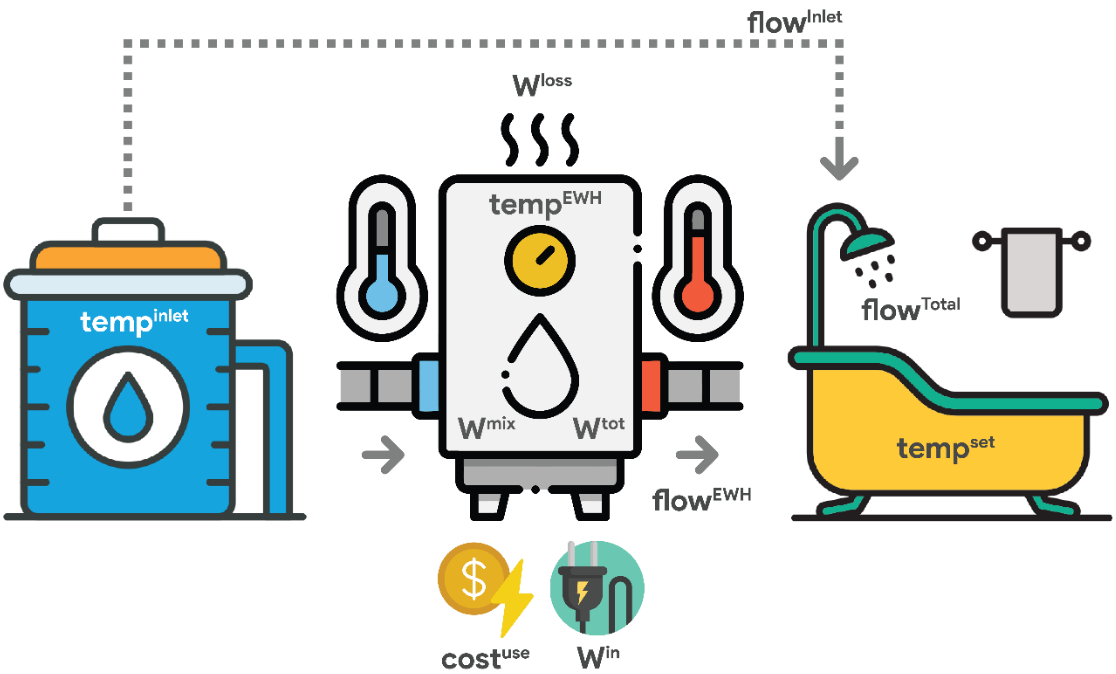
Indices and sets		
$t \in T$	Set of time intervals/slots over the operation period	-
Input parameters		
Δt	Length of time intervals	h
$\widehat{EWH}^{power}$	Heating power of the EWH	kW
$\widehat{EWH}^{height}$	Height of the EWH	cm
$\widehat{EWH}^{cap}$	Internal water capacity of the EWH	l
$\widehat{EWH}^{\max}$	Maximum allowable water temperature of the EWH	°C
$\widehat{EWH}^{min}$	Minimum allowable water temperature of the EWH	°C
$\hat{c}$	Specific heat capacity of water (4.18 J/g°C)	J/g°C
$\widehat{HC}$	Overall heat transfer coefficient of the EWH	kW/(m²K)
$\widehat{temp}^{set}$	User-defined comfort temperature for hot water usage	°C
$\widehat{temp}^{amb}$	EWH room ambient temperature	°C
$\widehat{M}$	Sufficiently big number (e.g., 1E3 if EWH's nominal capacity is in the tens of kWh)	-
$\widehat{flowrate}$	Water usage flow rate (fixed/constant value).	kg/min
$\widehat{tariff}$	Selection between energy price tariffs between simple (1) or bi-hourly (2), etc.	-
$\hat{\delta}_t^{use}$	Boolean parameter indicating periods of hot water usage (1 = using)	-
Variable inputs		
$\widehat{EWH}^{StartTemp}$	EWH internal water temperature at the beginning	°C
$\widehat{EWH}^{area}$	EWH Surface Area	m²
W^{init}	Total energy balance of the EWH at the beginning	kWh
W^{max}	Maximum energy balance of the EWH	kWh
W^{min}	Minimum energy balance of the EWH	kWh
$m_{\delta use}^{belowSet}$	Auxiliary m linearization parameter from *belowSet* regressor for usage	-
$m_{temp^{EWH}}^{belowSet}$	Auxiliary b linearization parameter from *belowSet* regressor for temperature	-
$b^{belowSet}$	Auxiliary b linearization parameter from *belowSet* regressor	
$m_{\delta use}^{aboveSet}$	Auxiliary m linearization parameter from *aboveSet* regressor for usage	-
$m_{temp^{EWH}}^{aboveSet}$	Auxiliary m linearization parameter from *aboveSet* regressor for temperature	
$b^{aboveSet}$	Auxiliary b linearization parameter from *aboveSet* regressor	-
Variables		
$temp_t^{EWH}$	Temperature of water at EWH outlet at the beginning of time interval t.	°C
W_t^{tot}	Total energy balance of prosumer's EWH at time interval t.	kWh
W_t^{in}	Energy into the prosumer's EWH at time interval t	kWh
W_t^{loss}	Thermal energy losses at time interval t	kWh
W_t^{mix}	Thermal energy stored in the EWH after usage and mixing with inlet water at time interval t	kWh
δ_t^{in}	Variable for EWH operation status, [0,1], (1 = ON, 0 = OFF, for the full period)	-
δ_t^{aux}	Boolean variable for if-else in constraint (7) at time interval t	-
$penalty_t^{comf}$	Penalty cost associated with water temperature reaching below comfort at time interval t	-
$cost_t^{use}$	Total period cost of that specific energy usage at time interval t	€
$price_t^{net}$	Network pricing of energy usage at time interval t	€
$tariff^m$	Energy daily tariff for each m price sets	€

