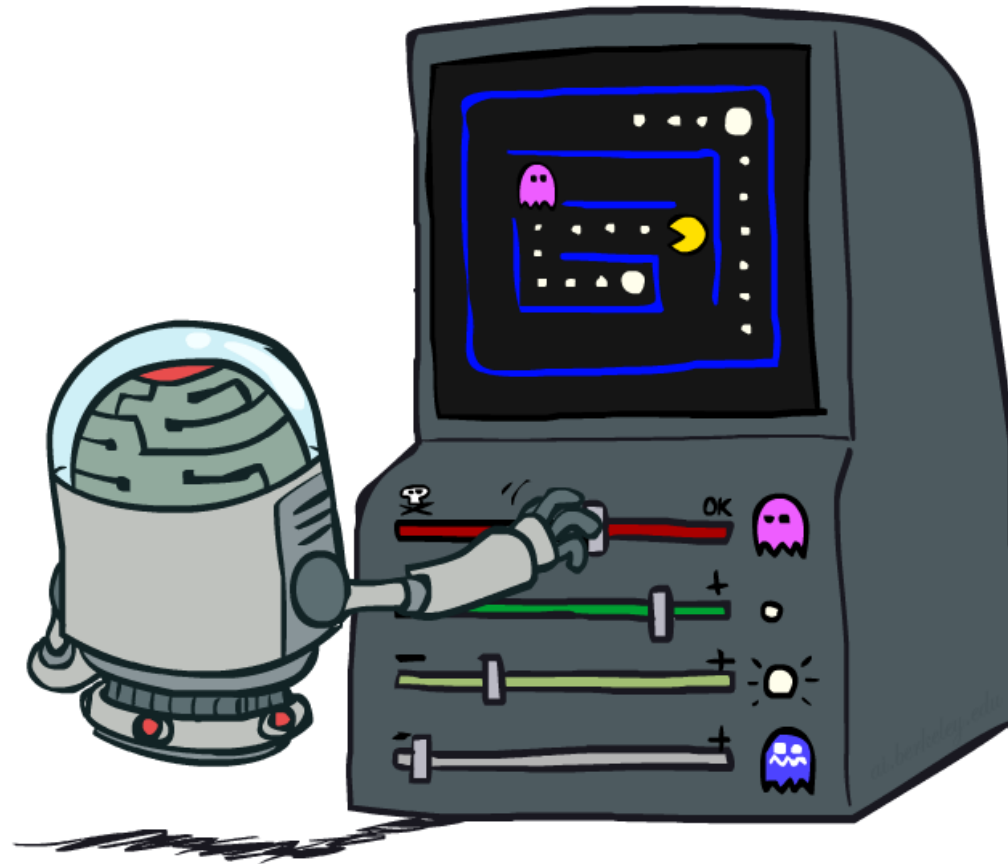


Artificial Intelligence

Reinforcement Learning II



Reinforcement Learning

- We still assume an MDP:
 - A set of states $s \in S$
 - A set of actions (per state) A
 - A model $T(s,a,s')$
 - A reward function $R(s,a,s')$
- Still looking for a policy $\pi(s)$
- New twist: don't know T or R , so must try out actions
- Big idea: Compute all averages over T using sample outcomes



The Story So Far: MDPs and RL

Known MDP: Offline Solution

Goal

Compute V^* , Q^* , π^*

Evaluate a fixed policy π

Technique

Value / policy iteration

Policy evaluation

Unknown MDP: Model-Based

Goal

Compute V^* , Q^* , π^*

Evaluate a fixed policy π

Technique

VI/PI on approx. MDP

PE on approx. MDP

Unknown MDP: Model-Free

Goal

Compute V^* , Q^* , π^*

Evaluate a fixed policy π

Technique

Q-learning

Value Learning

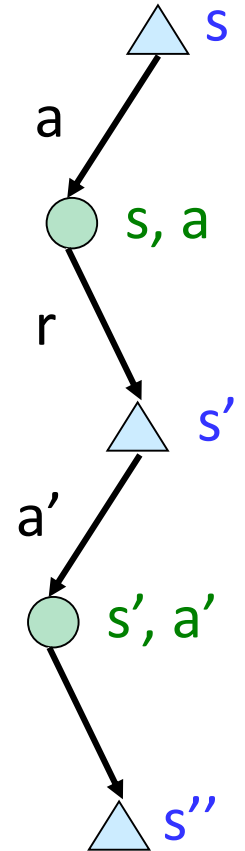
Model-Free Learning

- Model-free (temporal difference) learning

- Experience world through episodes

$$(s, a, r, s', a', r', s'', a'', r'', s'''' \dots)$$

- Update estimates each transition (s, a, r, s')
- Over time, updates will mimic Bellman updates



Q-Learning

- We'd like to do Q-value updates to each Q-state:

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

- But can't compute this update without knowing T, R

- Instead, compute average as we go

- Receive a sample transition (s,a,r,s')
- This sample suggests

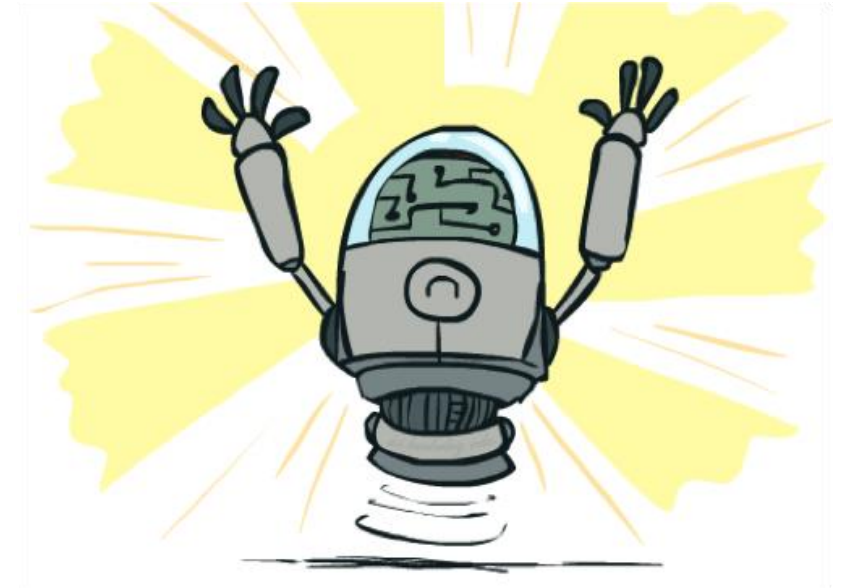
$$Q(s, a) \approx r + \gamma \max_{a'} Q(s', a')$$

- But we want to average over results from (s,a) (Why?)
- So keep a running average

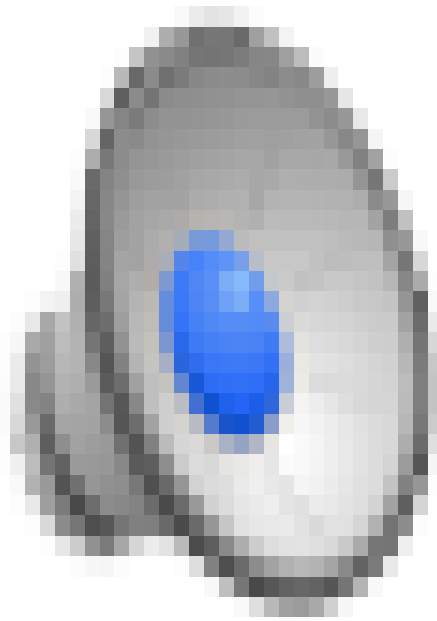
$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) \left[r + \gamma \max_{a'} Q(s', a') \right]$$

