AutoRegression Models

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Introduction

Autoregression (AR) is a statistical modeling technique to predict future values based on past values within a time series. It finds extensive applications in finance, weather forecasting, and signal processing.

Autoregression is a fundamental building block for more advanced time series models like ARIMA and state-space models. It is essential for creating predictive systems that address diverse real-world challenges.

Background

Time series is a sequence of data points indexed in time order (e.g., stock prices, temperature reading)

Basics of AR models

AR models express the current value of the series as a linear combination of its previous values and a noise term:

$$X_t=\phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + \epsilon_t$$

Where:

- ϕ_i: Coefficients of the AR model.
- ϵ_t: A white noise term representing random error.
- p: Order of AR model

Autoregression fundamentals

Stationarity:

- A stationary time series has a constant mean, variance, and autocorrelation over time.
- Stationarity is critical for AR models to be effective.
- Stationarity can be checked by methods like:
  - Augmented Dickey-Fuller (ADF) Test.
  - Visual inspection of the time series.

Root Conditions for Stationarity:

- An AR model is stationary if all roots of its characteristic equation have moduli greater than 1

Simulation of AR Models

AR(1) Model: An autoregressive model of order 1 (AR(1)) is defined as:

$$X_t=\phi_1 X_{t-1}+\epsilon_t$$

Where:

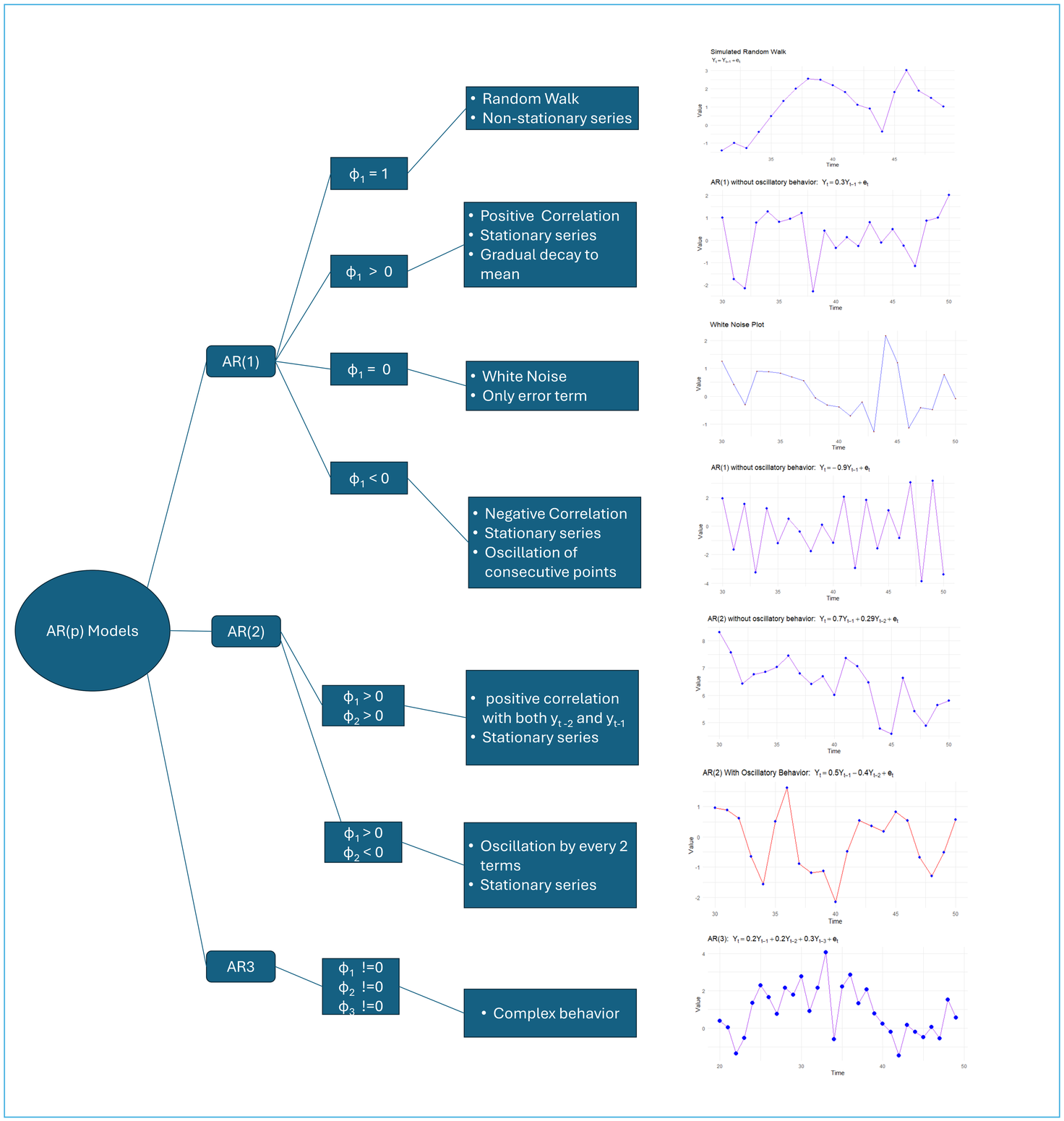
- ϕ_1: Coefficient of the lagged term.
- ϵ_t: White noise or error term.

$|\phi_1| < 0$ for stationary series.

Special Case ($\phi_1=1$):

When $\phi_1=1$, the AR(1) model simplifies to $X_t=X_{t-1}+\epsilon_t$

This is known as a **random walk**, where the current value X_t equals the previous value X_{t-1} plus a random noise term ϵ_t.



Application & Future perspective

Applications of AR Models:

- **Finance**: Stock price predictions.
- **Weather**: Temperature and rainfall forecasting.
- **Economics**: GDP and inflation rate modeling.

Conclusion

- AR models are foundational tools in time series analysis and have versatile applications across industries.
- Their simplicity makes them a great starting point for understanding more complex time series models.

Reference

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