Final Assignment

Part 1 - Employment

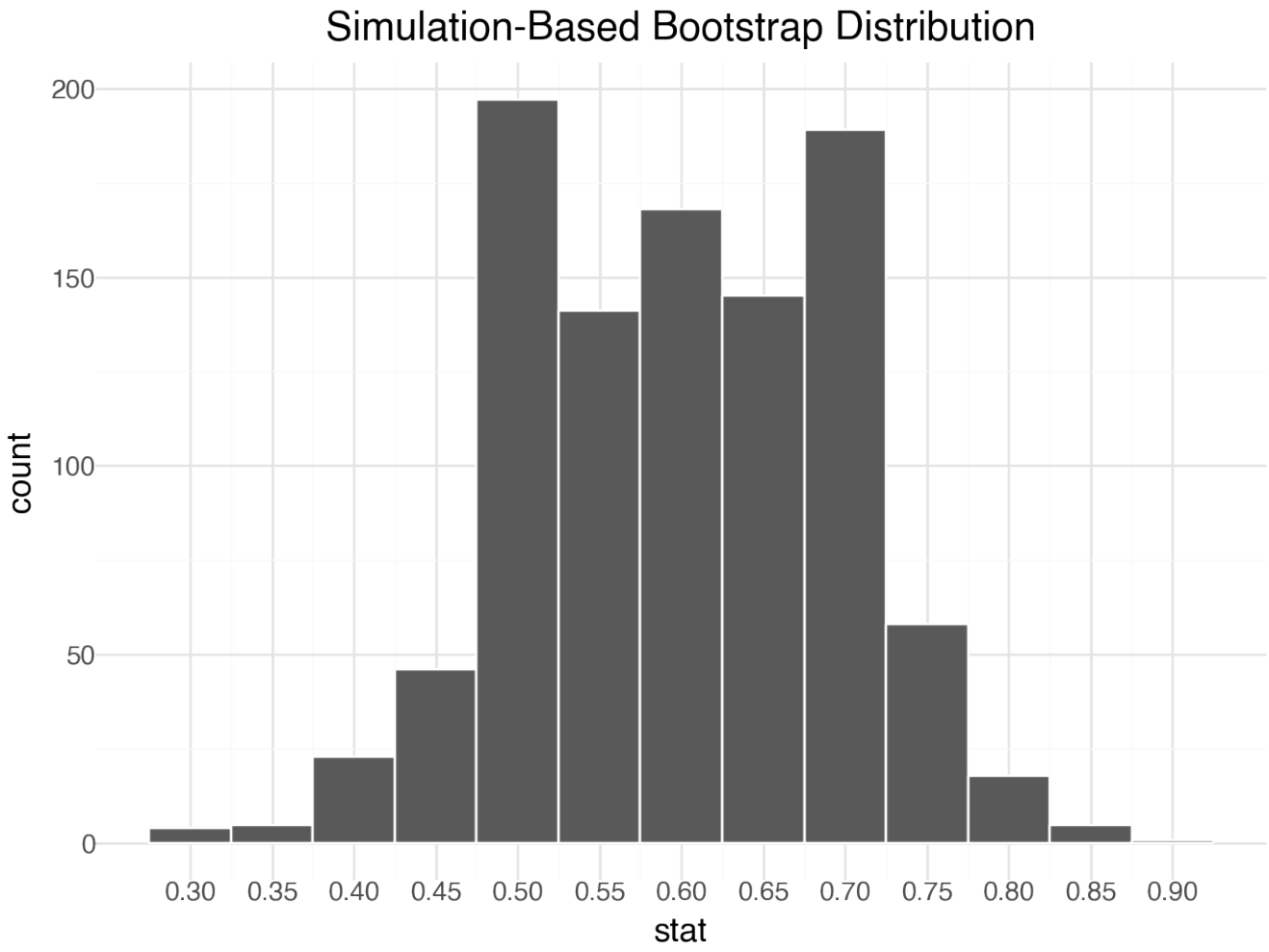
A large university knows that about 70% of the full-time students are employed at least 5 hours per week. The members of the Statistics Department wonder if the same proportion of their students work at least 5 hours per week. They randomly sample 25 majors and find that 15 of the students (60%) work 5 or more hours each week.

Question 1

Describe how you can set up a simulation to estimate the proportion of statistics majors who work 5 or more hours each week based on this sample.

