Reinforcement Learning

Georges Sakr
ESIB

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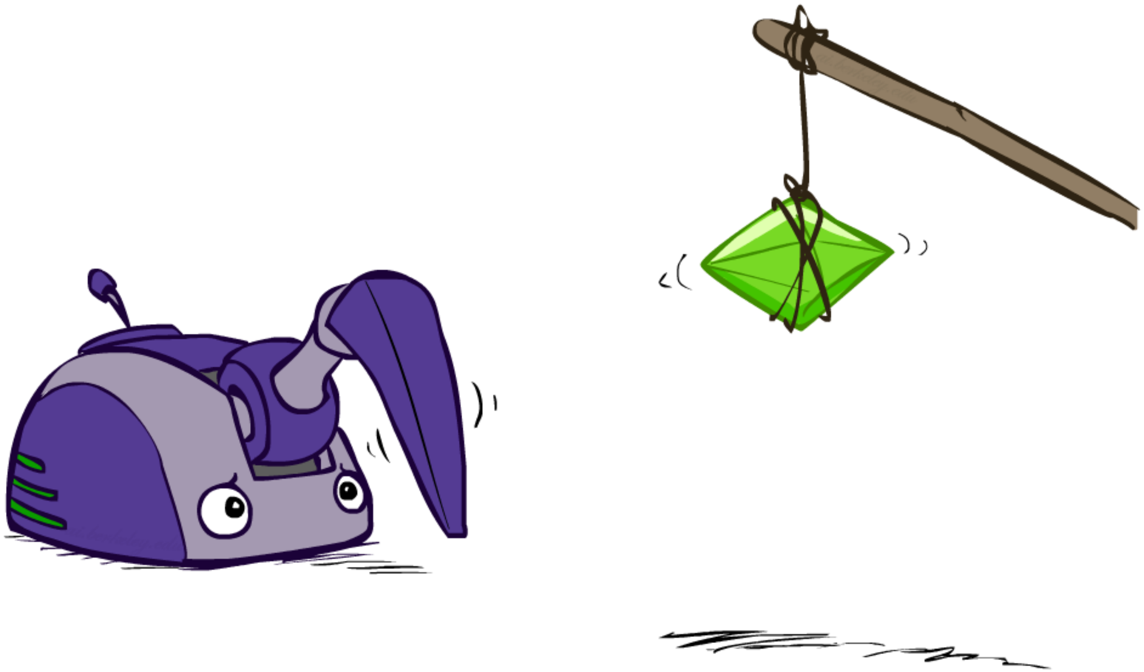

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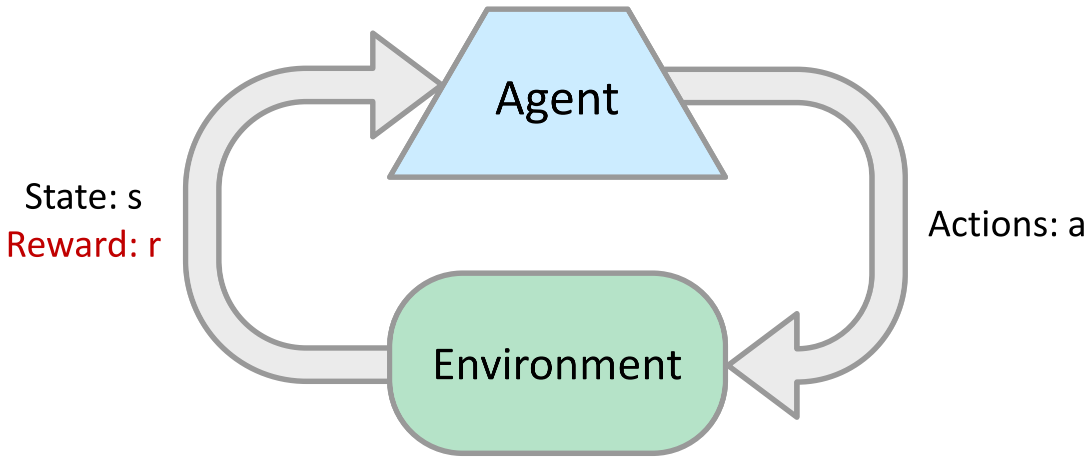
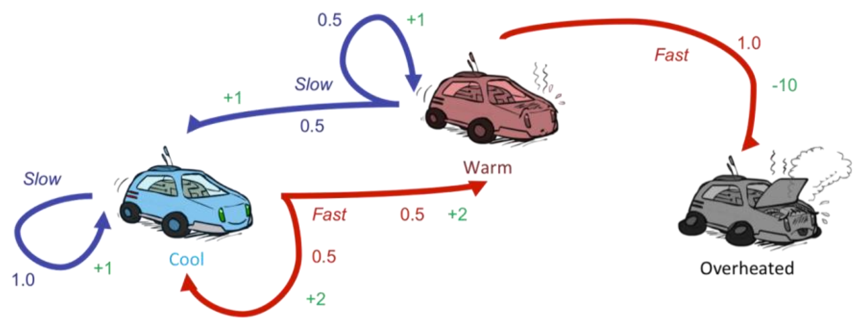


- **Basic idea:**
  - Receive feedback in the form of rewards
  - Agent's utility is defined by the reward function
  - Must (learn to) act so as to maximize expected rewards
  - All learning is based on observed samples of outcomes!

Reinforcement Learning

- Still assume a Markov decision process (MDP):
  - A set of states $s \in S$
  - A set of actions (per state) A
  - A model T(s,a,s')
  - A reward function R(s,a,s')
- Still looking for a policy $\pi(s)$



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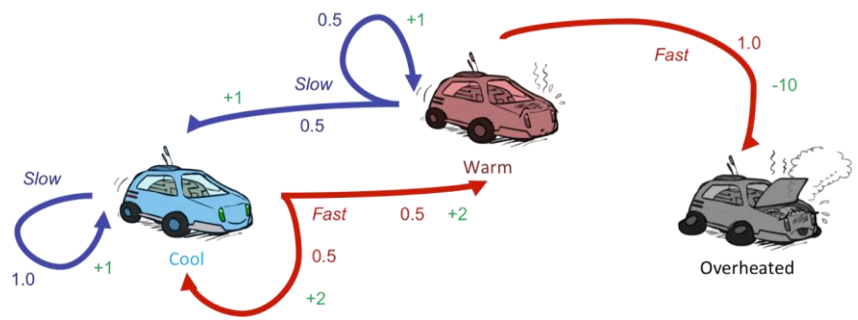

- New twist: don't know T or R
  - I.e. we don't know which states are good or what the actions do
  - Must actually try actions and states out to learn

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Offline (MDPs) vs. Online (RL)

