johalls-complexity

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1 Induktion

1.1

$$P(n) : \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Induktionssteg: Basfall $P(1)$

$$\sum_{i=1}^{1} i^2 = \frac{1(1+1)(2(1)+1)}{6} \iff 1^2 = \frac{1(2)(3)}{6} \iff 1 = 1$$

Induktionssteg, antar $P(k), k \in \mathbb{N}$

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$$

$$\iff (k+1)^2 + \sum_{i=1}^{k} i^2 = (k+1)^2 + \frac{k(k+1)(2k+1)}{6}$$

$$\iff \sum_{i=1}^{k+1} i^2 = \frac{6(k+1)^2}{6} + \frac{k(k+1)(2k+1)}{6}$$

$$\iff \sum_{i=1}^{k+1} i^2 = \frac{(k+2)(k+1)(2k+3)}{6}$$

$$\implies (P(k) \implies P(k+1))$$

Induktionsprincipen ger $P(n)$ är sann $\forall n \in \mathbb{N}$

1.2

$$P(n) : \sum_{i=1}^{n} 2i - 1 = n^2$$

Basfall $P(1)$

$$\sum_{i=1}^{1} 2i - 1 = 1^2 \iff 2(1) - 1 = 1 \iff 1 = 1$$

Induktionssteg, antar $P(k), k \in \mathbb{N}$

$$\sum_{i=1}^{k} 2i - 1 = k^2$$

$$\iff (2k + 1) + \sum_{i=1}^{k} 2i - 1 = (2k + 1) + k^2$$

$$\iff \sum_{i=1}^{k+1} 2i - 1 = (k + 1^2)$$

$$\implies (P(k) \implies P(k + 1))$$

2 Iterativ korrekthet

TC: $O(n)$ MC: $O(1)$

Börjar med $res_0 = 1$ (basfall) Sedan varje iteration $res_{i+1} = x * res_i$ (induktionssteg) $\implies res_i = x^i$ Resultatet är res_n alltså x^n

3 Rekursiv korrekthet

$T(n) = aT(n/b) + f(n)$ med $f(n) = 1, a = 2, b = 2$ Mästarsatsen ger $T(n) \in \nleq(n)$ TC: $O(n)$ MC: $O(log(n))$ (pga $log(n)$ rekursionsdjup, antar $O(1)$ minne per rekursion) Def:

$$e(n) = \begin{cases} e^n & n \leq 4 \\ e^{\lfloor n/2 \rfloor} e^{\lceil n/2 \rceil} & n > 4 \end{cases}$$

3.1 Visar $\lfloor n/2 \rfloor + \lceil n/2 \rceil = n$

3.1.1 Om $n = 2k + 1, k \in \mathbb{N}$

$$\lfloor n/2 \rfloor = \lfloor (2k + 1)/2 \rfloor = \lfloor k + \frac{1}{2} \rfloor = k$$

$$\lceil n/2 \rceil = \lceil (2k + 1)/2 \rceil = \lceil k + \frac{1}{2} \rceil = k + 1$$

$$\implies \lfloor n/2 \rfloor + \lceil n/2 \rceil = 2k + 1 = n$$

3.1.2 Om $n = 2k, k \in \mathbb{N}$

$$\lfloor n/2 \rfloor = \lfloor 2k/2 \rfloor = k$$
$$\lceil n/2 \rceil = \lceil 2k/2 \rceil = k$$
$$\implies \lfloor n/2 \rfloor + \lceil n/2 \rceil = 2k = n$$

$\implies e^{\lfloor n/2 \rfloor} e^{\lceil n/2 \rceil} = e^{\lfloor n/2 \rfloor + \lceil n/2 \rceil} = e^n$
$\implies e(n) = e^n$