Objective Function

$$min \sum_{t}^{T} cost_t^{use} \cdot a_1 + penalty_t^{comf} \cdot a_2$$

The objective of this problem will be to minimize the operating costs of using hot water, but always wanting to ensure that the comfort threshold is respected. However, as described in constraint (6), a penalty is added that allows the operation, even if this limit is not respected. In this sense, this parameter with a positive sign is adding an additional "cost", due to the use of water below comfort. The values of parameters a_1 and a_2 must be adjusted, according to the importance that the user may give to comfort or price (default as $a_1 = 100, a_2 = 1000$).

Graphical Visualization



$$\begin{cases} W_t^{tot} = W^{init}, & t = 0 \\ W_t^{tot} = W_{t-1}^{water} + W_{t-1}^{in} + W_{t-1}^{loss}, & t > 0 \end{cases} \quad (1)$$

$$W_t^{in} = \widehat{EWH}^{power} \cdot \Delta t \cdot \delta_t^{in} \cdot (\Delta t \cdot 60) \qquad \forall\, t \in T \quad (2)$$

$$cost_t^{use} = \delta_t^{in} \cdot \widehat{EWH}^{power} \cdot \Delta t \cdot price_t^{use} + tariff^m \cdot \frac{\Delta t}{24} \qquad \forall\, t \in T \quad (3)$$

$$\begin{cases} temp_t^{EWH} = \widehat{EWH}^{StartTemp}, & t = 0 \\ temp_t^{EWH} = \dfrac{W_t^{tot} \cdot 3600}{\widehat{EWH}^{cap} \cdot \hat{c}}, & t > 0 \end{cases} \quad (4)$$

$$W_t^{loss} = \widehat{EWH}^{height} \cdot \widehat{EWH}^{area} \cdot \Delta t \cdot \left(temp_t^{EWH} - \widehat{temp}^{amb}\right) \cdot (\Delta t \cdot 60) \qquad \forall\, t \in T \quad (5)$$

$$\begin{cases} W_t^{min} \le W_t^{tot} \\ W_t^{tot} \le W_t^{max} \\ \widehat{EWH}^{min} \le temp_t^{EWH} \\ temp_t^{EWH} \le \widehat{EWH}^{\max} \end{cases} \qquad \forall\, t \in T \quad (6)$$

