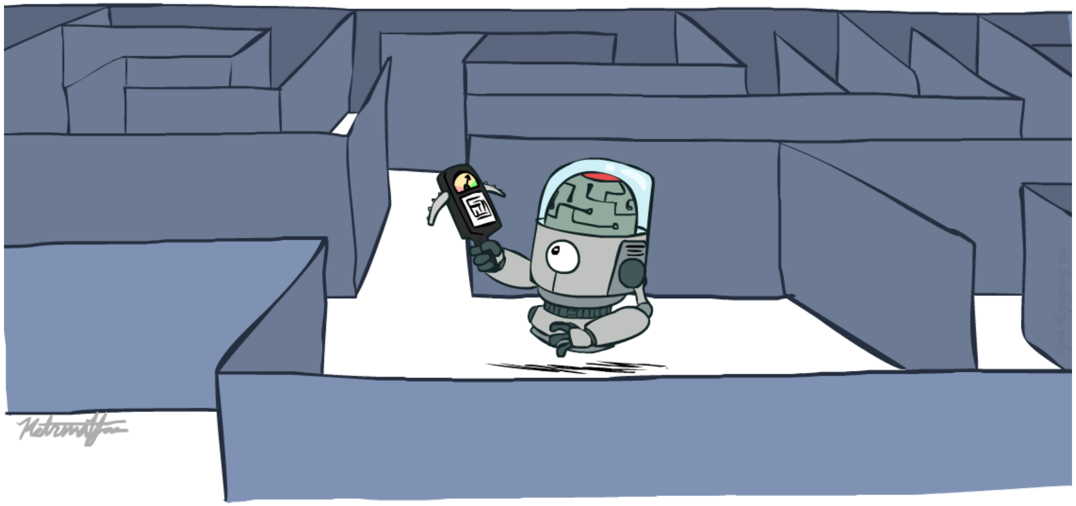
Informed Search

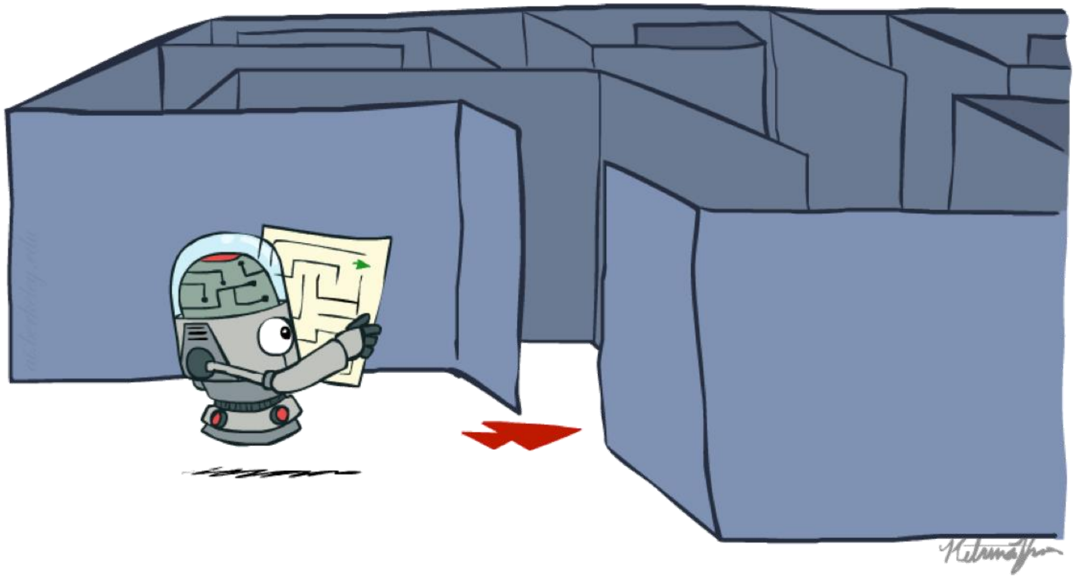


Georges Sakr
ESIB

Today

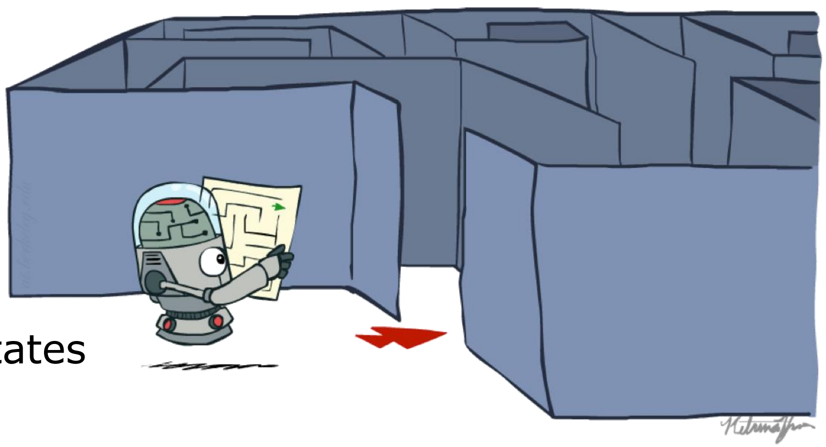
- Informed Search
  - Heuristics
  - Greedy Search
  - A* Search
- Graph Search

Recap: Search



Recap: Search

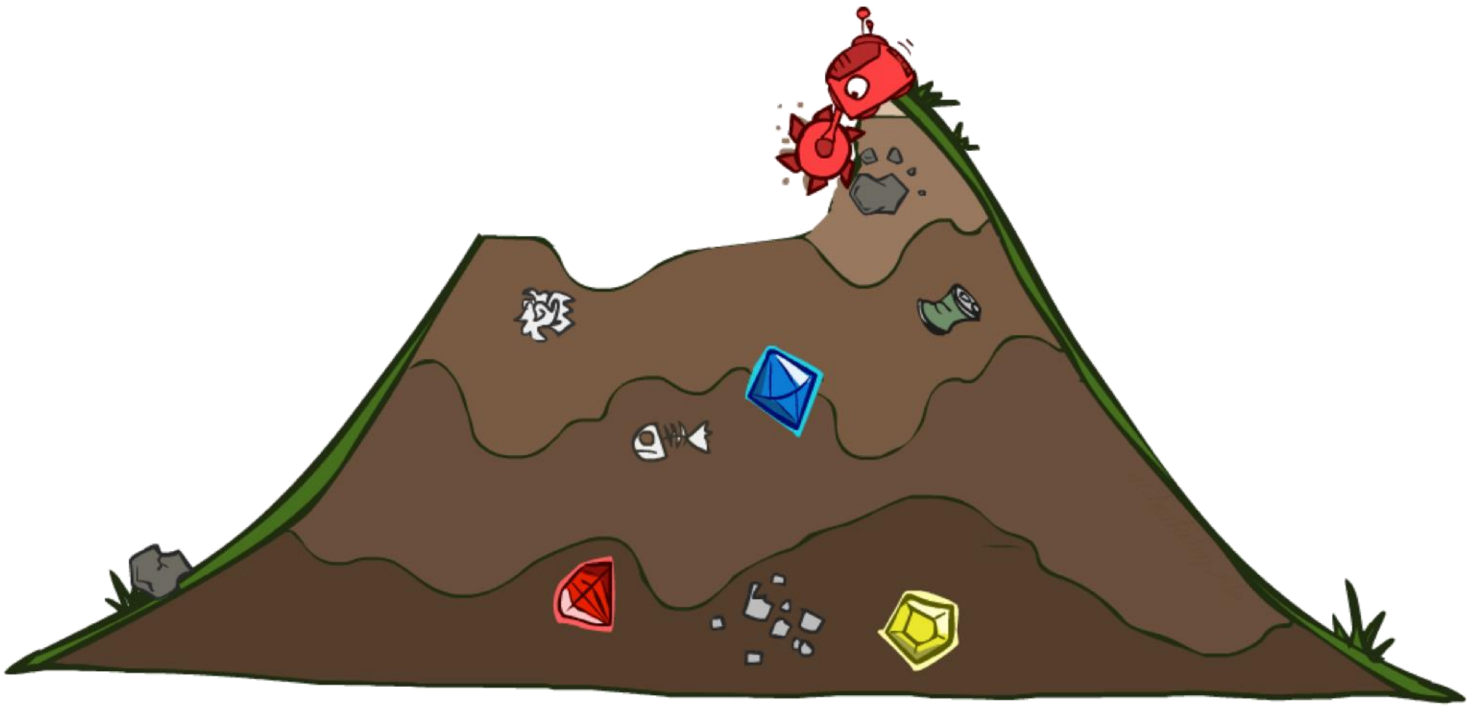
- Search problem:
  - States (configurations of the world)
  - Actions and costs
  - Successor function (world dynamics)
  - Start state and goal test



- Search tree:
  - Nodes: represent plans for reaching states
  - Plans have costs (sum of action costs)

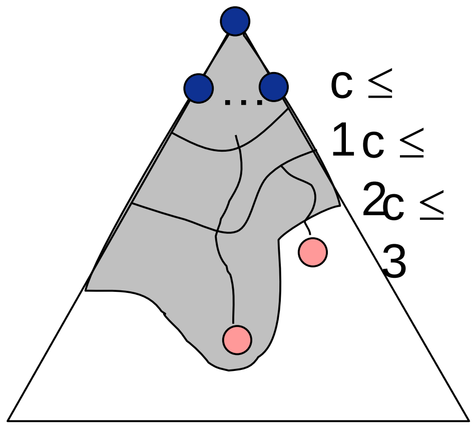
- Search algorithm:
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)
  - Optimal: finds least-cost plans

Uninformed Search



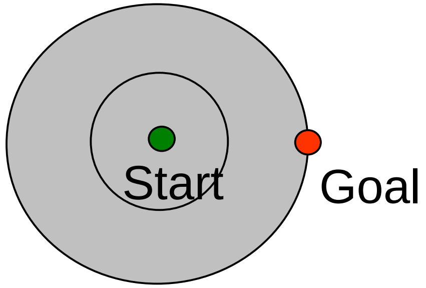
Uniform Cost Search

- Strategy: expand lowest path cost



- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location



[Demo: contours UCS empty (L3D1)]
[Demo: contours UCS pacman small maze (L3D3)]

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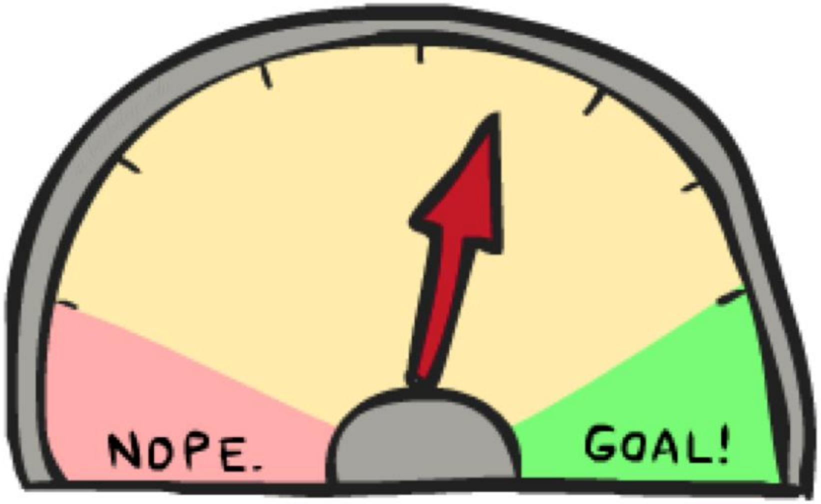

Search Heuristics

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- A heuristic is:
  - A function that *estimates* how close a state is to a goal
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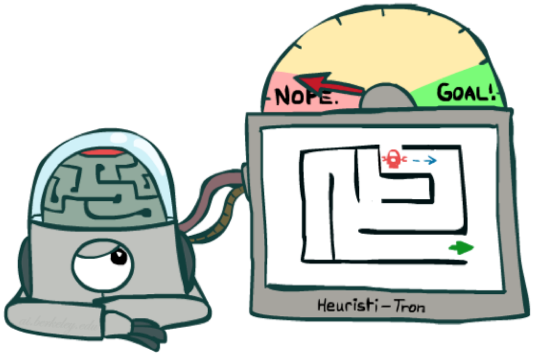

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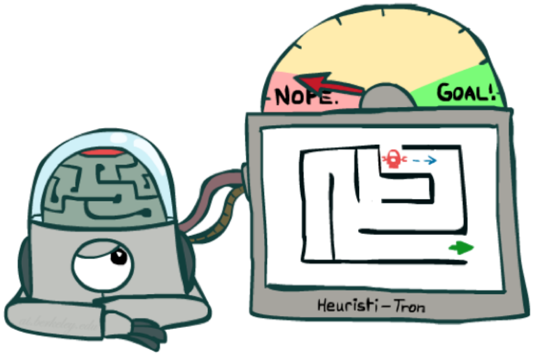

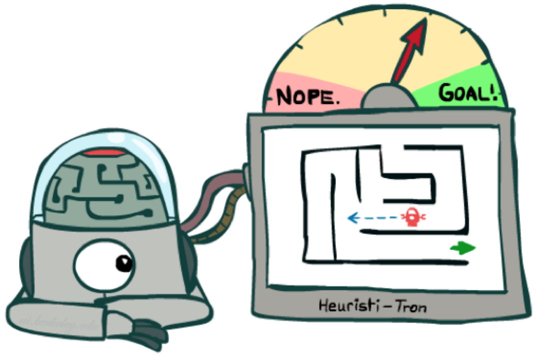

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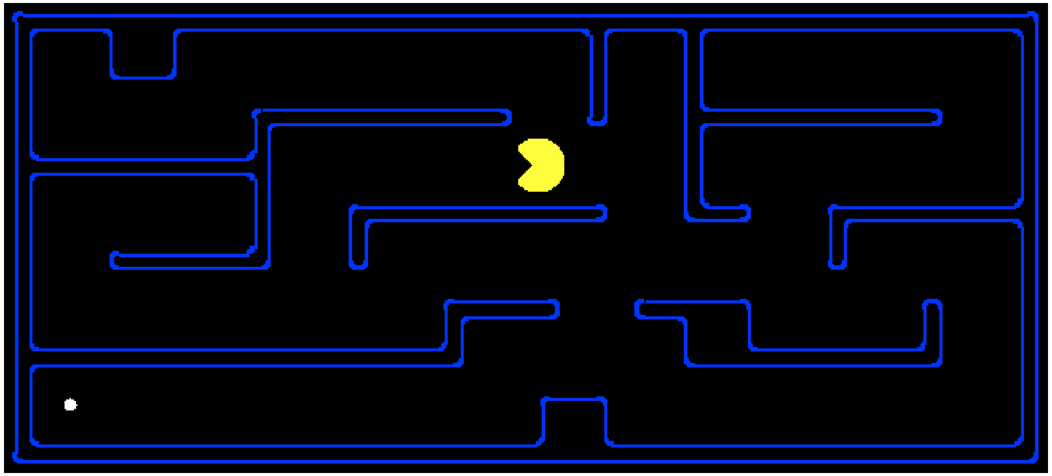

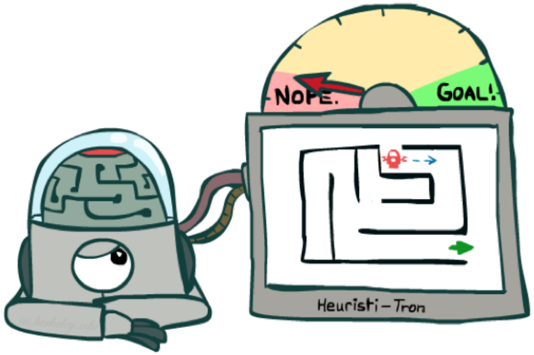

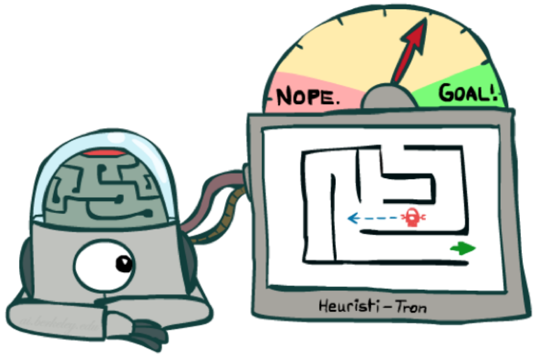

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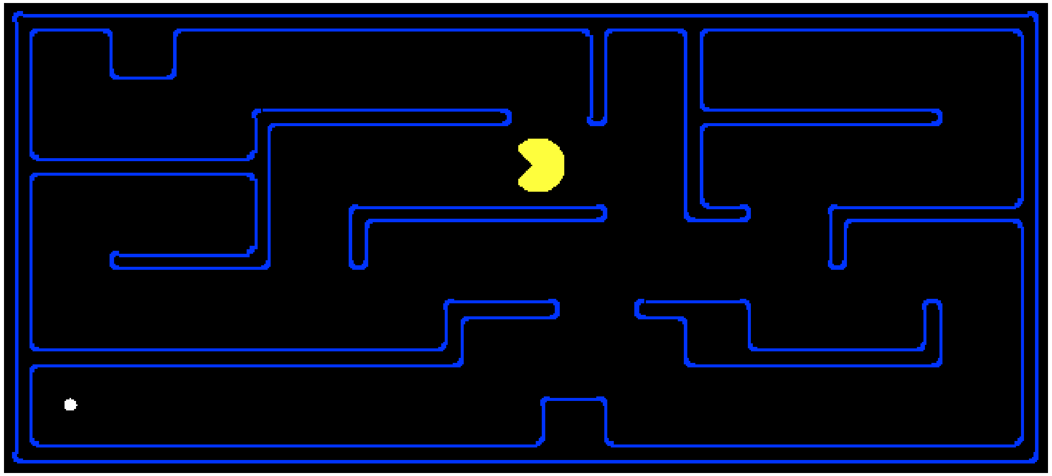

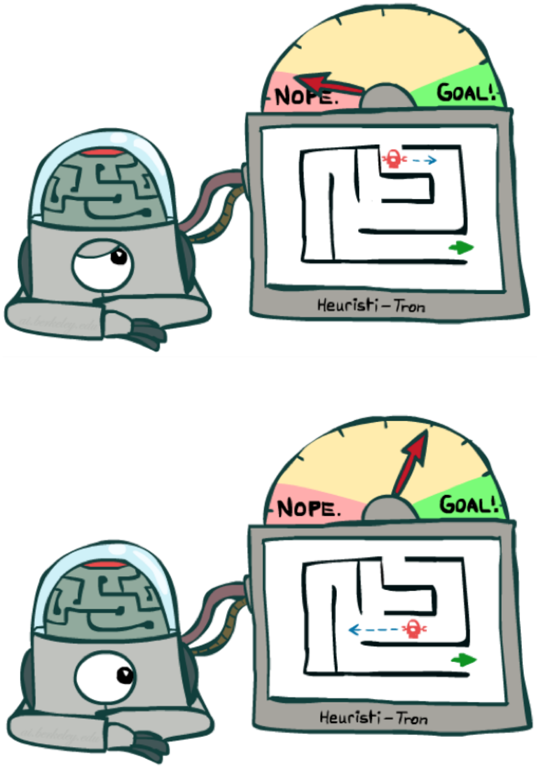

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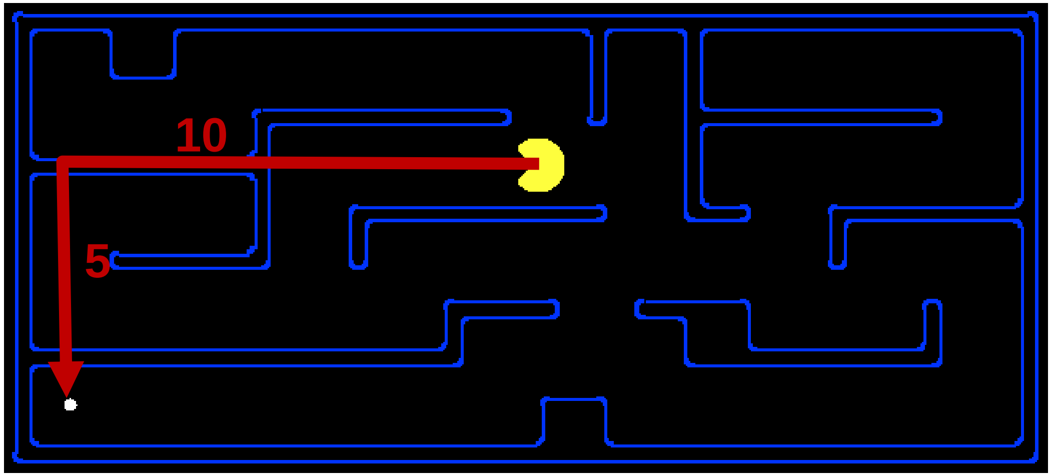

