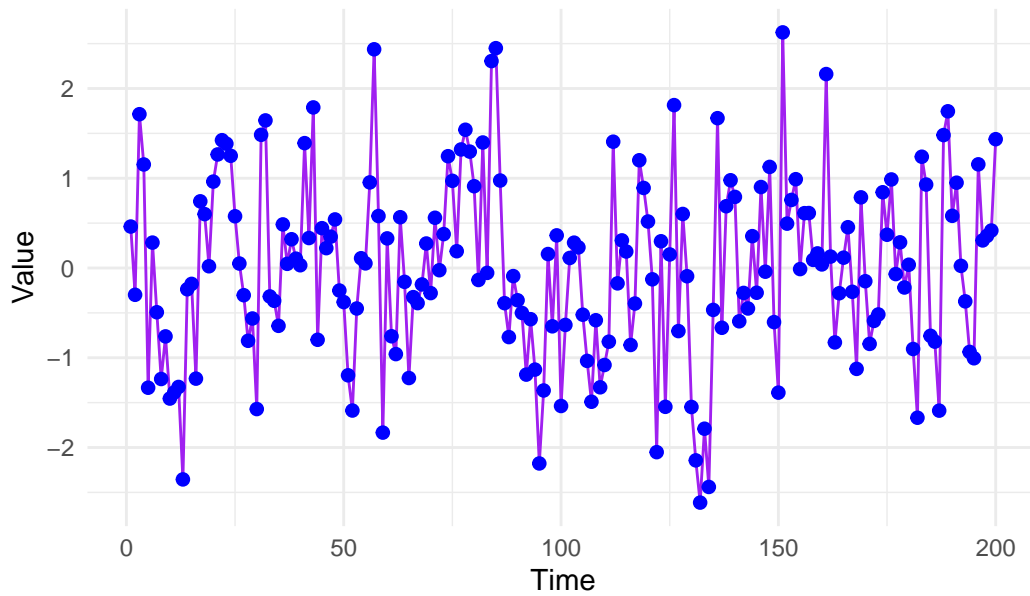


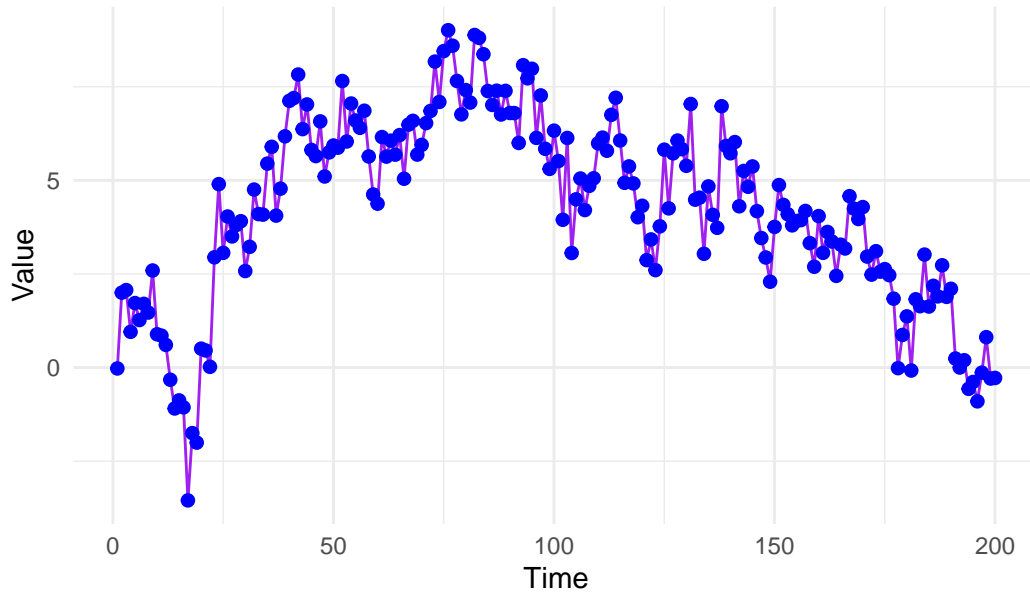
AR(2) Model with ($\phi_2 > 0$)

AR(2) Model without oscillatory behaviour. ($\phi_2 > 0$)

AR(2) without oscillatory behavior: $Y_t = 0.4Y_{t-1} + 0.1Y_{t-2} + e_t$



AR(2) without oscillatory behavior: $Y_t = 0.7Y_{t-1} + 0.29Y_{t-2} + e_t$



The behavior of the two AR(2) processes can be explained by examining their respective coefficients and their effect on the time series dynamics:

1. AR(2) Process with Coefficients (0.4, 0.1):

Equation:

$$Y_t = 0.4 \cdot Y_{t-1} + 0.1 \cdot Y_{t-2} + e_t$$

Key Observations:

- **Low coefficients (0.4, 0.1):**
 - These relatively small coefficients result in **weak persistence**, meaning the current value of Y_t depends only moderately on the previous two values.
 - Changes in Y_t are largely driven by random noise (e_t) rather than strong autoregressive patterns.
- **Behavior:**

- The process is **stable** with no oscillatory behavior, as both coefficients are positive and their sum ($0.4 + 0.1 = 0.5$) is less than 1.
- The values show gradual variations around the mean but return to the mean quickly after small deviations.
- This process exhibits **weak memory**, meaning the influence of past values fades quickly.

Overall:

- The time series appears **smooth** with mild fluctuations and no pronounced patterns of oscillation or persistence.

2. AR(2) Process with Coefficients (0.7, 0.29):

Equation:

$$Y_t = 0.7 \cdot Y_{t-1} + 0.29 \cdot Y_{t-2} + e_t$$

Key Observations:

- **Higher coefficients (0.7, 0.29):**
 - These larger coefficients result in **stronger persistence**, meaning Y_t is more heavily influenced by the recent past values.
 - The sum of the coefficients ($0.7 + 0.29 = 0.99$) is close to 1, leading to more pronounced autoregressive effects.
- **Behavior:**
 - The process is still **stable**, as the coefficients do not exceed the stability threshold.
 - Values exhibit **longer-lasting effects** from deviations, as changes in Y_t take longer to return to the mean.
 - No oscillations occur since both coefficients are positive and do not produce alternating signs in Y_t .

Overall:

- The time series shows **stronger and slower variations**, with values demonstrating a higher tendency to stay elevated or depressed for several time steps before converging back to the mean.

Comparison:

Aspect	Process 1 (0.4, 0.1)	Process 2 (0.7, 0.29)
Persistence	Weak	Strong
Stability	Stable	Stable
Effect of Past	Short-lived memory	Long-lasting memory
Oscillatory Behavior	None	None
Mean Reversion	Faster	Slower
Fluctuations	Small and smooth	Larger and more persistent

Conclusion:

- **Process 1** reflects a time series with weak dependencies and rapid convergence to the mean, resembling random noise with minor persistence.
- **Process 2** represents a process with stronger dependencies, where past values have a noticeable and prolonged influence on future values, although the process remains non-oscillatory and stable.