Q-Learning Properties

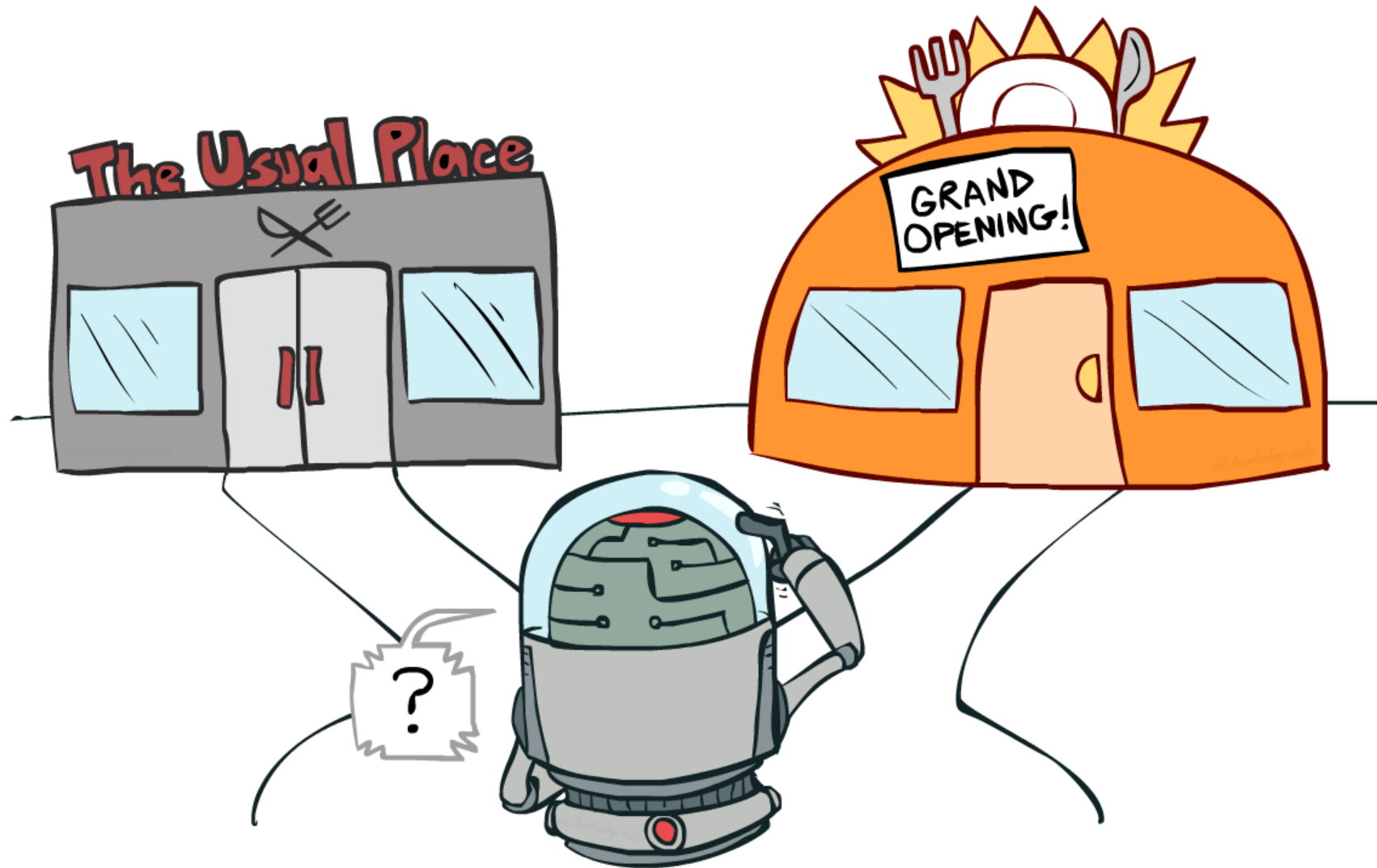
- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!
- This is called **off-policy learning**
- Caveats:
 - You have to explore enough
 - You have to eventually make the learning rate small enough
 - ... but not decrease it too quickly
 - Basically, in the limit, it doesn't matter how you select actions (!)



Video of Demo Q-Learning Auto Cliff Grid



Exploration vs. Exploitation



How to Explore?

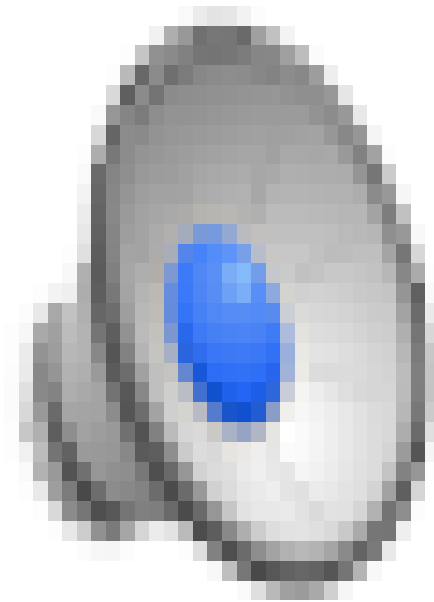
- Several schemes for forcing exploration
 - Simplest: random actions (ϵ -greedy)
 - Every time step, flip a coin
 - With (small) probability ϵ , act randomly
 - With (large) probability $1-\epsilon$, act on current policy
 - Problems with random actions?
 - You do eventually explore the space, but keep thrashing around once learning is done
 - One solution: lower ϵ over time
 - Another solution: exploration functions



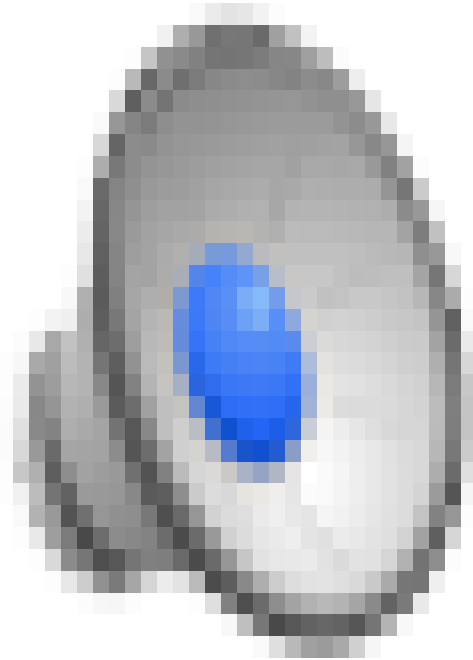
[Demo: Q-learning – manual exploration – bridge grid (L11D2)]

[Demo: Q-learning – epsilon-greedy -- crawler (L11D3)]

Video of Demo Q-learning – Manual Exploration – Bridge Grid



Video of Demo Q-learning – Epsilon-Greedy – Crawler



Exploration Functions

- When to explore?

- Random actions: explore a fixed amount
- Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

- Exploration function

- Takes a value estimate u and a visit count n , and returns an optimistic utility, e.g. $f(u, n) = u + k/n$

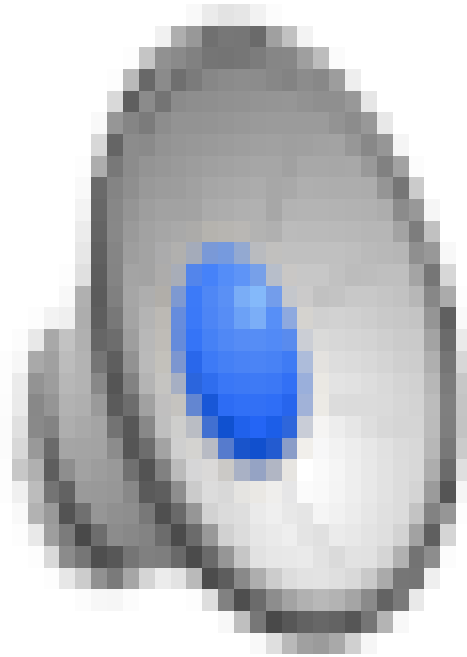
Regular Q-Update: $Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} Q(s', a')$

Modified Q-Update: $Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} f(Q(s', a'), N(s', a'))$

- Note: this propagates the “bonus” back to states that lead to unknown states as well!