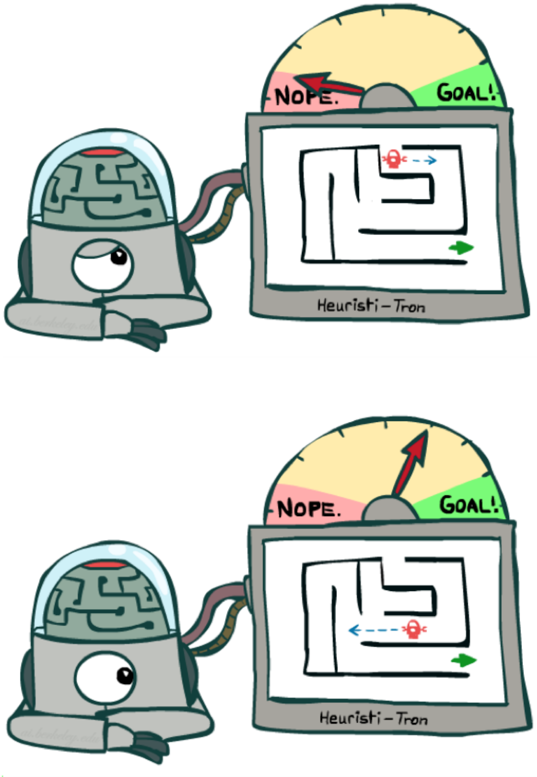

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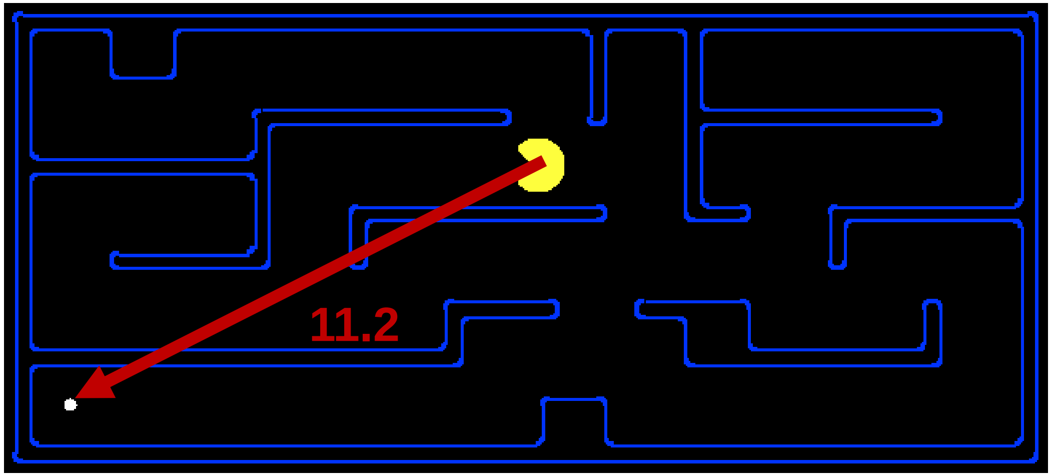
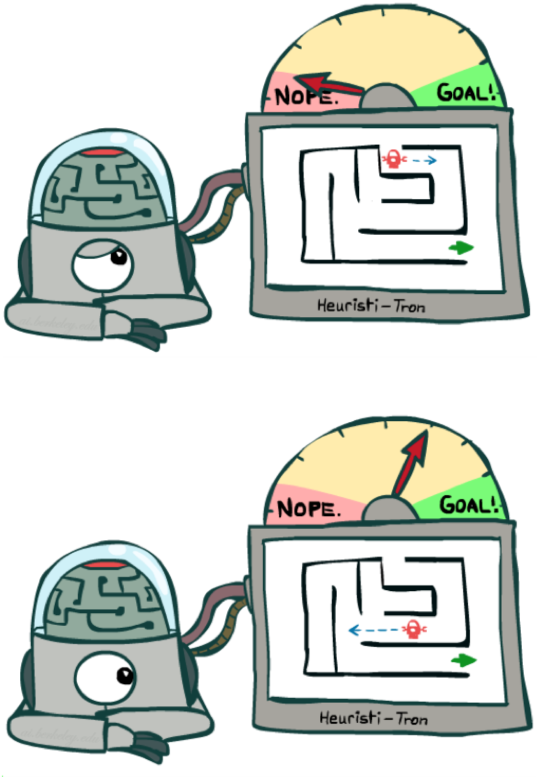
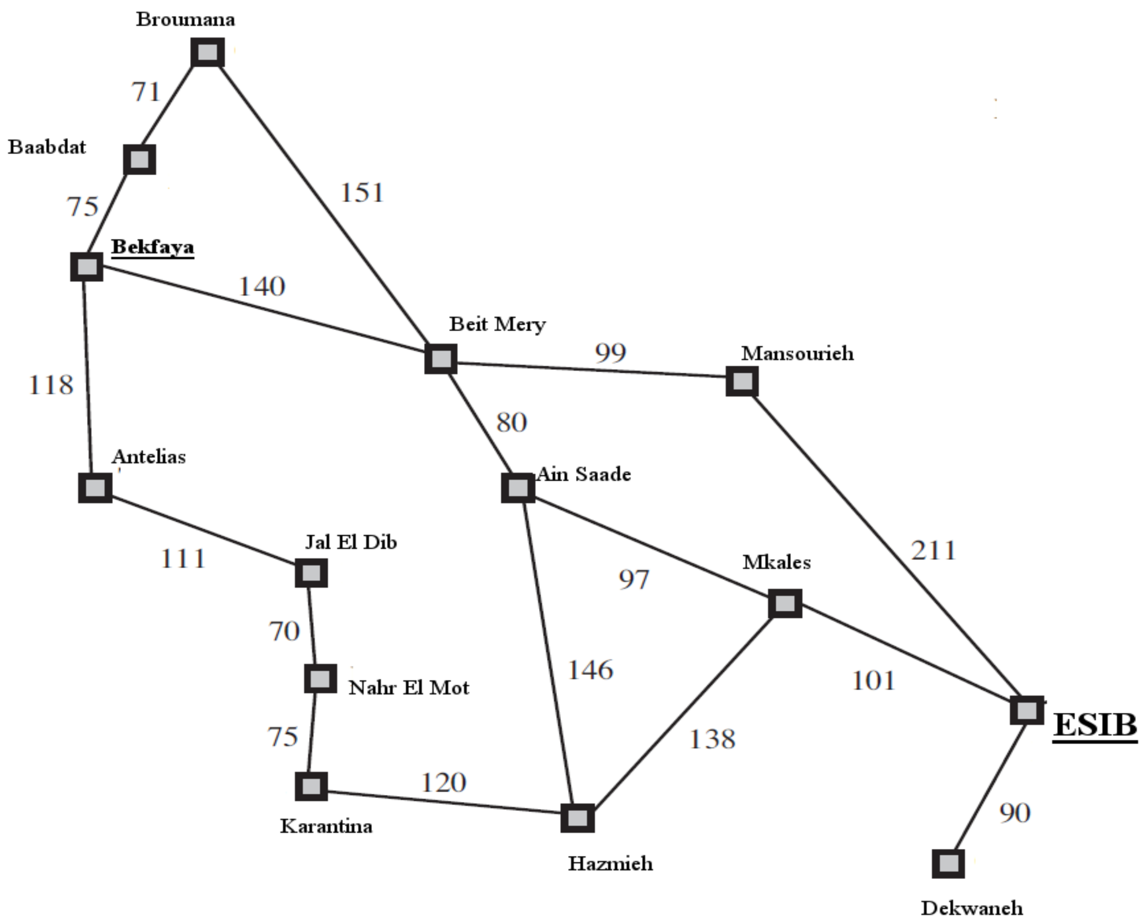




Example: Heuristic Function



Straight line distance to ESIB

Bekfaya	366
ESIB	0
Hazmieh	160
Karantina	242
Mansourieh	178
Dekwaneh	77
Jal El Dib	244
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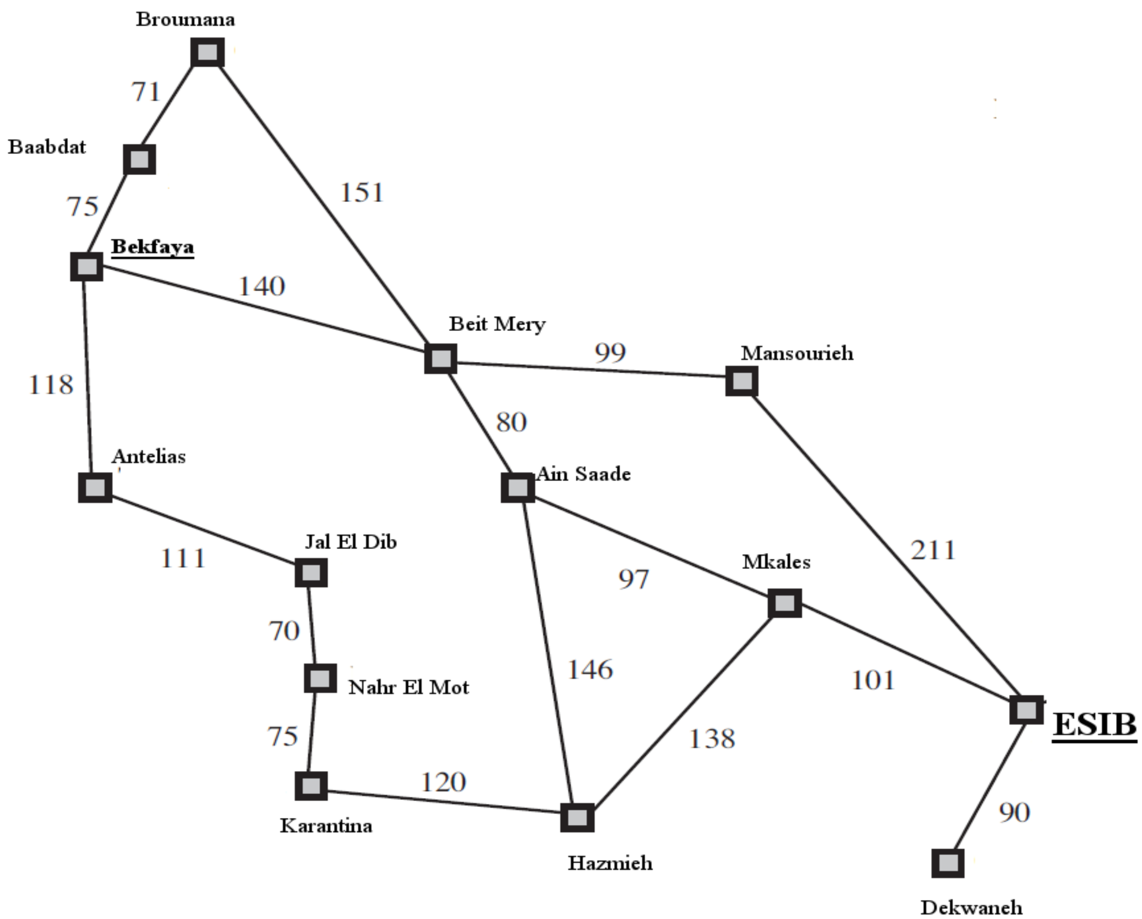
h(x)

Greedy Search

Example: Heuristic Function



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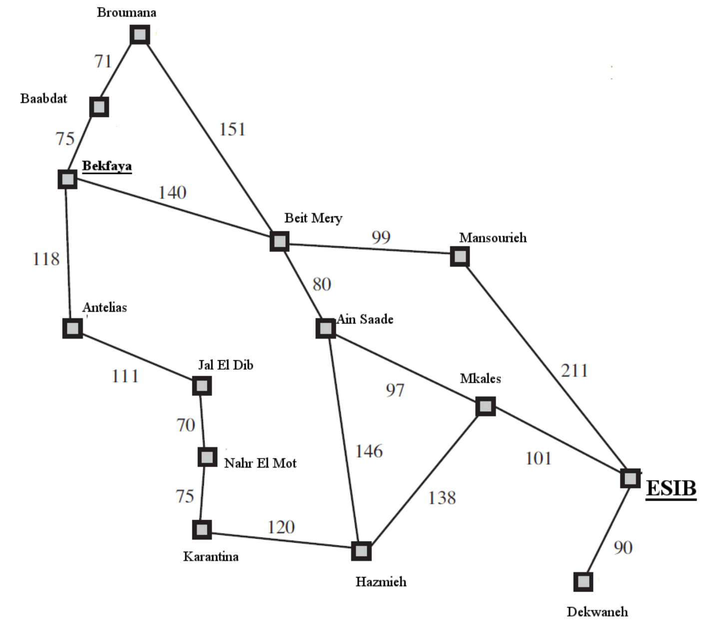
Greedy Search

Expand the node that seems closest…



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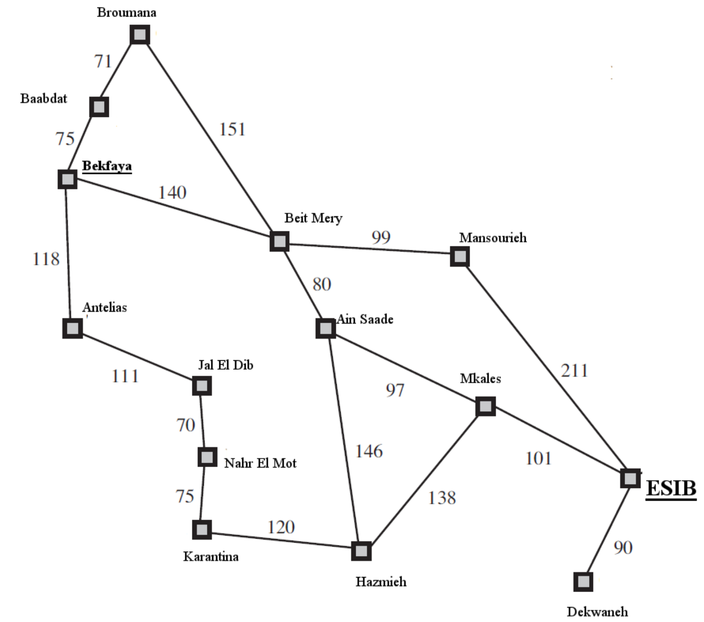

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Greedy Search

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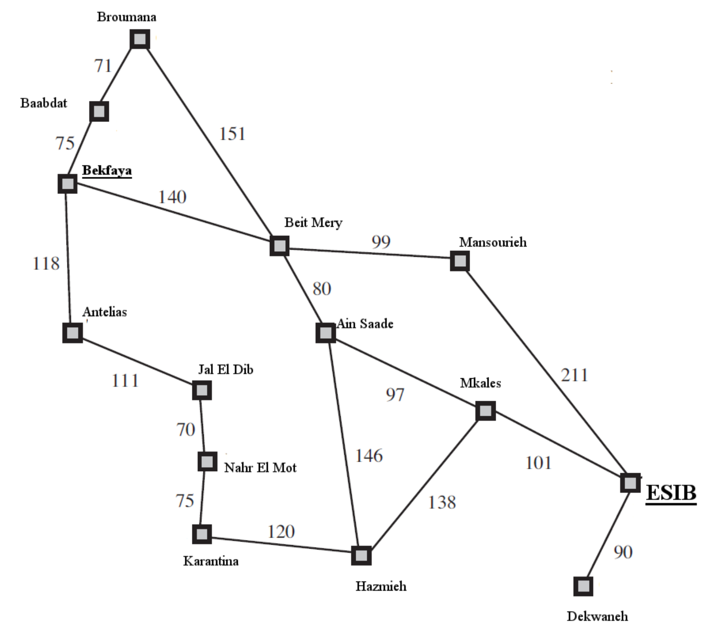

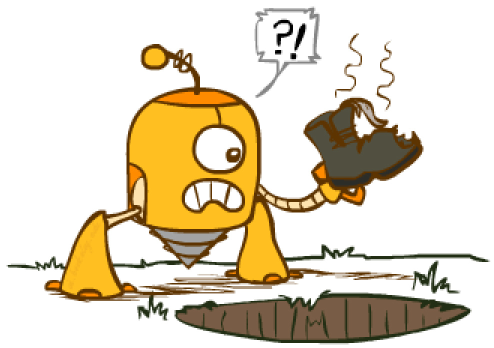

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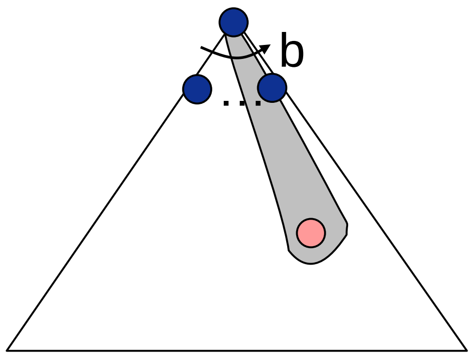
Greedy Search

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state



[Demo: contours greedy empty (L3D1)]
[Demo: contours greedy pacman small maze (L3D4)]

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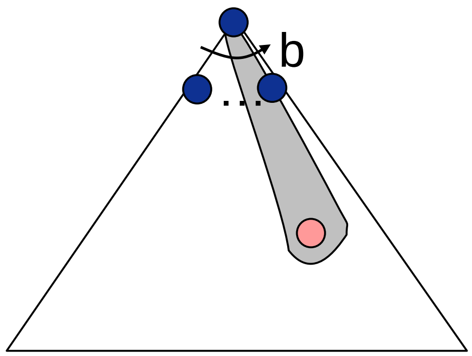

- A common case:
  - Best-first takes you straight to the (wrong) goal

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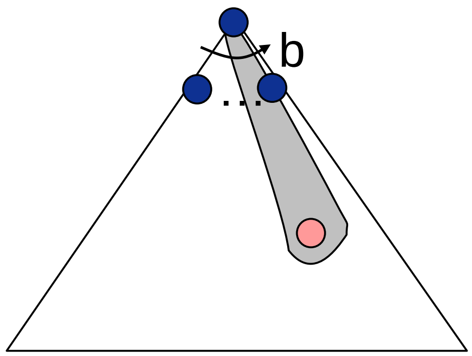

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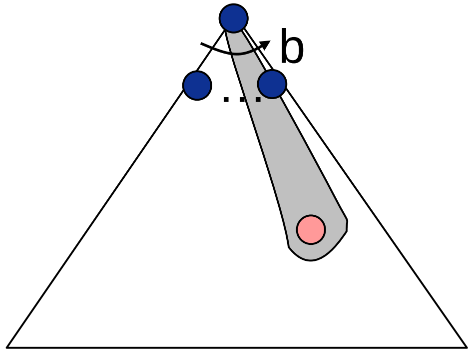

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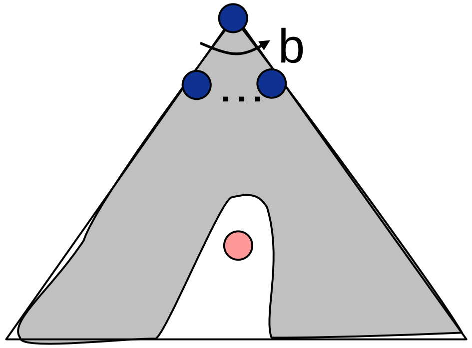

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A* Search

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UCS

A* Search

UCS

Greedy

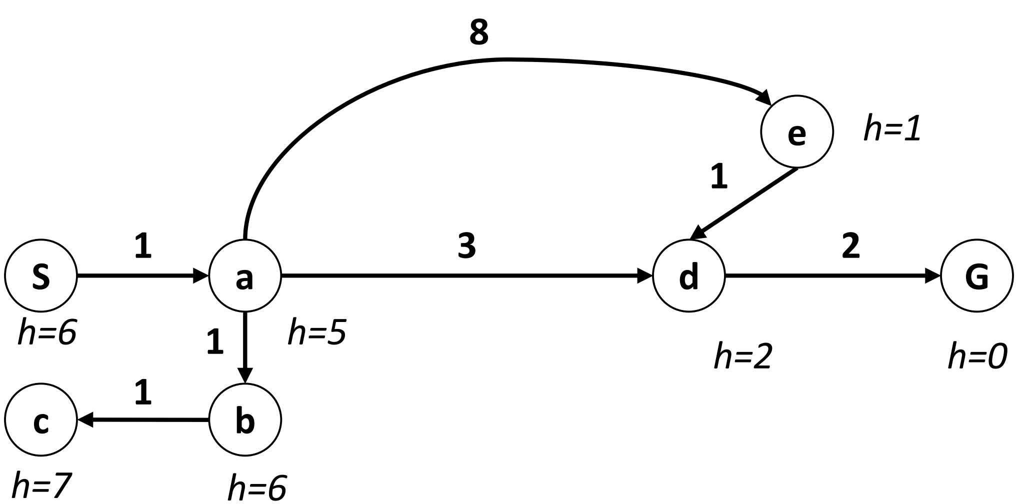
A* Search

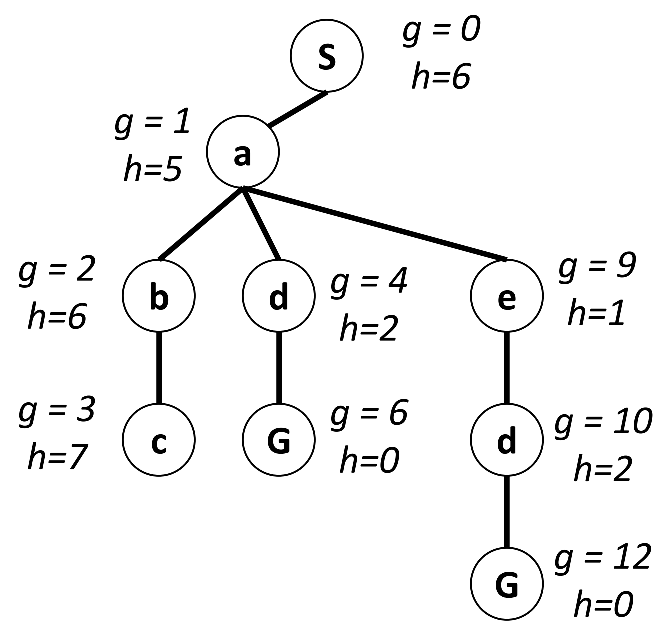
UCS

Greedy

A*

Combining UCS and Greedy

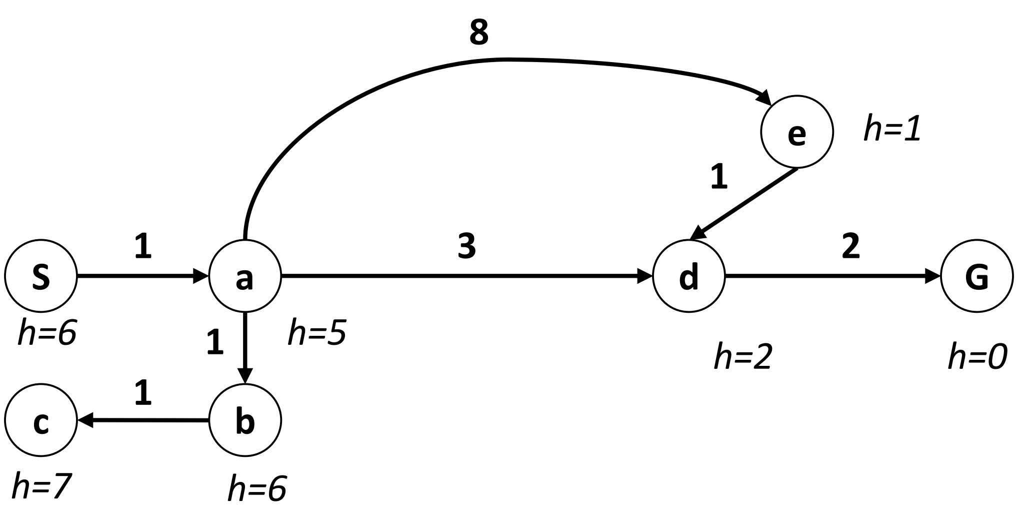


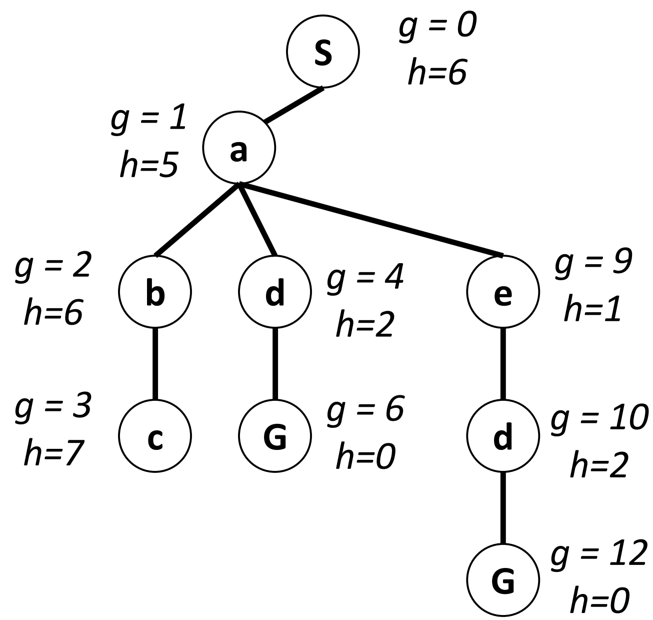


Example: Teg Grenager

Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)





