Bài tập. Tuần 3, 4

Tìm miền hội tụ của chuỗi hàm

1) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\ln^n x}$

2) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\alpha^n + n}$, $\alpha > 0$

3) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{x}}{2^{nx} + 1}$

4) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(x+n)(x+n+1)}$

5) $\displaystyle\sum_{n=1}^{\infty} n! \, x^{n!}$

6) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(nx)}{e^{nx}}$

7) $\displaystyle\sum_{n=1}^{\infty} \left[x^n + \frac{1}{2^n x^n} \right]$

8) $\displaystyle\sum_{n=1}^{\infty} \left(x^2 + \frac{1}{n} \right)^n$

9) $\displaystyle\sum_{n=1}^{\infty} \frac{x}{(1+x^2)^n}$

10) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{x^{2n} + 1}$

11) $\displaystyle\sum_{n=1}^{\infty} n^{\sqrt{x}} \arcsin\left(\frac{x}{2^{nx}} \right)$

12) $\displaystyle\sum_{n=1}^{\infty} 8^n n^2 \sin^{3n} x$

13) $\displaystyle\sum_{n=1}^{\infty} \sin^n \left(\frac{x \ln n}{x - n} \right)$

14) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\ln|x|}}$

15) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(nx)}{2^{nx}}$

16) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n^2+1}}{n^2+n+1} \left(\frac{4x-1}{x+3} \right)^n$

17) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2^n + 3^n}$

18) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{(n!)^2}$

19) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+5)^{2n-1}}{4^n n^2}$

20) $\displaystyle\sum_{n=1}^{\infty} \left[1 + \frac{(-1)^n}{n} \right] x^n$

21) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^{n^2}}{n^n}$

22) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{e^{n \sin x}}$

23) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\ln^n(x+e)}$

24) $\displaystyle\sum_{n=1}^{\infty} \frac{n^5 x^{2n+1}}{(n+1)!}$

Tính tổng của chuỗi hàm số

25) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \left(1 + \frac{1}{n} \right) x^{n-1}$

26) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}$

27) $\displaystyle\sum_{n=0}^{\infty} \left(1 + \frac{2}{3^{n+1}} \right) x^n$

28) $\displaystyle\sum_{n=1}^{\infty} n x^n$

29) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$

30) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n+2}}{(2n+1)(2n+2)}$

31) $\displaystyle\sum_{n=1}^{\infty} (2^n - n) x^{n+1}$

32) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{4n-3}}{4n-3}$

33) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n-2}$

34) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{5^n}$

35) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n)!!}$

36) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{(2n)!!}$

Khai triển hàm số thành chuỗi Maclaurin

37) $y = \dfrac{1}{1+x+x^2}$

38) $y = \dfrac{x}{4+x^4}$

39) $y = \displaystyle\int_0^x \dfrac{\sin t}{t}\, dt$

40) $y = \ln(x^2+3x+2)$

41) $y = \sqrt[3]{8+x}$

42) $y = x\ln(1+x)$

43) $y = \arcsin x$

44) $y = \ln(x+\sqrt{x^2+1})$

45) $y = \arctan x$

46) $y = xe^{x^2}$

47) $y = (\arcsin x)^2$

50) $y = \sin^3 x$

Khai triển hàm số thành chuỗi lũy thừa của $(x-1)$

51) $y = \dfrac{1}{x^2+3x+2}$

52) $y = \ln(2+x)$

53) $y = \sin \dfrac{\pi x}{3}$