Question 2

A bootstrap distribution with 1000 simulations is shown below. Approximate the bounds of the 95% confidence interval based on this distribution.



Question 3

Suppose the lower bound of the confidence interval from the previous question is L and the upper bound is U. Which of the following is correct?

a. Between L to U of statistics majors work at least 5 hours per week.

b. 95% of the time the true proportion of statistics majors who work at least 5 hours per week is between L and U.

c. Between L and U of random samples of 25 statistics majors are expected to yield confidence intervals that contain the true proportion of statistics majors who work at least 5 hours per week.

d. 95% of random samples of 25 statistics majors will yield confidence intervals between L and U.

e. None of the above.

Part 2 - Blizzard

In 2020, employees of Blizzard Entertainment circulated a spreadsheet to anonymously share salaries and recent pay increases amidst rising tension in the video game industry over wage disparities and executive compensation. (Source: Blizzard Workers Share Salaries in Revolt Over Pay)

The name of the data frame used for this analysis is `blizzard_salary` and the variables are:

- `percent_incr`: Raise given in July 2020, as percent increase with values ranging from 1 (1% increase to 21.5 (21.5% increase)
- `salary_type`: Type of salary, with levels `Hourly` and `Salaried`
- `annual_salary`: Annual salary, in USD, with values ranging from $50,939 to $216,856.
- `performance_rating`: Most recent review performance rating, with levels `Poor`, `Successful`, `High`, and `Top`. The `Poor` level is the lowest rating and the `Top` level is the highest rating.

The top ten rows of `blizzard_salary` are shown below:

	percent_incr	salary_type	annual_salary	performance_rating
0	1.0	year	1.0	High
1	1.0	year	1.0	Successful
2	1.0	year	1.0	High
3	1.0	Hourly	33987.2	Successful
4	NaN	Hourly	34798.4	High
5	NaN	Hourly	35360.0	NaN
6	NaN	Hourly	37440.0	NaN
7	0.0	Hourly	37814.4	NaN
8	4.0	Hourly	41100.8	Top
9	1.2	Hourly	42328.0	NaN

Question 4

Next, you fit a model for predicting raises (`percent_incr`) from salaries (`annual_salary`). We'll call this model `raise_1_fit`. An output of the model is shown below.

	Coef.	Std.Err.	t	P>|t|	[0.025	0.975]
Intercept	1.869965	0.432035	4.328268	0.000019	1.020397	2.719532
annual_salary	0.000016	0.000005	3.431459	0.000669	0.000007	0.000024

Which of the following is the best interpretation of the slope coefficient?

a. For every additional \$1,000 of annual salary, the model predicts the raise to be higher, on average, by 1.6%.
b. For every additional \$1,000 of annual salary, the raise goes up by 0.016%.
c. For every additional \$1,000 of annual salary, the model predicts the raise to be higher, on average, by 0.016%.
d. For every additional \$1,000 of annual salary, the model predicts the raise to be higher, on average, by 1.87%.

Question 5

You then fit a model for predicting raises (`percent_incr`) from salaries (`annual_salary`) and performance ratings (`performance_rating`). We'll call this model `raise_2_fit`. Which of the following is definitely true based on the information you have so far?

a. Intercept of `raise_2_fit` is higher than intercept of `raise_1_fit`.
b. Slope of `raise_2_fit` is higher than RMSE of `raise_1_fit`.
c. Adjusted R^2 of `raise_2_fit` is higher than adjusted R^2 of `raise_1_fit`.
d. R^2 of `raise_2_fit` is higher R^2 of `raise_1_fit`.

Question 6

The tidy model output for the `raise_2_fit` model you fit is shown below.

```
                                    Coef.  Std.Err.         t  \
Intercept                        2.617865  0.452366  5.787046
performance_rating_Poor[T.True] -3.499070  1.500230 -2.332356
performance_rating_Successful[T.True] -1.730554  0.361704 -4.784449
performance_rating_Top[T.True]   3.628454  0.730187  4.969215
annual_salary                    0.000013  0.000004  3.119855

                                       P>|t|    [0.025    0.975]
Intercept                       1.543384e-08  1.728293  3.507437
performance_rating_Poor[T.True] 2.022505e-02 -6.449249 -0.548892
performance_rating_Successful[T.True]  2.493372e-06 -2.441838 -1.019269
performance_rating_Top[T.True]  1.035401e-06  2.192554  5.064355
annual_salary                   1.953363e-03  0.000005  0.000021
```

When your teammate sees this model output, they remark "The coefficient for `performance_ratingSuccessful` is negative, that's weird. I guess it means that people who get successful performance ratings get lower raises." How would you respond to your teammate?