$$\begin{cases} W_t^{tot} \ge \widehat{temp}^{set} \cdot \widehat{EWH}^{cap} \cdot \dfrac{\hat{c}}{3600} \cdot (\Delta t \cdot 60) - penalty_t^{comf} \\ W_{t-1}^{tot} \ge \widehat{temp}^{set} \cdot \widehat{EWH}^{cap} \cdot \dfrac{\hat{c}}{3600} \cdot (\Delta t \cdot 60) - penalty_{t-1}^{comf} \end{cases} \qquad \hat{\delta}_t^{use} = 1\,,\ \ or: \ \begin{cases} \hat{\delta}_t^{use} - \hat{\delta}_{t-1}^{use} \ne 0 \\ \hat{\delta}_t^{use} - \hat{\delta}_{t-1}^{use} = -\hat{\delta}_{t-1}^{use} \end{cases} \quad (7)$$

$$\begin{cases} temp_t^{EWH} \ge \widehat{temp}^{set} - \widehat{M} \cdot (1 - \delta_t^{aux}), & \hat{\delta}_t^{use} > 0 \\ temp_t^{EWH} \le \widehat{temp}^{set} + \widehat{M} \cdot \delta_t^{aux}, & \hat{\delta}_t^{use} > 0 \\ W_t^{mix} \ge \left(m_{\delta^{use}}^{belowSet} \cdot \hat{\delta}_t^{use} + m_{temp^{EWH}}^{belowSet} \cdot temp_t^{EWH} + b^{belowSet}\right) - \widehat{M} \cdot \delta_t^{aux}, & \hat{\delta}_t^{use} > 0 \\ W_t^{mix} \le \left(m_{\delta^{use}}^{belowSet} \cdot \hat{\delta}_t^{use} + m_{temp^{EWH}}^{belowSet} \cdot temp_t^{EWH} + b^{belowSet}\right) + \widehat{M} \cdot \delta_t^{aux}, & \hat{\delta}_t^{use} > 0 \\ W_t^{mix} \ge \left(m_{\delta^{use}}^{aboveSet} \cdot \hat{\delta}_t^{use} + m_{temp^{EWH}}^{aboveSet} \cdot temp_t^{EWH} + b^{aboveSet}\right) - \widehat{M} \cdot (1 - \delta_t^{aux}), & \hat{\delta}_t^{use} > 0 \\ W_t^{mix} \le \left(m_{\delta^{use}}^{aboveSet} \cdot \hat{\delta}_t^{use} + m_{temp^{EWH}}^{aboveSet} \cdot temp_t^{EWH} + b^{aboveSet}\right) + \widehat{M} \cdot (1 - \delta_t^{aux}), & \hat{\delta}_t^{use} > 0 \\ W_t^{mix} = temp_t^{EWH} \cdot \widehat{EWH}^{cap} \cdot \dfrac{\hat{c}}{3600} \cdot \Delta t \cdot 60, & \hat{\delta}_t^{use} = 0 \end{cases} \quad (8)$$

- (1) represents the total energy stored within the EWH in each period. For the first instant, it is equal to the value stipulated as initial (or equal to energy associated with the tank filled with water at inlet temperature). For the remaining periods, it depends on the total energy resulting from mixing water (7), the energy gain from heating (2), and the energy thermal losses (5).
- (2) represents additional energy introduced by the heating element of the EWH. It depends on the power of the element, the length of the time period and if the EWH is operational or not.
- (3) represents the total cost of operating the EWH, depending on the energy cost, the selected daily tariff, the power of the element, the length of the time period and if the EWH is operational or not.

- (4) represents the temperature value of the internal water of the EWH. For the initial instant, it is equivalent to the established input, while for the following instants, it establishes a conversion to equivalent temperature through the value of the accumulated energy (1)
- (5) represents the total thermal losses of the EWH through its surface, depending on the length of the time period.
- (6) mandates adherence to prescribed upper and lower limits for both temperature and energy, requiring precise management to sustain system stability within defined boundaries.
- (7) represents the need for the internal water temperature of the EWH after use to be equal to or greater than the threshold defined by the user. It uses an auxiliary variable that allows the model to operate below the comfort threshold, but adds a penalty operating cost, which in turn is added to the objective function with a positive term.
- (8) Definition of the variable that represents the accumulated energy after using hot water, and after internal mixing with inlet water. As it is a conditional (if-else) and non-linear constraint, its formulation is built in such a way that it is programmable. Please refer to the **Mathematical Adjustments** section for a detailed description.

