AR Model-AR1, AR3

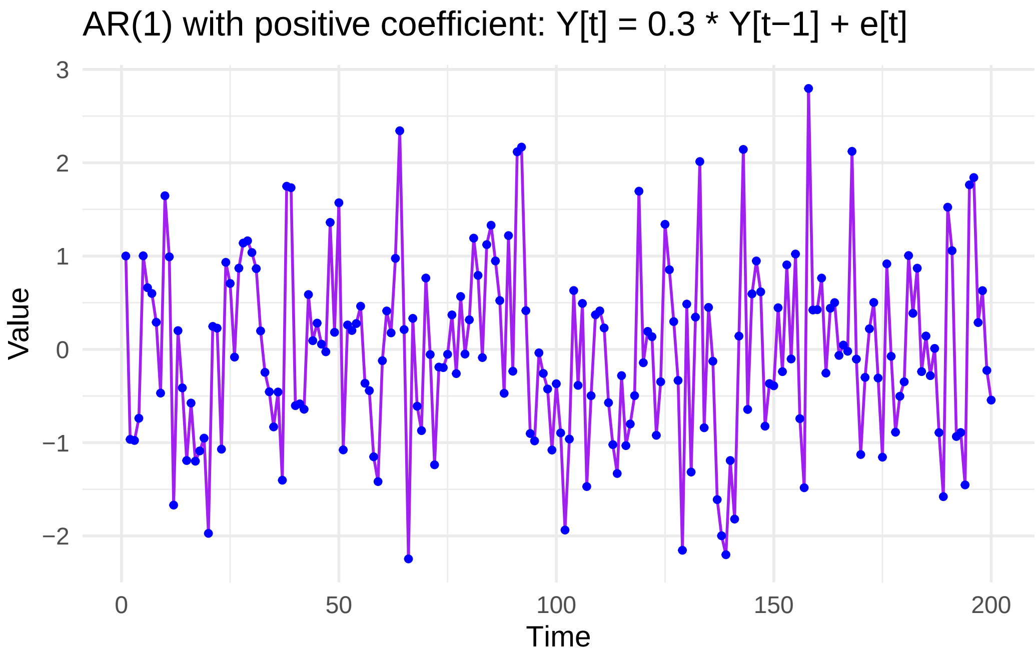
AR(1) with positive coefficients

```r
library(ggplot2)

set.seed(123)

ar_coeffs =0.3
title_expression = expression("AR(1) with positive coefficient: Y[t] = 0.3 * Y[t-1] + e[t]")

# Simulate AR(1) process
ar_process <- as.numeric(arima.sim(model = list(ar = ar_coeffs), n = 200))
plot_data <- data.frame(Time = 1:200, AR_Oscillatory = ar_process)
ggplot(plot_data, aes(x = Time, y = AR_Oscillatory)) +
  geom_line(color = "purple") +
  geom_point(color = "blue", size = 1.5, shape = 20) +
  labs(title = title_expression,
       x = "Time",
       y = "Value") +
  theme_minimal()
```



#The Behavior of the Model

Low Coefficient ($\phi = 0.3$)

The value of $\phi = 0.3$ is relatively low, which means the influence of Y_{t-1} is moderate. The series will exhibit limited persistence; therefore, the deviations from the mean will fade quickly as the lagged influence diminishes over time.

Stationarity

The process is stationary because $\phi < 1$. This implies that the time series will fluctuate around a constant mean (0 for this example). The variance of the series remains finite and does not grow over time.

White Noise Contribution

Random white noise (ϵ_t) plays a more significant role in shaping the series than lagged values. This results in a more stochastic (less deterministic) series.

