Revised PERSiST Equations

The new version of PERSiST (v 1.6.0) includes refinements to the stream flow velocity calculations and to the manner in which evapotranspiration (ET) is calculated. Both of these changes have been made to increase the physical realism of the model. The revised stream flow velocity calculations should help to give more realistic reach depth simulations, and when incorporated into INCA-P, more physically based simulations of instream sediment transport. The new ET calculations have been added so as to better represent the seasonal vegetation controls on ET, and to give model users greater flexibility in specifying evaporative process rates.

Example parameter files for PERSiST v 1.4.0 and 1.6.0 are providedin the accompanying Excel file.

1. **Flow Velocity Calculations**

The new version of the model is based on the presentation in Schulze et al. (2005). Instead of the old power law relationship between stream flow and velocity, the new version incorporates a Manning-Stickler flow velocity calculation (Equation 1)

$$v = n^{-1} \times R^{2/3} \times S^{1/2} \qquad (1)$$

In Eq. 1, the flow velocity (v; m/s) is calculated as a function of river bed roughness (n; dimensionless), hydraulic radius (R; m) and river slope (S; m/m).

Residence time in the reach (T; d) is calculated as follows:

$$T = \frac{l}{86400 \cdot v} \qquad (2)$$

Where *l* is the reach length (user specified parameter; m) and *v* is the stream velocity (equation 1; m/s)

Another big change is that the stream cross section is represented as a trapezoid instead of a rectangle. Thus, the dimensions of the channel cross section are defined by channel width at the sediment interface (W_B; m), channel width at the water surface (W_T; m) and depth of water in the channel (*d*; m).

The width of the top of the stream (W_T; m) is calculated as follows:

$$W_T = a \times Q^b \qquad (3)$$

Where *a* and *b* are empirical constants and *Q* is the streamflow (m^3/s; equation 9)

The depth of the stream channel (*d*; m) is calculated as follows:

$$d = c \cdot Q^f \qquad (4)$$

Where *c* and *f* are empirical constants and *Q* is the streamflow (m^3/s; equation 9). Allen et al. (1994) identified values of 2.71, 0.557, 0.349 and 0.341 for a,b,c and f when simulating bank full discharge.

The area of a trapezoidal stream channel cross section (*A*; m^2) is calculated as follows:

$$A = d \cdot \left(\frac{W_T + W_B}{2} \right) \qquad (5)$$

Where d is the depth of the channel (m; equation 4), W_T is the width of the channel at the water surface (m; equation 3) and W_B is the width of the channel at the sediment surface (m; user specified parameter).

A number of additional calculations are needed. The wetted perimeter of the stream channel (P; m) is calculated as follows:

$$P = W_B + 2 \cdot \left(\left(\frac{W_T - W_B}{2} \right)^2 + d^2 \right)^{\frac{1}{2}} \tag{6}$$

Where W_B is the width of the bottom of the channel (m; user specified parameter), W_T is the width of the top of the channel (m; equation 3) and d is the depth of the channel (m; equation 4)

The hydraulic radius (R; m) is calculated as follows:

$$R = \frac{A}{P} \tag{7}$$

Where A is the area of the stream cross section (m^2; equation 5) and P is the wetted perimeter (m; equation 6).

Reach volume or in-reach water storage (S; m^3) is calculated in a fairly complicated manner dependent on previous day's volume (S_0) and lateral and upstream inflows (Q_{IN}; m^3/d)

$$S = S_0 e^{\frac{-86400v}{l}} + \frac{l}{86400 \cdot v} \cdot Q_{IN} \cdot \left(1 - e^{\frac{-86400v}{l}} \right) \tag{8}$$

Where v is the stream velocity (m/s; equation 1), l is the reach length (m; user specified parameter), S_0 is the previous day's storage (m^3) and Q_{IN} (m^3) is the daily sum of upstream and lateral inflows.