Mathematical Adjustments

1. Mixing Fluids

The final temperature when mixing fluids is calculated below.

$$t_{out} = \frac{m_1 \cdot c_1 \cdot t_1 + m_2 \cdot c_2 \cdot t_2}{m_1 \cdot c_1 + m_2 \cdot c_2}$$

2. Temperature to Energy

The final energy (kWh) is calculated below.

$$w_{out} = \frac{t_{out} \cdot m_{out} \cdot c}{3600}$$

A. Estimate the total hot water flow rate.

For a single hot water usage ($\widehat{flow}^{Total}$), the final usage temperature ($\widehat{temp}^{set}$) is given by the mixture of both EWH hot water flow ($flow_t^{EWH}$) at the current EWH temperature ($temp_t^{EWH}$) and Inlet flow ($flow_t^{Inlet}$) at the respective temperature ($\widehat{temp}^{inlet}$).

The objective is to estimate the necessary EWH flow rate that, at the current EWH temperature, should be mixed with inlet water. The inlet water flow should represent the difference between the usage total flow rate and the estimated EWH flow rate.

Assuming,

$$flow_t^{inlet} = \widehat{flow}^{Total} - flow_t^{EWH}$$

$$c^{EWH} = c^{inlet} = \hat{c}$$

Using the mixing fluid expression, gives:

$$\widehat{temp}^{set} = \frac{flow_t^{EWH} \cdot \hat{c} \cdot temp_t^{EWH} + \left(\widehat{flow}^{Total} - flow_t^{EWH}\right) \cdot \hat{c} \cdot \widehat{temp}^{inlet}}{flow_t^{EWH} \cdot \hat{c} + \left(\widehat{flow}^{Total} - flow_t^{inlet}\right) \cdot \hat{c}}$$

Solving to $flow_t^{EWH}$:

$$flow_t^{EWH} = \widehat{flow}^{Total} \frac{\widehat{temp}^{set} - \widehat{temp}^{inlet}}{temp_t^{EWH} - \widehat{temp}^{inlet}}$$

B. Estimate the EWH internal temperature after usage.

After a single hot water usage, the final EWH internal temperature ($temp_t^{mix}$) can be estimated using the fluid mixture expression. In this scenario, the final water mixture should address the remainder of the hot water inside the EWH after the usage ($\widehat{EWH}^{cap} - flow_t^{EWH}$) at the current temperature ($temp_{EWH}$) being mixed with the necessary inlet water to fill the amount that was used ($flow_t^{EWH}$) at inlet temperature ($\widehat{temp}^{inlet}$). Using the mixing fluid expression, gives:

$$temp_t^{mix} = \frac{\left(\widehat{EWH}^{cap} - flow_t^{EWH}\right) \cdot c \cdot temp_t^{EWH} + flow_t^{EWH} \cdot c \cdot \widehat{temp}^{inlet}}{\left(\widehat{EWH}^{cap} - flow_t^{EWH}\right) \cdot c + flow_t^{EWH} \cdot c}$$

Simplifying:

$$temp_t^{mix} = \frac{\left(\widehat{EWH}^{cap} - flow_t^{EWH}\right) \cdot temp_t^{EWH} + flow_t^{EWH} \cdot \widehat{temp}^{inlet}}{\widehat{EWH}^{cap}}$$

C. Estimating the total stored energy after a hot water usage

After estimating an expression to find the final temperature of the water inside the EWH after a single hot water usage, the $flow_{EWH}$ value can be replaced by the expression found in (A), thus resulting in an expression that relies only on the internal EWH temperature to estimate the total stored energy after a single hot water usage. By using the expression that allows to convert temperature to stored energy, the amount of stored energy (kWh), after a hot water usage, is given by:

$$W_t^{mix} = \frac{temp_t^{mix} \cdot \widehat{EWH}^{cap} \cdot \hat{c}}{3600}$$

Replacing the $temp_t^{mix}$ with the respective expression gives:

$$W_t^{mix} = \frac{\left(\frac{\left(\widehat{EWH}^{cap} - flow_t^{EWH}\right) \cdot temp_t^{EWH} + flow_t^{EWH} \cdot \widehat{temp}^{inlet}}{\widehat{EWH}^{cap}}\right) \cdot \widehat{EWH}^{cap} \cdot \hat{c}}{3600}$$