Smoothness and Short-Term Correlation

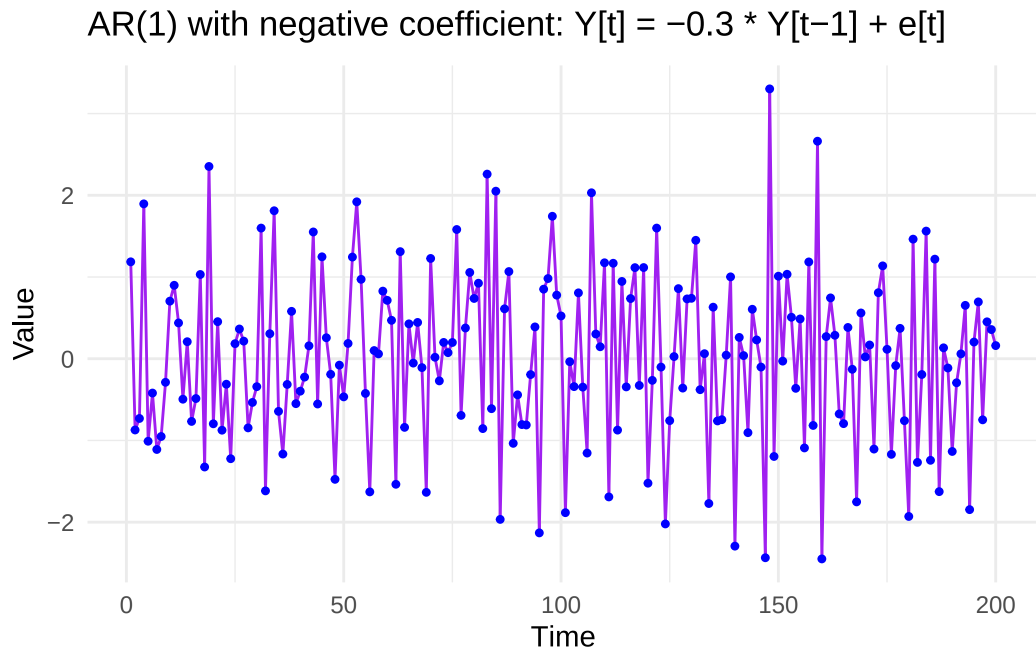
The small ϕ value results in limited smoothness; Y_t is only weakly correlated with Y_{t-1}. Short-term trends are minimal, and the series appears more random.

Mean-Reverting Property

The series will revert to the mean quickly after a deviation because the influence of the lagged term diminishes rapidly.

AR(1) with negative coefficients

```r
neg_ar1_process <- function(ar_coeffs = -0.3,
                            title_expression = expression("AR(1) with negative coefficient: Y
  #Simulate AR(1) process
  ar_process <- as.numeric(arima.sim(model = list(ar = ar_coeffs), n = 200))
  # Create a data frame for plotting
  plot_data <- data.frame(Time = 1:200, AR_Oscillatory = ar_process)
  # Generate the plot using ggplot2
  ggplot(plot_data, aes(x = Time, y = AR_Oscillatory)) +
    geom_line(color = "purple") +
    geom_point(color = "blue", size = 1.5, shape = 20) +
    labs(title = title_expression,
         x = "Time",
         y = "Value") +
    theme_minimal() }
neg_ar1_process()
```



#The Behavior of the Model Low Negative Coefficient ($\phi = -0.3$)

A small negative coefficient indicates: A weak negative correlation between Y_t and Y_{t-1}. Y_t tends to move in the opposite direction to Y_{t-1} but with low persistence.

Oscillatory Behavior

The negative coefficient introduces an alternating pattern: If Y_{t-1} is positive, Y_t is more likely to be negative, and vice versa. However, the oscillations are not pronounced due to the small magnitude of ϕ.

Stationarity

The process is stationary because $\phi < 1$. This ensures the series fluctuates around a constant mean (0 for this example). The variance remains stable over time, preventing explosive growth or decay.

Randomness and White Noise

Random white noise (ϵ_t) dominates the series: The weak ϕ value means that the current value is heavily influenced by random noise rather than the lagged term.

Limited Persistence

The impact of deviations from the mean dissipates quickly because ϕ is small. The series lacks strong trends or long-term dependencies.

AR(3) model with complex cycle

```
# Function to plot AR(1) process
plot_ar3_process <- function(ar_coeffs, title_expression) {
  # Generate AR(1) process
  ar_process <- as.numeric(arima.sim(model = list(ar = ar_coeffs), n = 200))
  # Create data frame for plotting
  plot_data <- data.frame(Time = 1:200, AR_Oscillatory = ar_process)
  ggplot(plot_data, aes(x = Time, y = AR_Oscillatory)) +
    geom_line(color = "purple") +
    geom_point(color = "blue", size = 3, shape = 20) +
    labs(
      title = title_expression,
      x = "Time",
      y = "Value") +
    theme_minimal() }
```

