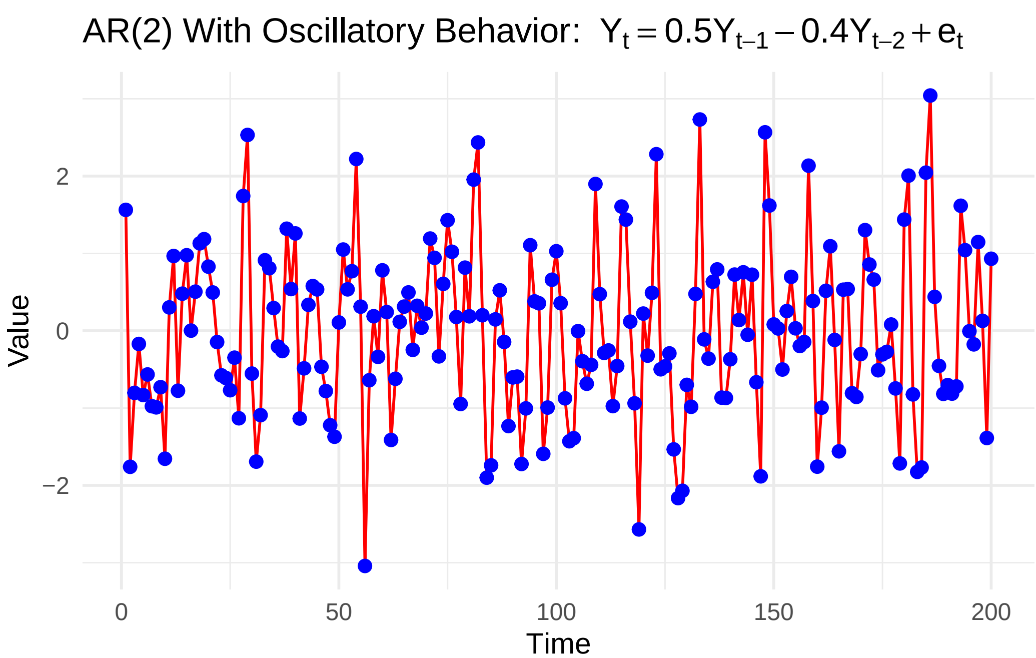
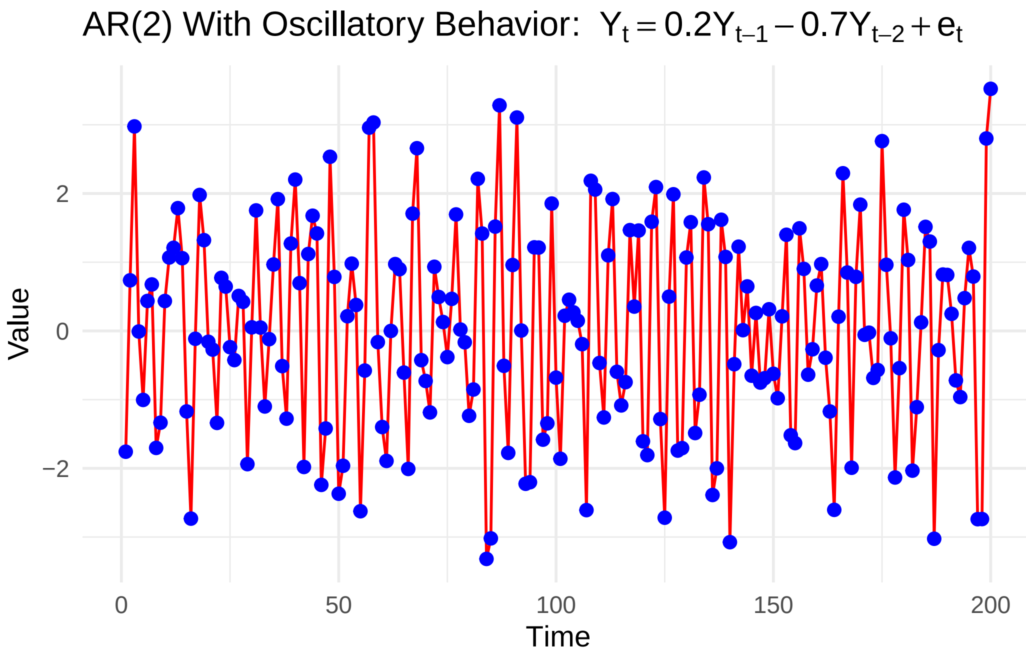
AR(2) Model with ($\phi_2 < 0$) demonstrating Oscillatory Behaviour

AR(2) Model With Oscillatory Behaviour. ($\phi_2 < 0$)





The behavior of the two AR(2) processes can be explained by examining their respective coefficients and their effect on the time series dynamics:

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1. AR(2) Process with Coefficients (0.5, -0.4):

Equation:

$$Y_t = 0.5 \cdot Y_{t-1} - 0.4 \cdot Y_{t-2} + e_t$$

Key Observations:

Moderate Oscillatory Behavior:

- The negative $\phi_2 = -0.4$ introduces oscillations in the time series, with values alternating around the mean.
- The oscillations are moderate because $\phi_1 = 0.5$ is not too large, and the influence of past values decays relatively quickly.