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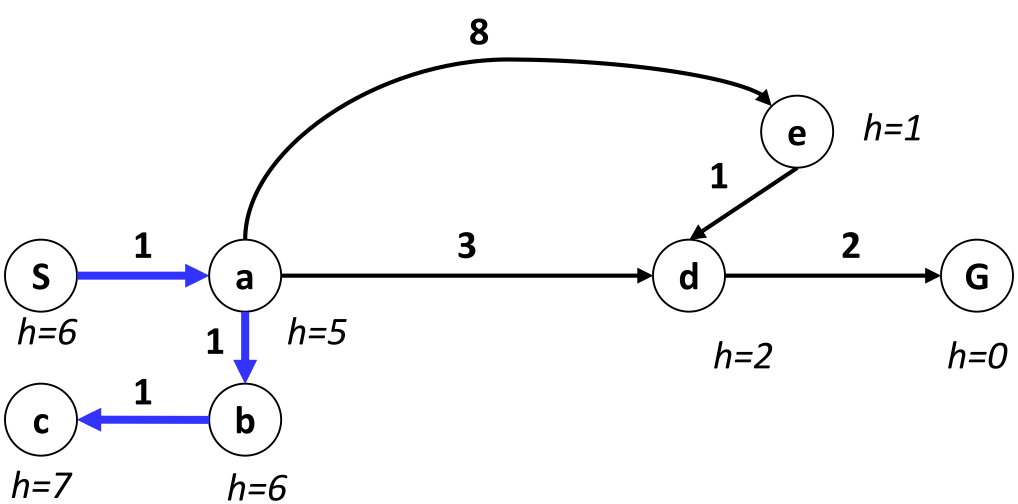

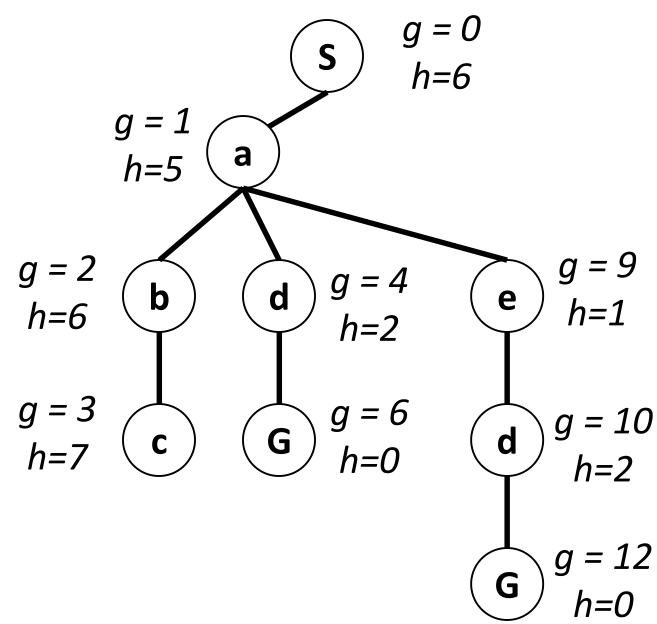

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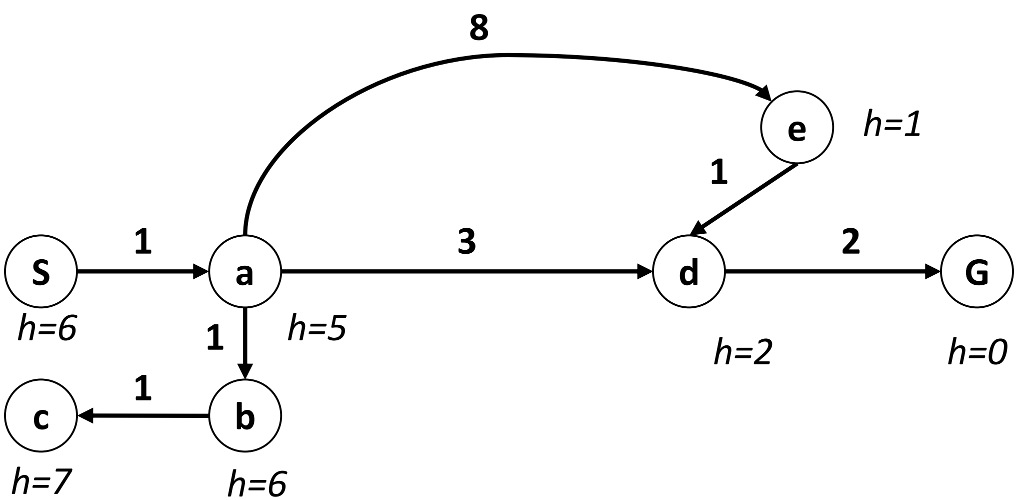

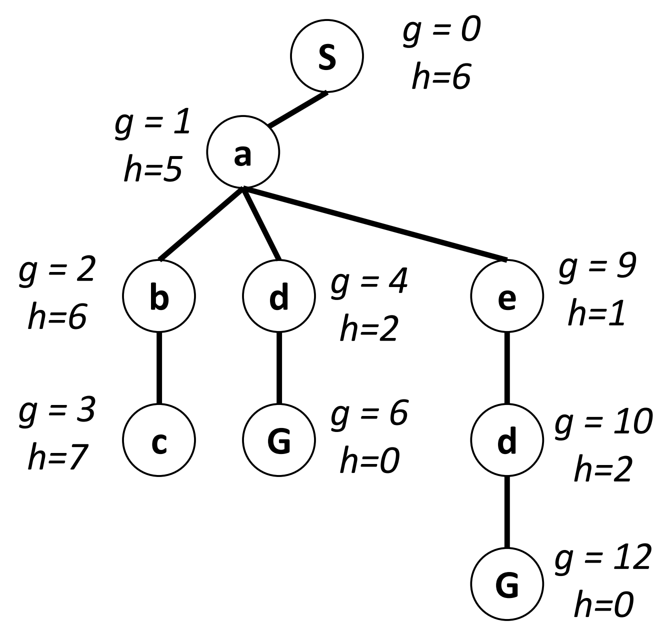

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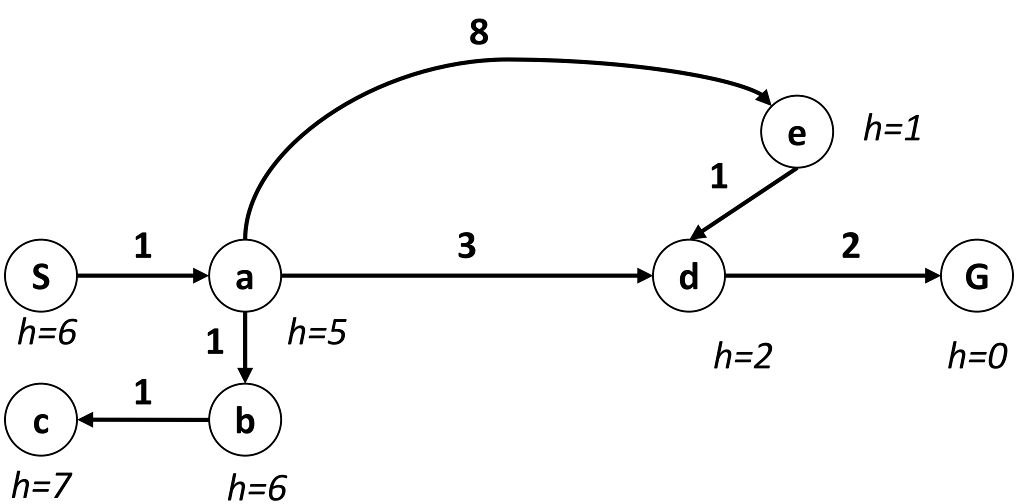

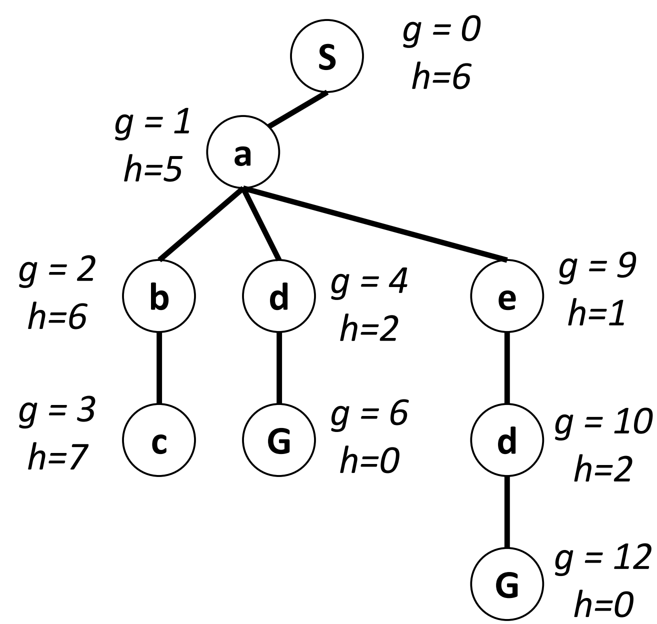

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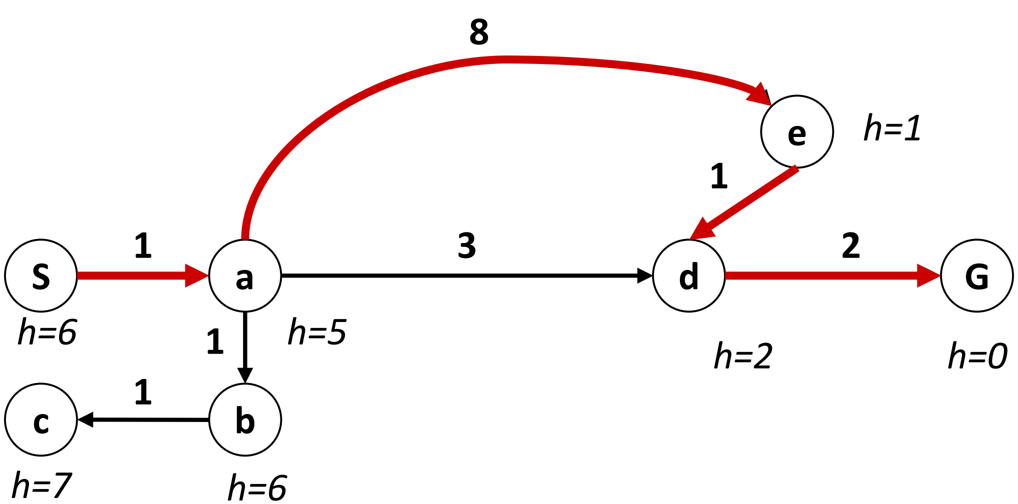

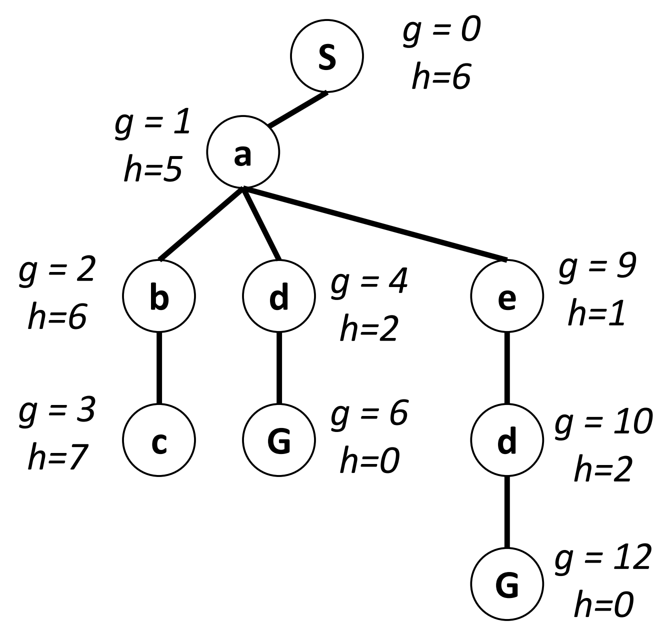

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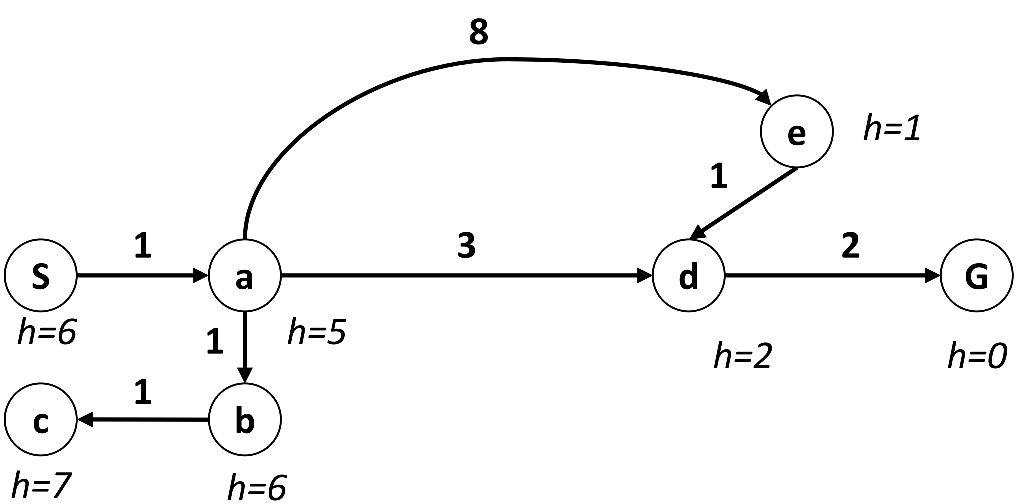

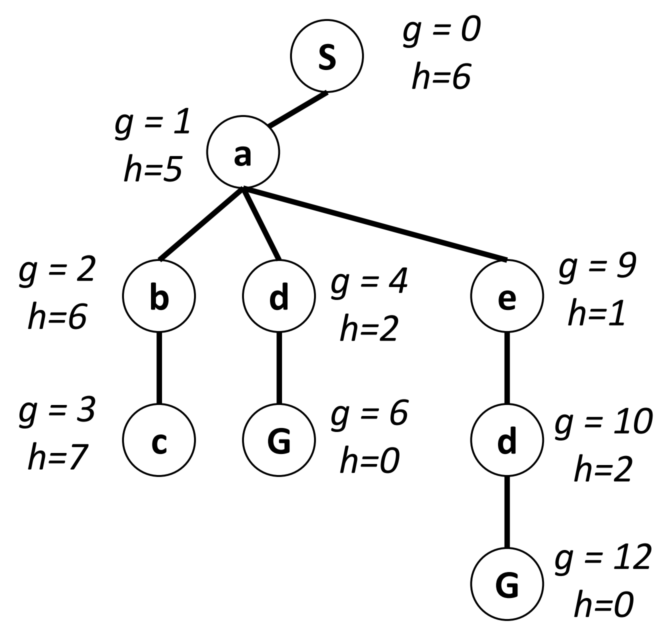

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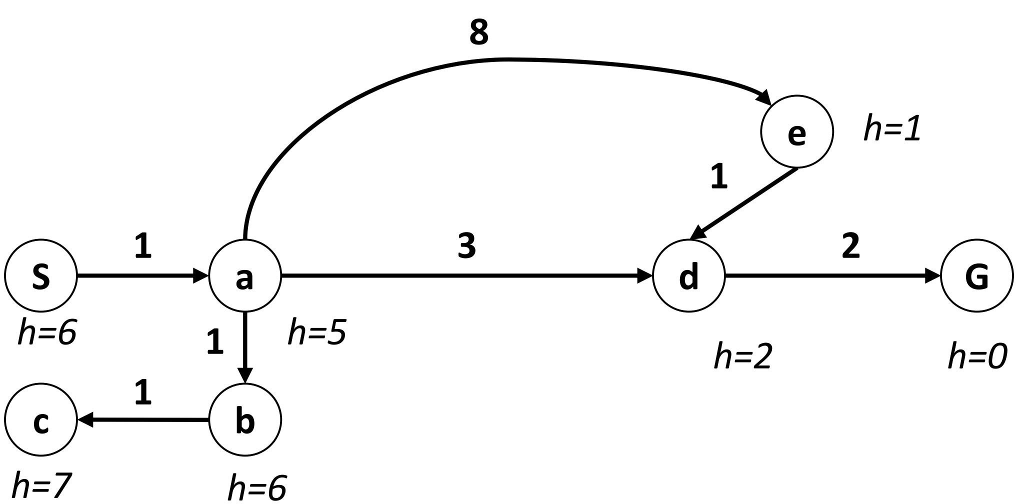

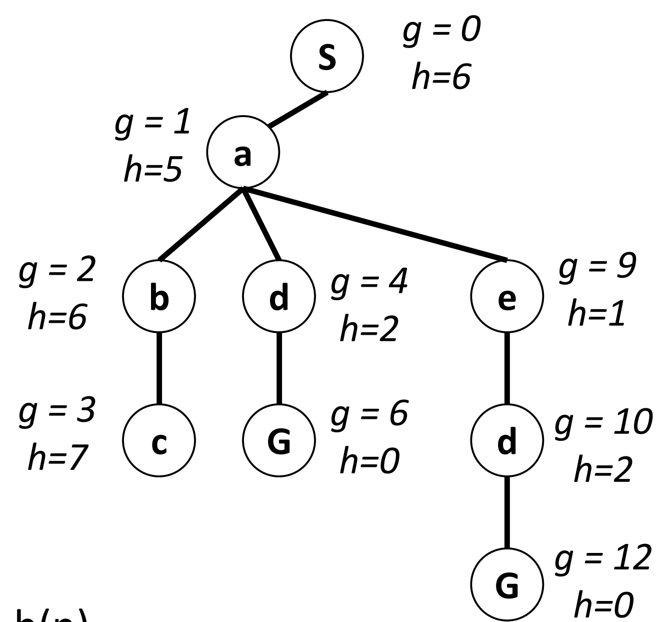

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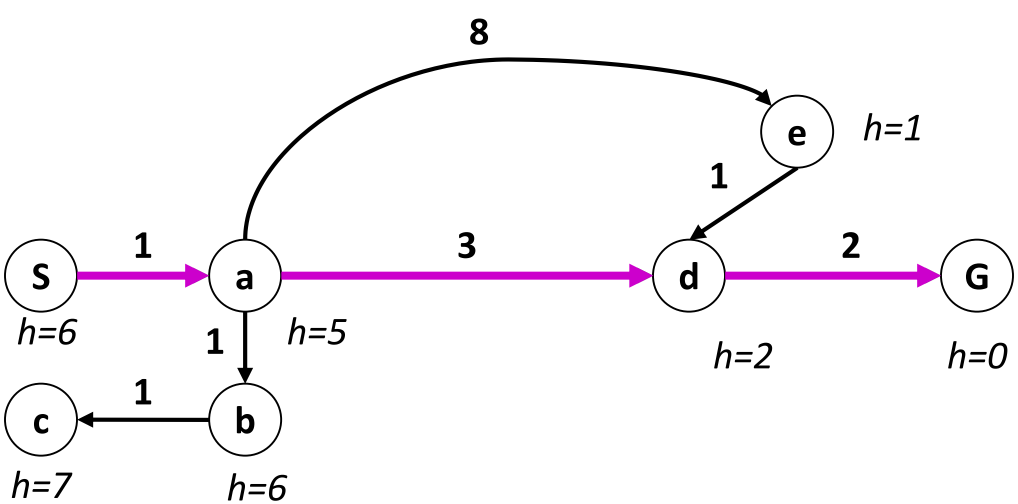

