Faculty of Engineering and Physical Sciences
Department of Physics

UNIVERSITY OF SURREY

Year 3, Financial Derivatives
Assessed Coursework Assignment

1. Analysis of Share Price and Option Pricing

The first part of your coursework uses the *daily* closing share prices of a personally allocated share. High frequency financial data is generally more expensive to get, though it does contain interesting correlations. Your allocated share is contained in a separate document on SurreyLearn.

You can download historical data for your share through the Yahoo:Finance website

`http://uk.finance.yahoo.com/`

Enter your share code into the search box on the web site and then click "historical prices" on the left-hand margin. Download to spreadsheet (button at the bottom of the page) and select the largest range of data that is available.

(Note you can use python packages to download the data directly if you wish.)

These data may be read direclty into an EXCEL spreadsheet, or converted to csv and read into other software.

You should undertake a statistical analysis of the share data. Please use the *Adjusted Closing Price* to avoid complications from dividends and other events.

(a) (i) Plot a histogram of the daily returns over the last few years.
(ii) Determine the fit of the daily returns to a normal distribution.
(iii) Present a normal quantile plot to establish the extent to which your data follows a normal distribution.
(iv) Repeat this analysis for 5 day and 10 day intervals and investigate any change in the probability density function.

(b) (i) Calculate the annual drift and volatility of the share price using a 60 day rolling average over the last year, plotting your results on a graph as a function of time.
(ii) Repeat the calculation above, but use the more robust estimate of the median to find the drift, and the median absolute deviation (MAD) to find the volatility:

$$MAD = 1.4826\text{Median}\left(|x_i - \text{Median}(x)|\right).$$

This formula gives a more robust estimate for the standard deviation of the data points x_i.

(c) (i) Using LIBOR as the risk free borrowing rate calculate the value of 1 month and 6 month at-the-money call option, stating any assumptions you make.
(ii) Estimate the uncertainty in these option prices.
(iii) Comment on how you might modify this price if you were a banker selling options on the share in light of your answer to a).

(d) Challenging additional question: Investigate the following two strategies for your share.

(i) Buy and hold for one year.

(ii) If the share price moves up by $x\%$ then buy and hold until it moves down by at least $x\%$ from the subsequent high. At this point sell and go short 1 share. Maintain this position until the price rises by at least $x\%$, whereupon you cover the short and buy one share.

2. Statistical Arbitrage

A hedge fund has bought some data that shows a particular share will rise in value above the risk free rate r. The fund would like to take advantage of this opportunity by using the Black-Scholes model of the share price. Hence, they assume that the share price, S, will evolve according to

$$\frac{dS}{S} = \mu dt + \sigma dW \tag{1}$$

where dS is the change in the share price in a time interval dt, μ the growth rate of the share price, σ the volatility. We will assume that μ and σ are constant.

The proposed strategy to make a profit is as follows. At $t = 0$ we start with no assets, and the share price is $S_0 = 100$. We sell a bond and buy one share. We will hold this portfolio until the share price hits $B = S_0(1 + k)e^{rt}$ at a time t^*. At this point we invest the proceeds in a risk free bond. Hence the portfolio's discounted value to today is

$$v(t) = \begin{cases} S(t)e^{-rt} - S_0 & t \leq t^* \\ S_0 k & t > t^* \end{cases} \tag{2}$$

where t^* is the time at which the share price first touches the value $S(t^*) = B$.

In this question you will write a `FORTRAN 90` program to analyse the long term profitability and risk of this strategy.

a) Derive the present day value of the portfolio given in Eq. (2).

b) If x_1 and x_2 are independent, uniformly distributed random numbers on $(0, 1)$ then they can be used to find two random numbers from a standard normal distribution z_1 and z_2 using the Box-Muller transform:

$$z_1 = \sqrt{-2 \ln x_1} \cos(2\pi x_2) \tag{3}$$

$$z_2 = \sqrt{-2 \ln x_1} \sin(2\pi x_2). \tag{4}$$

Using the intrinsic uniform random numbers from `rand` write a function generate a normally distributed random number with mean zero and variance one from the Box-Muller transform.

c) Write a program that produces a realisation of the daily share price given in Eq. (1) for an annual drift $\mu = 0.16$ and volatility $\sigma = 0.2$. Assume that there are 250 trading days in a year. Use your program to produce a simulation of the daily share price for one year.

d) Using your answer to the previous part calculate the value of the portfolio for 1000 realisations of the stock price at $t = 1$ year, 2 years and 5 years. Use this data to plot a histogram of the value $v(t)$ of the portfolio. Assume that $r = 0.04$ and $k = 0.2$.

[Note: `gnuplot` can produce a histogram from a series of values as follows:

```
binwidth=5
bin(x,width)=width*floor(x/width) + binwidth/2.0
plot 'data' using (bin($1,binwidth)):(1.0) smooth freq
```
]

e) Plot the mean and variance of $v(t)$ as a function of time from $t = 0$ up to 20 years.

f) Plot the probability of a loss as a function of time from $t = 0$ up to 20 years.

g) New data reveals that the volatility is $\sigma = 0.6$. Analyse the trading strategy in this case, and comment.

You should present your answers to these two questions in a structured report. Your structured report should contain a maximum of about **2000** words. Your report should include:

- an abstract
- a section containing a clear, *concise*, and critical analysis of your share from the first question. Include any additional observations you have made. Derivations of key equations are not required but equations should be stated and referenced. Any assumptions should be justified.
- a section containing your answers to the second question, including a commentary on any figures you have produced.
- a reference list (see library resources for correct referencing methods)
- **text files** of your `FORTRAN` programs for the calculations in the second part. Your program should compile using `gfortran`.

You may attempt this piece of coursework using Matlab/python/C/C++/excel if you wish. You still need to submit any programs that demonstrate how you arrived at your results. **Warning:** If your program doesn't produce the correct result is this case then you may miss out on method marks.