Stability:

- The process is **stable** because the coefficients satisfy the stability condition:

$\phi_2 > -1, \quad \phi_1 + \phi_2 < 1, \quad \phi_1 - \phi_2 < 1$

Here: $-0.4 > -1 \quad 0.5 + (-0.4) = 0.1 < 1 \quad \text{and} \quad 0.5 - (-0.4) = 0.9 < 1$

Behavior:

- The oscillations decay over time, meaning the process eventually reverts to the mean.
- The persistence is moderate, with past values influencing the current value for a few time steps before fading.

Overall:

- This process introduces **oscillatory behavior** while remaining stable and moderately persistent.
- It is a good choice if you want to observe oscillations without excessive persistence.

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2. AR(2) Process with Coefficients (0.2, -0.7)

Equation:

$Y_t = 0.2 \cdot Y_{t-1} - 0.7 \cdot Y_{t-2} + e_t$

Stronger Oscillatory Behavior:

- The larger negative $\phi_2 = -0.7$ results in **stronger oscillations**, with more pronounced alternations around the mean.
- The smaller $\phi_1 = 0.2$ means the influence of the most recent past value (Y_{t-1}) is weaker, while the second lag (Y_{t-2}) dominates the dynamics.

Stability:

- The process is also **stable** because the coefficients satisfy the stability condition:

$\phi_2 > -1, \quad \phi_1 + \phi_2 < 1, \quad \phi_1 - \phi_2 < 1$

Here: $-0.7 > -1 \quad 0.2 + (-0.7) = -0.5 < 1 \quad \text{and} \quad 0.2 - (-0.7) = 0.9 < 1$

Behavior:

- The oscillations are more persistent compared to $(0.5, -0.4)$, as the larger magnitude of ϕ_2 causes the influence of past values to decay more slowly.
- The process takes longer to revert to the mean, and the oscillations are more pronounced.

Overall:

- This process introduces **stronger and more persistent oscillatory behavior**.
- It is a good choice if you want to emphasize oscillations and persistence in the time series.

Comparison of (0.5, -0.4) and (0.2, -0.7)

Aspect	Process 1 (0.5, -0.4)	Process 2 (0.2, -0.7)
Oscillatory Behavior	Moderate	Strong
Persistence	Moderate	High
Stability	Stable	Stable
Effect of Past	Balanced influence of $ Y_{t-1} $ and $ Y_{t-2} $	Dominated by $ Y_{t-2} $
Mean Reversion	Faster	Slower
Fluctuations	Smaller and decaying	Larger and more persistent

Comparison with Non-Oscillatory Cases

Let's compare these oscillatory processes to the non-oscillatory cases:

Aspect	Non-Oscillatory (0.4, 0.1)	Non-Oscillatory (0.7, 0.29)	Oscillatory (0.5, -0.4)	Oscillatory (0.2, -0.7)
Oscillatory Behavior	None	None	Moderate	Strong

Aspect	Non-Oscillatory (0.4, 0.1)	Non-Oscillatory (0.7, 0.29)	Oscillatory (0.5, -0.4)	Oscillatory (0.2, -0.7)
Persistence	Weak	Strong	Moderate	High
Stability	Stable	Stable	Stable	Stable
Effect of Past	Short-lived memory	Long-lasting memory	Moderate memory	Dominated by $ Y_{t-2} $
Mean Reversion	Faster	Slower	Moderate	Slow
Fluctuations	Small and smooth	Larger and more persistent	Alternating and decaying	Alternating and persistent

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Conclusion:

Both options are better for demonstrating oscillatory behavior. The choice depends on whether you want **moderate** or **strong oscillations**.

- **Process 1:** If you want moderate oscillations with faster mean reversion use $ (0.5, -0.4) $. This process is stable, introduces oscillations, and has moderate persistence.
- **Process 2**: If you want stronger oscillations with slower mean reversion use $ (0.2, -0.7) $. This process emphasizes oscillatory behavior and persistence, making it more suitable for scenarios where past values have a prolonged influence.