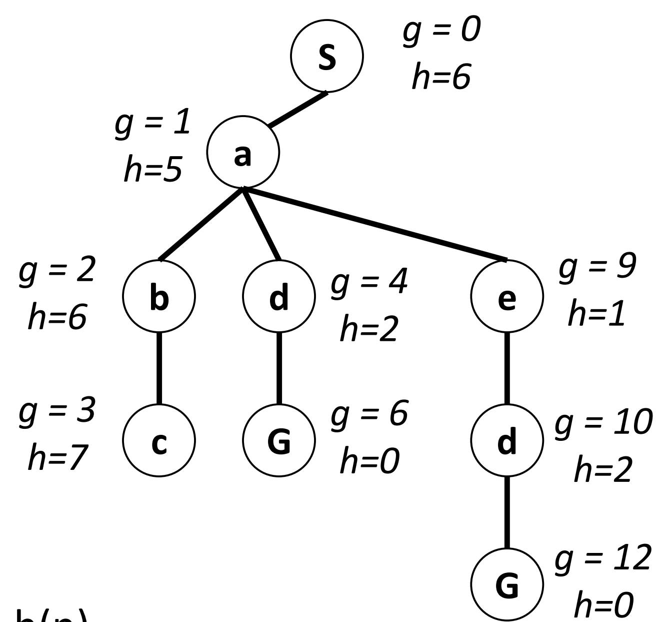

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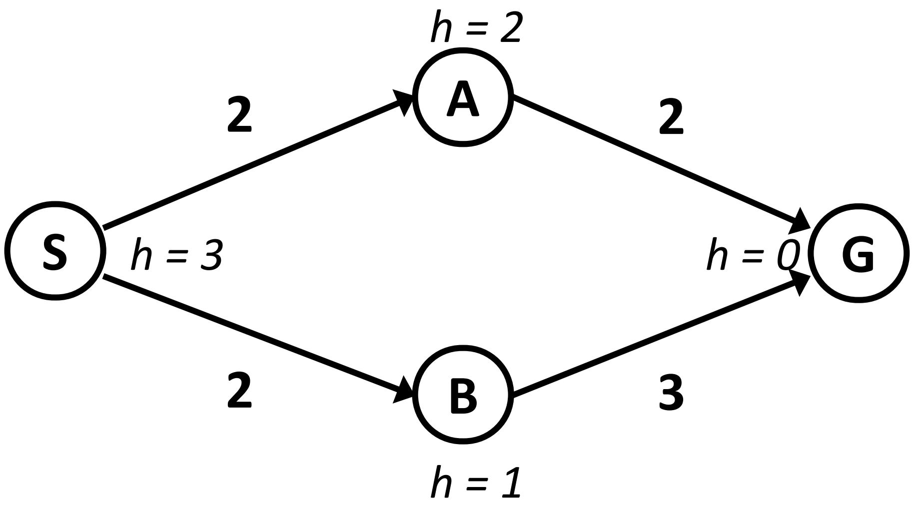
Example: Teg Grenager

When should A* terminate?

- Should we stop when we enqueue a goal?



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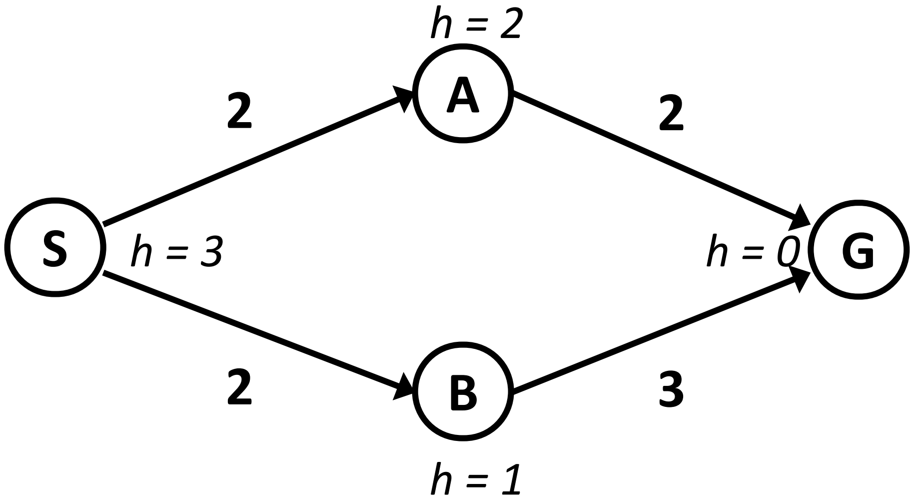
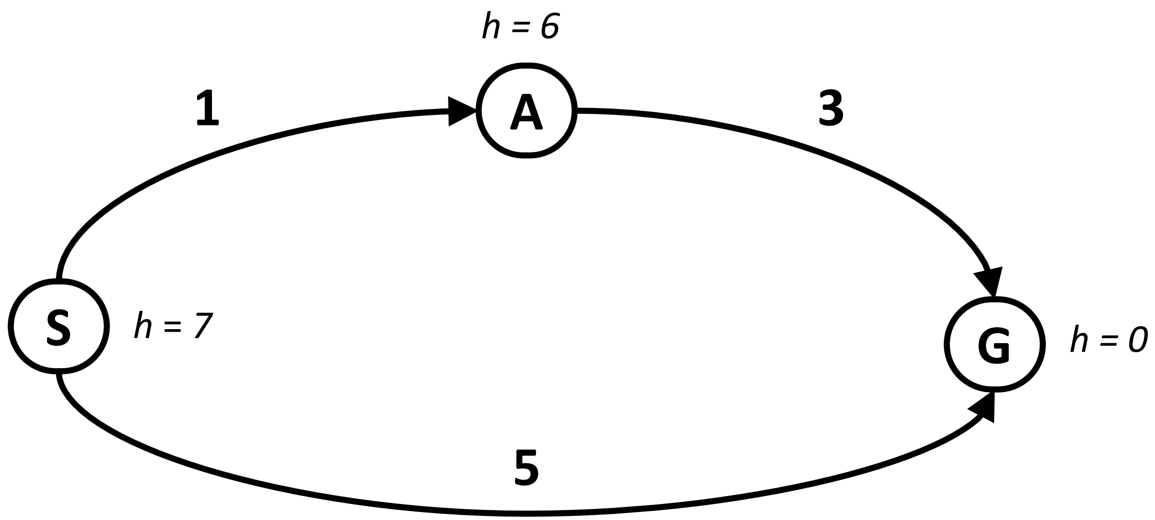


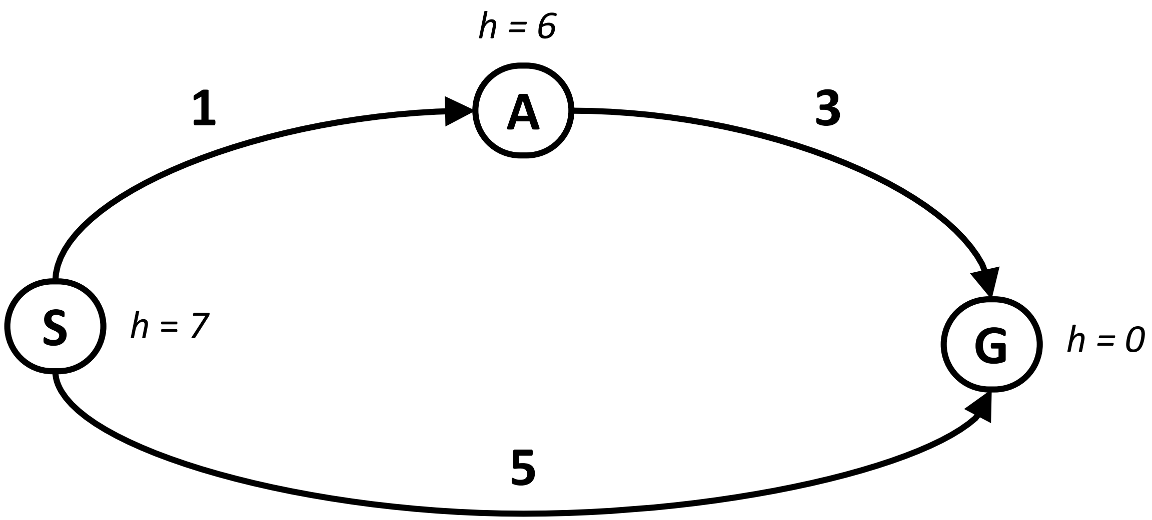
- No: only stop when we dequeue a goal

Is A* Optimal?



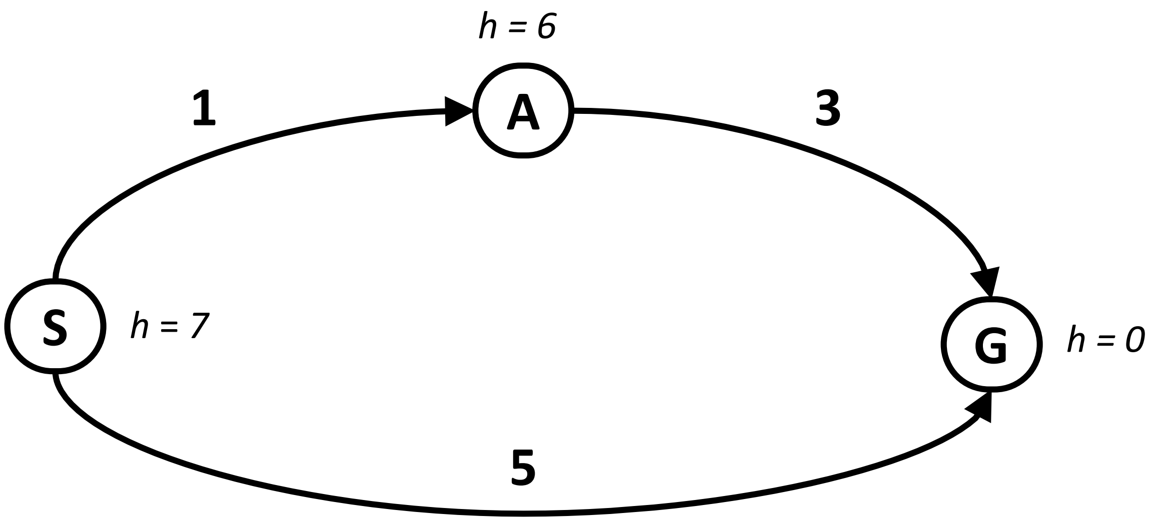
- What went wrong?

Is A* Optimal?



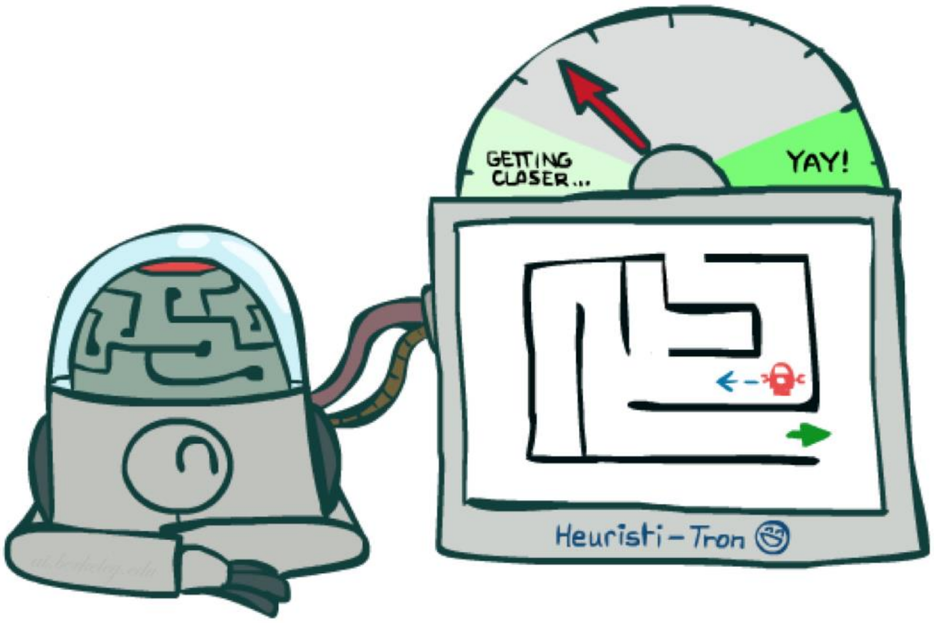
- What went wrong?
- Actual bad goal cost < estimated good goal cost

Is A* Optimal?

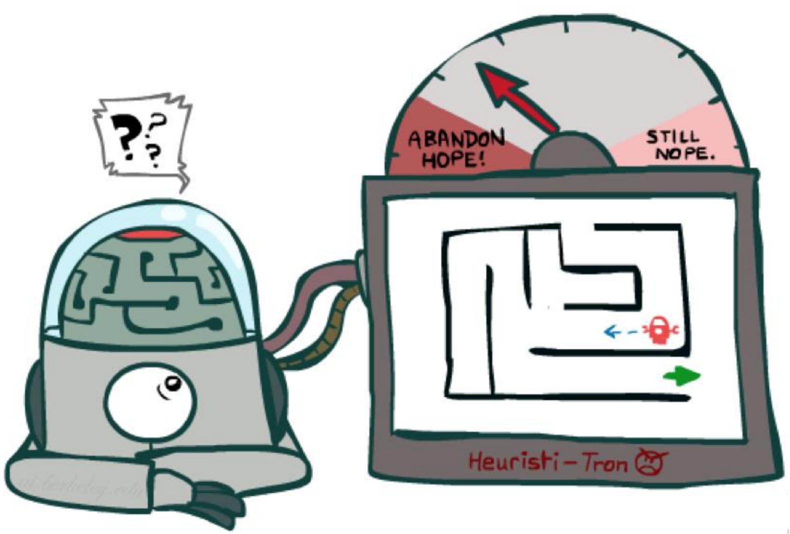


- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

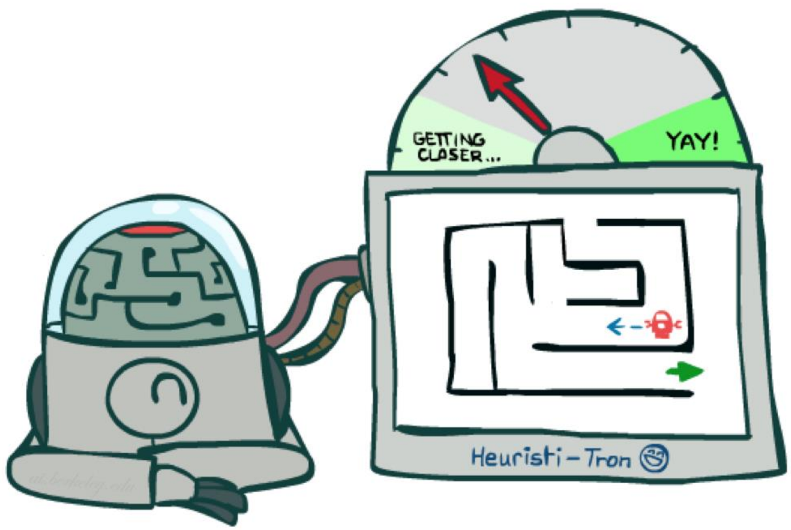
Admissible Heuristics



Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

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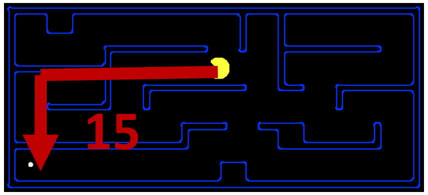

Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

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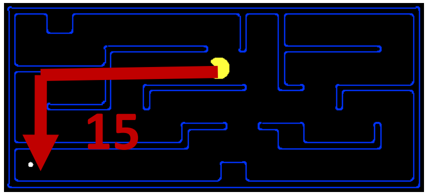
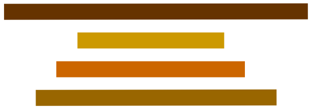


4



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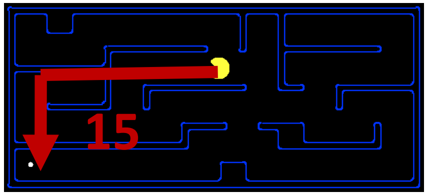

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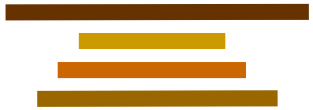
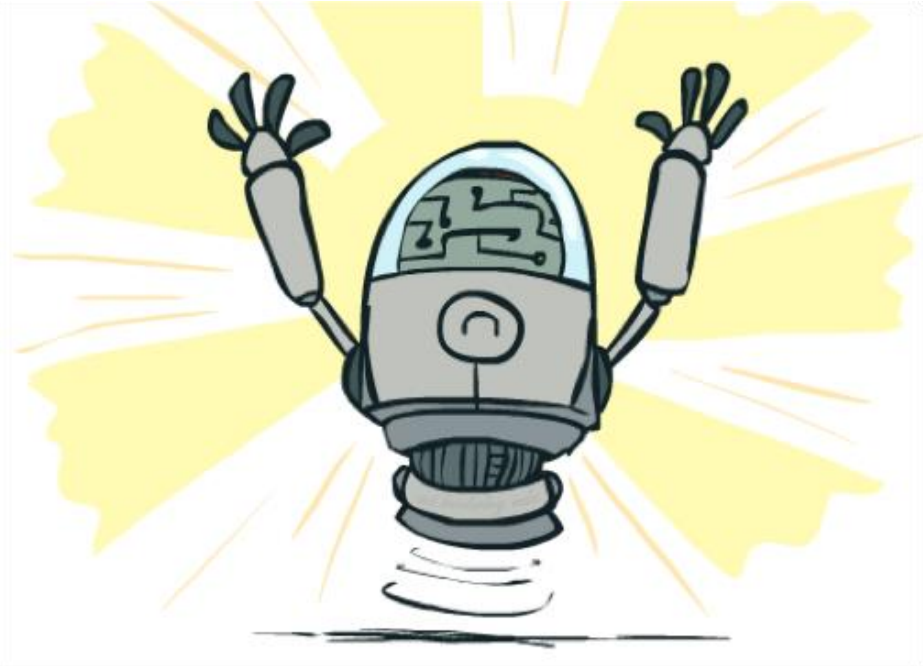


- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Optimality of A* Tree Search



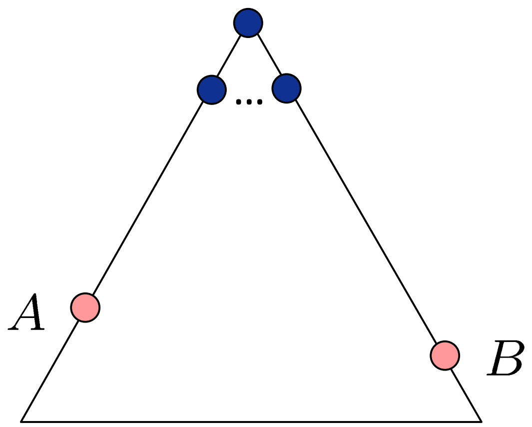
Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:

- A will exit the fringe before B



Optimality of A* Tree Search: Blocking

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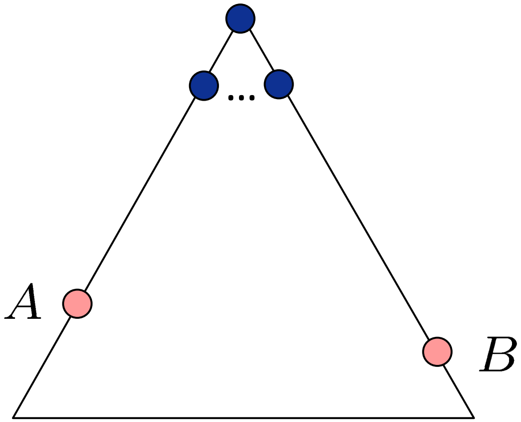

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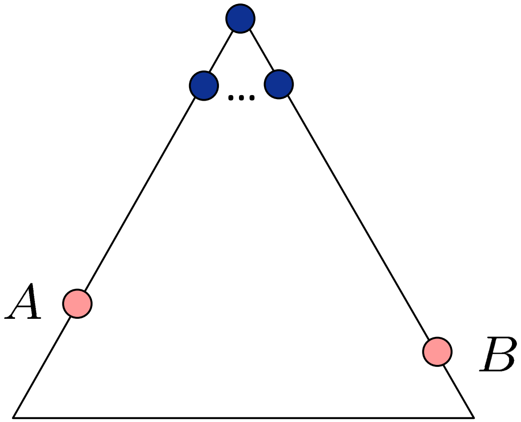