Question 7

Ultimately, your teammate decides they don't like the negative slope coefficients in the model output you created (not that there's anything wrong with negative slope coefficients!), does something else, and comes up with the following model output. *Note however that the coefficient is still negative, but this satisfies your friend…*

```
                                      Coef.  Std.Err.          t  \
Intercept                          1.785333  0.509233  3.505927
performance_rating_Successful[T.True] -0.800356  0.439216 -1.822238
performance_rating_High[T.True]        1.574196  0.475552  3.310248
performance_rating_Top[T.True]         4.569528  0.768132  5.948886
annual_salary                          0.000012  0.000004  2.854297

                                             P>|t|     [0.025    0.975]
Intercept                          5.116857e-04  0.783935  2.786732
performance_rating_Successful[T.True]  6.923704e-02 -1.664068  0.063355
performance_rating_High[T.True]        1.025064e-03  0.639030  2.509362
performance_rating_Top[T.True]         6.327904e-09  3.059009  6.080047
annual_salary                          4.559386e-03  0.000004  0.000020
```

Unfortunately they didn't write their code in a Quarto document, instead just wrote some code in the Console and then lost track of their work. They remember using the `fct_relevel()` function and doing something like the following:

```
blizzard_salary['performance_rating'] = pd.Categorical(
    blizzard_salary['performance_rating'],
    categories=[____],
    ordered=True
)
```

What should they put in the blanks to get the same model output as above?

a. “Poor”, “Successful”, “High”, “Top”
b. “Successful”, “High”, “Top”
c. “Top”, “High”, “Successful”, “Poor”
d. Poor, Successful, High, Top

Question 8

Suppose we fit a model to predict `percent_incr` from `annual_salary` and `salary_type`. A tidy output of the model is shown below.

```
                            Coef.  Std.Err.         t     P>|t|    [0.025  \
Intercept                1.242597  0.570278  2.178932  0.029972  0.121174
salary_type_year[T.True] 0.913343  0.543715  1.679819  0.093844 -0.155845
annual_salary            0.000014  0.000005  2.958979  0.003287  0.000005

                           0.975]
Intercept                2.364020
salary_type_year[T.True] 1.982532
annual_salary            0.000023
```

Which of the following visualizations represent this model? Explain your reasoning.

Visualizations of the relationship between percent increase, annual salary, and salary type

a. Figure 1
b. Figure 2
c. Figure 3
d. Figure 4


20%
15%
10%
5%
0%
Percent increase
$0.00
$50000.00
$100000.00
$150000.00
$200000.00
Annual salary
Salary type
Hourly
year

Figure 1


20%
15%
10%
5%
0%
Percent increase
$0.00
$50000.00
$100000.00
$150000.00
$200000.00
Annual salary
salary_type
Hourly
year