That simplifies to:

$$W_t^{mix} = \frac{\left(\left(\widehat{EWH}^{cap} - flow_t^{EWH}\right) \cdot temp_t^{EWH} + flow_t^{EWH} \cdot \widehat{temp}^{inlet}\right) \cdot \hat{c}}{3600}$$

Replacing $flow_t^{EWH}$ with the expression found in (A), when $temp_t^{EWH} \geq \widehat{temp}^{set}$:

$$W_t^{mix \mid temp_t^{EWH} \geq \widehat{temp}^{set}} = \frac{\left(\left(\widehat{EWH}^{cap} - \widehat{flow}^{Total} \frac{\widehat{temp}^{set} - \widehat{temp}^{inlet}}{temp_t^{EWH} - \widehat{temp}^{inlet}}\right) \cdot temp_t^{EWH} + \widehat{flow}^{Total} \frac{\widehat{temp}^{set} - \widehat{temp}^{inlet}}{temp_t^{EWH} - \widehat{temp}^{inlet}} \cdot \widehat{temp}^{inlet}\right) \cdot \hat{c}}{3600}$$

However, when $temp_t^{EWH} < \widehat{temp}^{set}$, the EWH flow rate must be equal to the total flow rate, guaranteeing that the output temperature is actual temperature inside of the EWH, and hence:

$$W_t^{mix \mid temp_t^{EWH} < \widehat{temp}^{set}} = \frac{\left(\left(\widehat{EWH}^{cap} - \widehat{flow}^{Total}\right) \cdot temp_t^{EWH} + \widehat{flow}^{Total} \cdot \widehat{temp}^{inlet}\right) \cdot \hat{c}}{3600}$$

D. **Use-case Scenario & Linearization**

Having described the two expressions, the next step involves the creation of two linear expressions, for each of the time intervals described above. To do so, all non-repeating $temp_{Inlet}$ input values are used, individually paired with $temp_t^{EWH}$ and δ_t^{use} values from each of the intervals and for each of the expressions. For example, for a single observation where $\widehat{temp}^{inlet} = 15°C$, assuming $\widehat{temp}^{set} = 40°C$, and $\widehat{temp}^{Max|EWH} = 80°C$, 41 final W_t^{mix} values are calculated (with $temp_t^{EWH}$ values from 40 up to and including 80, and for every possible δ^{use}, dependent on selected resolution). This procedure is repeated for each non-repeated $temp_{Inlet}$ value, for each of the expressions, with the respective ranges of $temp_t^{EWH}$ values). The range of possible δ_t^{use} values depend on the used sampling rate. For the original 1-minute resolution, there is only one possible value ($\delta^{use} = 1$), while for the 15-min resampled resolution, there are 15 different values that should cover all adjustment possibilities (i.e., $\frac{1}{15}, \frac{2}{15}, \ldots, \frac{15}{15}$).

Table 1 - Auxiliary dataset for calculating W_{mix}, for the two $temp^{EWH}$ levels.

EWH^{cap}	$temp^{inlet}$	$temp^{EWH}$	δ^{use}	$temp^{set}$	$flow^{Total}$	$flow^{EWH}$	W^{out}	$temp^{EWH}$	W^{mix}
100	20	20	1	40	8.5	8.500	0.198	20.0	**2.326**
100	20	21	1	40	8.5	8.500	0.208	20.9	**2.432**
100	20	22	1	40	8.5	8.500	0.217	21.8	**2.538**
100	20	23	1	40	8.5	8.500	0.227	22.7	**2.645**
…	…	…	…	…	…	…	…	…	**…**
100	20	37	1	40	8.5	8.500	0.366	35.6	**4.134**
100	20	38	1	40	8.5	8.500	0.376	36.5	**4.241**
100	20	39	1	40	8.5	8.500	0.385	37.4	**4.347**
100	20	40	1	40	8.5	8.500	0.395	38.3	**4.453**
100	20	40	1	40	8.5	8.500	0.395	38.3	**4.453**
100	20	41	1	40	8.5	8.095	0.386	39.3	**4.570**
100	20	42	1	40	8.5	7.727	0.377	40.3	**4.686**
100	20	43	1	40	8.5	7.391	0.370	41.3	**4.802**
…	…	…	…	…	…	…	…	…	**…**
100	20	77	1	40	8.5	2.982	0.267	75.3	**8.756**
100	20	78	1	40	8.5	2.931	0.266	76.3	**8.872**
100	20	79	1	40	8.5	2.881	0.265	77.3	**8.988**
100	20	80	1	40	8.5	2.833	0.264	78.3	**9.105**