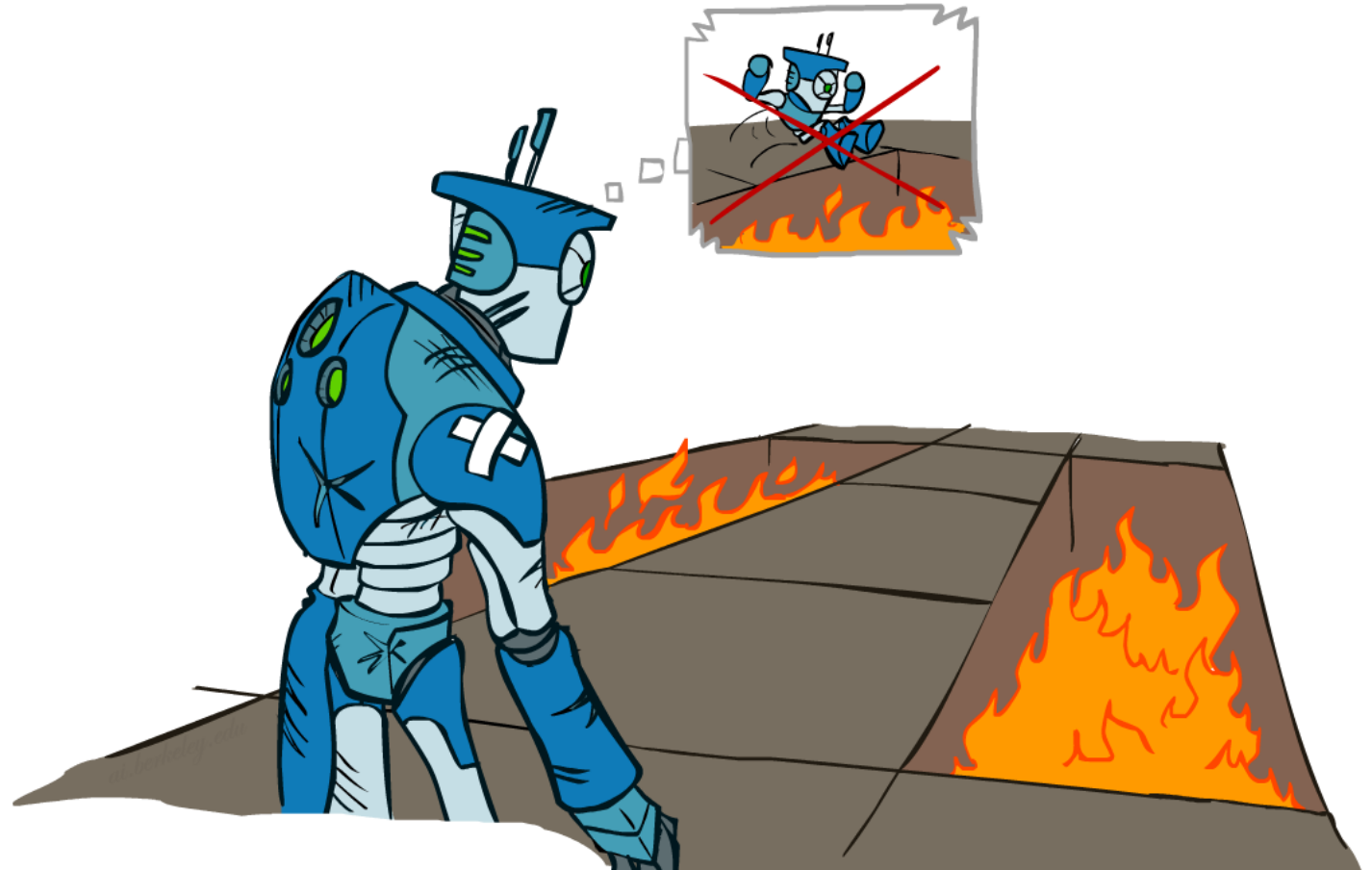


Video of Demo Q-learning – Exploration Function – Crawler

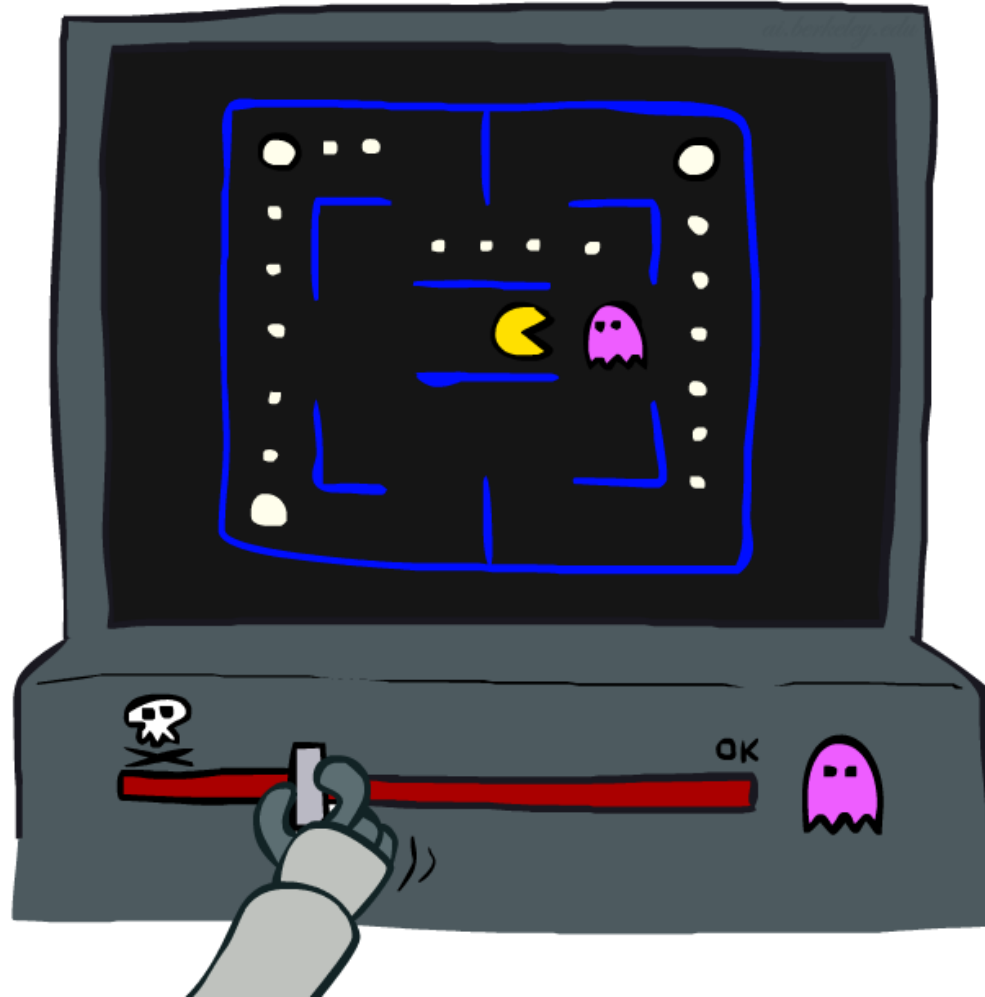


Regret

- Even if you learn the optimal policy, you still make mistakes along the way
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