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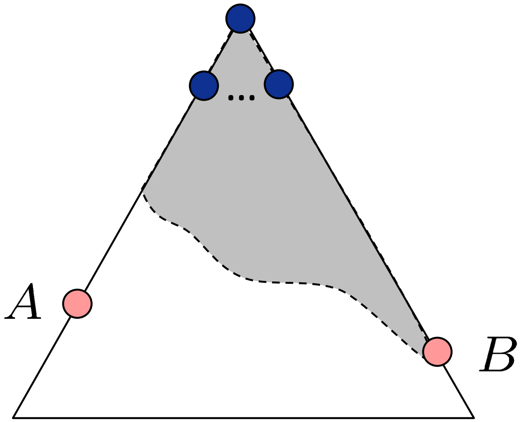

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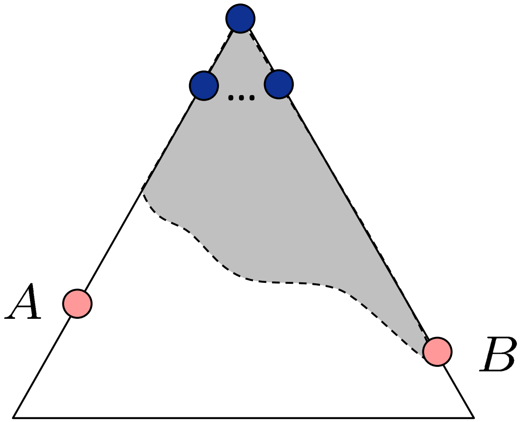

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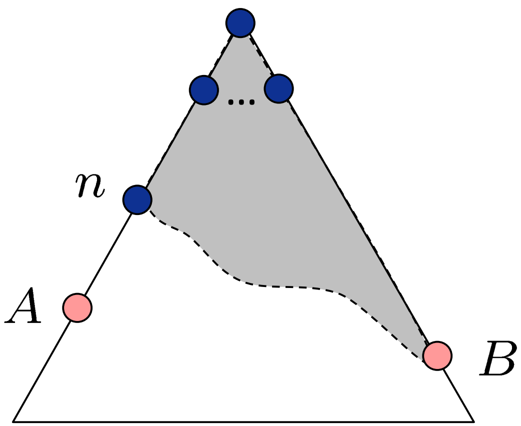

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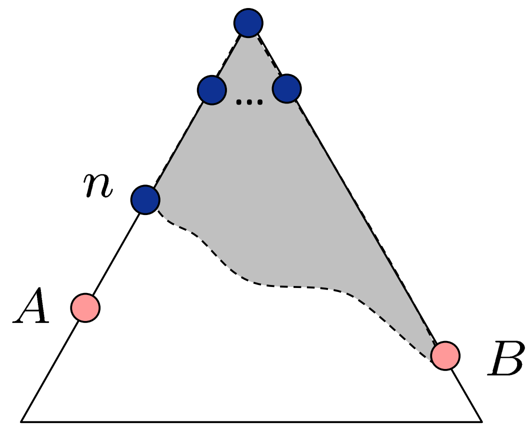

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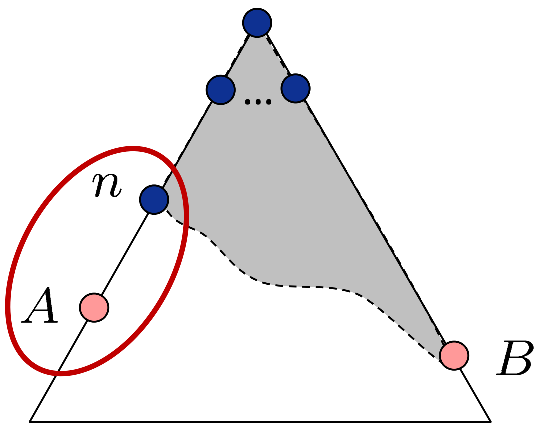

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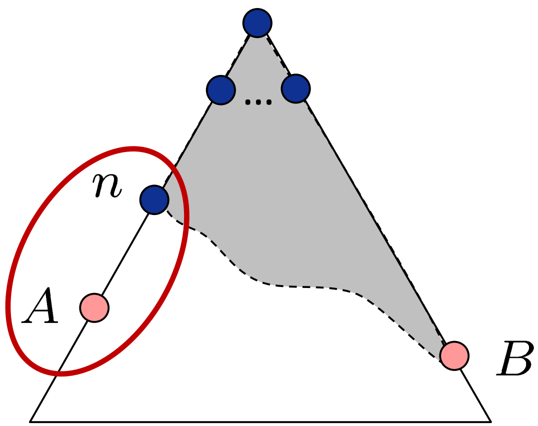

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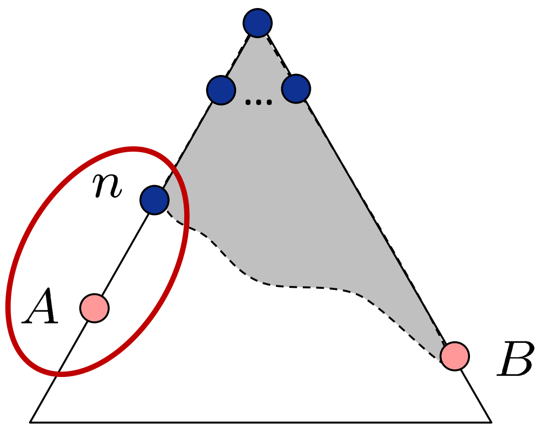

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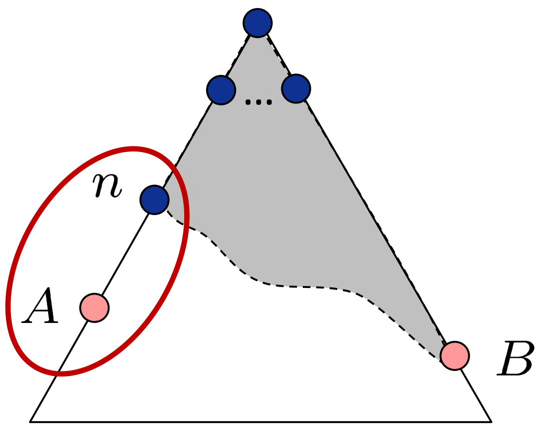

$$f(n) = g(n) + h(n)$$ Definition of f-cost

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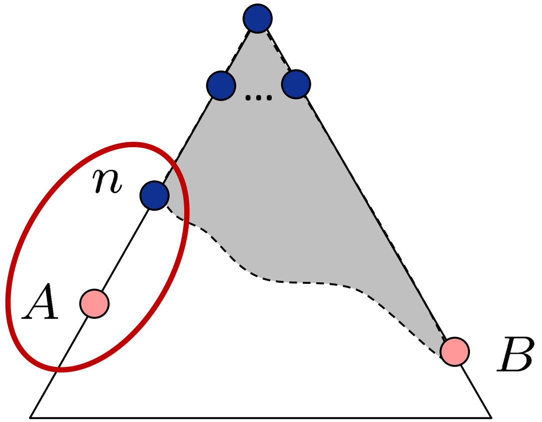

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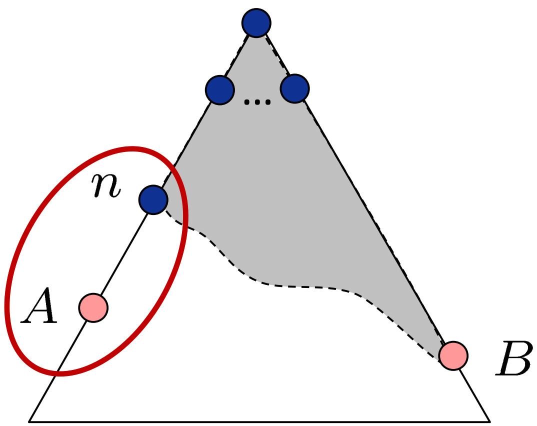

$f(n) = g(n) + h(n)$	Definition of f-cost
$f(n) \leq g(A)$	Admissibility of h
$g(A) = f(A)$	h = 0 at a goal

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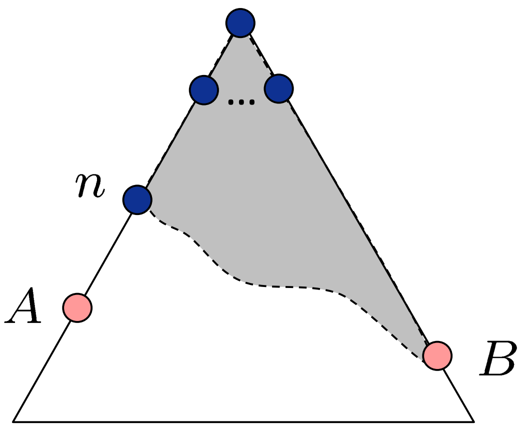

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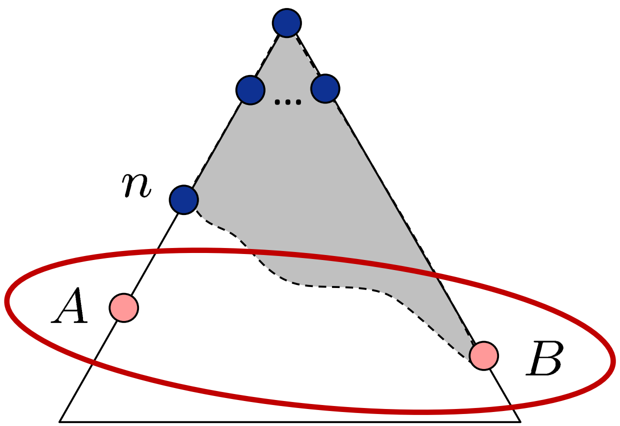

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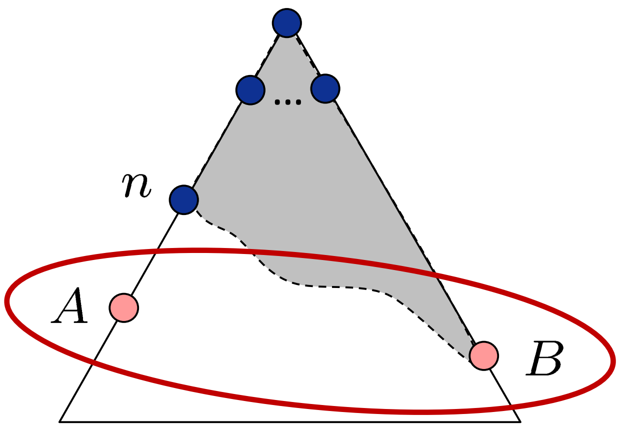

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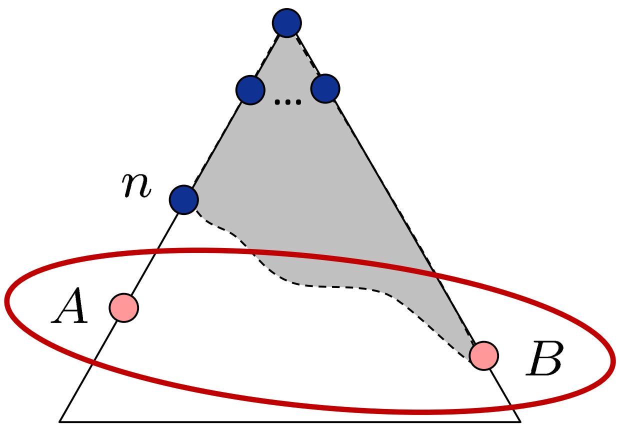

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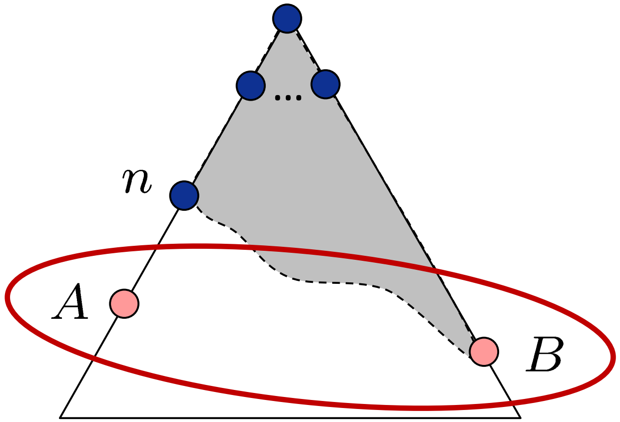

$g(A) < g(B)$ B is suboptimal

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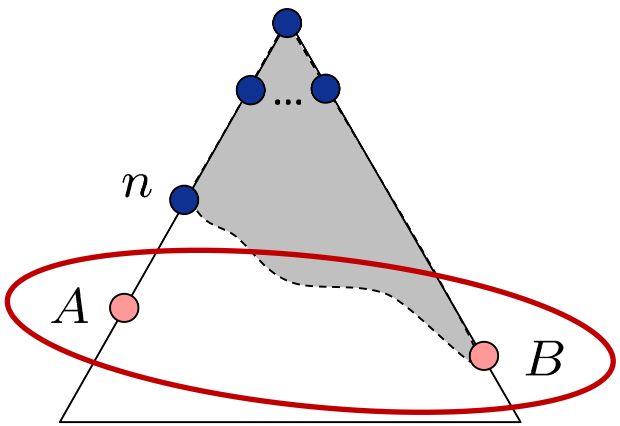

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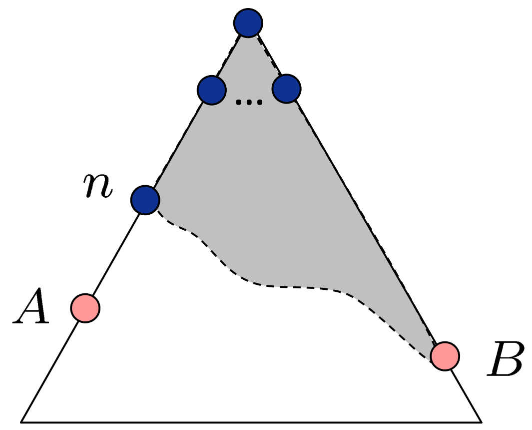

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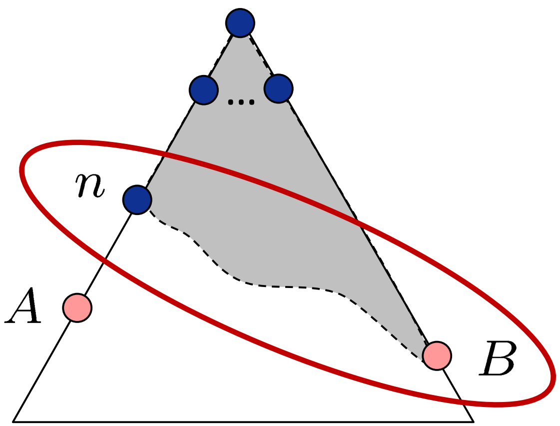

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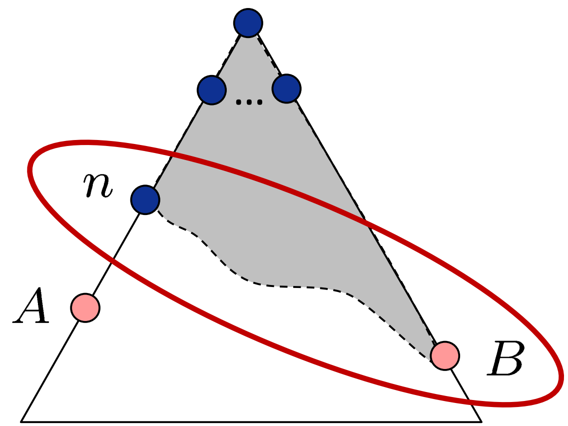

$$f(n) \leq f(A) < f(B)$$

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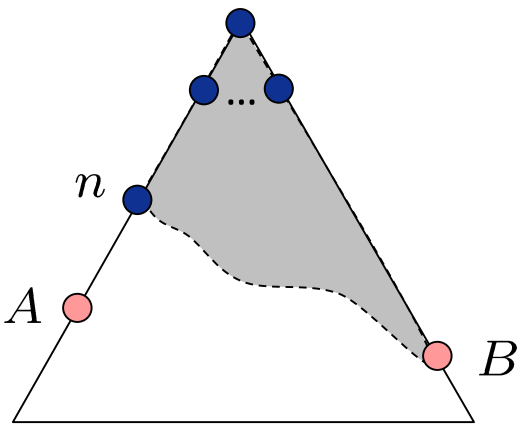

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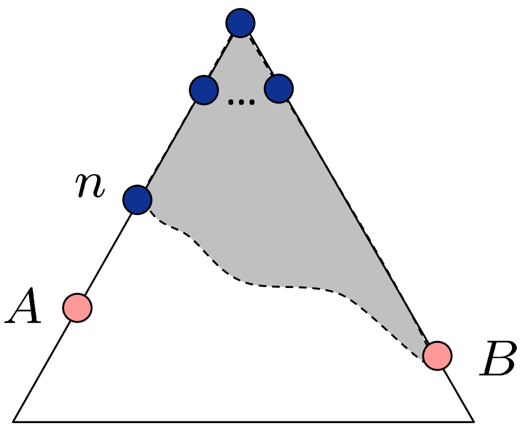

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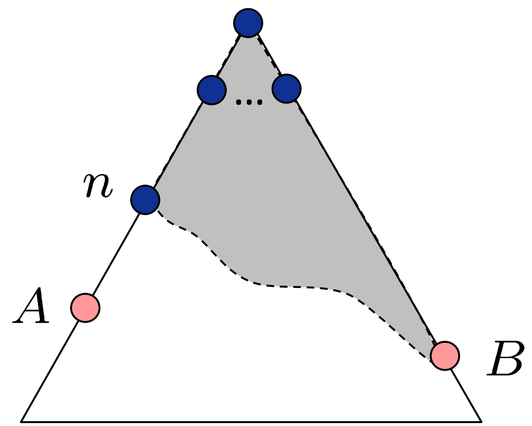

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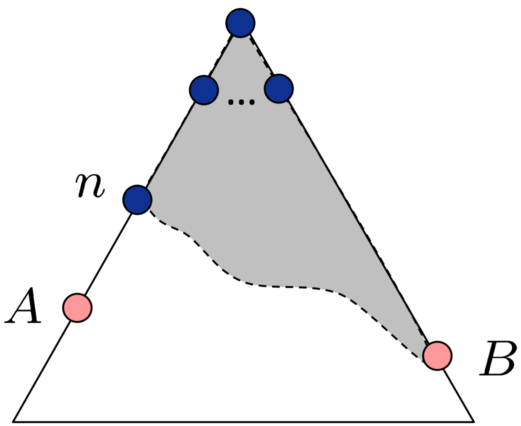
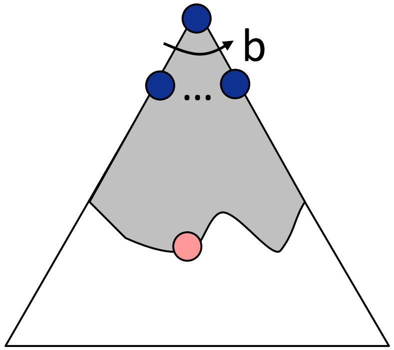
- All ancestors of A expand before B
- A expands before B
- A* search is optimal



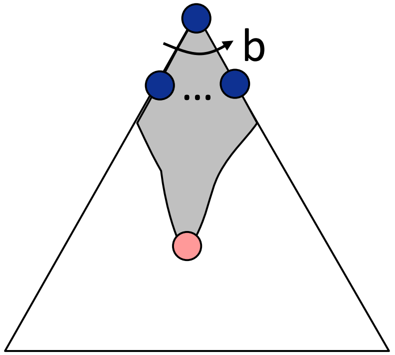
Properties of A*

Properties of A*

Uniform-Cost

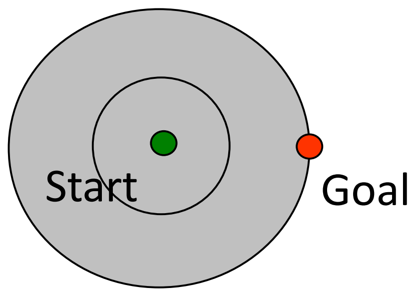


A*

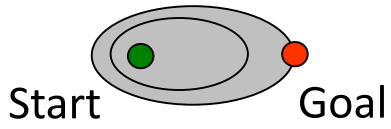


UCS vs A* Contours

- Uniform-cost expands equally in all "directions"

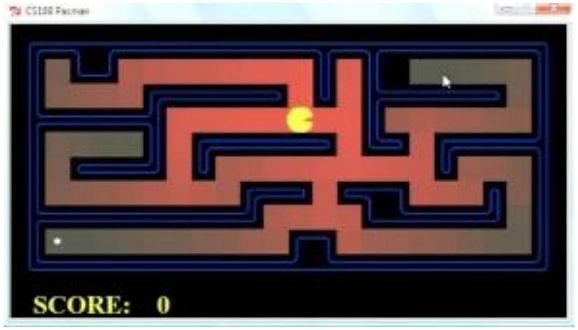


- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



[Demo: contours UCS / greedy / A* empty (L3D1)]
[Demo: contours A* pacman small maze (L3D5)]