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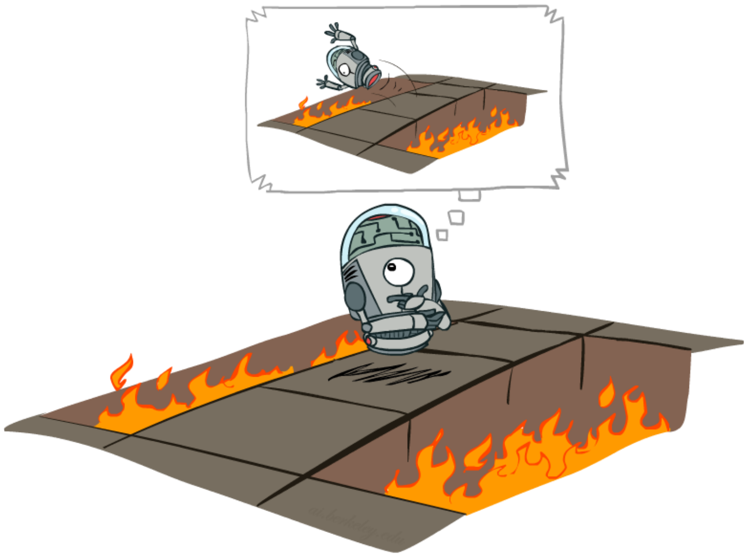
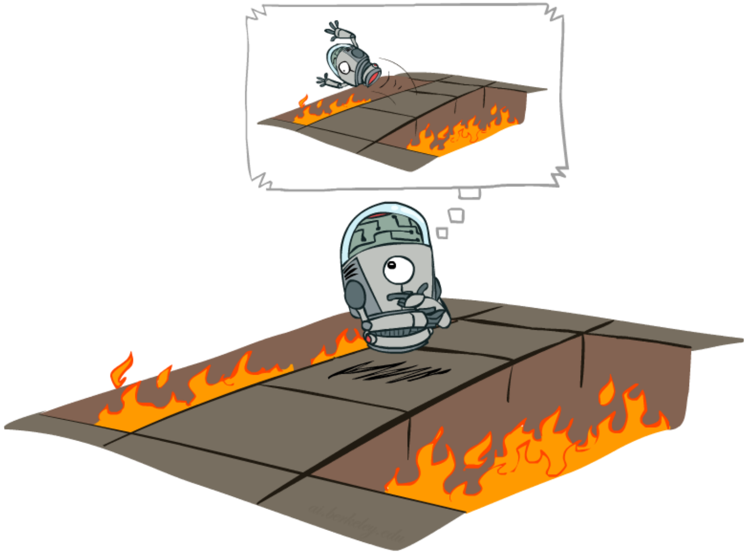


Offline Solution

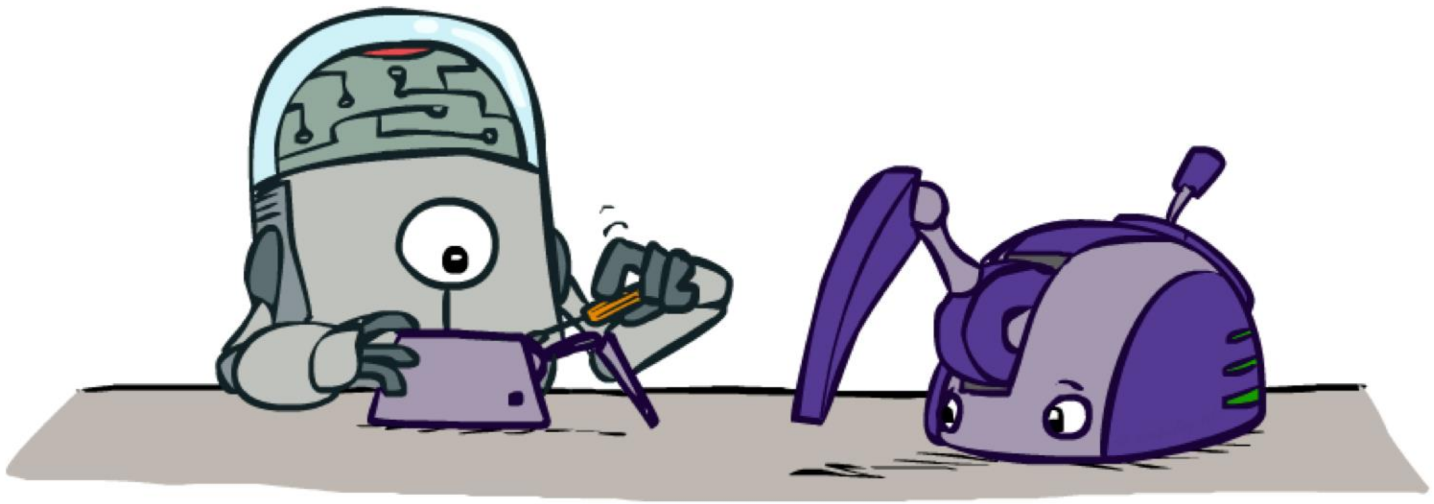
Offline (MDPs) vs. Online (RL)



Offline Solution

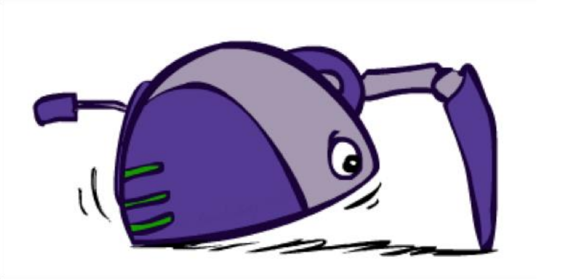
Online Learning

Model-Based Learning



Model-Based Learning

- Model-Based Idea:
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct



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- Step 1: Learn empirical MDP model
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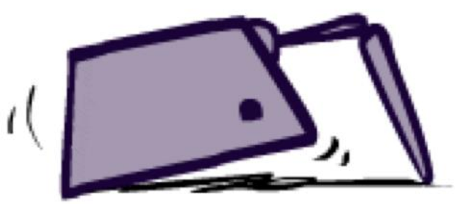
  - Normalize to give an estimate of $\hat{T}(s,a,s')$
  - Discover each $\hat{R}(s,a,s')$ when we experience (s, a, s′)



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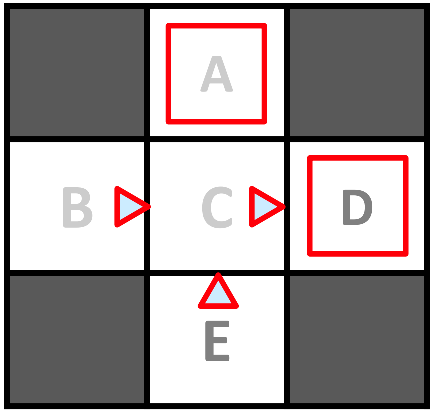
- Step 2: Solve the learned MDP
  - For example, use value iteration, as before

Example: Model-Based Learning

Input Policy π



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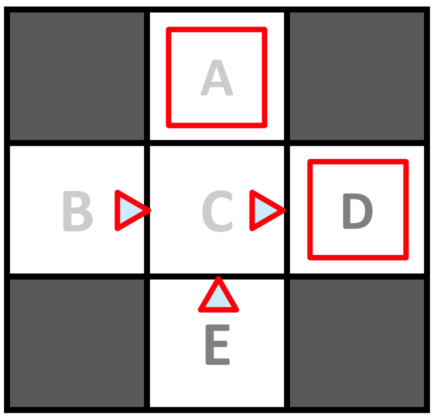

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C, east, D, -1
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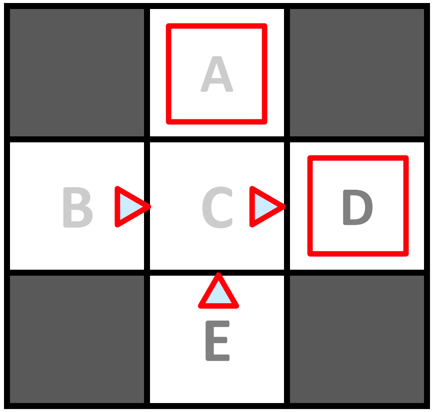

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Learned Model

$\widehat{T}(s, a, s')$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
…

$\widehat{R}(s, a, s')$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Example: Expected Age