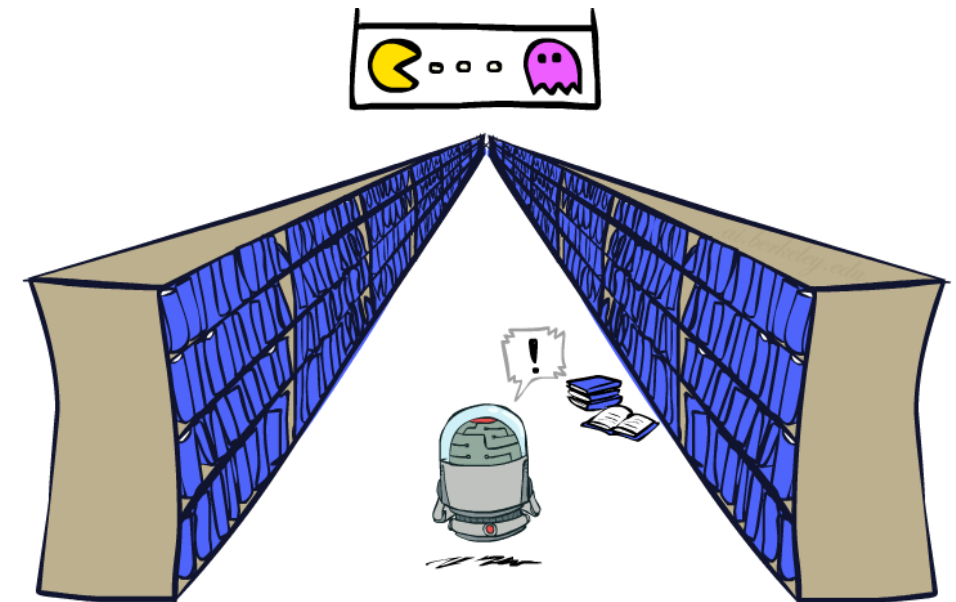
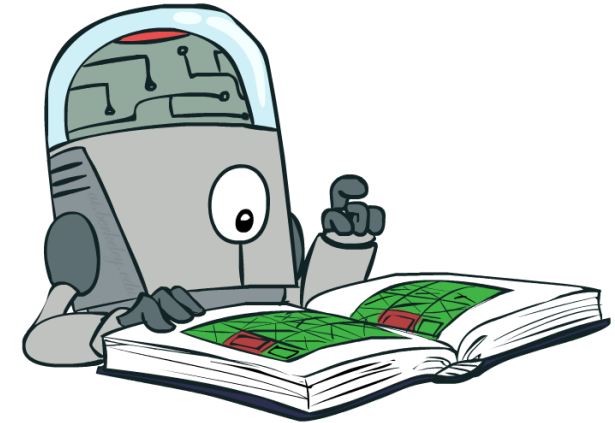


Approximate Q-Learning



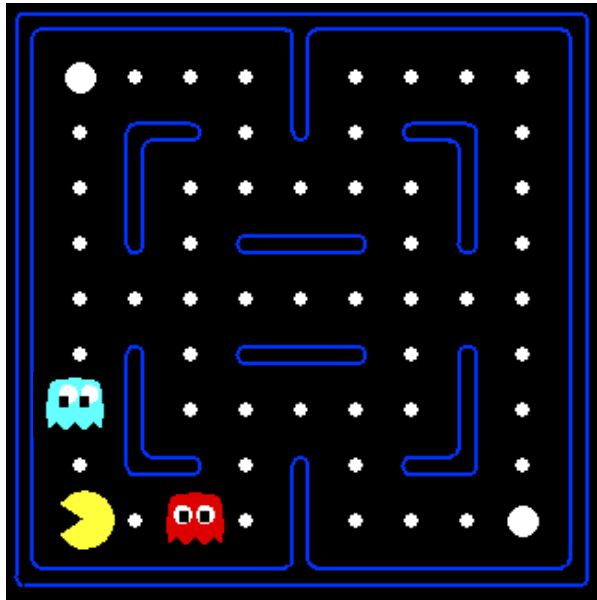
Generalizing Across States

- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
 - Too many states to visit them all in training
 - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
 - Learn about some small number of training states from experience
 - Generalize that experience to new, similar situations
 - This is a fundamental idea in machine learning, and we'll see it over and over again

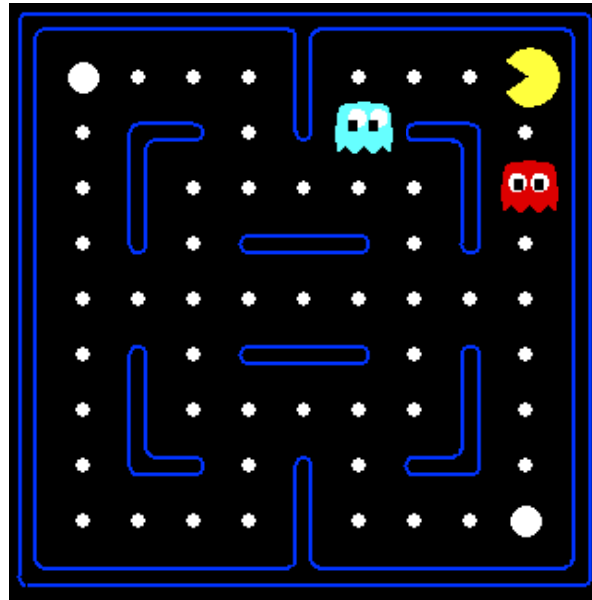


Example: Pacman

Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:

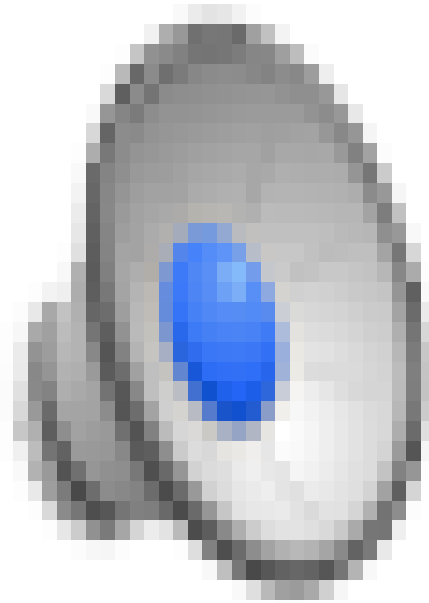


Or even this one!