Comparison

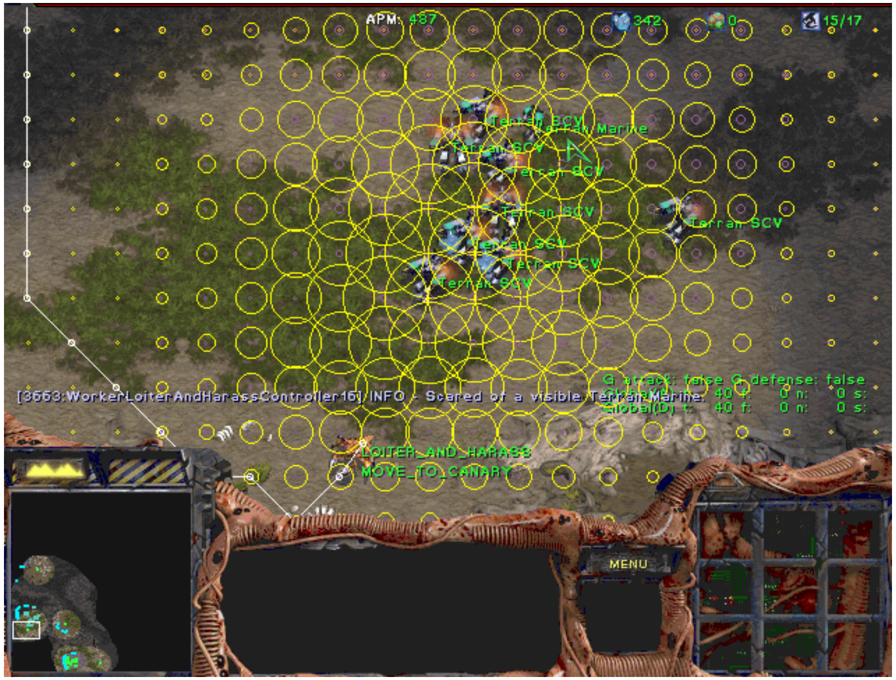


Greedy

Uniform Cost

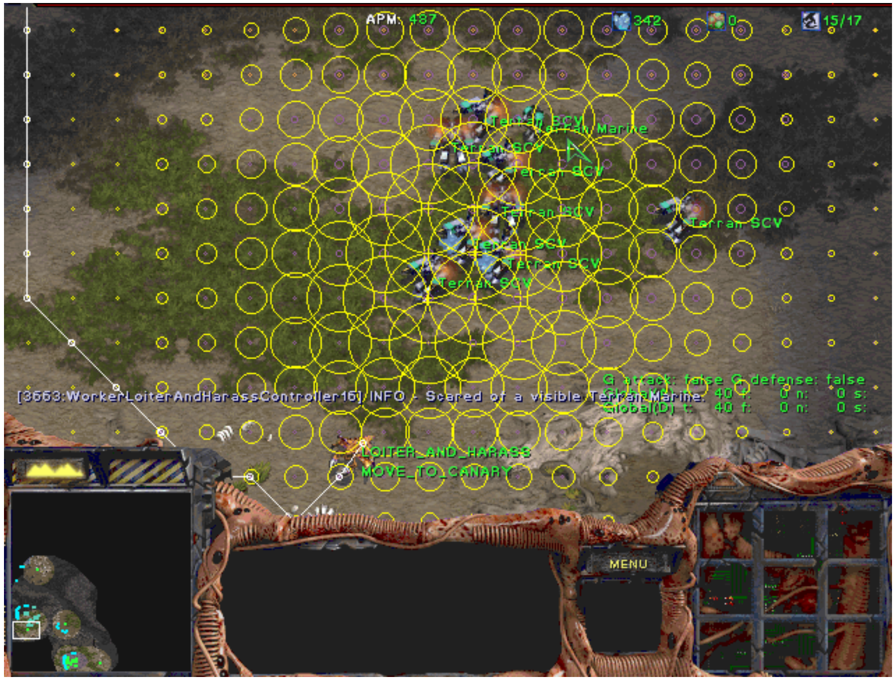
A*

A* Applications



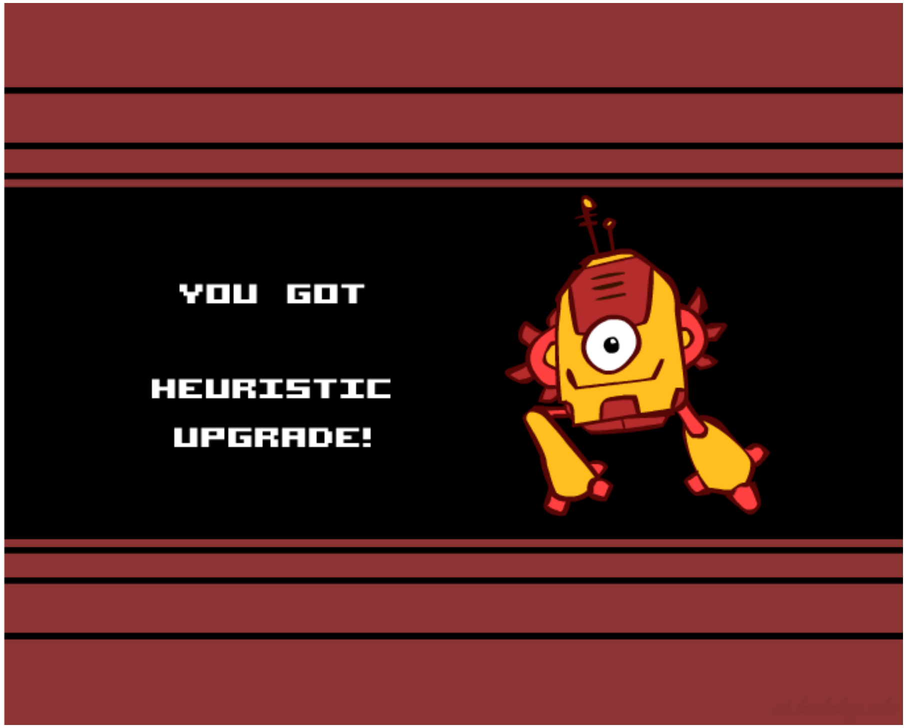
A* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …



[Demo: UCS / A* pacman tiny maze (L3D6,L3D7)]
[Demo: guess algorithm Empty Shallow/Deep (L3D8)]

Creating Heuristics



Creating Admissible Heuristics

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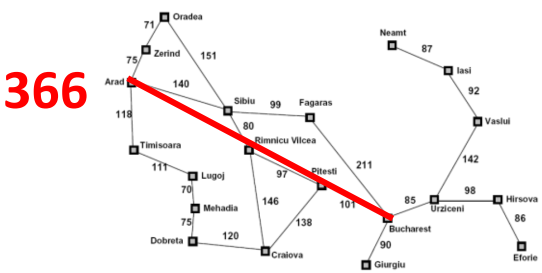
- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available



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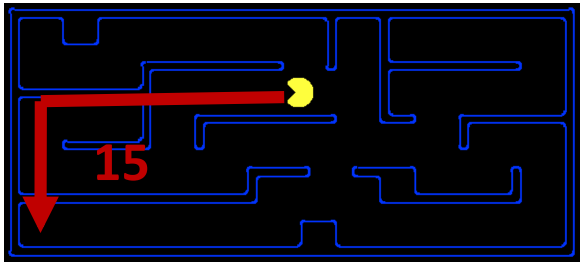

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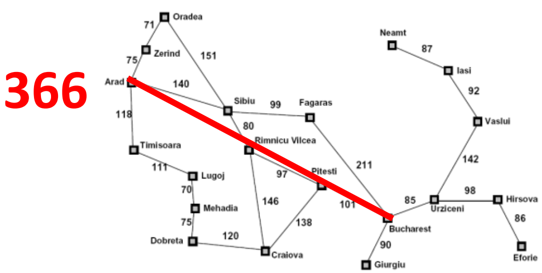
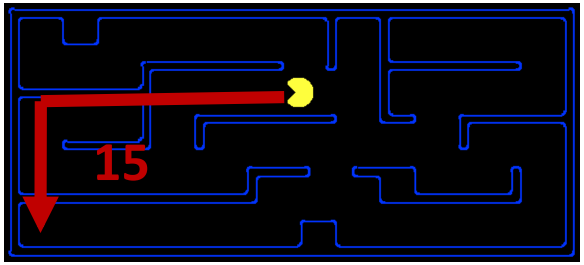
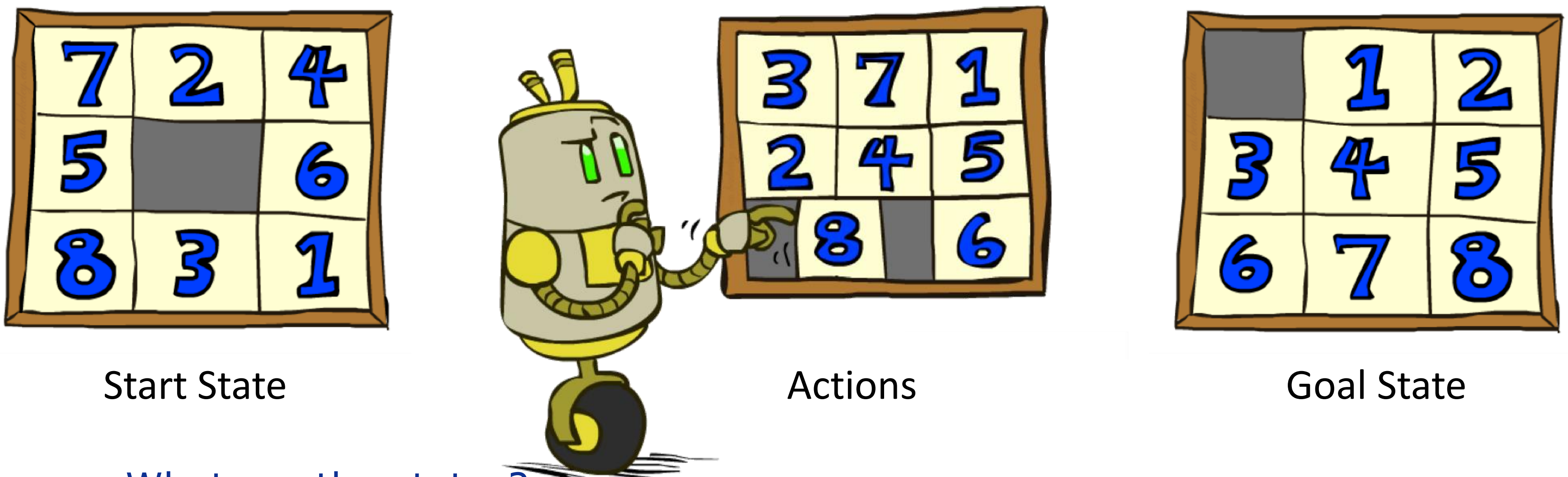


- Inadmissible heuristics are often useful too



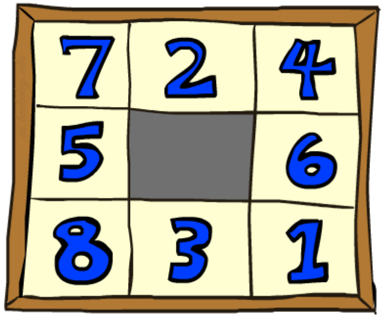
Example: 8 Puzzle



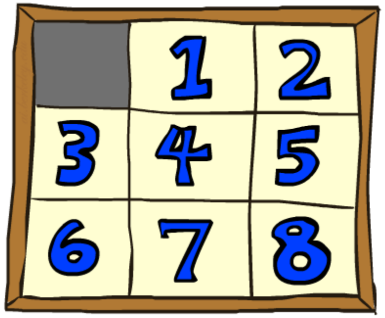
- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

8 Puzzle I

- Heuristic: Number of tiles misplaced



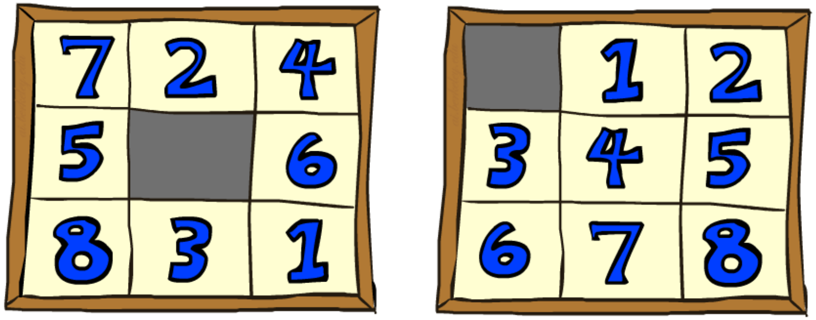
Start State



Goal State

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?

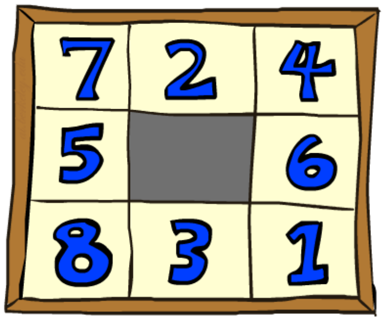


Start State

Goal State

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) =



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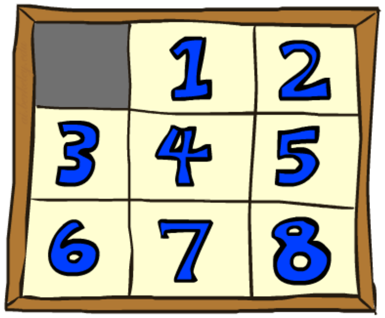

Goal State

8 Puzzle I

- Heuristic: Number of tiles misplaced
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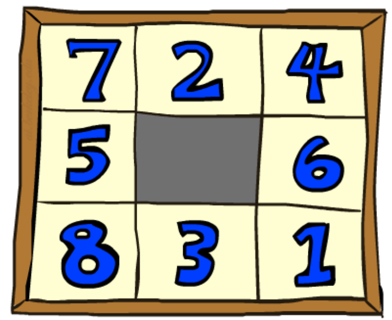

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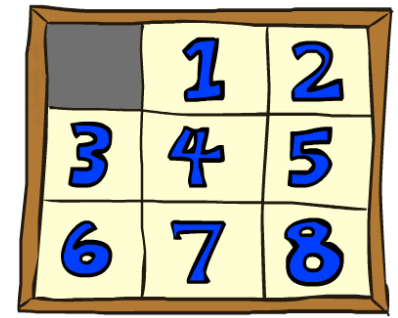

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	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

Statistics from Andrew Moore

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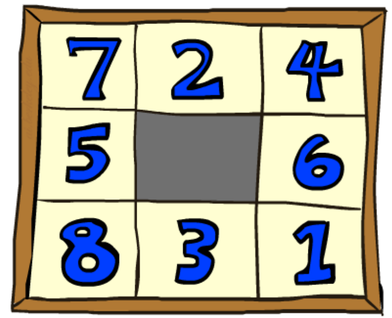

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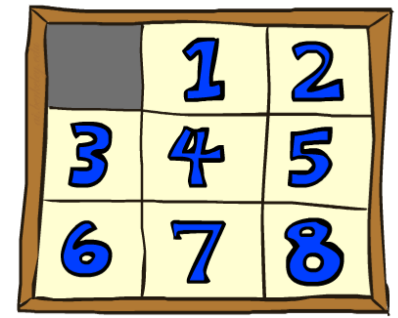

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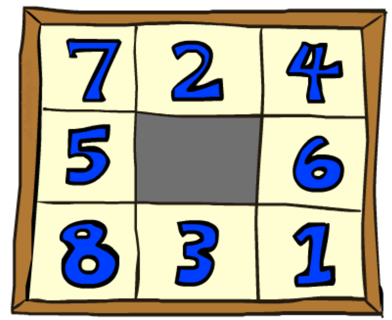

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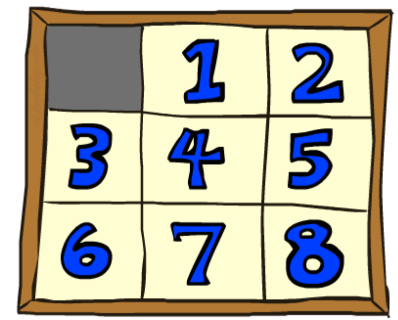

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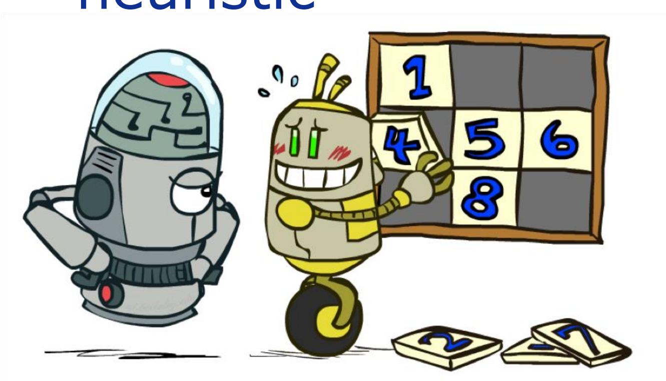

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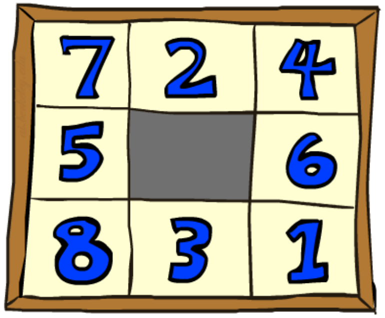

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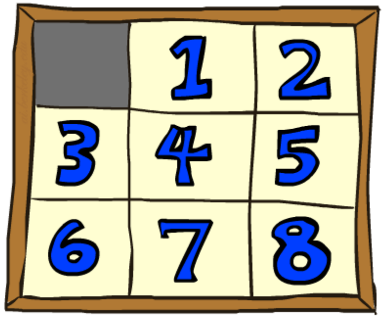

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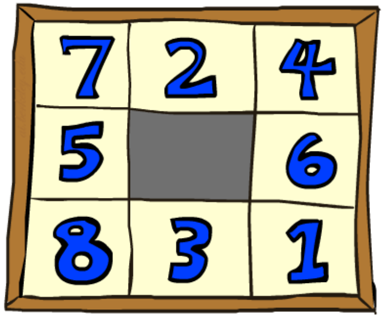

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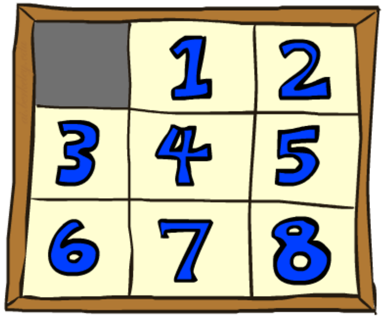

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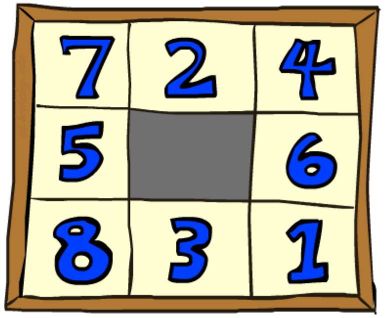

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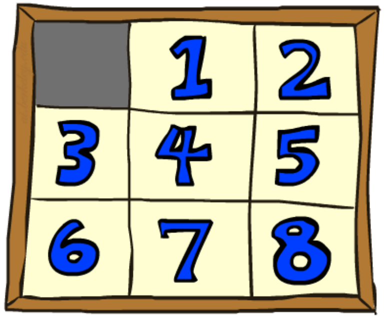

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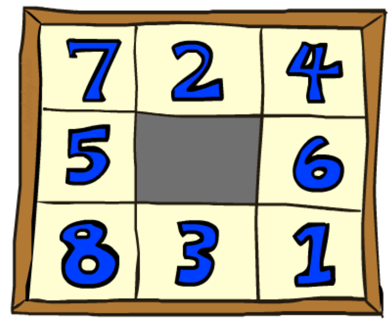

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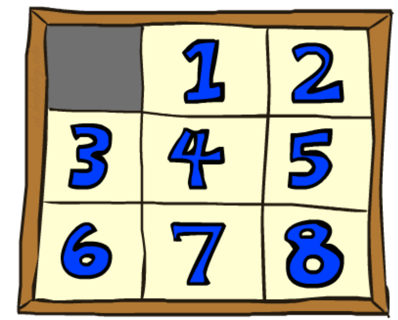

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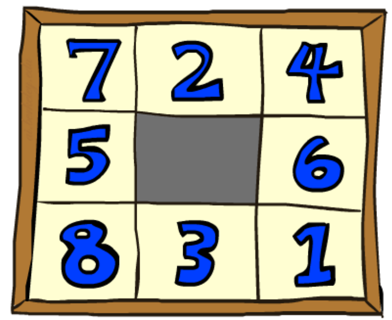

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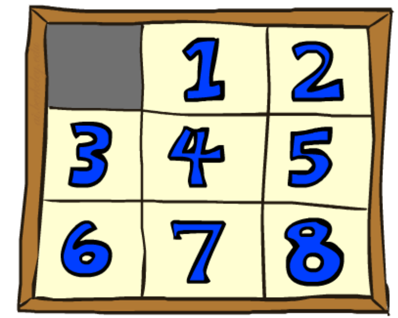

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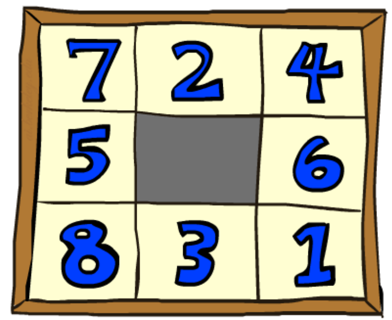