Reach outflow (Q; m^3/s) is calculated as follows:

$$Q = \frac{S_0 + Q_{IN} - S}{86400} \tag{9}$$

Where S is the reach storage volume (m^3; equation 8), S_0 is the previous day's storage (m^3) and Q_{IN} Q_{IN} (m^3) is the daily sum of upstream and lateral inflows.

There are extensive tabulations of Manning's roughness (n), but generally values in natural stream channels vary between 0.01 and 0.1, which higher values possible for overbank flow.

The new values must be specified for each reach (Figure 1).

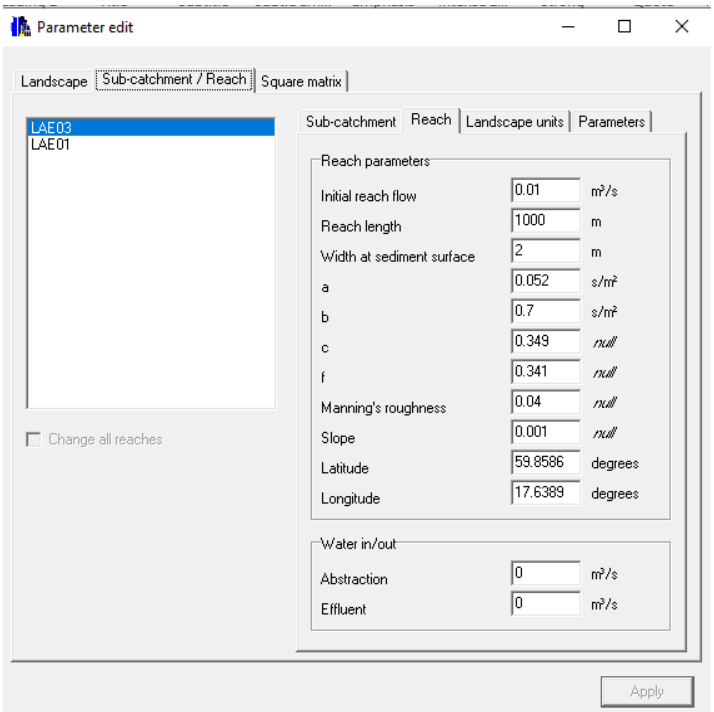

Figure 1; PERSiST 1.6.0 Reach dialog box showing the new parameters to be specified for flow velocity calculations and to estimate extraterrestrial solar radiation. The width at the sediment surface is the width at the bottom of the stream channel, a,b,c, and f are the coefficients for calculating width and depth from stream flow. Manning's roughness and slope are self-explanatory.

2. Evapotranspiration Calculations

The old version of PERSiST used a simple air temperature based model of potential evapotranspiration (PET) based on a plant growth threshold offset and a multiplier. This approach led to unrealistically high PET estimates, especially late in the growing season. There are a number of simple models to calculate PET which do not rely on additional meteorological time series and which perform as well or better than more data intensive methods such as the Penman-Monteith approach (see Oudin et al. 2005 for a review and recommendations).

The new version of PERSiST offers three ways to specify PET: (i) use a Jensen-Haise / McGuinness type model (equation 10) forced by air temperature and modelled extra-solar radiation, (ii) use equation 10 with a user-specified solar radiation time series, (iii) use input time series of land-cover specific daily PET.

Oudin et al. (2005) review a range of PET specifications for rainfall-runoff models and conclude that simple PET representations based on daily air temperature and extraterrestrial radiation perform as well or better than more complex, and data intensive methods. They suggest that an equation of the following form is appropriate when the air temperature (T_A °C) plus the plant growth offset (K_2; °C) is greater than 0:

$$E = \frac{R_e}{\lambda\rho} \times \frac{T_A + K_2}{K_1} \qquad (10)$$

In equation 10, the potential evapotranspiration (E; mm/d) is estimated from extraterrestrial radiation (E_R; W/m^2), the latent heat flux (λ; 28.36 W d /kg), the density of water (ρ; 1000 kg/m^3) and a solar radiation scaling factor (K_1). The latent heat flux is represented as the latent heat of vaporization (2.45 MJ/kg) multiplied by a scaling factor for units conversion (11.57 W/(MJ/d)).