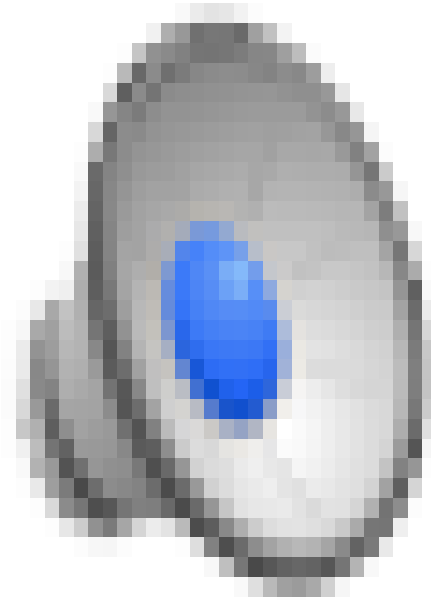


[Demo: Q-learning – pacman – tiny – watch all (L11D5)]
[Demo: Q-learning – pacman – tiny – silent train (L11D6)]
[Demo: Q-learning – pacman – tricky – watch all (L11D7)]

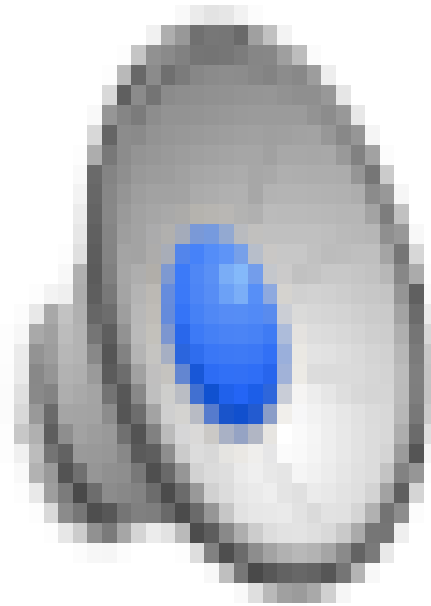
Video of Demo Q-Learning Pacman – Tiny – Watch All



Video of Demo Q-Learning Pacman – Tiny – Silent Train

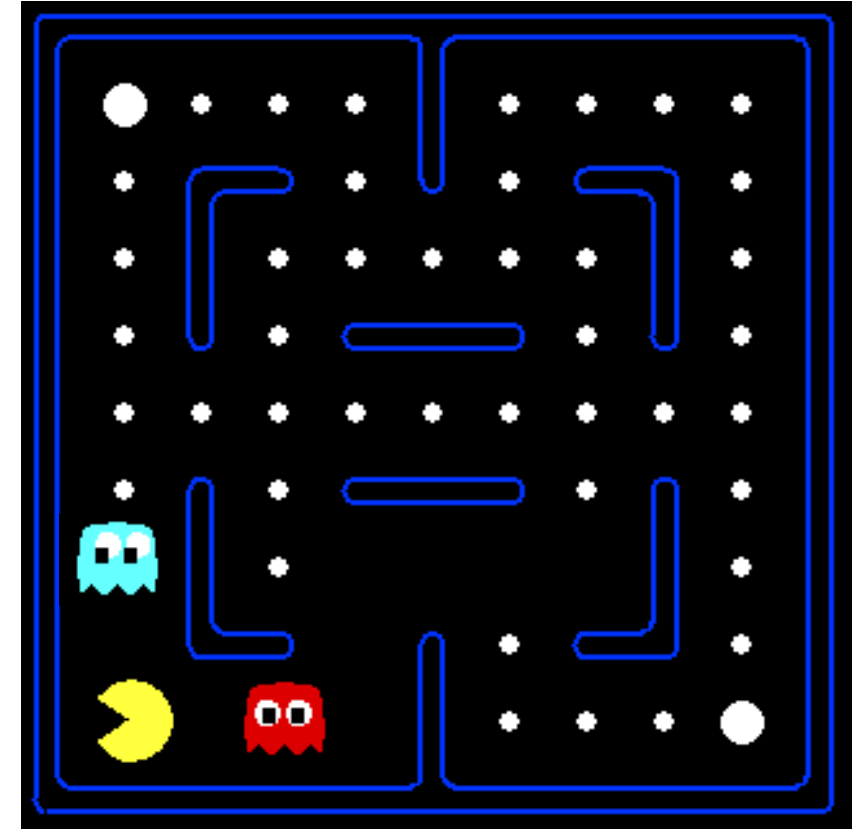


Video of Demo Q-Learning Pacman – Tricky – Watch All



Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
 - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
 - Example features:
 - Distance to closest ghost
 - Distance to closest dot
 - Number of ghosts
 - $1 / (\text{dist to dot})^2$
 - Is Pacman in a tunnel? (0/1)
 - etc.
 - Is it the exact state on this slide?
 - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Q-learning with linear Q-functions:

$$\text{transition} = (s, a, r, s')$$

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s', a') \right] - Q(s, a)$$

$$Q(s, a) \leftarrow Q(s, a) + \alpha [\text{difference}]$$

$$w_i \leftarrow w_i + \alpha [\text{difference}] f_i(s, a)$$

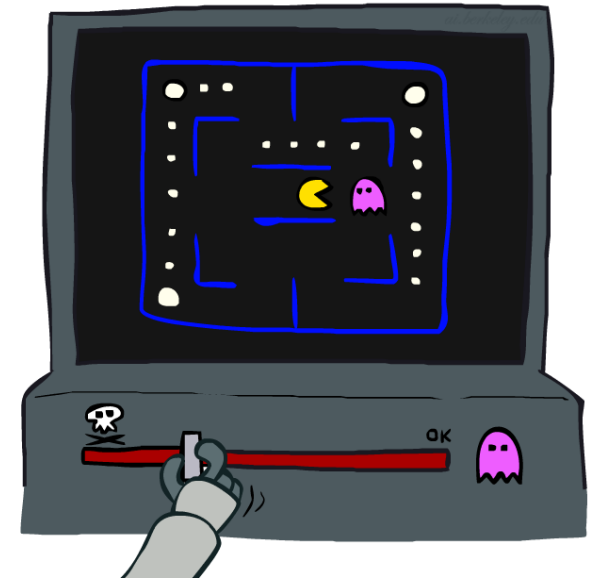
Exact Q's

Approximate Q's

- Intuitive interpretation:

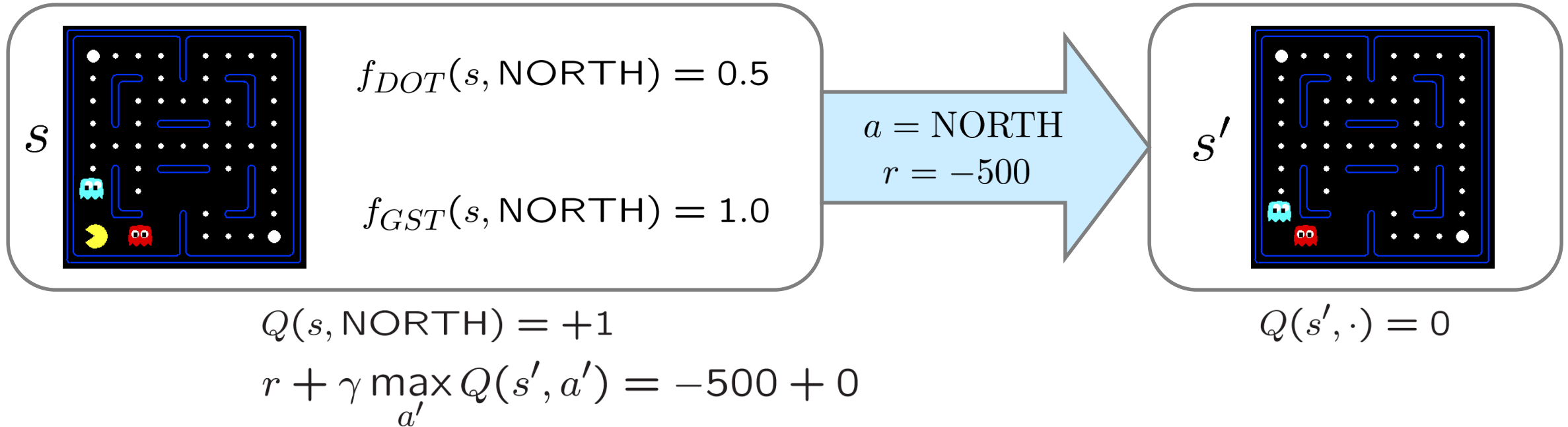
- Adjust weights of active features
- E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features