Goal: Compute expected age of students

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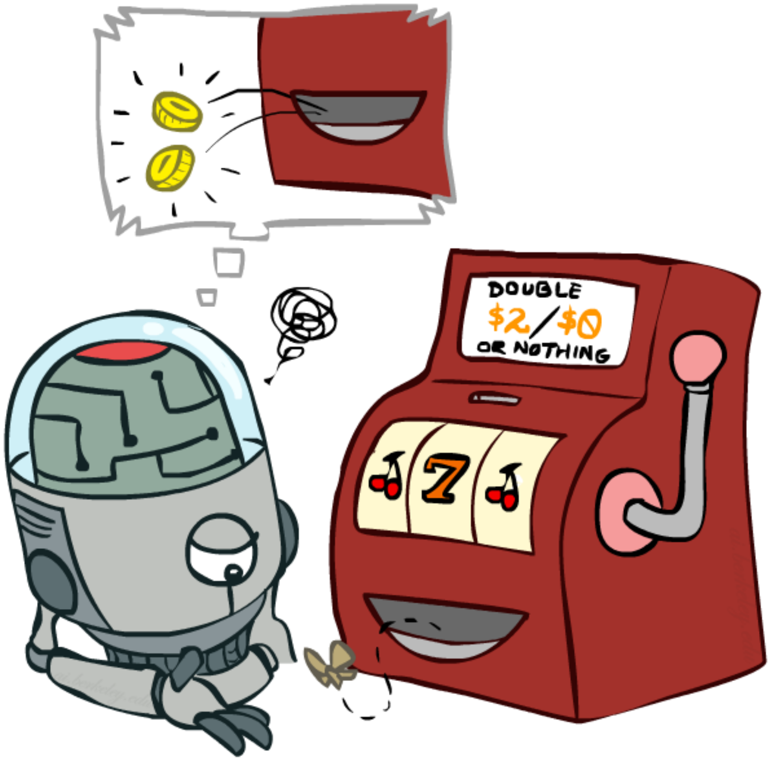
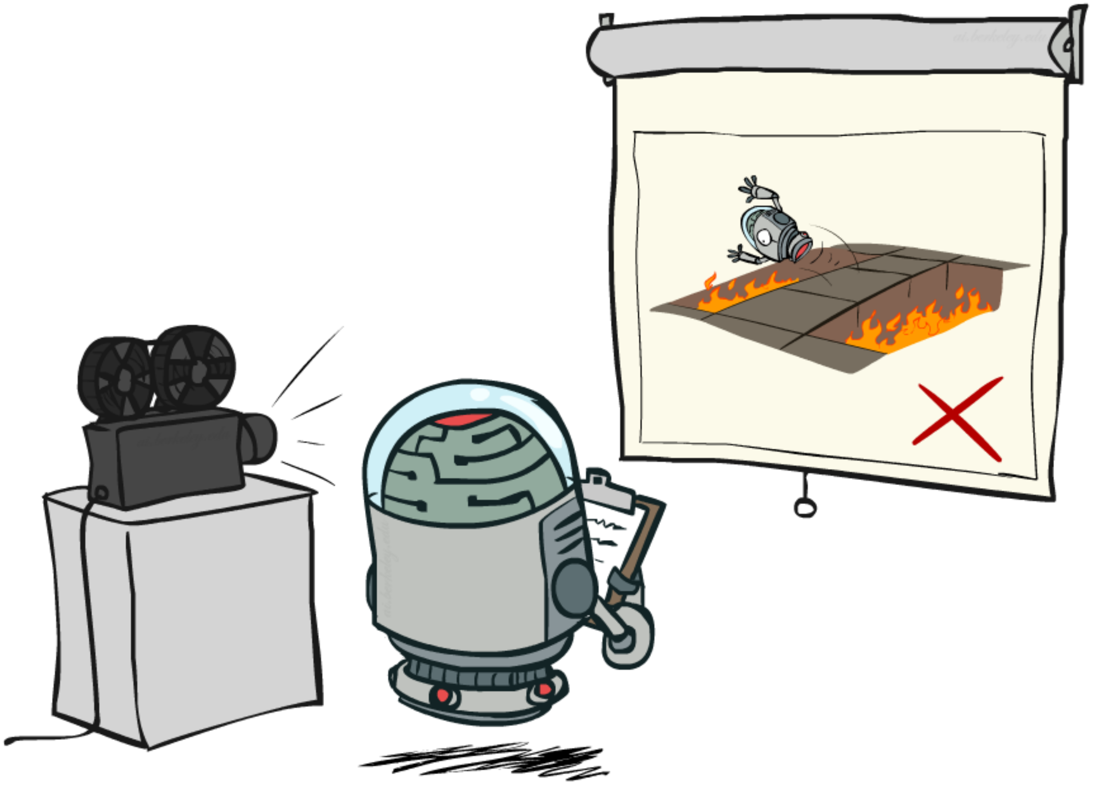
Why does this work? Because samples appear with the right frequencies.

Model-Free Learning

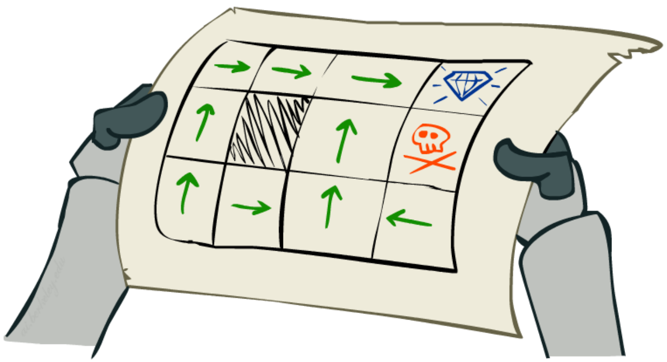


Passive Reinforcement Learning



Passive Reinforcement Learning

- Simplified task: policy evaluation
  - Input: a fixed policy $\pi(s)$
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - Goal: learn the state values



- In this case:
  - Learner is "along for the ride"
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - This is NOT offline planning! You actually take actions in the world.

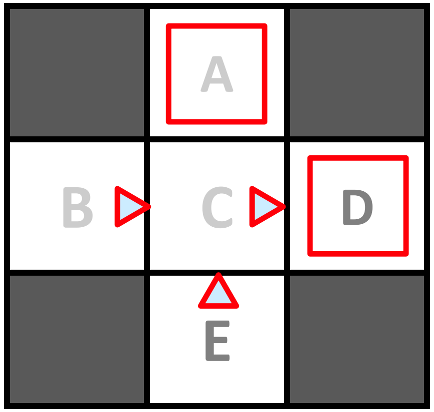
Direct Evaluation

- Goal: Compute values for each state under π
- Idea: Average together observed sample values
  - Act according to π
  - Every time you visit a state, write down what the sum of discounted rewards turned out to be
  - Average those samples
- This is called direct evaluation

Example: Direct Evaluation

Input Policy π

Output Values



Assume: $\gamma = 1$

Example: Direct Evaluation

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Observed Episodes (Training)

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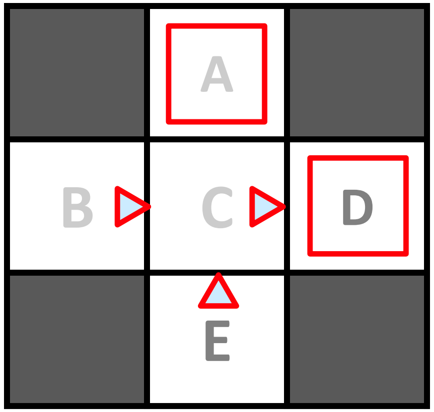

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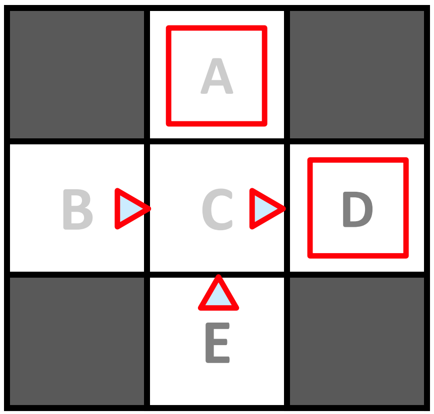