Once this auxiliary dataset is created (see Table 1), trend curves must be created for each of the levels (see Figure 1). The expressions of these trend curves will be those associated with the restriction that allows calculating the stored energy value after using hot water. In this way, the three linear parameters ($m_{\delta^{use}}, m_{temp^{EWH}}$, and b) are calculated for each of the levels. In this specific scenario, since it refers to the original 1-min resolution, δ^{use} has no weight on the regression. Furthermore, since $\widehat{temp}^{inlet}$ is constant, the m parameter is very close. Ultimately, the regressor strategy is fully prepared to deal with different values from both δ^{use} and $\widehat{temp}^{inlet}$.

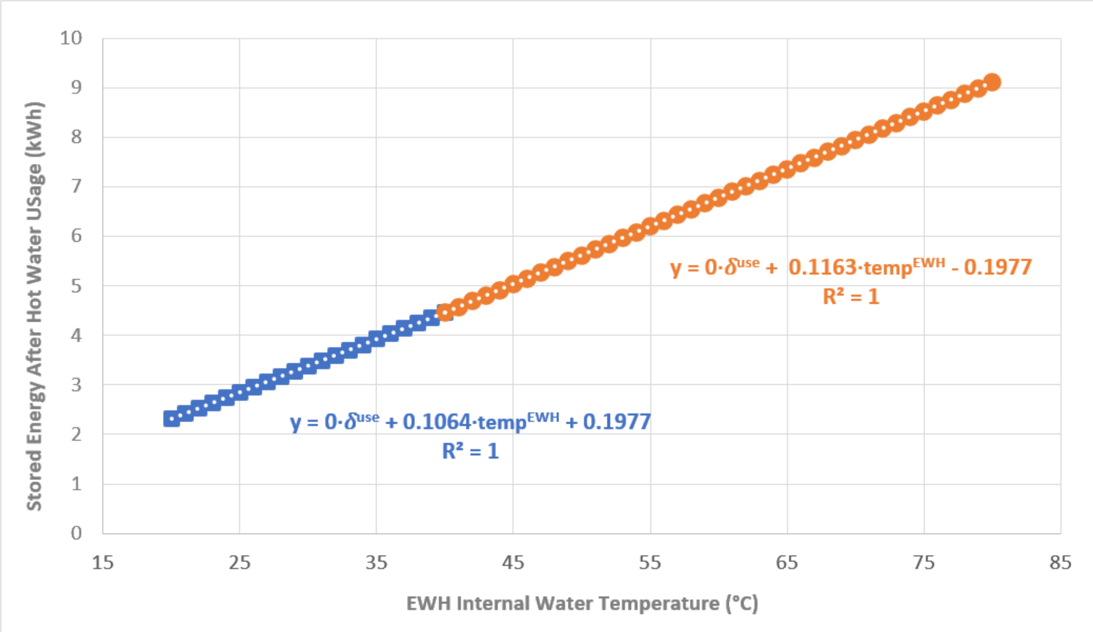


Figure 1 – Example trendlines for the two EWH water temperature scenarios

With the new values of the parameters ($m_{\delta^{use}}^{aboveSet}, m_{temp^{EWH}}^{aboveSet}, b^{aboveSet}, m_{\delta^{use}}^{belowSet}, m_{temp^{EWH}}^{belowSet}, b^{belowSet}$), it is possible to write the final restriction that allows to obtain the stored energy after a single use of hot water ($\hat{\delta}_t^{use} > 0$):

$$If\ \hat{\delta}_t^{use} > 0:$$

$$If\ temp_t^{EWH} < \widehat{temp}^{set}$$

$$W_t^{mix} = m_{\delta^{use}}^{belowSet} \cdot \hat{\delta}_t^{use} + m_{temp^{EWH}}^{belowSet} \cdot temp_t^{EWH} + b^{belowSet}$$

$$If\ temp_t^{EWH} \geq \widehat{temp}^{set}$$

$$W_t^{mix} = m_{\delta^{use}}^{aboveSet} \cdot \hat{\delta}_t^{use} + m_{temp^{EWH}}^{aboveSet} \cdot temp_t^{EWH} + b^{aboveSet}$$