- Formal justification: online least squares



Example: Q-Pacman

$$Q(s, a) = 4.0 f_{DOT}(s, a) - 1.0 f_{GST}(s, a)$$



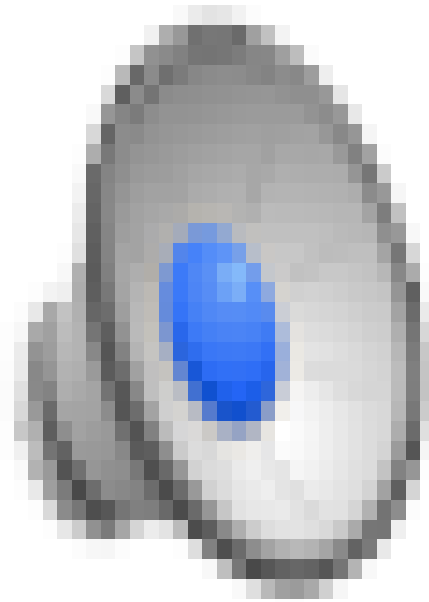
difference = -501

$$\begin{aligned} w_{DOT} &\leftarrow 4.0 + \alpha [-501] 0.5 \\ w_{GST} &\leftarrow -1.0 + \alpha [-501] 1.0 \end{aligned}$$

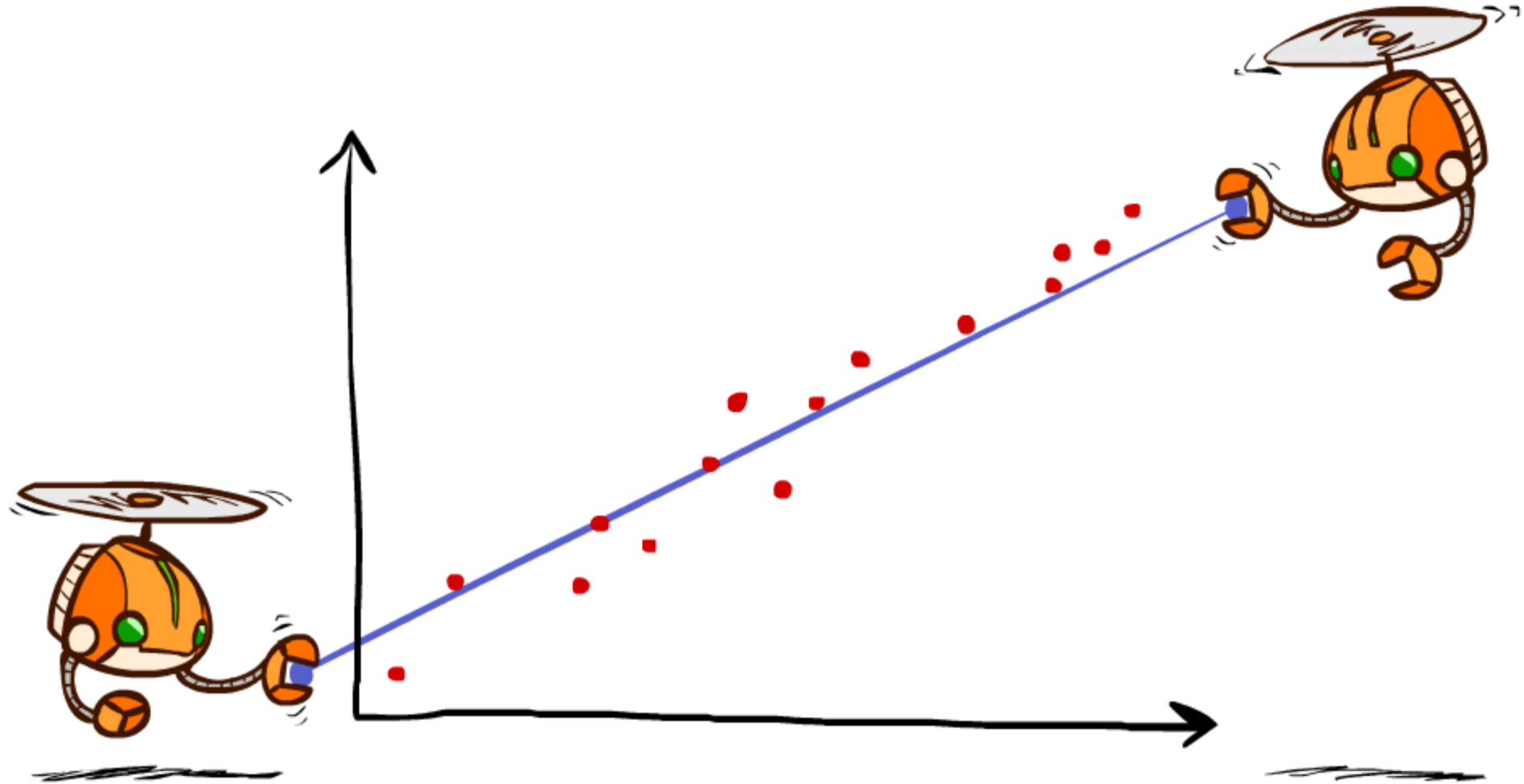
$$Q(s, a) = 3.0 f_{DOT}(s, a) - 3.0 f_{GST}(s, a)$$

[Demo: approximate Q-learning pacman (L11D10)]

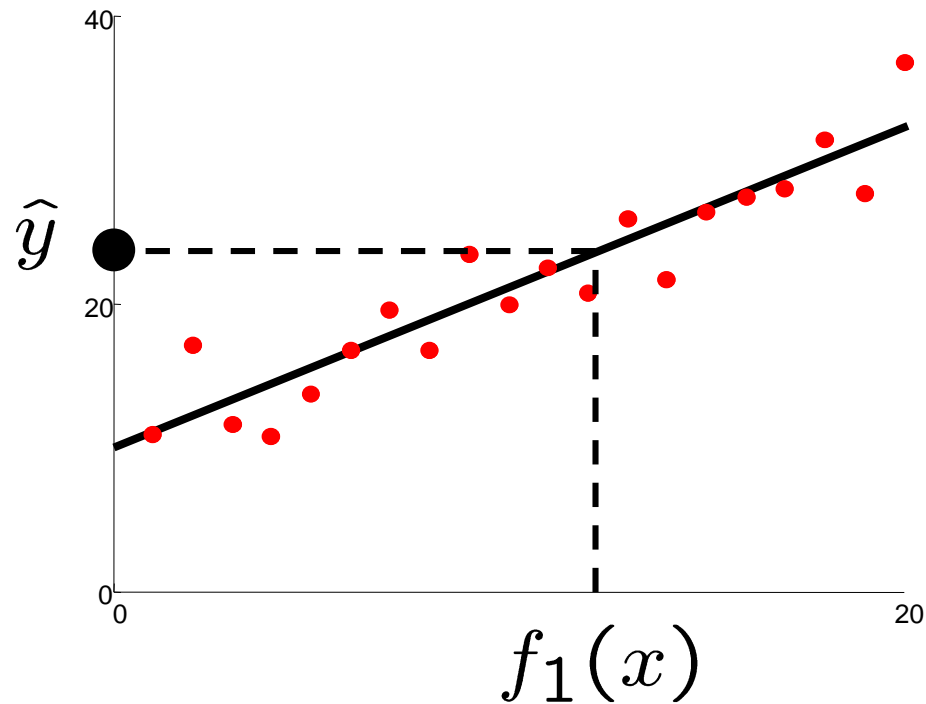
Video of Demo Approximate Q-Learning -- Pacman