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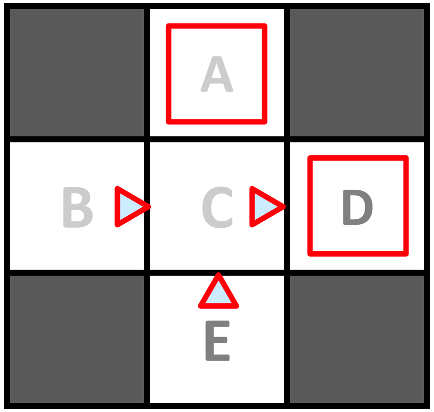

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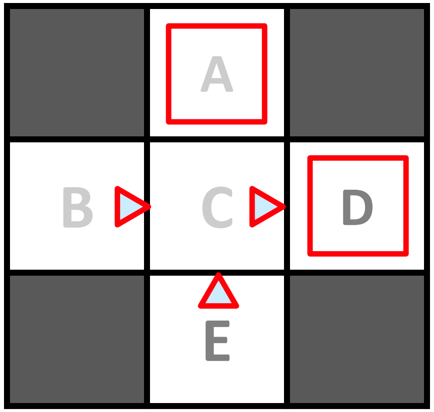

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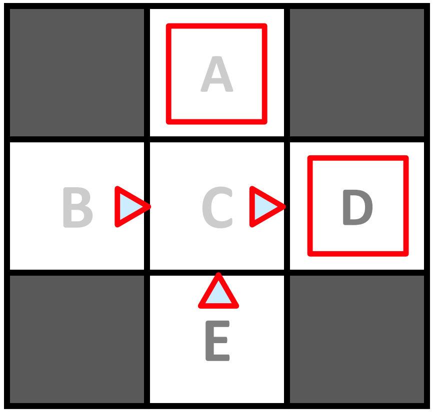

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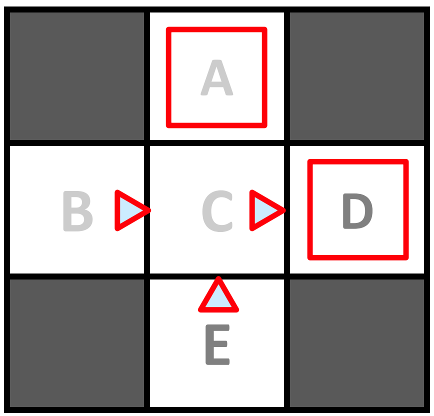

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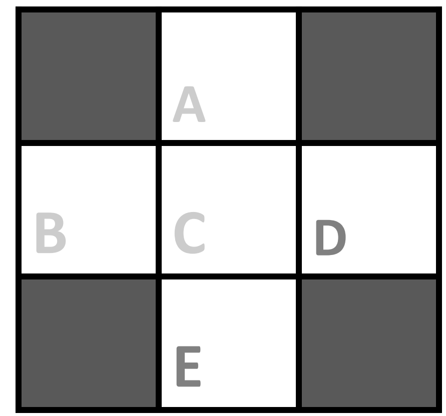

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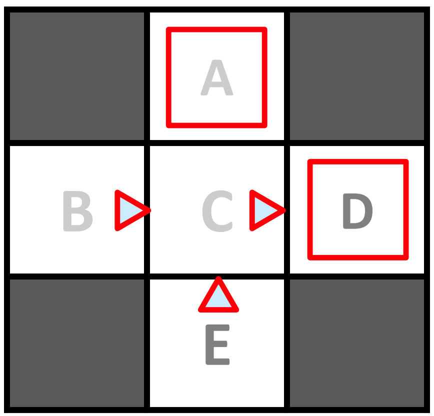

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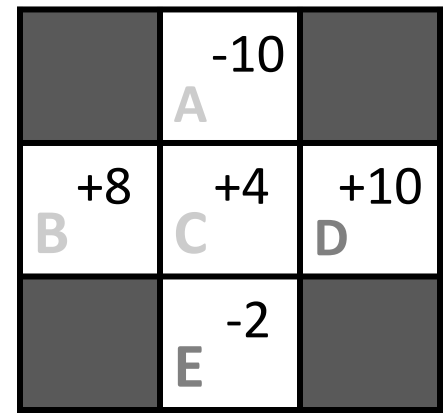

Problems with Direct Evaluation

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  - It's easy to understand
  - It doesn't require any knowledge of T, R
  - It eventually computes the correct average values, using just sample transitions

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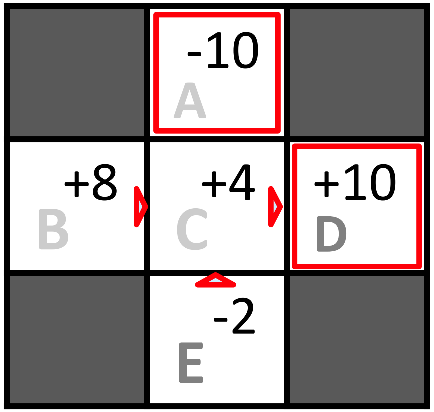

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  - Each state must be learned separately
  - So, it takes a long time to learn

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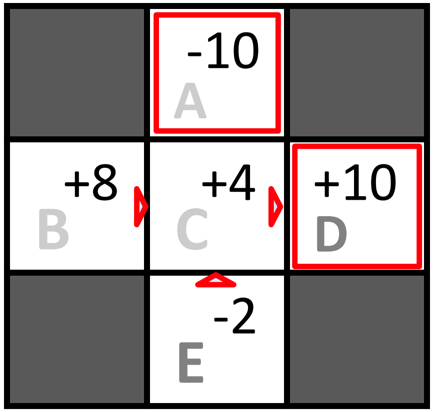

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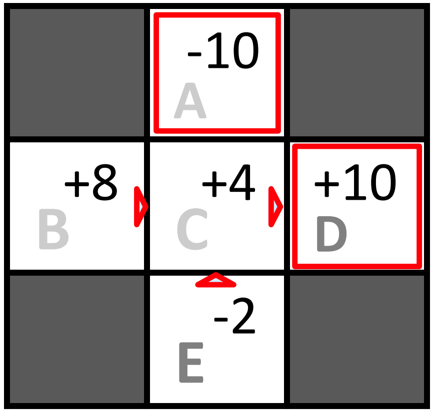
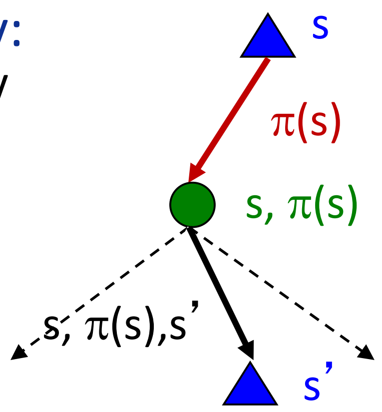


If B and E both go to C under this policy, how can their values be different?

Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate V for a fixed policy:
  - Each round, replace V with a one-step-look-ahead layer over V



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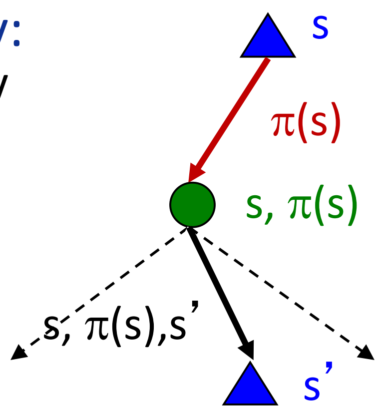

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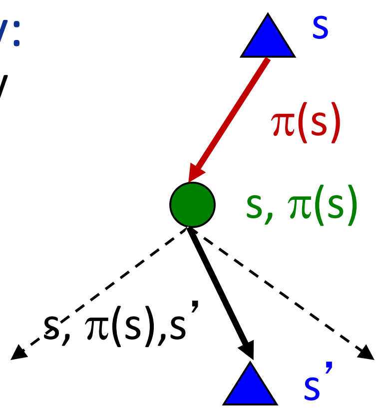

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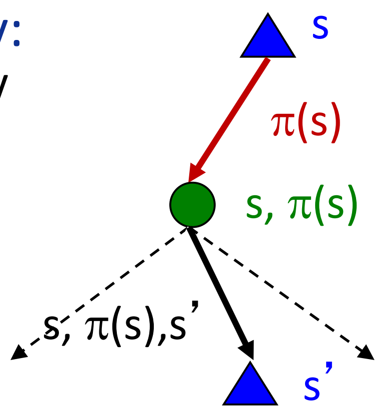

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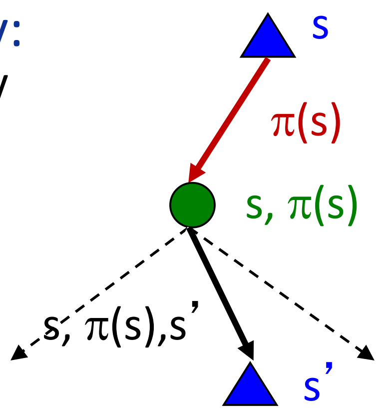

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- Key question: how can we do this update to V without knowing T and R?
  - In other words, how do we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

- We want to improve our estimate of V by computing these averages:

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- Idea: Take samples of outcomes s’ (by doing the action!) and average

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$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')]$$

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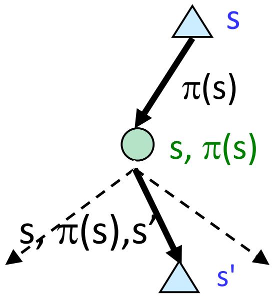

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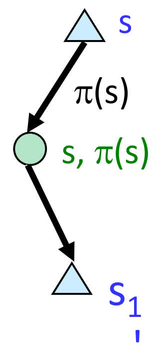

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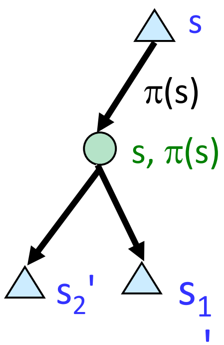

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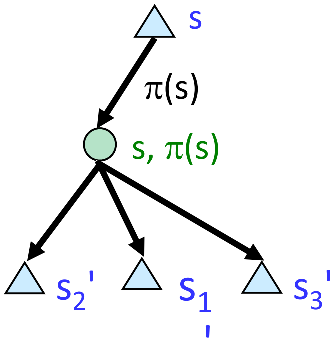

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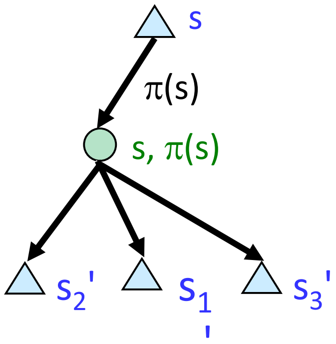

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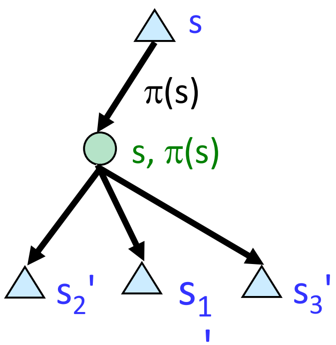

Almost! But we can't rewind time to get sample after sample from state s.

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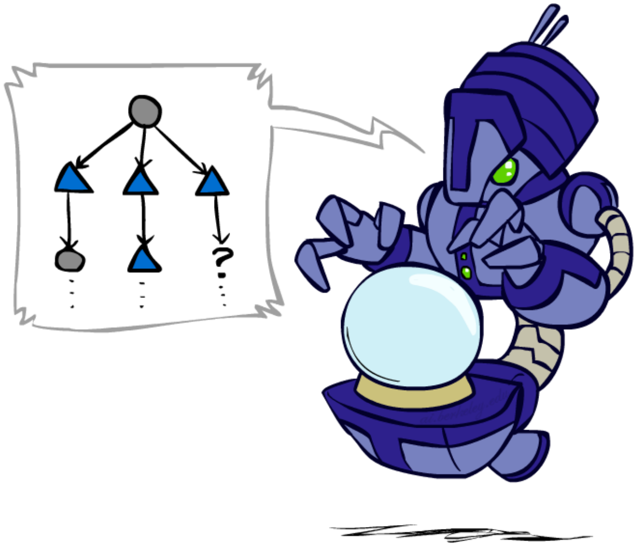

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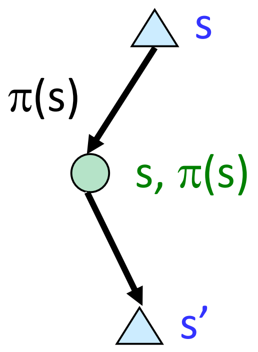
Temporal Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience a transition (s, a, s', r)
  - Likely outcomes s' will contribute updates more often



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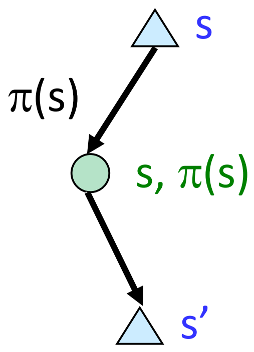
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average



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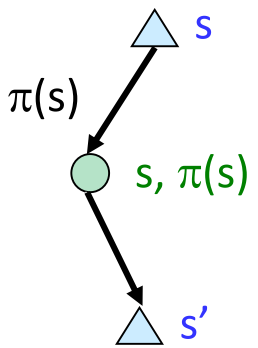

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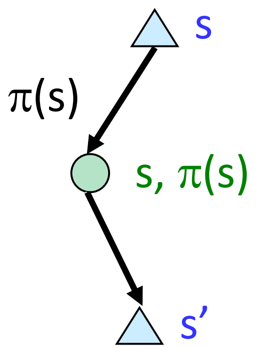

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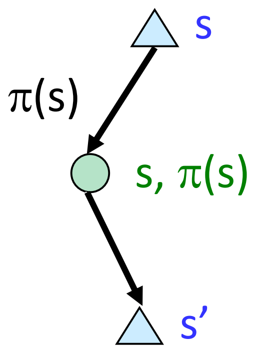

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Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$

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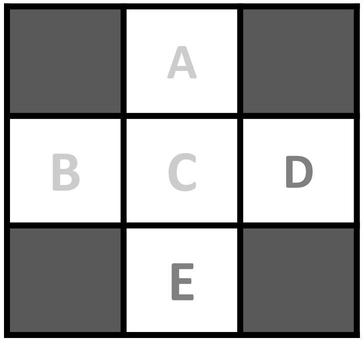
  - Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning

States



Assume: γ = 1, α = 1/2

Example: Temporal Difference Learning

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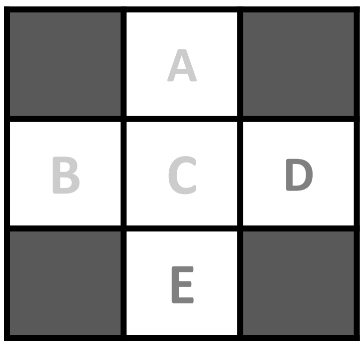

