Lösningar HW 13

Question 1

Induktion 1: Bevisa följande:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \tag{1}$$

$$\sum_{j=1}^{n} (2j-1) = n^2 \tag{2}$$

Solution: (1)
Basfall $n = 1$:

$$\text{V.L: } \sum_{i=1}^{1} i^2 = 1; \quad \text{H.L: } \frac{1(2)(3)}{6} = 1 \quad \text{(OK!)}$$

Induktionsantagande:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Induktionssteget:

$$P_{V.L}(n+1) \implies \sum_{i=1}^{n+1} i^2 \iff \sum_{i=1}^{n} i^2 + (n+1)^2 \iff P_{V.L}(n) + (n+1)^2$$

Enligt induktionsantagandet kan vi skriva om $P_{V.L}(n)$ till $\frac{n(n+1)(2n+1)}{6}$, s.a.:

$$P_{V.L}(n) + (n+1)^2 \iff \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$P_{H.L}(n+1) \implies \frac{(n+1)(n+2)(2n+3)}{6} \iff \frac{(n+1)(2n^2+7n+6)}{6} \iff \frac{(n+1)(2n^2+n+6(n+1))}{6}$$

$$P_{H.L}(n+1) \implies \frac{(n+1)(2n^2+n+6(n+1))}{6} \iff \frac{n(n+1)(2n+1)+6(n+1)^2}{6} \iff \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

Det sista raden implicerar att $P_{H.L}(n+1) = P_{V.L}(n+1)$. Alltså **V.S.B**!

Solution: (2)
Basfall $n = 1$:

$$\text{V.L: } \sum_{i=j}^{1} (2j-1) = 1; \quad \text{H.L: } 1^2 = 1 \quad \text{(OK!)}$$

Induktionsantagande:

$$\sum_{j=1}^{n} (2j-1) = n^2$$

Induktionssteget:

$$P_{V.L}(n+1) \implies \sum_{j=1}^{n+1} (2j-1) \iff \sum_{j=1}^{n} (2j-1) + 2(n+1) - 1 \iff P_{V.L}(n) + 2n + 1$$

Enligt induktionsantagandet kan vi skriva om $P_{V.L}(n)$ till n^2, s.a.:

$$P_{V.L}(n) + 2n + 1 \iff n^2 + 2n + 1 \iff (n+1)^2 = P_{V.L}(n+1)$$

$$P_{H.L}(n+1) \implies (n+1)^2$$

Alltså $P_{H.L}(n+1) = P_{V.L}(n+1)$ **V.S.B**!