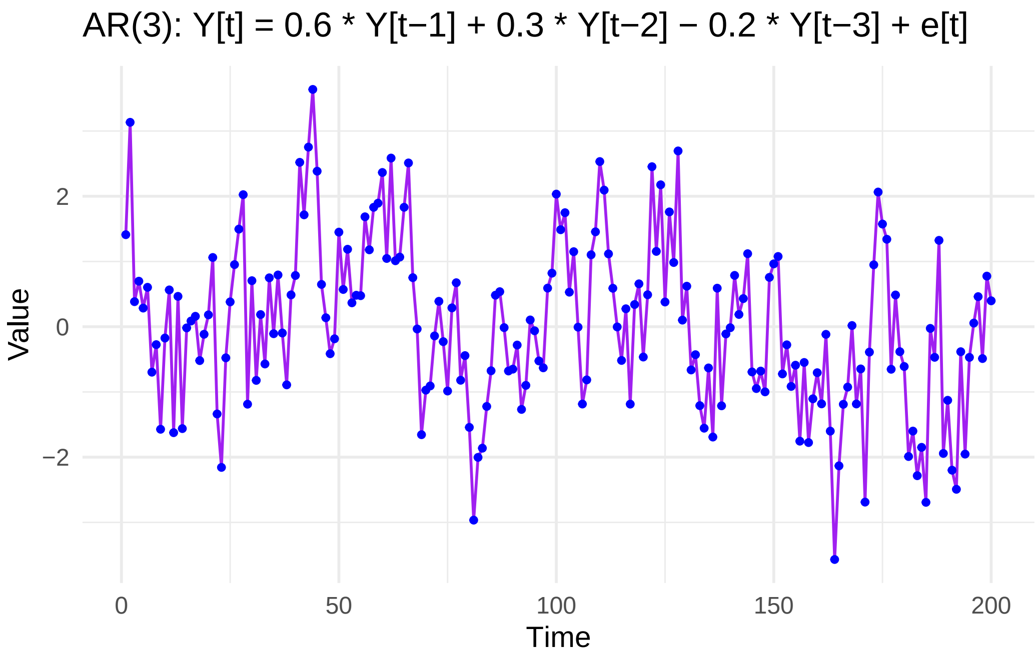
```
# Function to plot AR(3) process
plot_ar3_process <- function(ar_coeffs = c(0.6, 0.3, -0.2),
                             title_expression = expression("AR(3): Y[t] = 0.6 * Y[t-1] + 0.3
  # Generate AR(3) process
  ar_process <- as.numeric(arima.sim(model = list(ar = ar_coeffs), n = 200))

  # Create data frame for plotting
  plot_data <- data.frame(Time = 1:200, AR_Oscillatory = ar_process)

  # Generate the plot
  ggplot(plot_data, aes(x = Time, y = AR_Oscillatory)) +
    geom_line(color = "purple") +
    geom_point(color = "blue", size = 1.5, shape = 20) +
    labs(
      title = title_expression,
      x = "Time",
      y = "Value"
    ) +
```

```
    theme_minimal()
}

# Call the function to plot the AR(3) process
plot_ar3_process()
```



#The Behavior of the Model Positive and Negative Coefficients

The AR(3) model has a combination of both positive and negative coefficients.

$\phi_1 = 0.6$: The current value Y_t has a positive dependence on the previous value Y_{t-1}, meaning that if Y_{t-1} is large, Y_t will likely also be large.

$\phi_2 = 0.3$: The current value Y_t has a moderate positive dependence on Y_{t-2} but is weaker than Y_{t-1}.

$\phi 3 = -0.2$: The current value Y_t has a negative dependence on Y_{t-3}, meaning that if Y_{t-3} is large, Y_t will likely be smaller.

Oscillatory Behavior

The process exhibits oscillations when it has positive and negative coefficients. Positive coefficients contribute to values moving in the same direction, while negative coefficients introduce reversals. This can lead to a complex, oscillatory pattern, where the influence of earlier terms (both positive and negative) generates alternating cycles. The size of the coefficients determines the amplitude of these oscillations. Large positive coefficients (like $\phi_1 = 0.6$) cause the series to follow the previous term closely, while the negative coefficient $\phi_3 = -0.2$ creates some counteracting force.

Stationarity

As long as the roots of the characteristic equation (derived from the AR coefficients) are outside the unit circle, the process is stationary. Given that the coefficients here are moderate in size, this process is stationary, meaning that the mean and variance will remain constant over time.

Impact of White Noise

The white noise term ϵ_t adds randomness to the model, making the series unpredictable and variable. While the autoregressive terms provide structure, the random noise plays a major role in shaping the series, especially when the influence of past values is not very strong.

Persistence of Shocks

Because of the combination of positive and negative coefficients, shocks (deviations from the mean) may persist longer than in models with only one or two coefficients, as multiple lags influence the series. However, the persistence of shocks is still limited by the relatively small size of the negative coefficient ($\phi_3 = -0.2$).