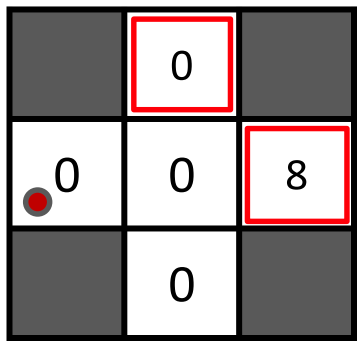

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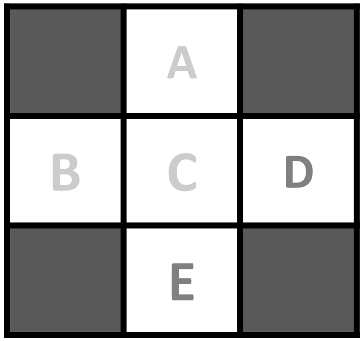

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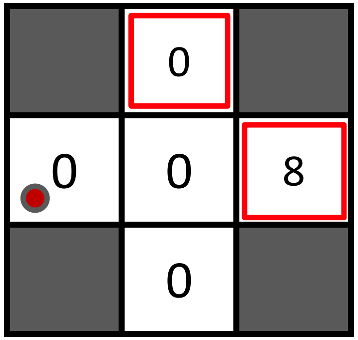

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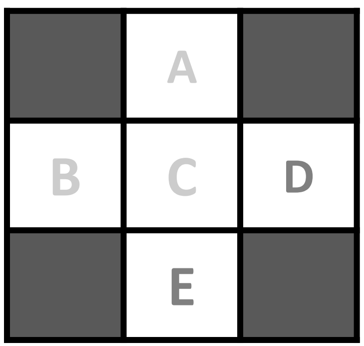

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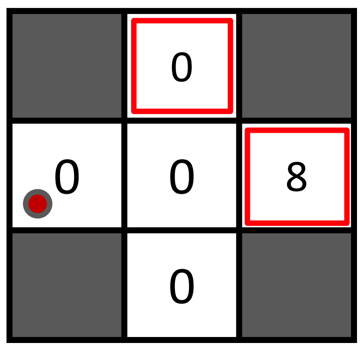
Observed Transitions

B, east, C, -2



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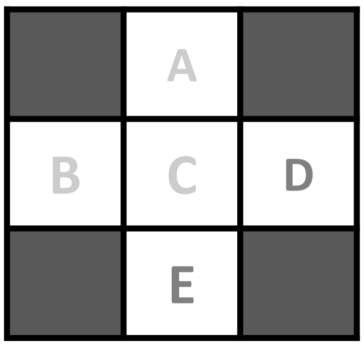

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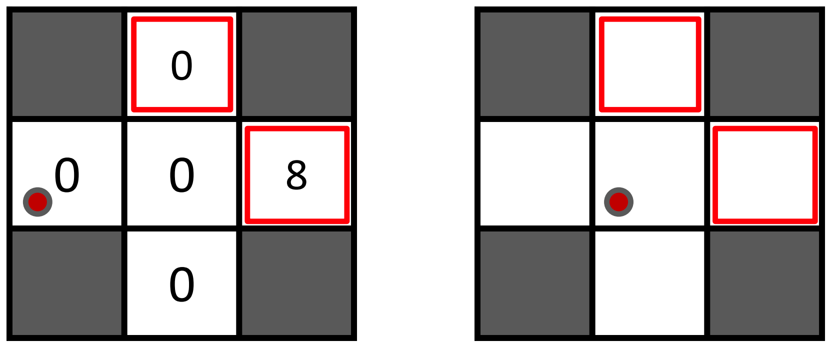

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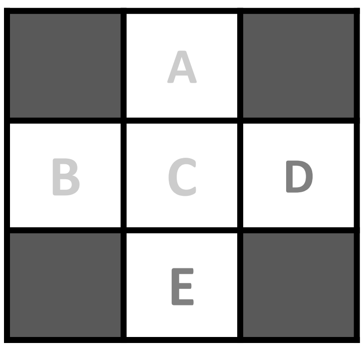

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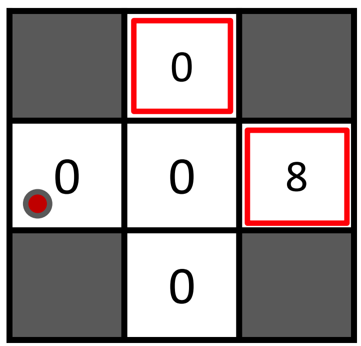

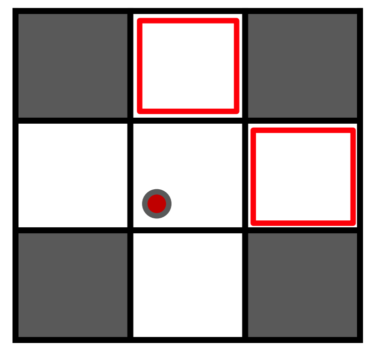

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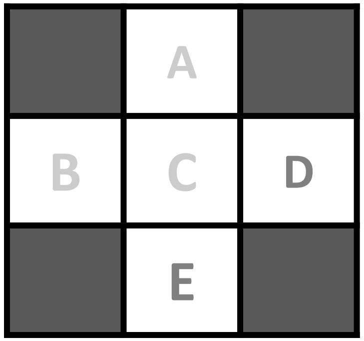

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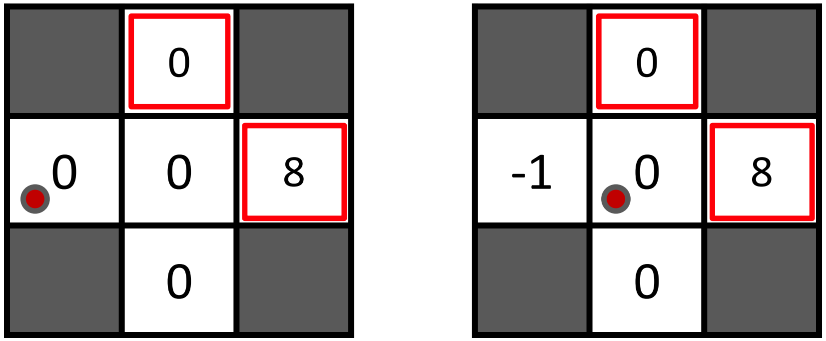

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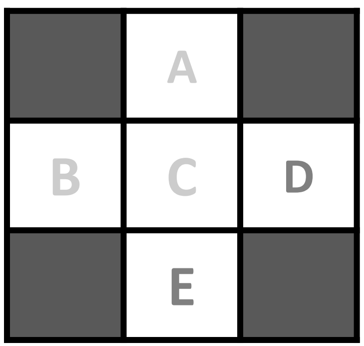

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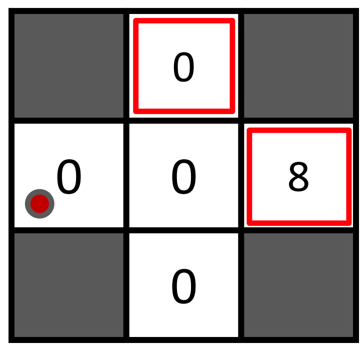
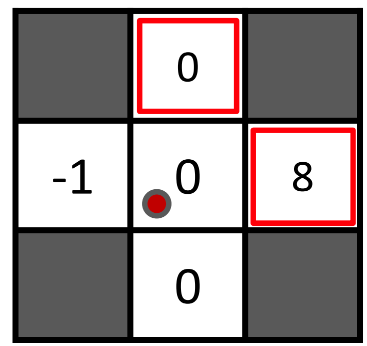
C, east, D, -2