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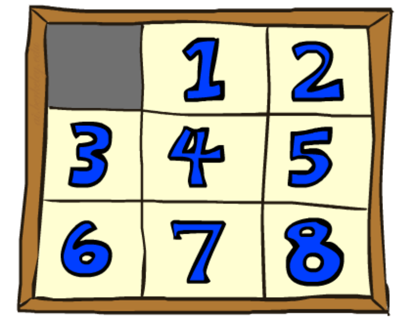

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TILES	13	39	227
MANHATTAN	12	25	73

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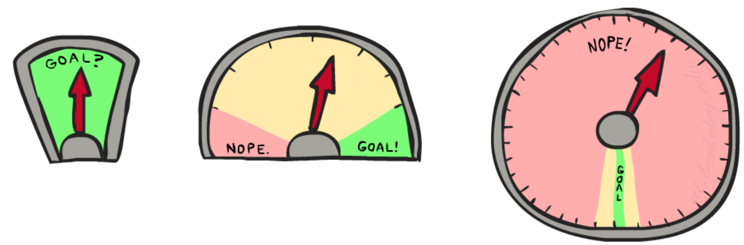

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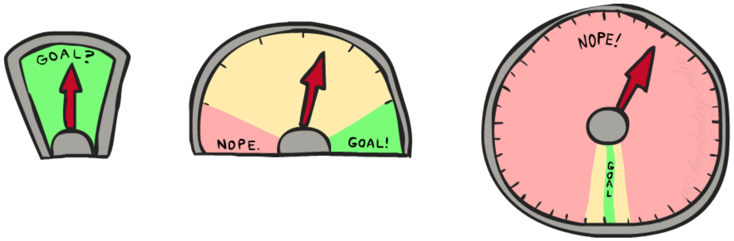


- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Semi-Lattice of Heuristics

Trivial Heuristics, Dominance

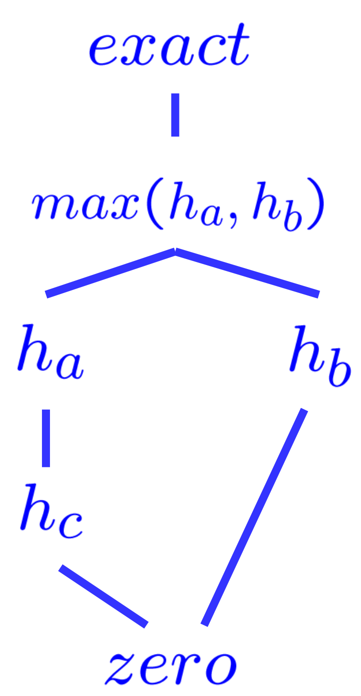
- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

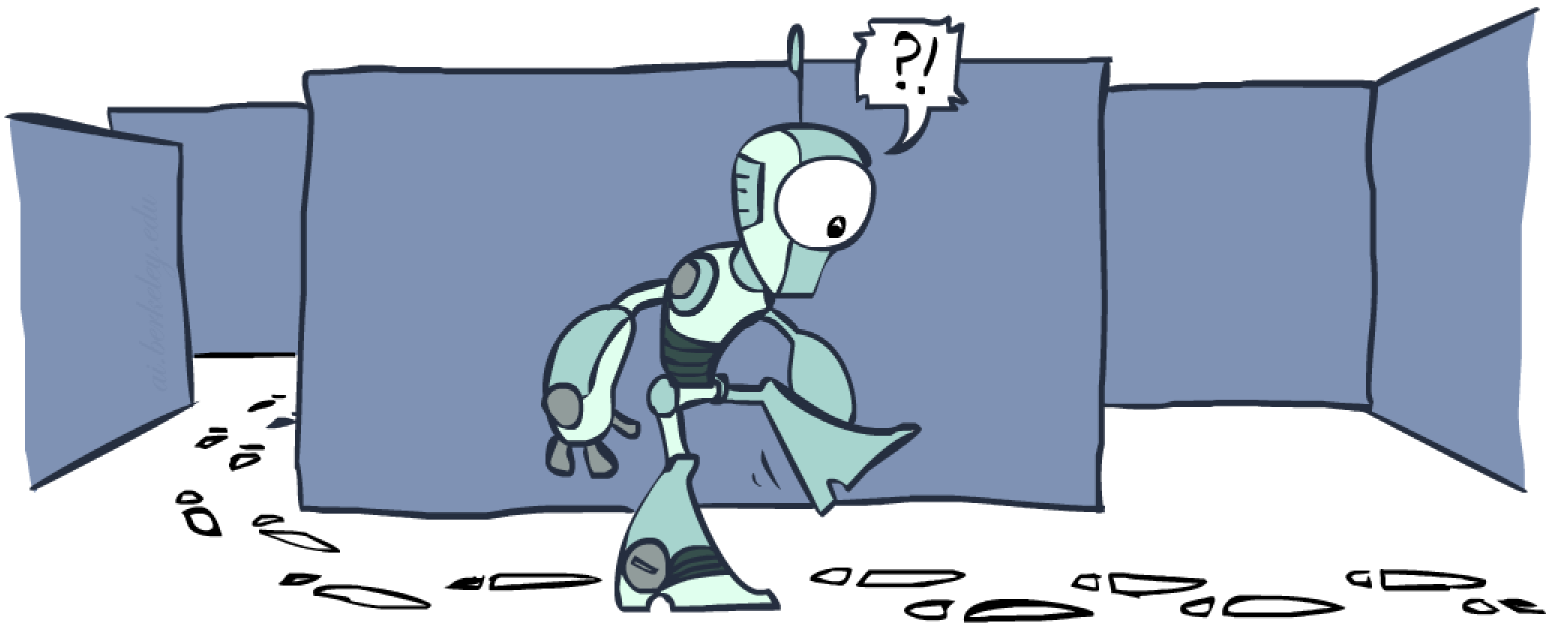
- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic

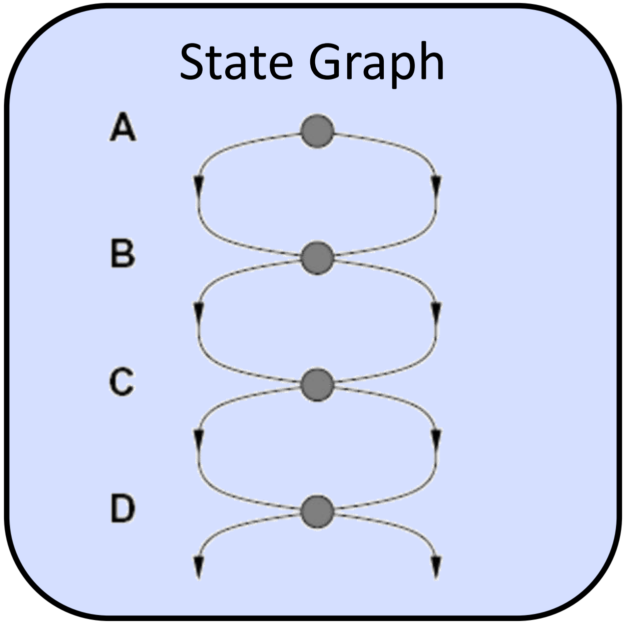


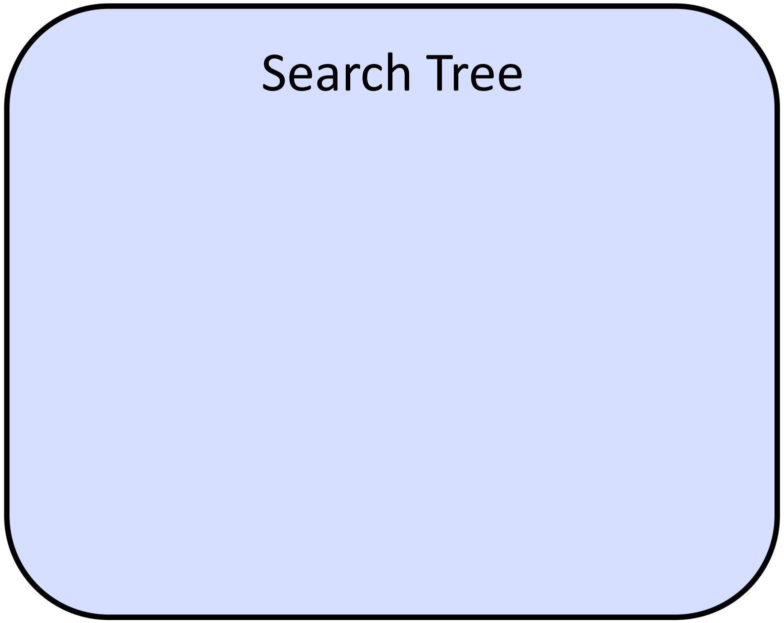
Graph Search



Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.





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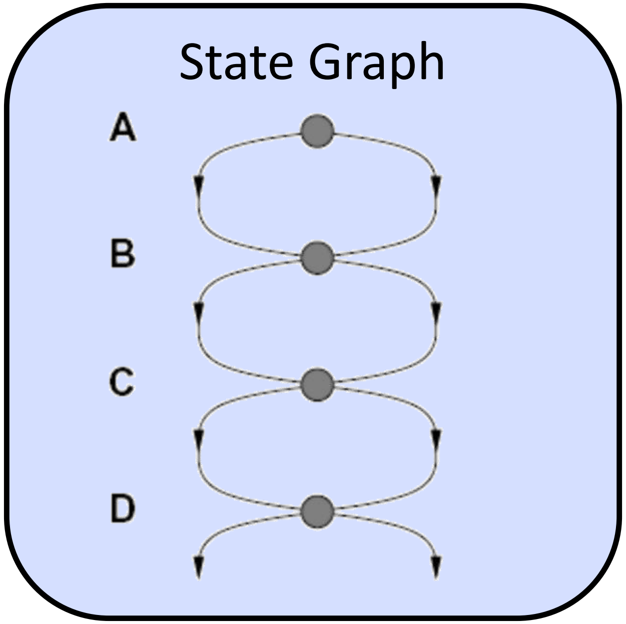

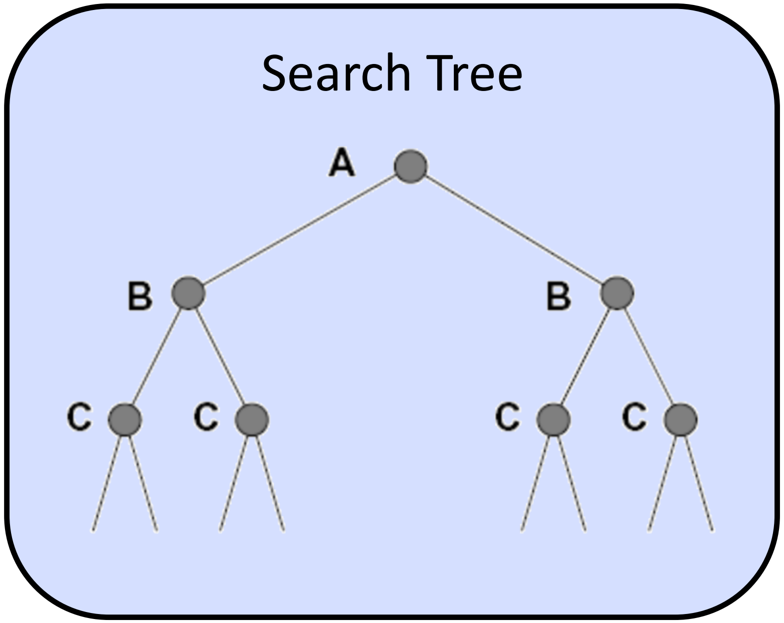

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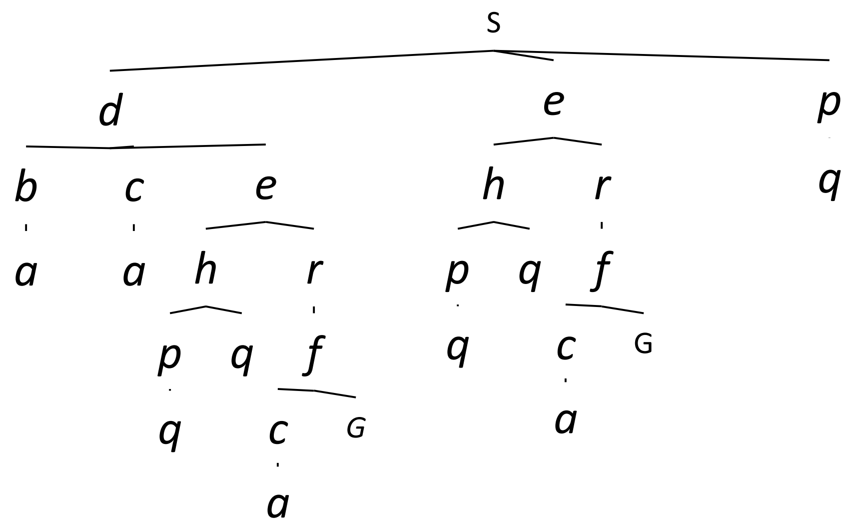

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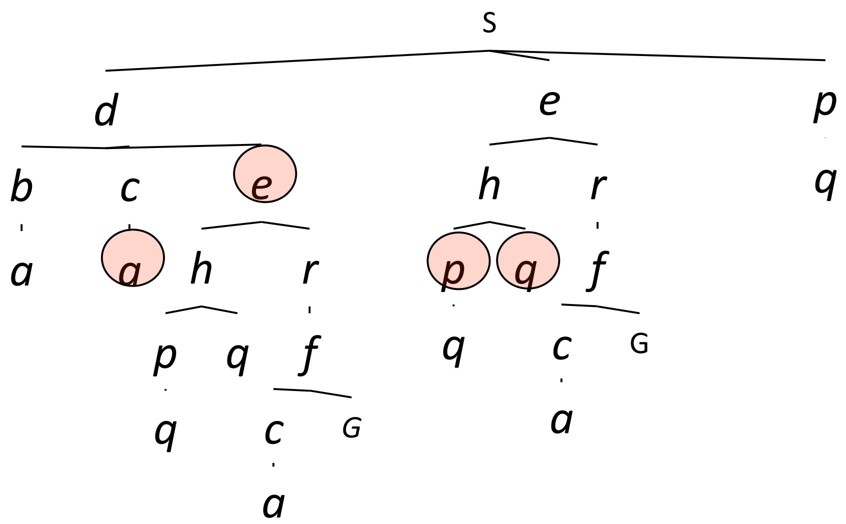

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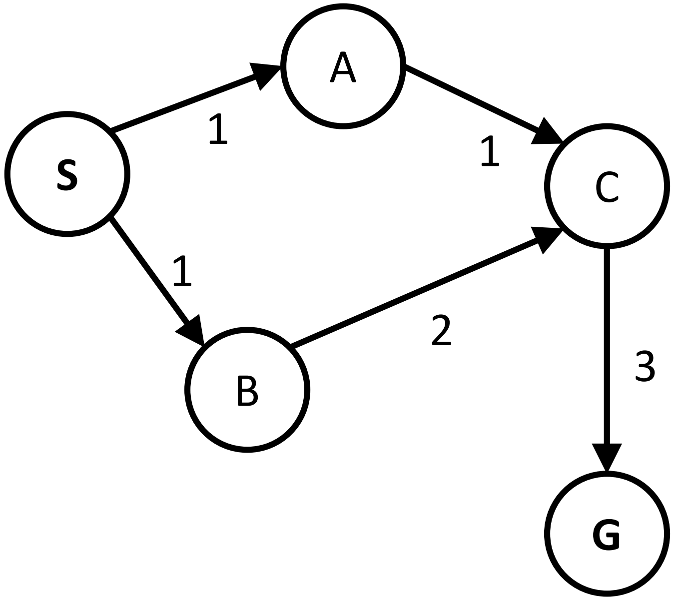
A* Graph Search Gone Wrong?

State space graph



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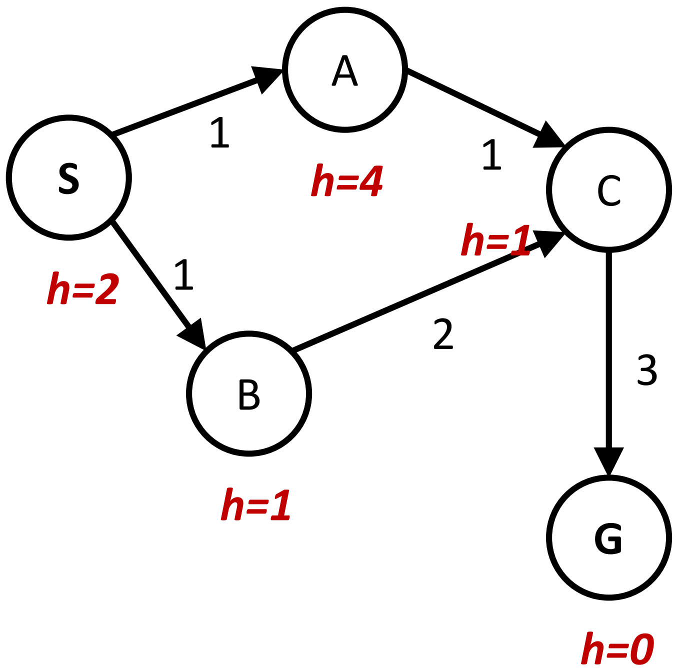

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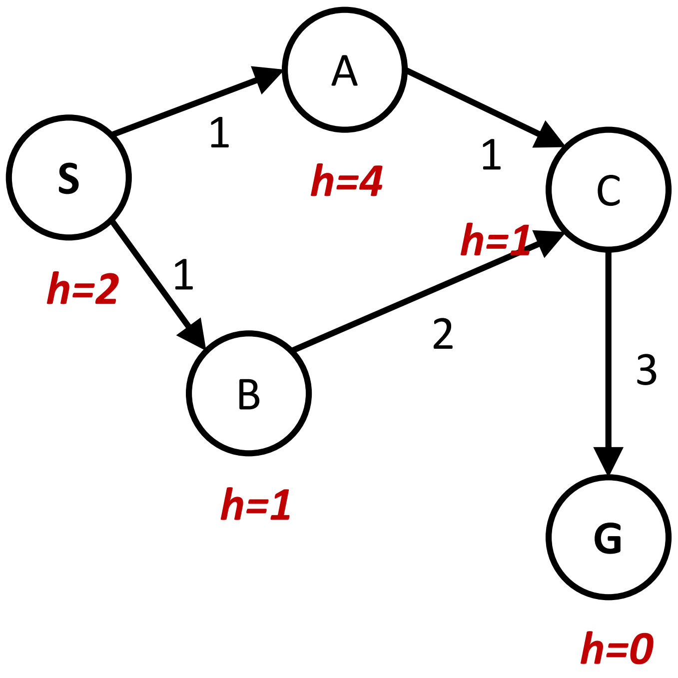

Search tree

S (0+2)

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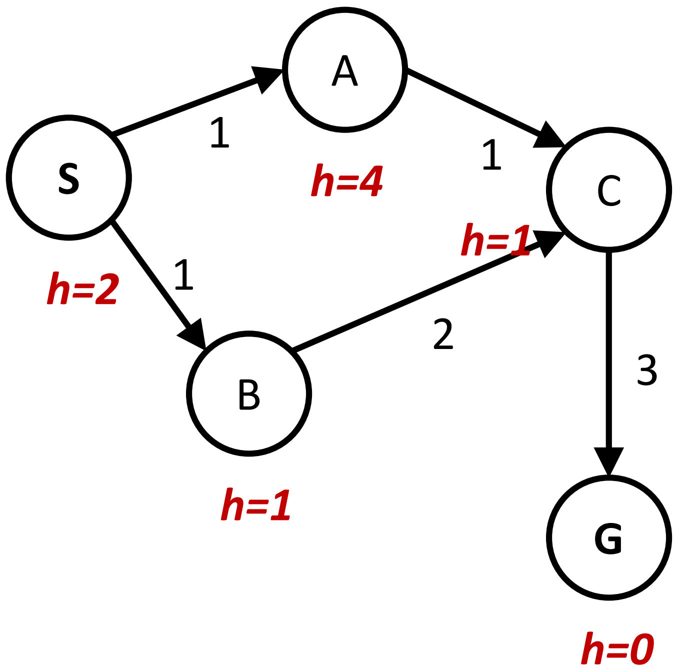

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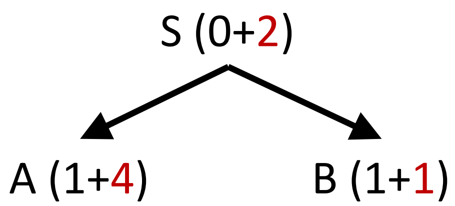

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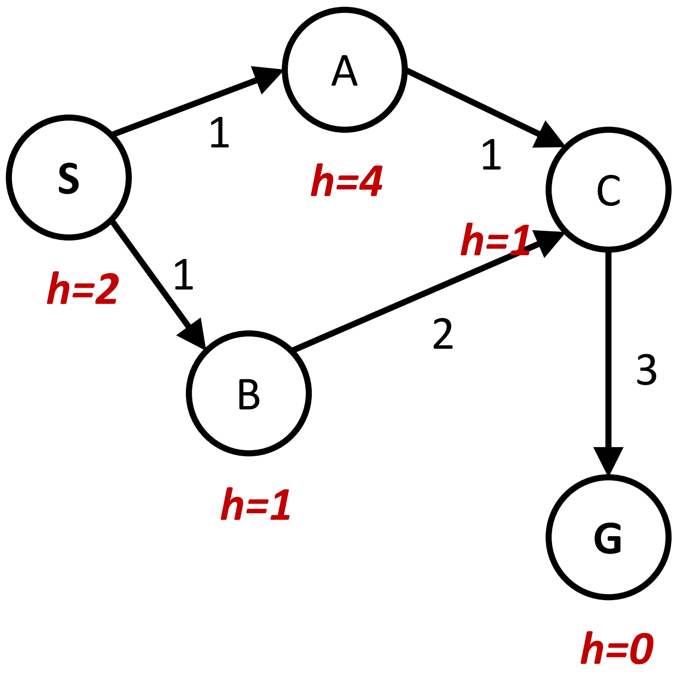

