcal{U}(t-1))$

$\boxed{\mathcal{U}(t) - 2(\mathcal{U}(t-1))}$

②

ⓐ Si $f(t) = t\,\text{sen}\,5t \Rightarrow \mathcal{L}\{t\,\text{sen}\,5t\} = \boxed{\cancel{\frac{5}{s^4+25s^2}}}$

$\frac{1}{s^2} \qquad \frac{5}{s^2+25} = \frac{5}{s^4+25s^2} = \boxed{\frac{10s}{(s^2+25)^2}}$

ⓑ Si $h(t) = \int_0^t \text{Sen}(t-\tau)\cos(\tau)\,d\tau$ entonces $\mathcal{L}\{h(t)\} = \boxed{-\frac{1}{s^2+1}}$

$\cancel{(\cos(0) - \cos(t))}$

$\cos(t-t)(-\text{Sen}\,t) - \cancel{\cos(t-0)(-\text{Sen}(0))}$

$-\text{Sen}\,t$

$\mathcal{L}\{-\text{sen}\,t\} = -\frac{1}{s^2+1}$

ⓒ $\mathcal{L}\left(\int_0^t e^{a\tau} f(\tau)\,d\tau\right) = \boxed{\dfrac{2}{s^4 - as^3}}$ mientras que $\mathcal{L}\left(e^{a\tau}\int_0^t f(\tau)\,d\tau\right)$

$= \boxed{\dfrac{1}{s^3 - as^2}}$

$\tau e^{a\tau} \tau \Big|_0^t$

$t^2 e^{at} - 0^2 e^{a0}$

$\mathcal{L}\left\{t^2 e^{at}\right\} = \dfrac{2}{s^3} \cdot \dfrac{1}{s-a} = \dfrac{2}{s^4 - as^3}$

$e^{a\tau}\ t = \dfrac{1}{s-a} \cdot \dfrac{1}{s^2} = \dfrac{1}{s^3 - as^2}$

ⓓ Si $\mathcal{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+1}\right\}$ entonces $f(t)$ $\quad \mathcal{U}(t-\pi)$

~~$\mathcal{L}^{-1}\{e^{-\pi s}\}\,\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+1}\right\}$~~

$\boxed{f(t) = \mathcal{U}(t-\pi)\,\mathrm{sen}(t-\pi)}$

$\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+1}\right\} = \mathrm{sen}\ 1t$

(e) $\mathcal{L}^{-1}\left\{\ln\left(\frac{s-3}{s+1}\right)\right\} = \boxed{-\frac{e^{3t}-e^{-t}}{t}}$

$-\frac{1}{t}\ \mathcal{L}^{-1}\left\{\frac{4}{(s-3)(s+1)}\right\}$

$-\frac{1}{t}\ e^{3t}-e^{-t} = -\frac{e^{3t}-e^{-t}}{t}$

(3)

(a) $\mathcal{L}\{t e^{3t} \operatorname{Sen} t\} =$ $\boxed{\frac{2(s-3)}{((s-3)^2+1)^2}}$

(e)

$\frac{1}{s^3}\left(\frac{2}{(s+3)^2+1}\right) = \frac{2(s-3)}{((s-3)^2+1)^2}$

(b) $\mathcal{L}^{-1}\left\{\frac{5-10s}{s^2+9}\right\} = \boxed{\frac{5}{3}\operatorname{Sen} 3t - 10\operatorname{Cos} 3t}$

$\frac{5}{3}\operatorname{Sen} 3t \rightarrow \frac{3}{s^2+9}\left(\frac{5}{3}\right)$ (a)

$10\operatorname{Cos} 3t \rightarrow \frac{10s}{s^2+9}$

(c) $\mathcal{L}\{y'' + 4y = f(t)\}$ $\quad y(0) = 1 \quad y'(0) = 0$

$$s^2Y - s(1) - 0 + 4Y = f(t)$$

$$s^2Y - s + 4Y = f(t)$$

$$s^2Y - s + 4Y = \operatorname{Sen} t\, \mathcal{U}(t-2\pi)$$

$$f(t) = \begin{cases} 0, & 0 \leq t \leq 2\pi \\ \operatorname{Sen} t, & t \geq 2\pi \end{cases}$$

$$Y(s^2+4) - s = \operatorname{Sen} t\, \mathcal{U}(t-2\pi$$

$$\operatorname{Sen} t\, \mathcal{U}(t-2\pi)$$

$$Y(s^2+4) = \frac{e^{-2\pi s}}{s^2+1} + \frac{8}{}$$

$$\boxed{Y = \frac{e^{-2\pi s}}{(s^2+1)(s^2+4} + \frac{8}{(s^2+4)}}$$

(b)

(d) $y'' + y = \mathcal{U}(t-2\pi)$ $\quad y = 0, \; y' = 1$

$$s^2Y - \cancel{s(0)} - 1 + Y = \mathcal{U}(t-2\pi)$$

$$s^2Y - 1 + Y$$

$$Y(s^2+1) - 1 = \mathcal{U}(t-2\pi)$$

$$Y = \frac{\mathcal{U}(t-2\pi) + 1}{(s^2+1)}$$

(c)

$$\boxed{Y = \operatorname{Sen} t + \operatorname{Sen} t\, \mathcal{U}_{2\pi}(t)}$$

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$$Ri + \frac{1}{C}\int_0^t i(\tau)\, d\tau = E(t)$$

$R = 10\Omega$

$C = 0{,}5F$

$E(t) = 2(t^2 + t)$

$$10 + \frac{1}{0{,}5}\int_0^t i(\tau)\, d\tau = 2(t^2 + t)$$

$$10 + \frac{1}{2}\, i(t) = 2(t^2 + t)$$

$$i(t) = \frac{2t^2 + 2t + 10}{2}$$

$$i(t) = \frac{\cancel{2}(t^2 + t + 5)}{\cancel{2}}$$

$$\boxed{i(t) = t^2 + t + 5}$$