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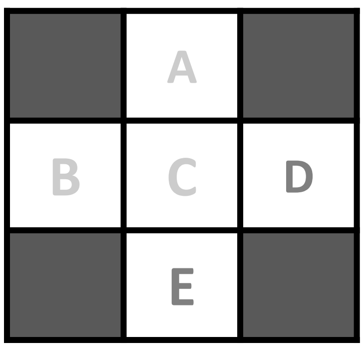

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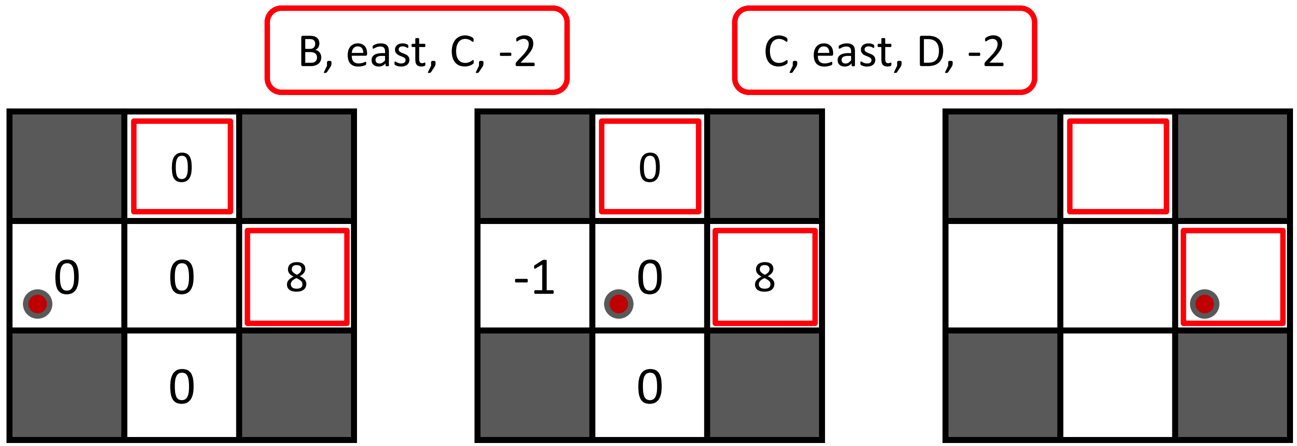

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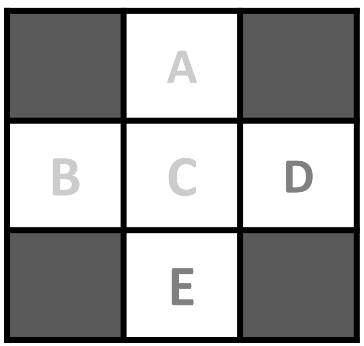

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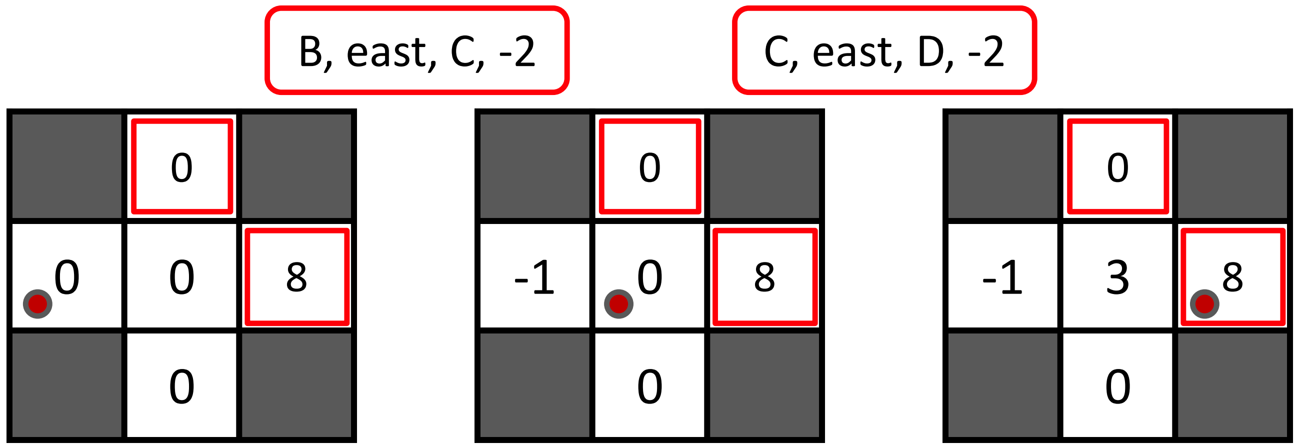

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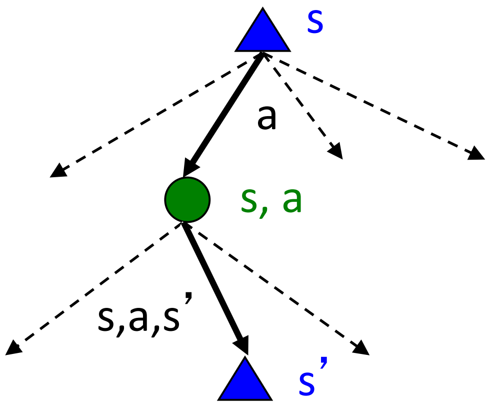
Problems with TD Value Learning