Figure 2

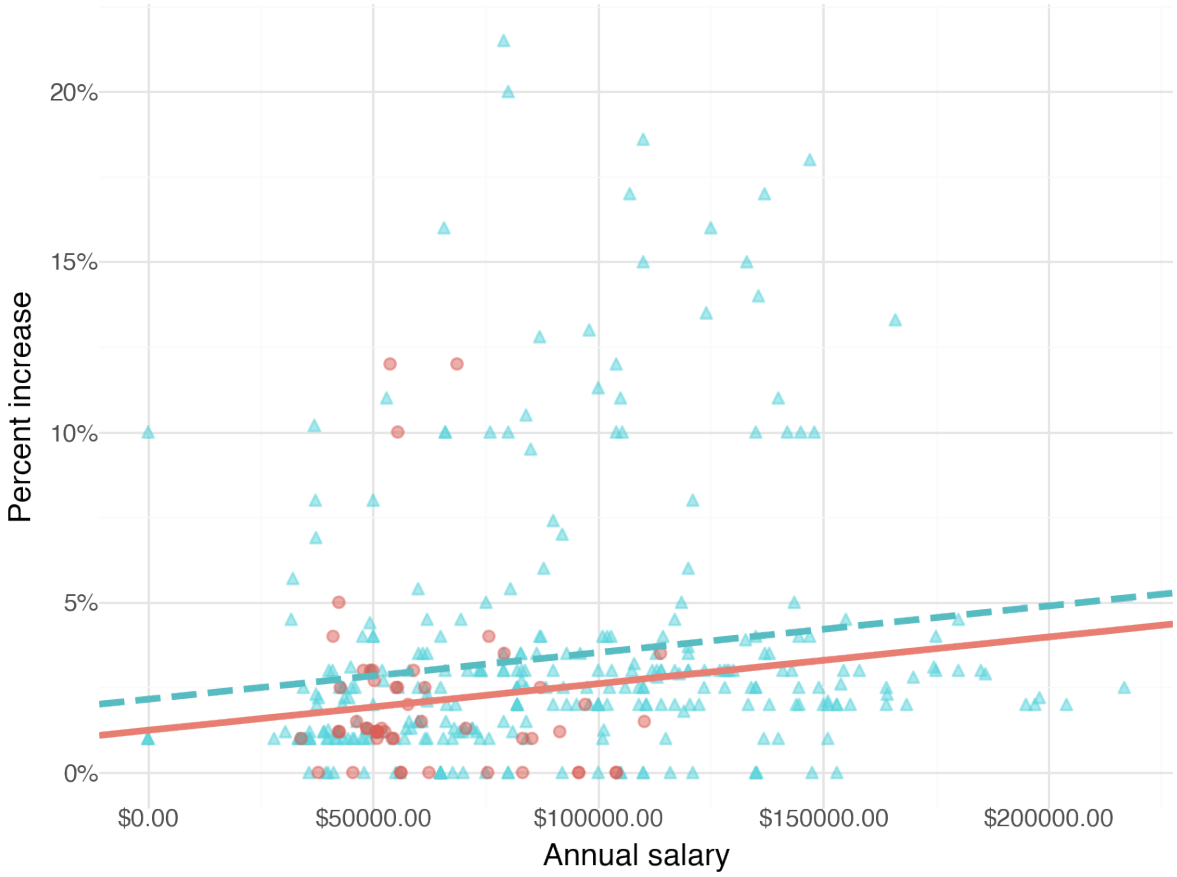

20%
15%
10%
5%
0%
Percent increase
$0.00
$50000.00
$100000.00
$150000.00
$200000.00
Annual salary