When there is no hot water usage ($\hat{\delta}_t^{use} = 0$), the constraint is direct, and it uses an equivalent temperature to energy conversion using:

$$If\ \hat{\delta}_t^{use} = 0:$$

$$W_t^{mix} = temp_t^{EWH} \cdot \widehat{EWH}^{cap} \cdot \frac{\hat{c}}{3600} \cdot \Delta t \cdot 60$$

E. **Conditional Constraint**

As shown in (D), the formulation results in a set of conditional constraints. In this type of problem, if the condition is based on an input variable, where it is possible, a priori, to verify its value, it is possible to program the conditional restriction. If the condition is imposed by a decision variable, it is not possible to program the restriction, since, internally, the restrictions, for all instants of time, are written in a file. In this sense, as the conditional variable has not yet been assigned a value, it is not possible to implement it in a conditional constraint. In this problem, we have two degrees of condition: the first one, based on the input variable $\hat{\delta}_t^{use}$, can be introduced directly in the formulation. However, the second condition is based on the $temp_t^{EWH}$ variable, which, being a decision variable, prevents the implementation from being straightforward. In this sense, for this second part, it is necessary to transform the conditional constraint, through the introduction of sufficiently wide tolerance intervals. Through the formulations indicated in (Bisschop, 2023), the restrictions formulated in (D) are then transformed into:

$If\ \widehat{\delta}_t^{use} = 1:$

$$
\begin{gathered}
temp_t^{EWH} \geq \widehat{temp}^{set} - \widehat{M} \cdot \left(1 - \delta_t^{aux}\right) \\
temp_t^{EWH} \leq \widehat{temp}^{set} + \widehat{M} \cdot \delta_t^{aux} \\
W_t^{mix} \geq \left(m_{\delta^{use}}^{belowSet} \cdot \hat{\delta}_t^{use} + m_{temp^{EWH}}^{belowSet} \cdot temp_t^{EWH} + b^{belowSet}\right) - \widehat{M} \cdot \delta_t^{aux} \\
W_t^{mix} \leq \left(m_{\delta^{use}}^{belowSet} \cdot \hat{\delta}_t^{use} + m_{temp^{EWH}}^{belowSet} \cdot temp_t^{EWH} + b^{belowSet}\right) + \widehat{M} \cdot \delta_t^{aux} \\
W_t^{mix} \geq \left(m_{\delta^{use}}^{aboveSet} \cdot \hat{\delta}_t^{use} + m_{temp^{EWH}}^{aboveSet} \cdot temp_t^{EWH} + b^{aboveSet}\right) - \widehat{M} \cdot (1 - \delta_t^{aux}) \\
W_t^{mix} \leq \left(m_{\delta^{use}}^{aboveSet} \cdot \hat{\delta}_t^{use} + m_{temp^{EWH}}^{aboveSet} \cdot temp_t^{EWH} + b^{aboveSet}\right) + \widehat{M} \cdot (1 - \delta_t^{aux})
\end{gathered}
$$

$If\ \widehat{\delta}_t^{use} = 0:$

$$
W_t^{mix} = temp_t^{EWH} \cdot \widehat{EWH}^{cap} \cdot \frac{\hat{c}}{3600} \cdot \Delta t \cdot 60
$$

F. **Comfort**

Since it is already possible to calculate the volume of energy stored after using hot water, the aim is now to ensure that the comfort temperature is respected. To this end, a restriction is added that indicates that, at the beginning of the period following the use of water, the stored energy of the water in the EWH is at least equal to the energy associated with the comfort temperature. In this sense, the energy value associated with the comfort temperature is calculated using the temperature to energy expression, as follows:

$$
W_{confort} = \frac{\widehat{temp}^{set} \cdot \widehat{EWH}^{cap} \cdot \hat{c}}{3600} \cdot \Delta t \cdot 60
$$

There is also the need to use an auxiliary variable that allows the model to operate below the comfort threshold, but adding a penalty operating cost, which in turn is added to the objective function with a positive term. Hence, the final constraint:

$$
If\ \delta_{usage}[t] - \delta_{usage}[t-1] < 0
$$

$$
W_{t-1}^{tot} \geq W_{confort} - penalty_t^{comf}
$$

References

Bisschop, J. (2023). *AIMMS - Optimization Modeling.*