- TD value leaning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

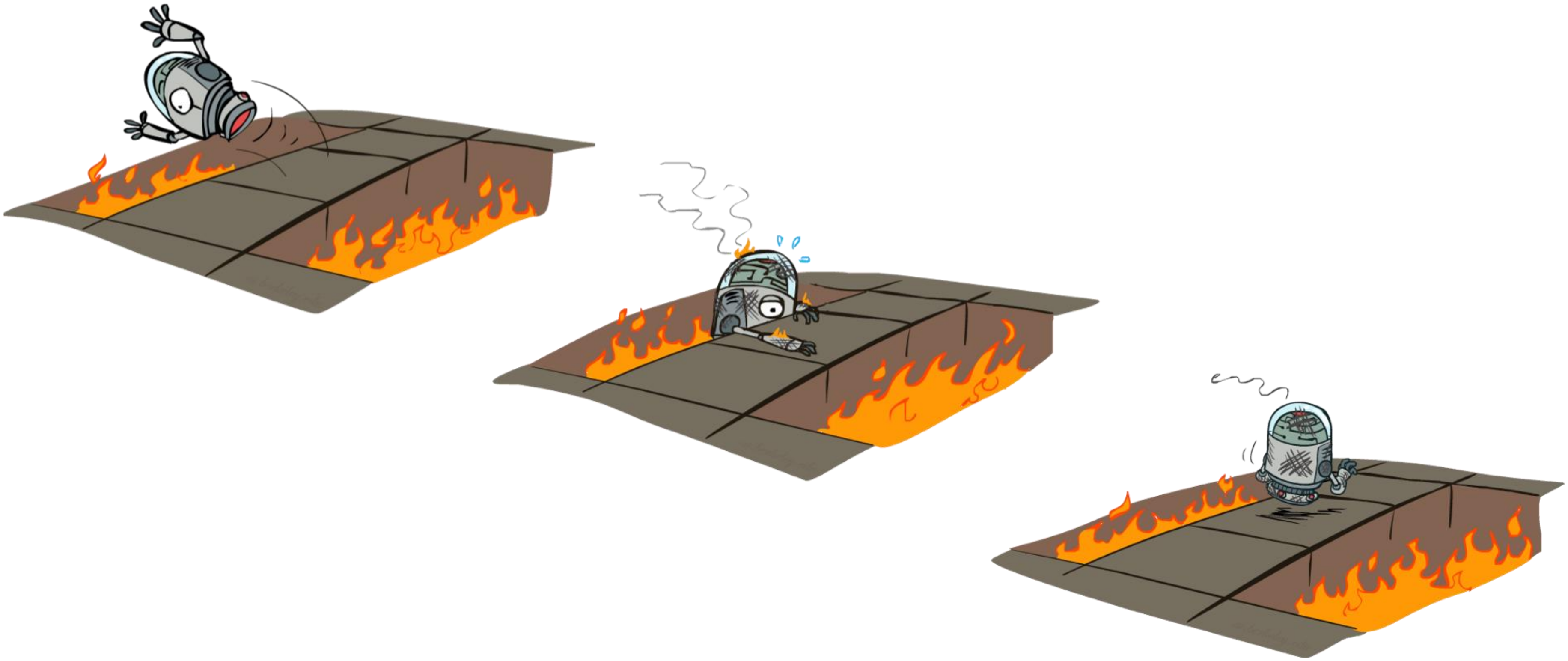
$$\pi(s) = \arg\max_a Q(s,a)$$

$$Q(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V(s') \right]$$



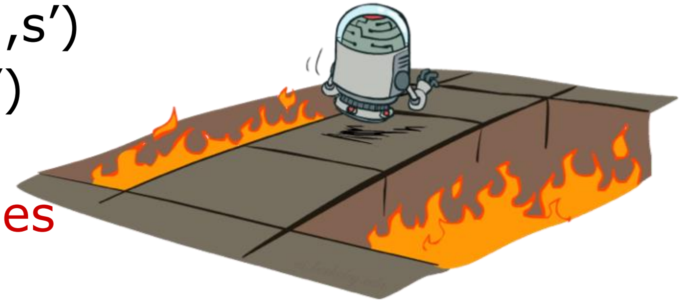
- Idea: learn Q-values, not values
- Makes action selection model-free too!

Active Reinforcement Learning



Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - You choose the actions now
  - Goal: learn the optimal policy / values



- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens…

Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values
  - Start with $V_0(s) = 0$, which we know is right
  - Given V_k, calculate the depth k+1 values for all states:

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Q-Learning

- Q-Learning: sample-based Q-value iteration

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[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

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- Learn Q(s,a) values as you go

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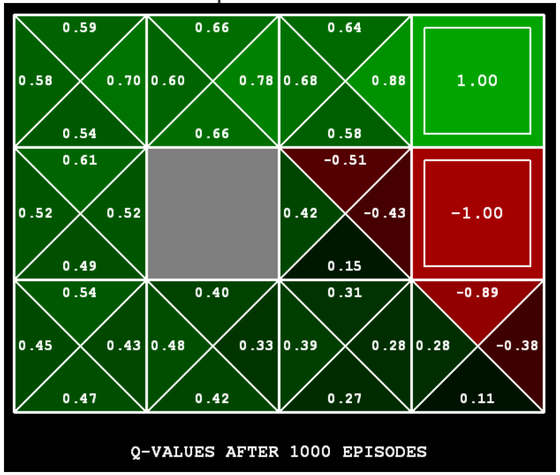

[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

- Learn Q(s,a) values as you go
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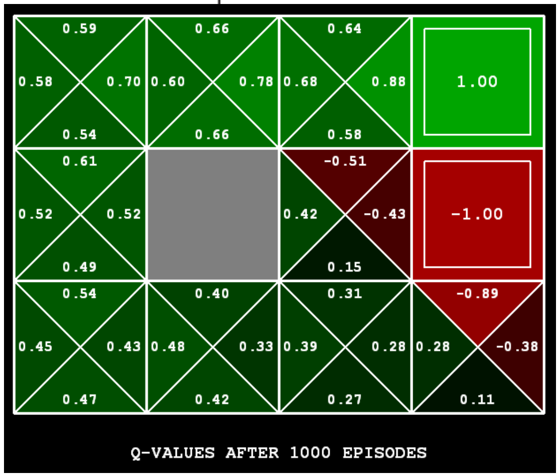

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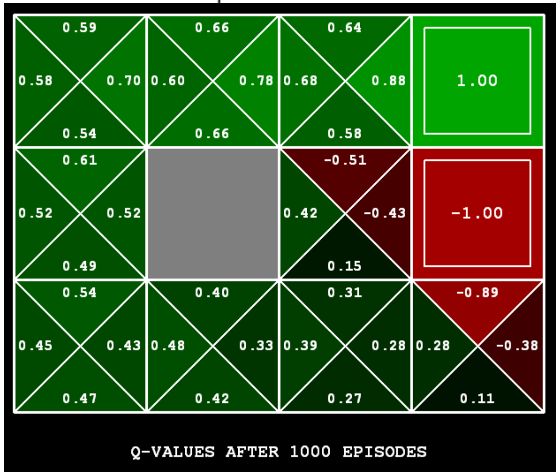
  - Consider your old estimate: $Q(s,a)$



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- Learn Q(s,a) values as you go
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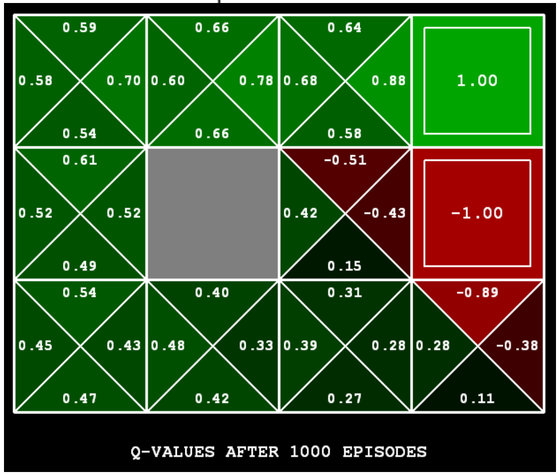
  - Consider your new sample estimate:

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$



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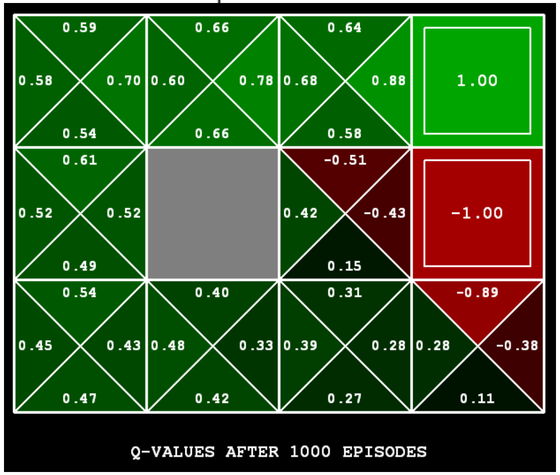

  - Incorporate the new estimate into a running average:

[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

Q-Learning

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  - Receive a sample (s,a,s',r)
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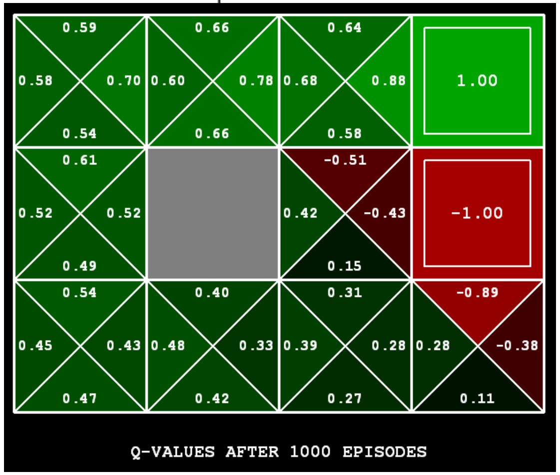
  - Consider your new sample estimate:

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

  - Incorporate the new estimate into a running average:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\,[sample]$$



[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!

- This is called off-policy learning

- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - … but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions (!)