Q-Learning and Least Squares

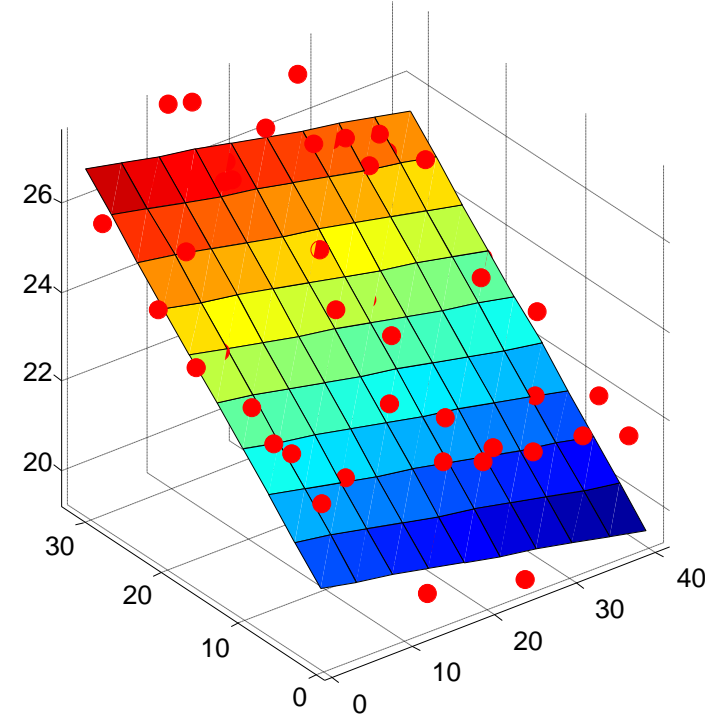


Linear Approximation: Regression*



Prediction:

$$\hat{y} = w_0 + w_1 f_1(x)$$

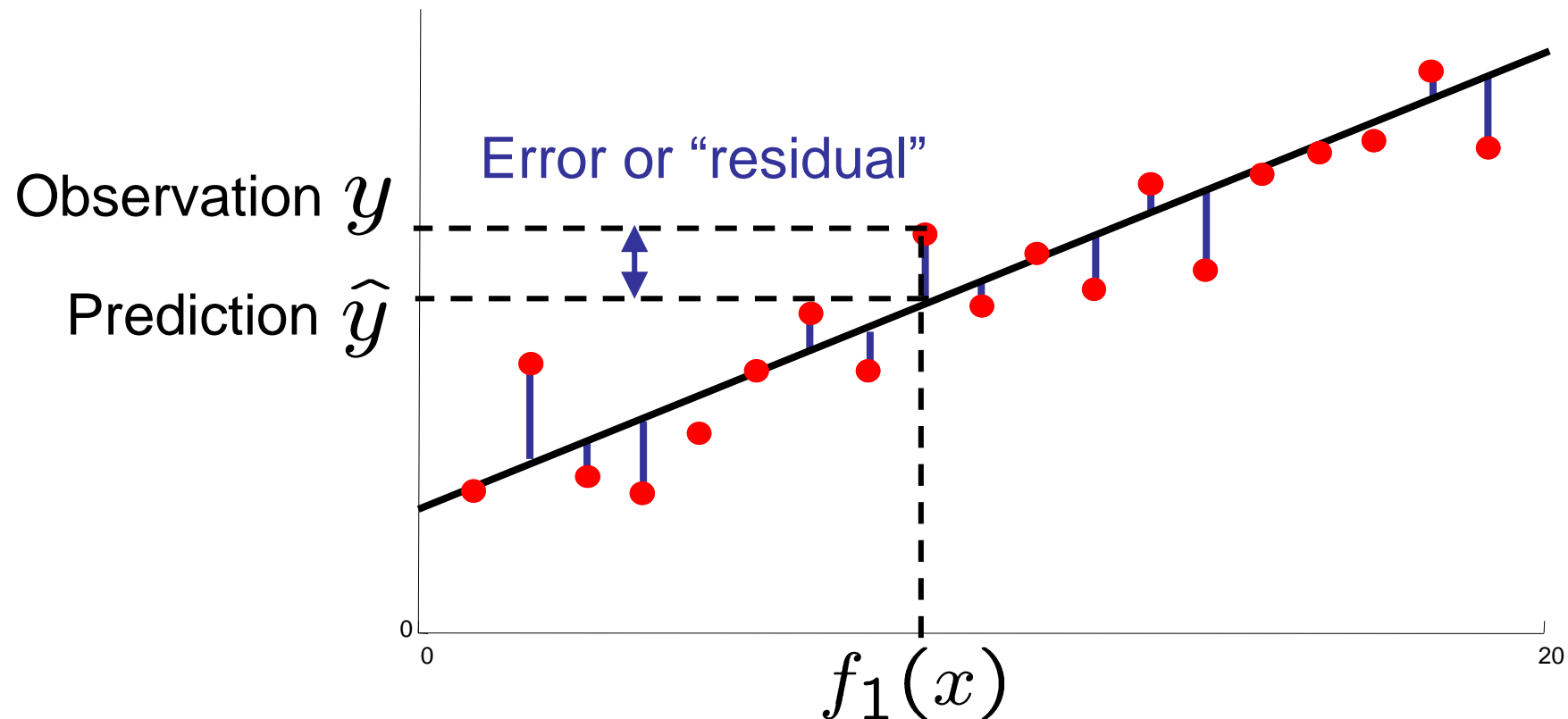


Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares*

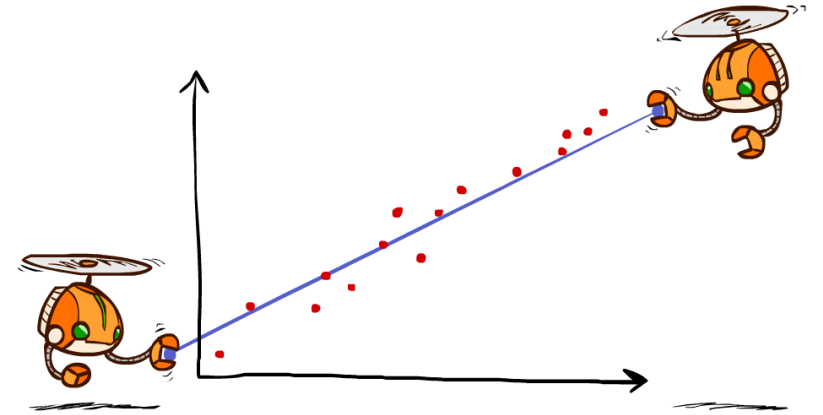
$$\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$



Minimizing Error*

Imagine we had only one point x , with features $f(x)$, target value y , and weights w :

$$\begin{aligned}\text{error}(w) &= \frac{1}{2} \left(y - \sum_k w_k f_k(x) \right)^2 \\ \frac{\partial \text{error}(w)}{\partial w_m} &= - \left(y - \sum_k w_k f_k(x) \right) f_m(x) \\ w_m &\leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x) \right) f_m(x)\end{aligned}$$