The manner in which actual evapotranspiration (AET) is calculated from PET has not changed. AET is equal to PET when the depth of water exceeds the retained water depth and is constrained by the evapotranspiration adjustment factor when the depth of water is less than the retained water depth (see Futter et al. 2014). It is still possible to specify canopy interception if needed.

Extraterrestrial solar radiation can be calculated based on site latitude and day of year. Thus, PERSiST now requires the user to specify the outflow coordinates (in decimal degrees) for each reach (Figure 1). The mode l will also accept a time series of solar radiation (W/m^2) as the third column of a .dat file. The radiation scaling factor (K_1) and the growing degree threshold, or plant growth offset (K_2) must be specified for each land cover type (Figure 2).

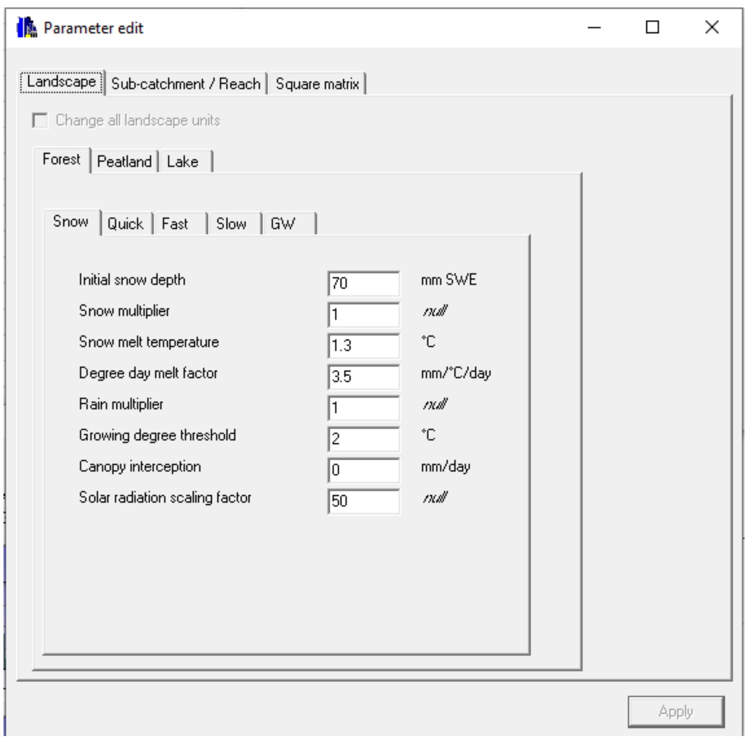

Figure 2: Land cover-specific parameter inputs for PERSiST 1.6. The growing degree threshold (K_2) and the solar radiation scaling factor (K_1) must be provided to estimate potential evapotranspiration.

If the user provides an input .pet file containing daily, land-cover specific values for PET, these will override model calculated values.

References

Allen, P.M., Arnold, J.C. and Byars, B.W., 1994. Downstream channel geometry for use in planning-level models 1. Journal of the American Water Resources Association, 30(4), pp.663-671.

Futter, M.N., Erlandsson, M.A., Butterfield, D., Whitehead, P.G., Oni, S.K. and Wade, A.J., 2014. PERSiST: a flexible rainfall-runoff modelling toolkit for use with the INCA family of models. Hydrology and Earth System Sciences, 18(2), pp.855-873.

Oudin, L., Andréassian, V., Mathevet, T., Perrin, C. and Michel, C., 2006. Dynamic averaging of rainfall-runoff model simulations from complementary model parameterizations. Water Resources Research, 42(7).

Schulze, K., Hunger, M. and Döll, P., 2005. Simulating river flow velocity on global scale. Advances in Geosciences, 5, pp.133-136.