Figure 3

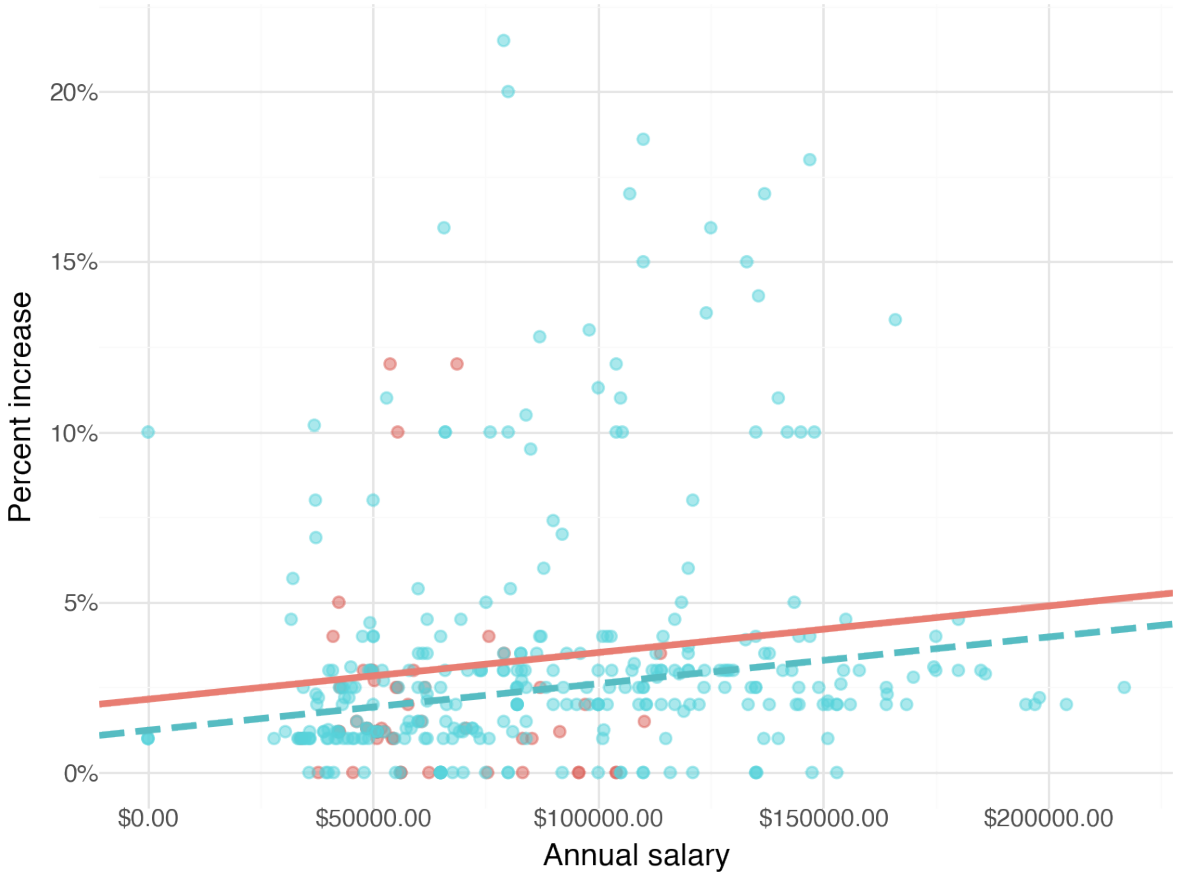

20%
15%
10%
5%
0%
Percent increase
$0.00
$50000.00
$100000.00
$150000.00
$200000.00
Annual salary

Figure 4

Question 9

Define the term parsimonious model.

Question 10

Suppose you now fit a model to predict the natural log of percent increase, `log(percent_incr)`, from performance rating. The model is called `raise_4_fit`.

You're provided the following:

```
raise_4_fit_coefs['exp_estimate'] = np.exp(raise_4_fit_coefs['estimate'])
print(raise_4_fit_coefs)
```

```
                           term  estimate  exp_estimate
0                     Intercept -2.088594      0.123861
1  performance_rating_Successful  1.872176      6.502427
2        performance_rating_High  3.110229     22.426176
3         performance_rating_Top  3.854360     47.198390
```

Based on this, which of the following is true?

a. The model predicts that the percentage increase employees with Successful performance get, on average, is higher by 10.25% compared to the employees with Poor performance rating.

b. The model predicts that the percentage increase employees with Successful performance get, on average, is higher by 6.93% compared to the employees with Poor performance rating.

c. The model predicts that the percentage increase employees with Successful performance get, on average, is higher by a factor of 6.502427 compared to the employees with Poor performance rating.

d. The model predicts that the percentage increase employees with Successful performance get, on average, is higher by a factor of 1.872176 compared to the employees with Poor performance rating.

Question 11

Which of the following is the definiton of a regression model? Select all that apply.

a. $\hat{y} = \beta_0 + \beta_1 X_1$

b. $y = \beta_0 + \beta_1 X_1$

c. $\hat{y} = \beta_0 + \beta_1 X_1 + \epsilon$

d. $y = \beta_0 + \beta_1 X_1 + \epsilon$

Part 3 - Calculus

Question 12

Compute the derivative ($\frac{d}{dx}$) of the following function:

$$g(x) = \left(\sin(x^2) + \cos(ax)\right)^k$$

Question 13

Compute the following integral:

$$\int_a^b \left(e^{cx} + \frac{1}{x^n}\right) dx$$

Part 4 - Linear algebra

Question 14

Given a vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, write down its transpose $x^\top$.

Question 15

Given the following matrix N:

$$N = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \\ n_{31} & n_{32} \\ n_{41} & n_{42} \end{bmatrix}$$

Write down its transpose, $N^\top$.

Question 16

Consider the following matrices C and D:

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}, \quad D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \end{bmatrix}$$

1. What are the dimensions of C?
2. What are the dimensions of D?
3. For the matrix product CD:
    1. Determine if the product is valid, and explain why.
    2. If the product is valid, write down the dimensions of the resulting matrix without computing the product.

Question 17

Given the matrices E and F:

$$E = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \\ e_{31} & e_{32} \end{bmatrix}, \quad F = \begin{bmatrix} f_{11} \\ f_{21} \end{bmatrix}$$

1. What are the dimensions of E?
2. What are the dimensions of F?
3. For the matrix product EF:
    1. Determine if the product is valid, and explain why.
    2. If the product is valid, compute the resulting matrix.

Bonus

Pick a concept we introduced in class so far that you've been struggling with and explain it in your own words.