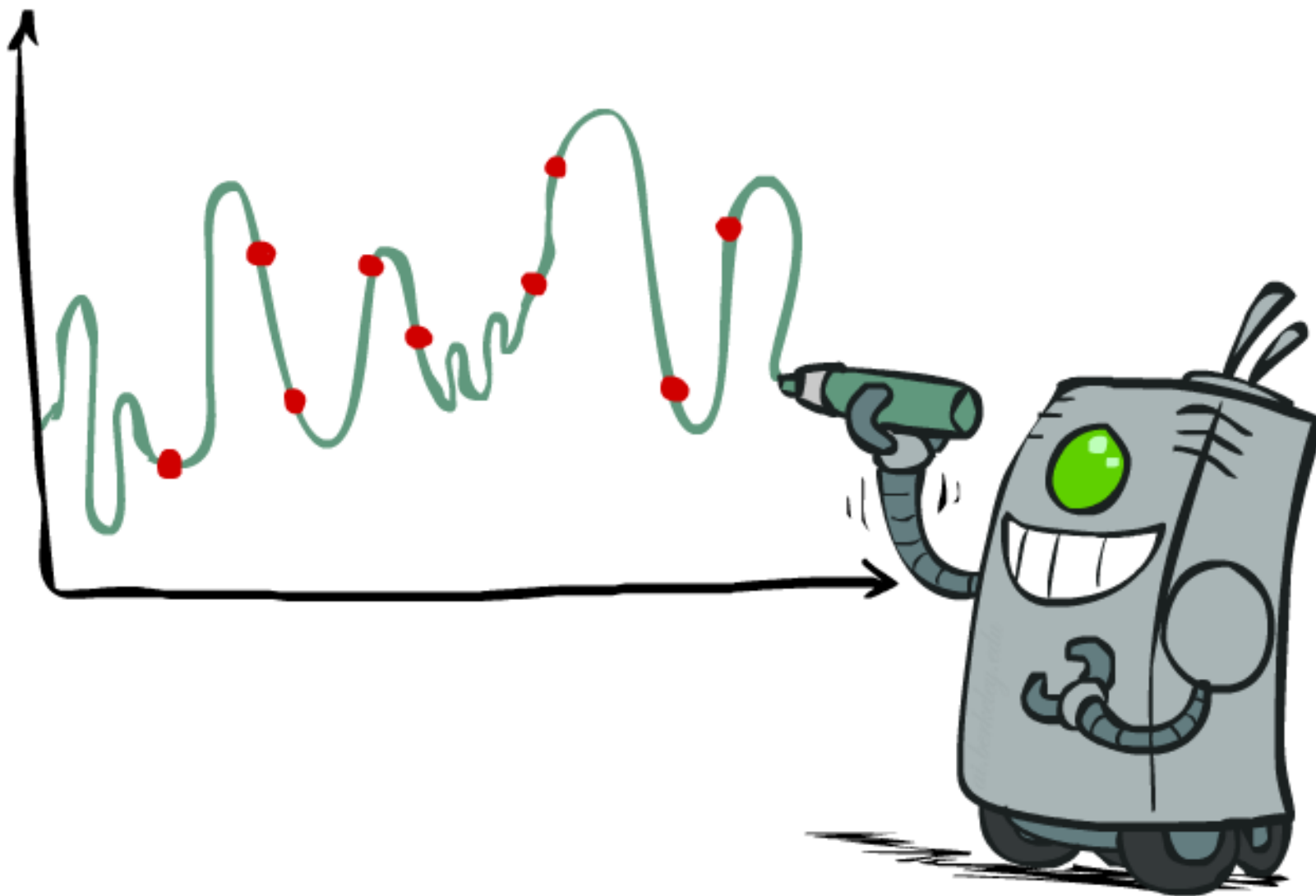
Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[\underbrace{r + \gamma \max_{a'} Q(s', a')}_{\text{“target”}} - \underbrace{Q(s, a)}_{\text{“prediction”}} \right] f_m(s, a)$$

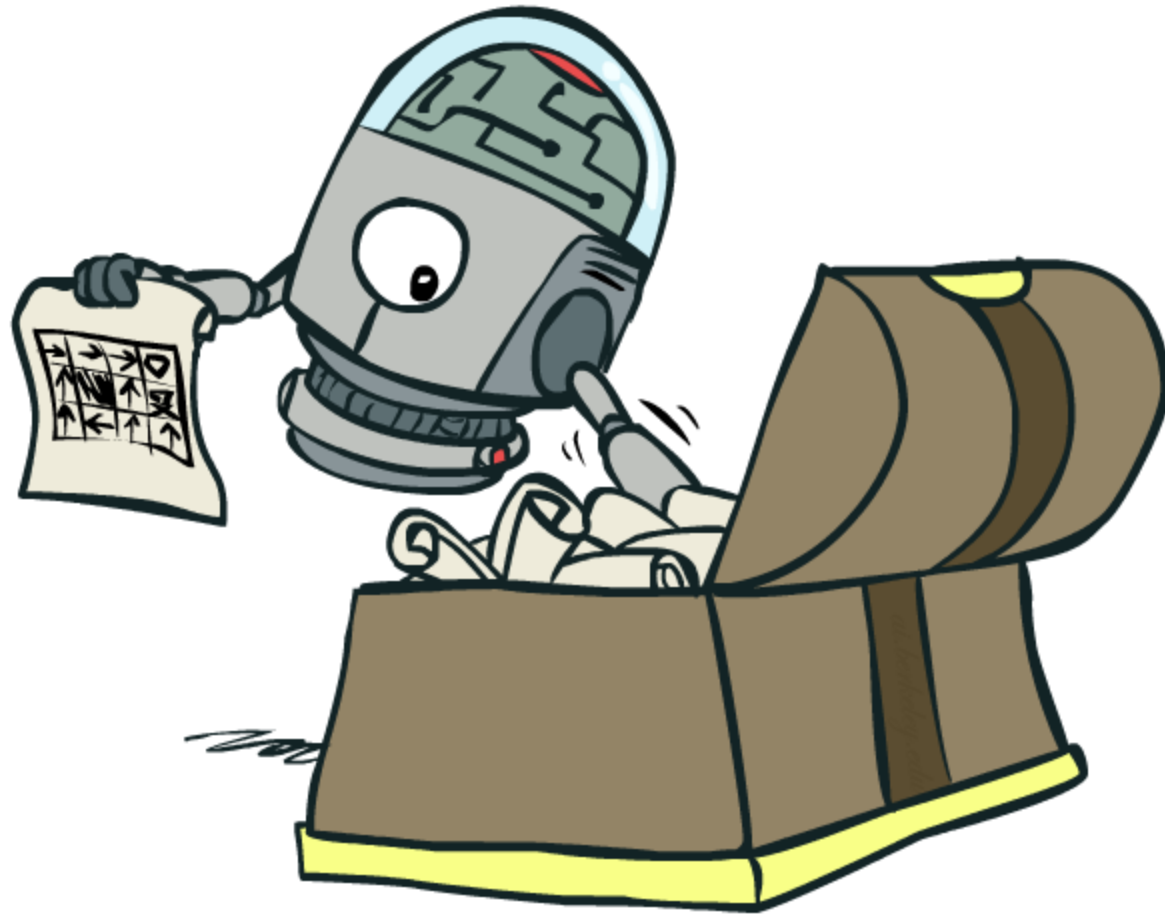
“target”

“prediction”

Overfitting: Why Limiting Capacity Can Help*



Policy Search



Policy Search

- Problem: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
 - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
 - Q-learning's priority: get Q-values close (modeling)
 - Action selection priority: get ordering of Q-values right (prediction)
 - We'll see this distinction between modeling and prediction again later in the course
- Solution: learn policies that maximize rewards, not the values that predict them
- Policy search: start with an ok solution (e.g. Q-learning) then fine-tune by hill climbing on feature weights

Policy Search

- Simplest policy search:
 - Start with an initial linear value function or Q-function
 - Nudge each feature weight up and down and see if your policy is better than before
- Problems:
 - How do we tell the policy got better?
 - Need to run many sample episodes!
 - If there are a lot of features, this can be impractical
- Better methods exploit lookahead structure, sample wisely, change multiple parameters...

Policy Search



Conclusion

- We're done with Part I: Search and Planning!
- We've seen how AI methods can solve problems in:
 - Search
 - Constraint Satisfaction Problems
 - Games
 - Markov Decision Problems
 - Reinforcement Learning
- Next up: Part II: Uncertainty and Learning!

