Nov-04:

Z-test:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{25 - 24}{\frac{1.5}{\sqrt{36}}}$$

$$= \frac{1}{\frac{1.5}{6}}$$

$$= \frac{6}{1.5}$$

$$\boxed{z = 4}$$

$\alpha = 0.05$

In z table check score for 4 with CI/ $\alpha = 0.05$ is 0.99997.

Area Under the Curve (AUC) = 1

$P = 1 - 0.99997$

$P = \frac{0.00003}{2}$

$$\boxed{P = 0.000015}$$

Here, p value is less than α value. Hence we reject the null hypothesis.
$(0.000015 < 0.05)$.

2. In the population the avg. IQ is 100 with SD of 15. Researchers want to test a new medication to see if there is +ve or -ve effect on intelligence or no effect at all, a sample of 30 participants who have taken the medicine has a mean of 140 did the ~~median~~ medication offers the intelligence or not with a CI 95% ?

Ho: The avg. IQ is 100
H1: The avg. IQ is not 100.

$\mu = 100$, $\sigma = 15$, $n = 30$, $\bar{x} = 140$, CI = 95%, $\alpha = 0.05$

Here, we go with Z-test ~~here~~ as they have given population SD.

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{140-100}{\frac{15}{\sqrt{30}}}$$

$$= \frac{40}{\frac{15}{5.477}}$$

$$= 40 \times \frac{5.477}{15}$$

$$= 40 \times 0.3651$$

$$\boxed{Z = 14.605}.$$

For $Z = 14.605$ and $\alpha = 0.05$ check z table, it is 1.

Area under curve $= 1$

$$P = 1 - 1$$

$$\boxed{P = 0}$$

$p < \alpha$.

$\therefore$ We reject null hypothesis.

For the same scenario, $\mu = 100$, $n = 30$, $\bar{x} = 140$, $S = 20$, $\alpha = 0.05$

Here, we are given sample's SD so we proceed with t-test.

$$\boxed{t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}}}$$

$$t = \frac{140-100}{\frac{20}{\sqrt{30}}}$$

$$= \frac{40}{\frac{20}{5.47}}$$

$$= \frac{\overset{2}{\cancel{40}} \times 5.47}{\cancel{20}}$$

$$\boxed{t = 10.95}$$

If t value > t table value then we can reject H_0.

If t value < t table value then we have to accept H_0.

Degree of freedom $= n-1$

$DOF = 30-1$

$\boxed{DOF = 29}$

In t table check the value with DOF = 29 with CI = 95%

$\alpha = 0.05$ (for one tail)

$\alpha = 0.025$ (for 2 tail)

t-table value is 2.045

Here t value > t-table value (10.95 > 2.045)

Hence, we reject null hypothesis.

3. Credit Card Launch Example:

population (N) = 1,00,000

Sample (n) = 140

$\bar{x} = \$1990$

$S = \$2833$

$\sigma = \$2500$

We have to calculate the range / Interval values always the default CI is 95%

$$\boxed{\text{For calculating Intervals CI} = \bar{x} \pm Z_\alpha \frac{\sigma}{\sqrt{n}}}$$

Lower bound

$\bar{x} - Z_\alpha \frac{\sigma}{\sqrt{n}}$

$= 1990 - Z_{0.025} \frac{2500}{\sqrt{140}}$

$= 1990 - Z_{0.025} (211.28)$

Upper bound

$\bar{x} + Z_\alpha \frac{\sigma}{\sqrt{n}}$

$= 1990 + Z_{0.025} \frac{2500}{\sqrt{140}}$

$= 1990 + Z_{0.025} (211.28)$

$= 1990 - (1.96)(211.28)$

$= 1990 - 414.10$

$= 1575.9$

$LB = 1575.9$

$\boxed{LB = 1576}$

$= 1990 + (1.96)(211.28)$

$= 1990 + 414.10$

$= 2404.1$

$UB = 2404.1$

$\boxed{UB = 2404}$

The avg. balance they are going to maintain on full fledged launch of credit card is [1576, 2404]

4. On a quant test of CAT exam of a sample of 25 test takers has a sample mean of 520 with sample S.D of 80. Construct 95% CI about the Mean?

Sample $(n) = 25$

$\bar{x} = 520$

$s = 80$

We need to calculate range.

$$\boxed{\bar{x} \pm t_\alpha \frac{s}{\sqrt{n}}}$$

Lower bound

$\bar{x} - t_\alpha \frac{s}{\sqrt{n}}$

$= 520 - t_{0.025} \frac{80}{\sqrt{25}}$

$= 520 - t_{0.025}(16)$

for $t_{0.025}$ dof $= n-1 = 24$

$= 520 - (2.064)(16)$

$= 520 - 33.024$

$= 486.976$

$\boxed{LB = 487.}$

Upper Bound

$\bar{x} + t_\alpha \frac{s}{\sqrt{n}}$

$= 520 + t_{0.025} \frac{80}{\sqrt{25}}$

$= 520 + t_{0.025}(16)$

for $t_{0.025}$ dof $= n-1 = 24$

$= 520 + (2.064)(16)$

$= 520 + 33.024$

$= 553.024$

$\boxed{UB = 553}$

The mean of the ~~poputies~~ papulation lies in the interval [487 553]