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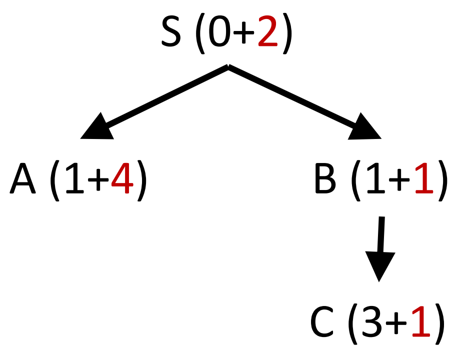

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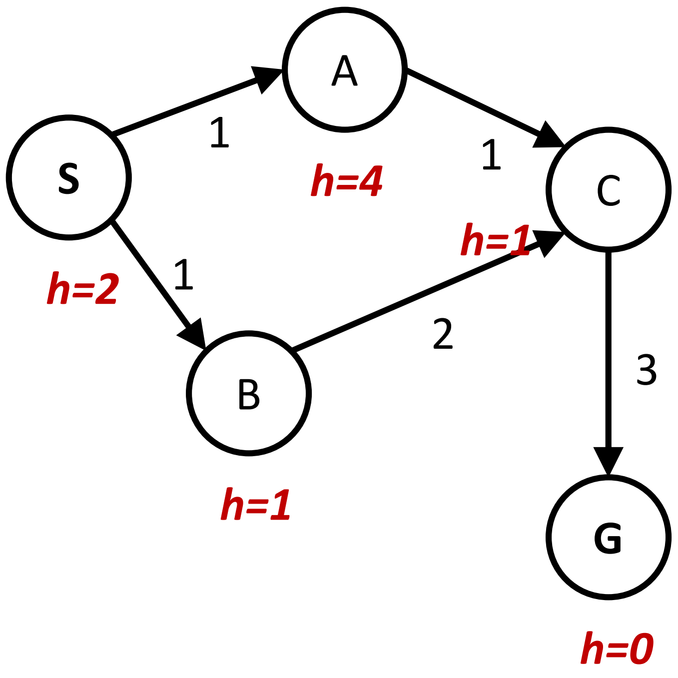

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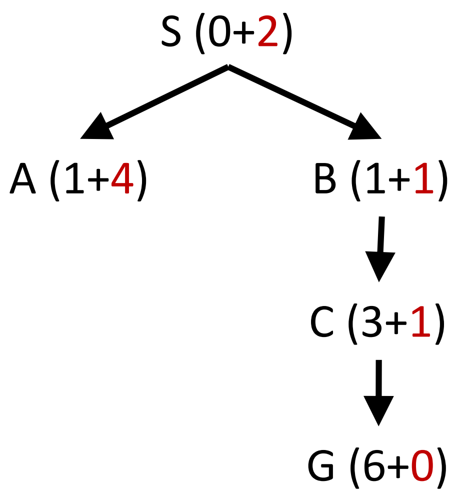

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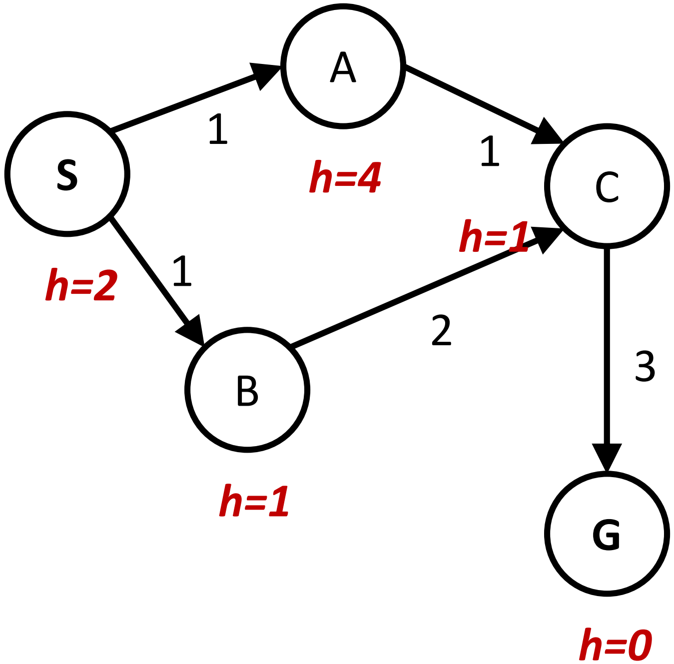

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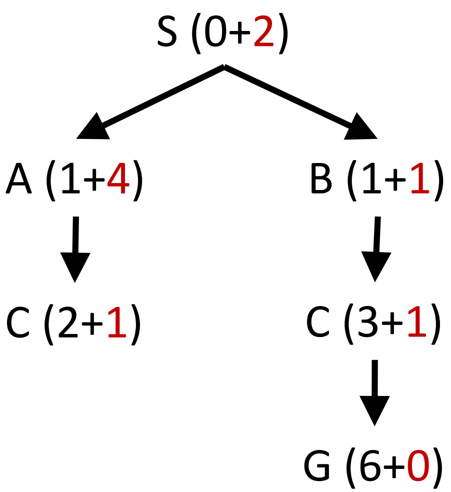

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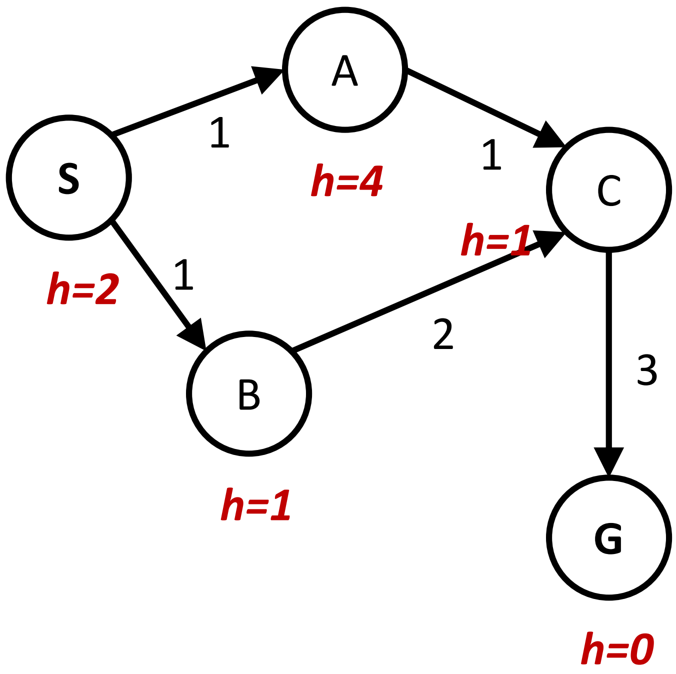

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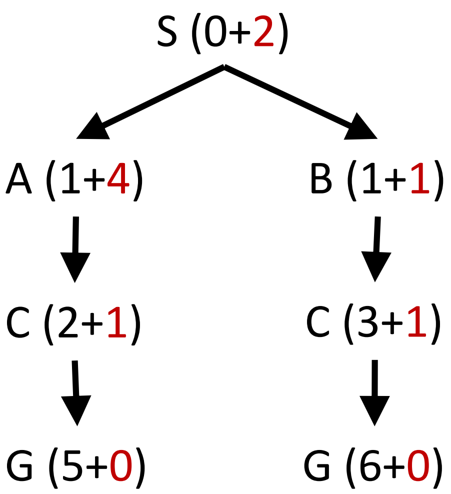

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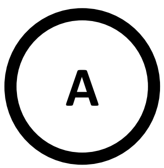

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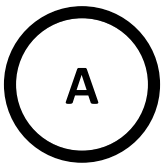

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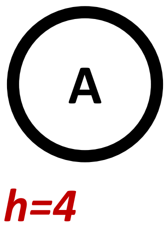

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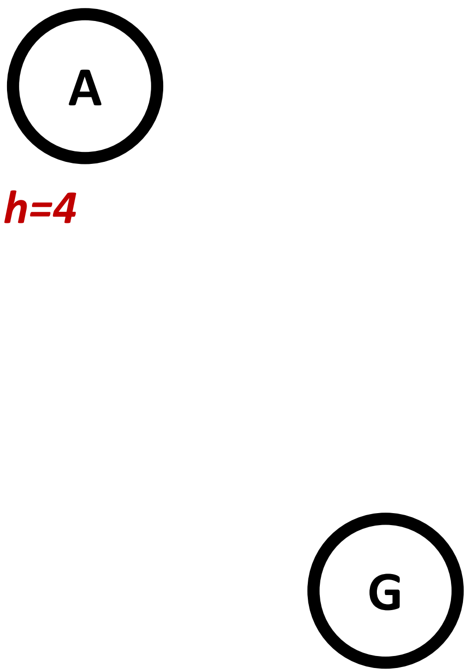

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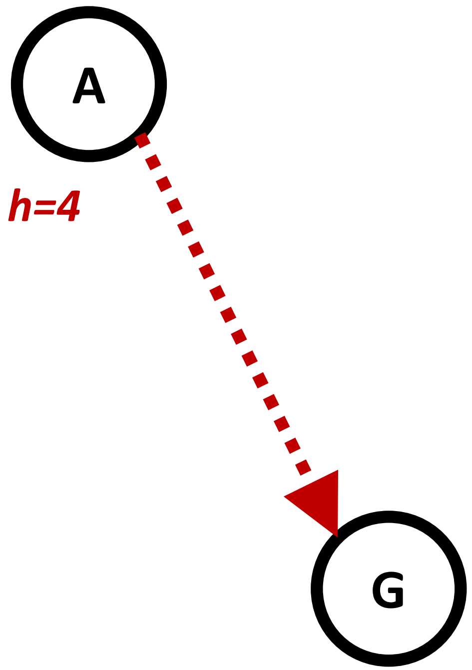

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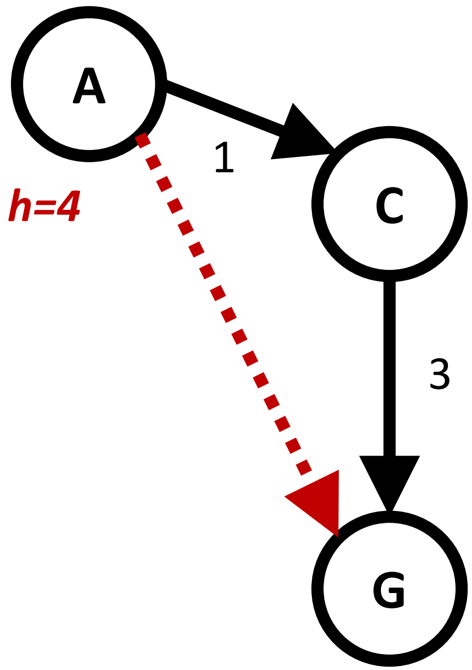

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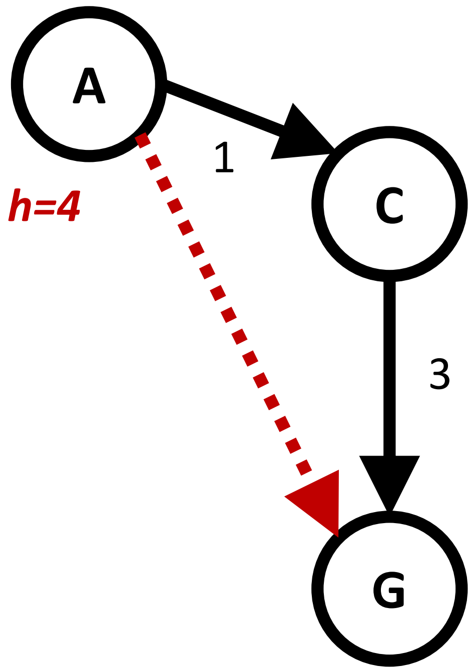

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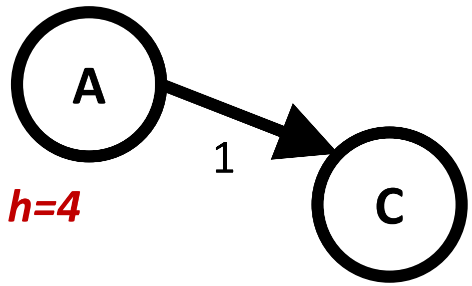

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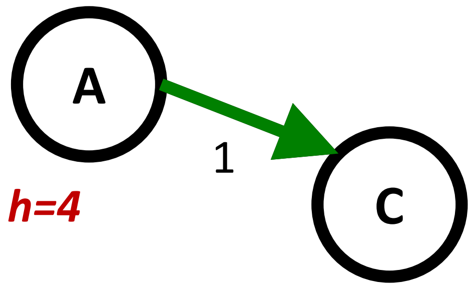

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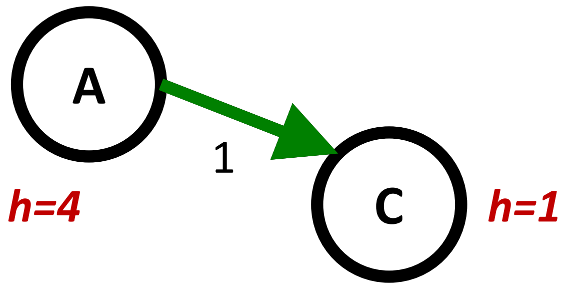

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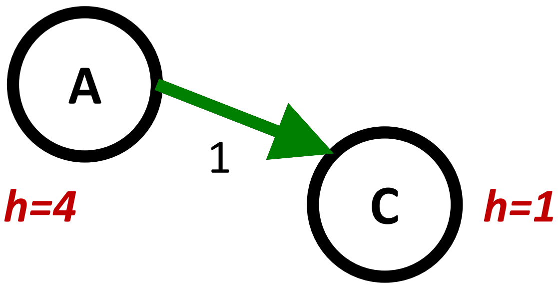
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G
  - Consistency: heuristic “arc” cost ≤ actual cost for each arc

Consistency of Heuristics



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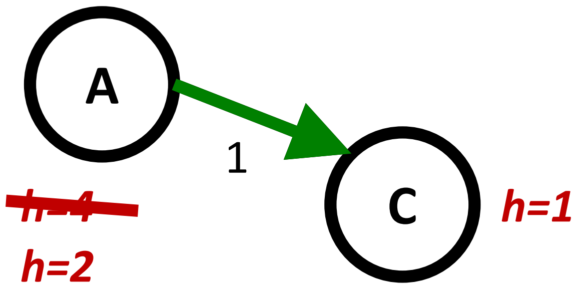

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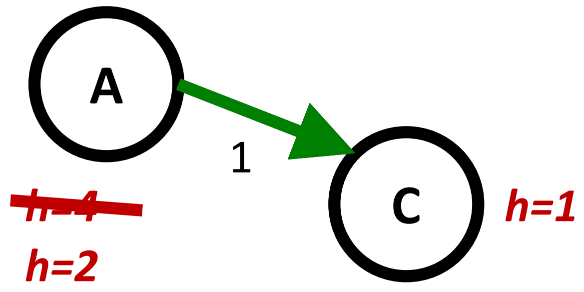

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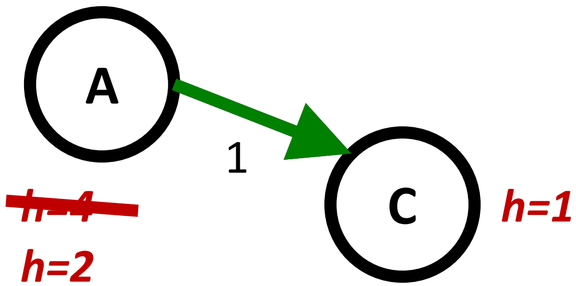
Consistency of Heuristics



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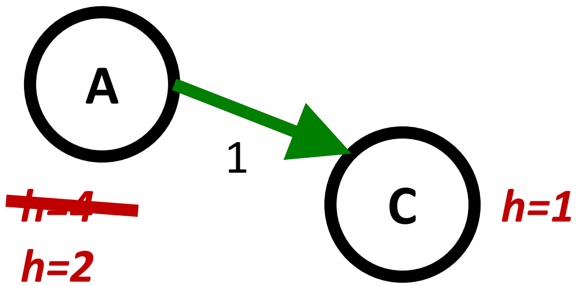
- Consequences of consistency:
  - The f value along a path never decreases

Consistency of Heuristics



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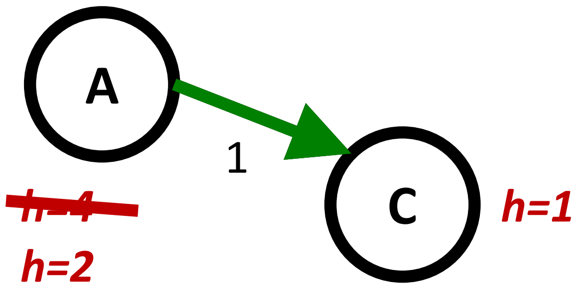
- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)

Consistency of Heuristics



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- Consequences of consistency:
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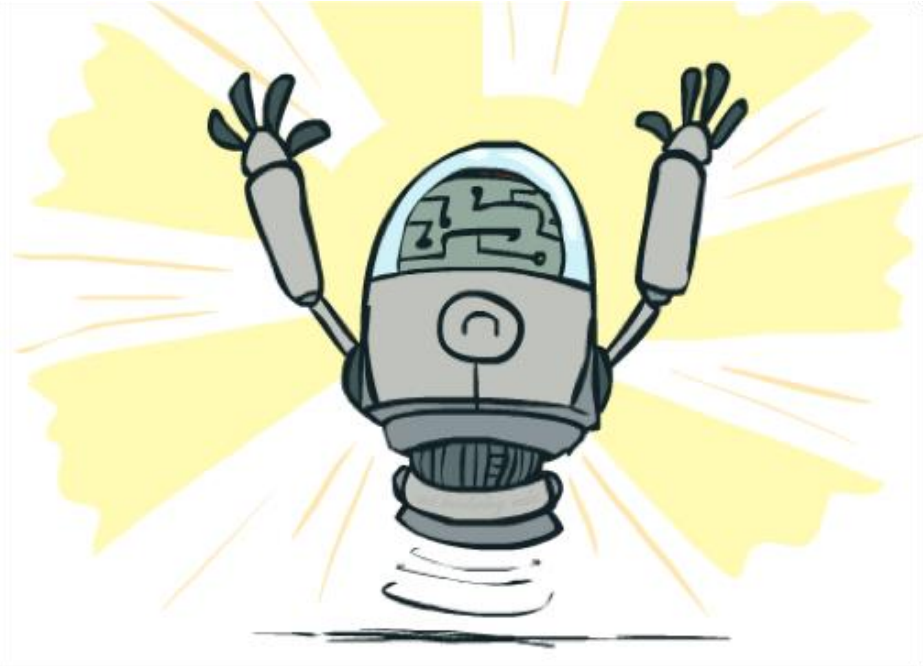
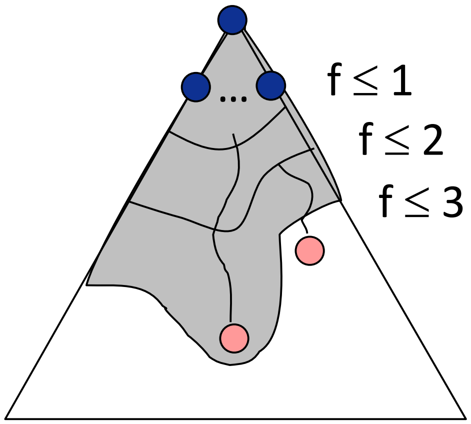
    h(A) ≤ cost(A to C) + h(C)
  - A* graph search is optimal

Optimality of A* Graph Search



Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal



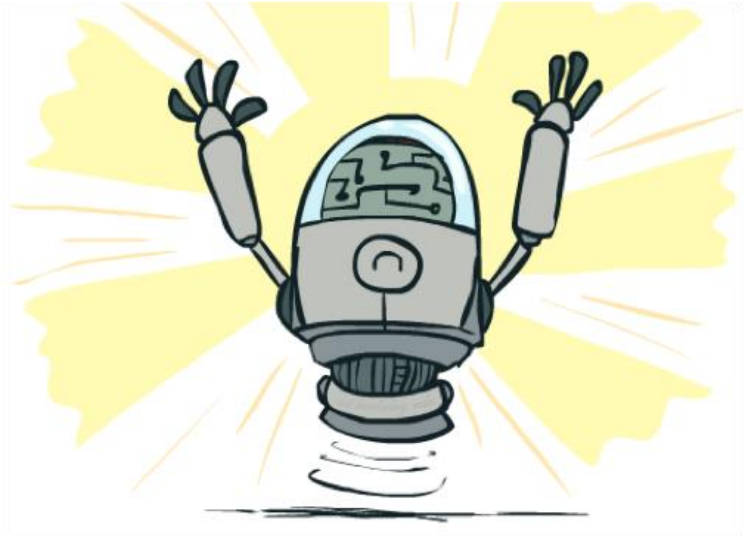
Optimality

- Tree search:
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)

- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)

- Consistency implies admissibility

- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems



A*: Summary

A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems