Time Series Analysis

INFO 523 - Lecture 12

Dr. Greg Chism

Lesson 2:

Processing Timeseries Data

Lagging Values

- While analyzing time series, we often refer to values that our time series took **1**, **2**, **3**, etc., time steps in the past
- These are known as lagged values and denoted:

$$X_{t-l}$$

- where l is the value of the lag we are considering.

Lagging Values

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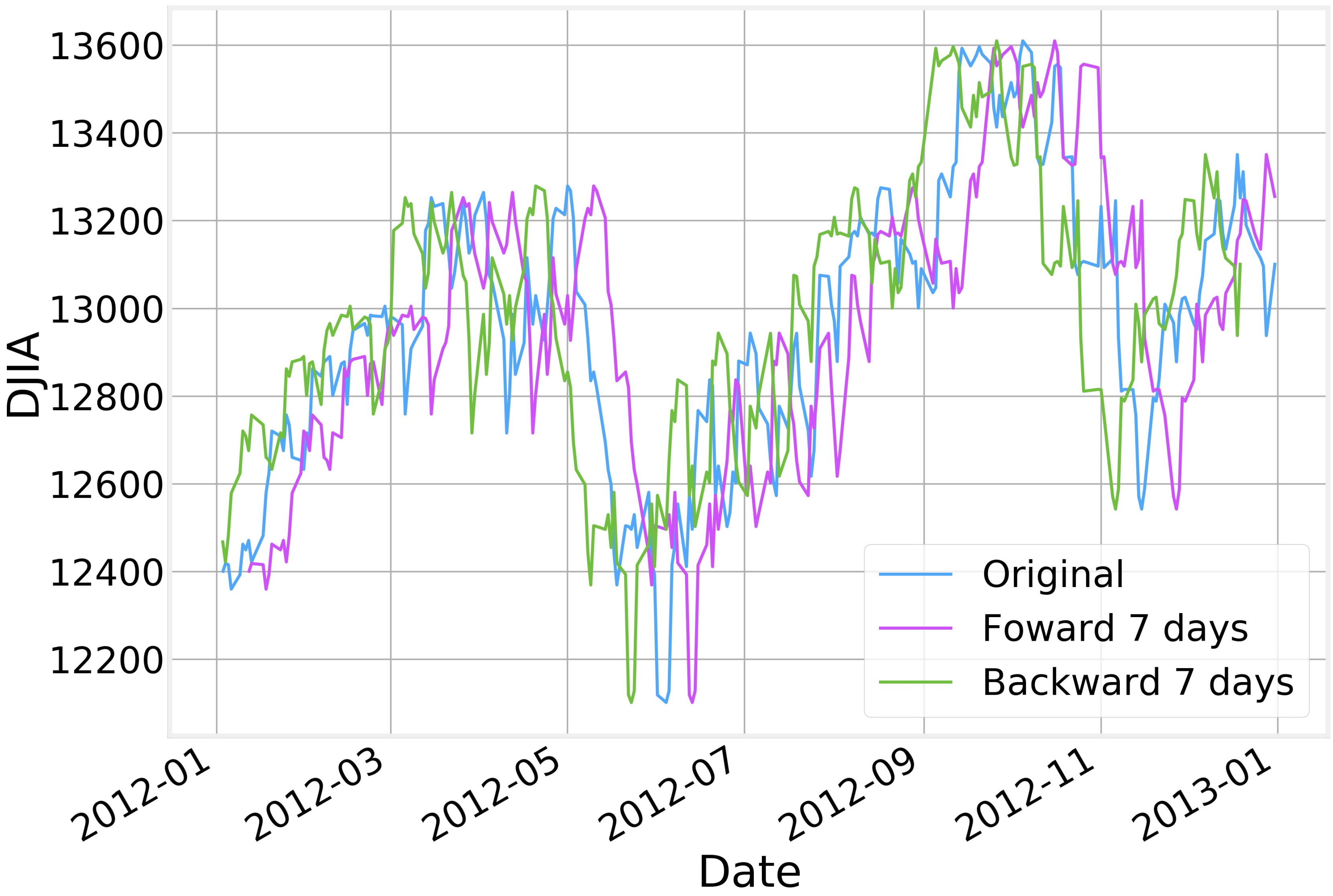


Differences

- Perhaps the most common use case for lagged values is for the calculation of differences of the form:

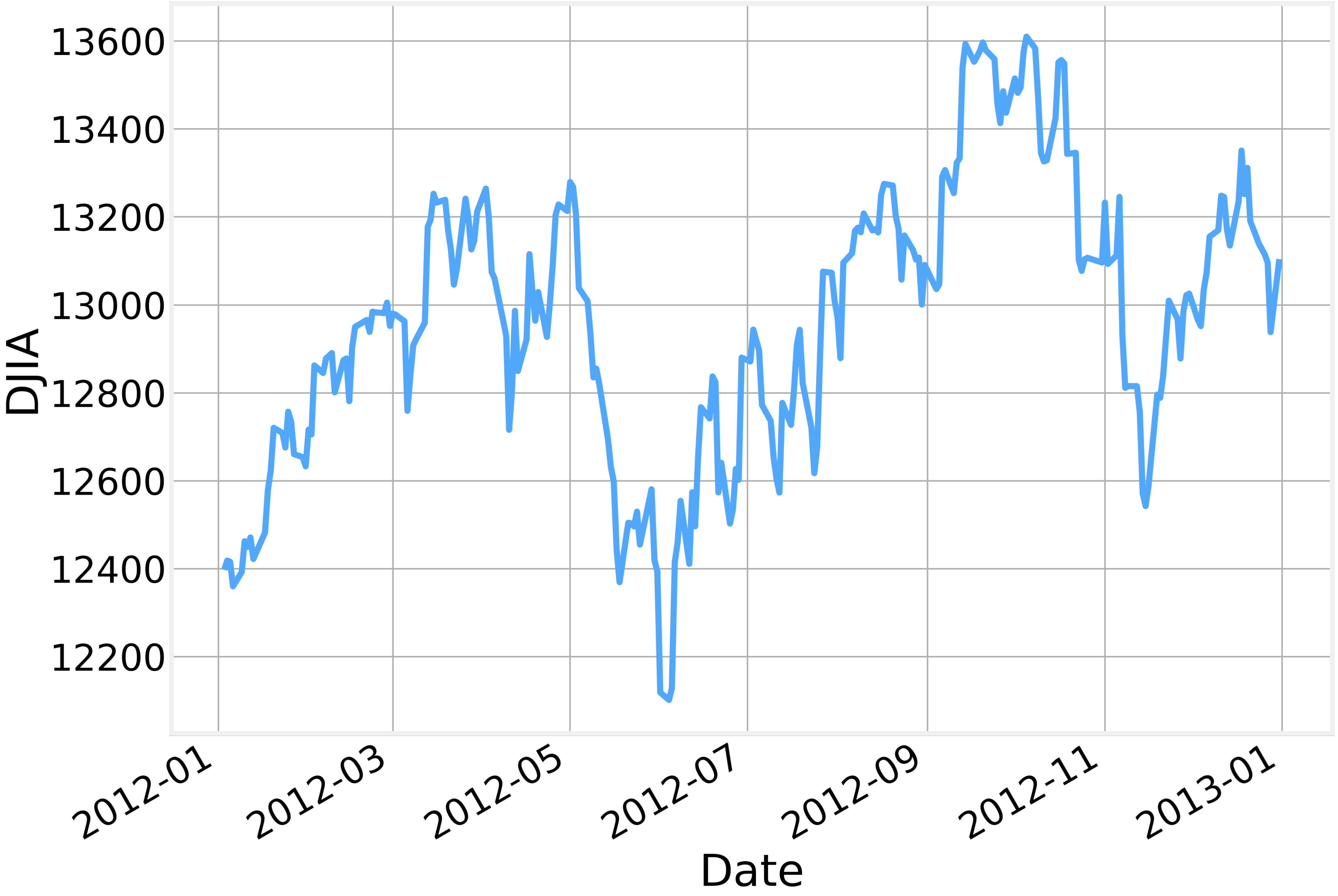
$$X_t - X_{t-l}$$

- Where $l \geq 1$ is the value of the lag we are interested in.
- Naturally, higher order differences can also be used, in which case, the difference of the difference is calculated:

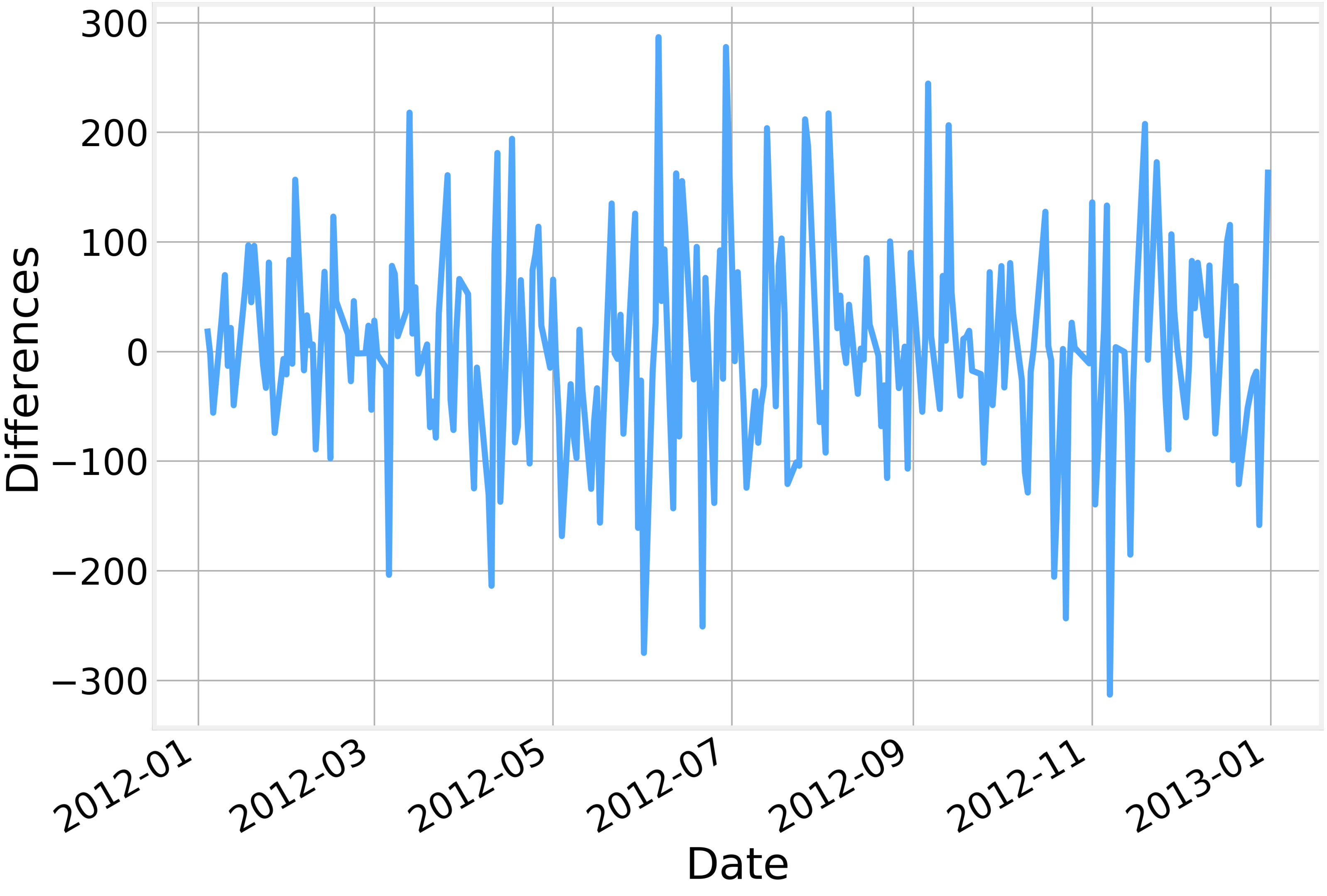
$$y_t = x_t - x_{t-l}$$
$$z_t = y_t - y_{t-l} \equiv x_t - 2x_{t-l} + x_t - 2l$$

- This can be thought of as a discrete version of the usual derivative of a function.
- Differences are also a particularly simple way to detrend a time series

Differences



Differences



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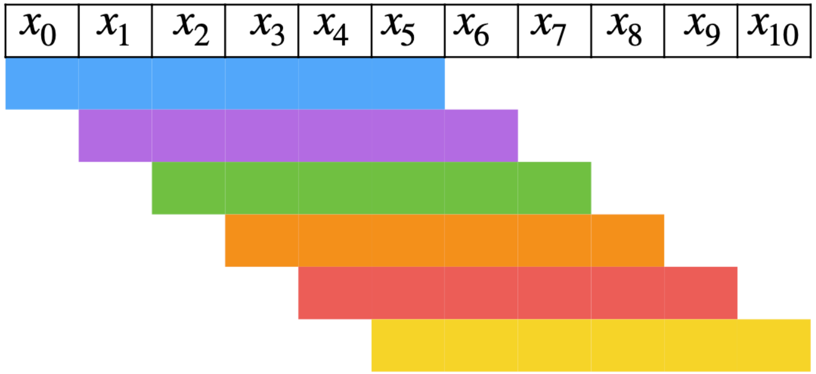
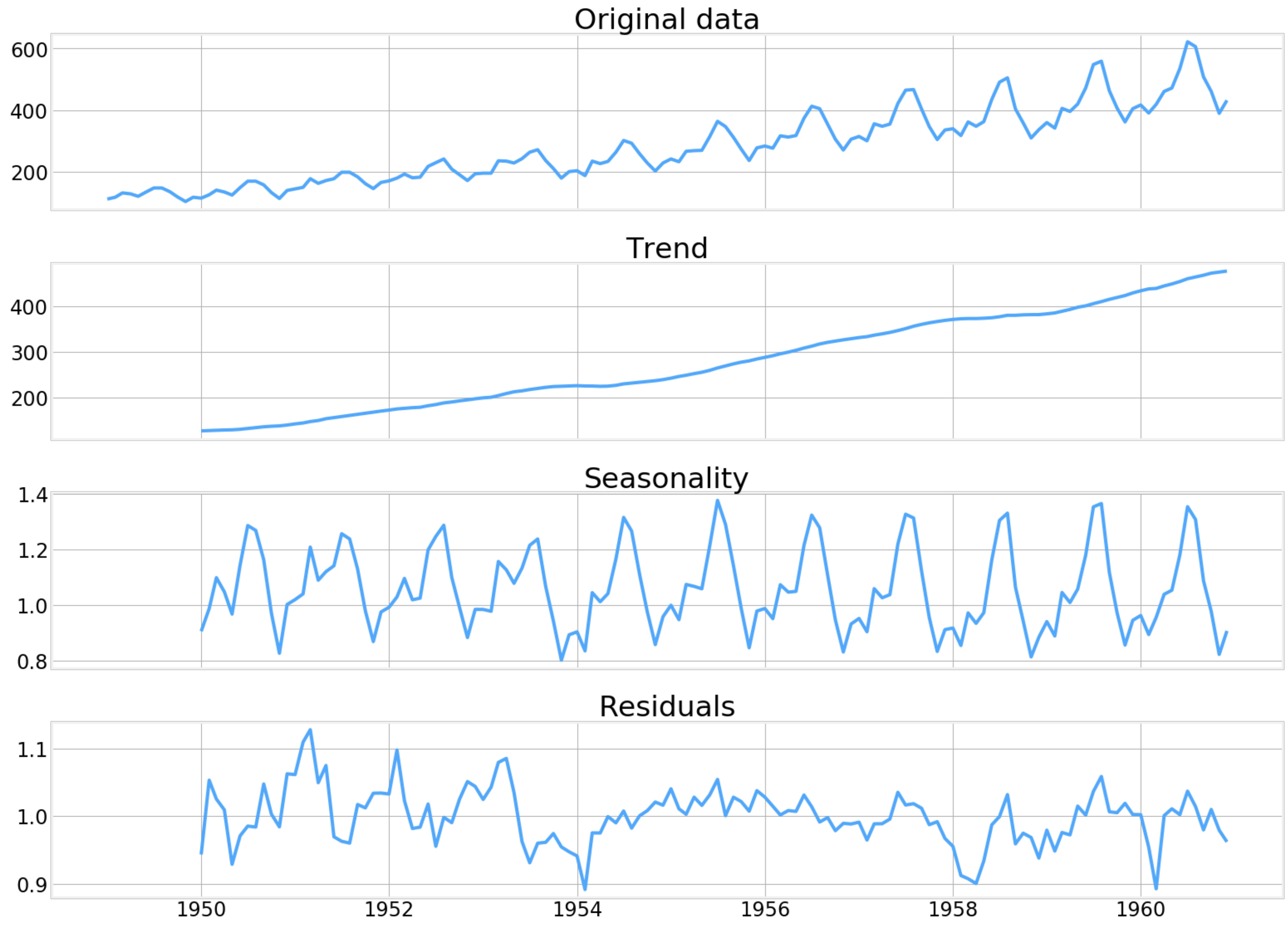


$$= \begin{matrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \\ x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \end{matrix}$$

Windowing



Original data
600
400
200
Trend
400
300
200
Seasonality
1.4
1.2
1.0
0.8
Residuals
1.1
1.0
0.9
1950
1952
1954
1956
1958
1960

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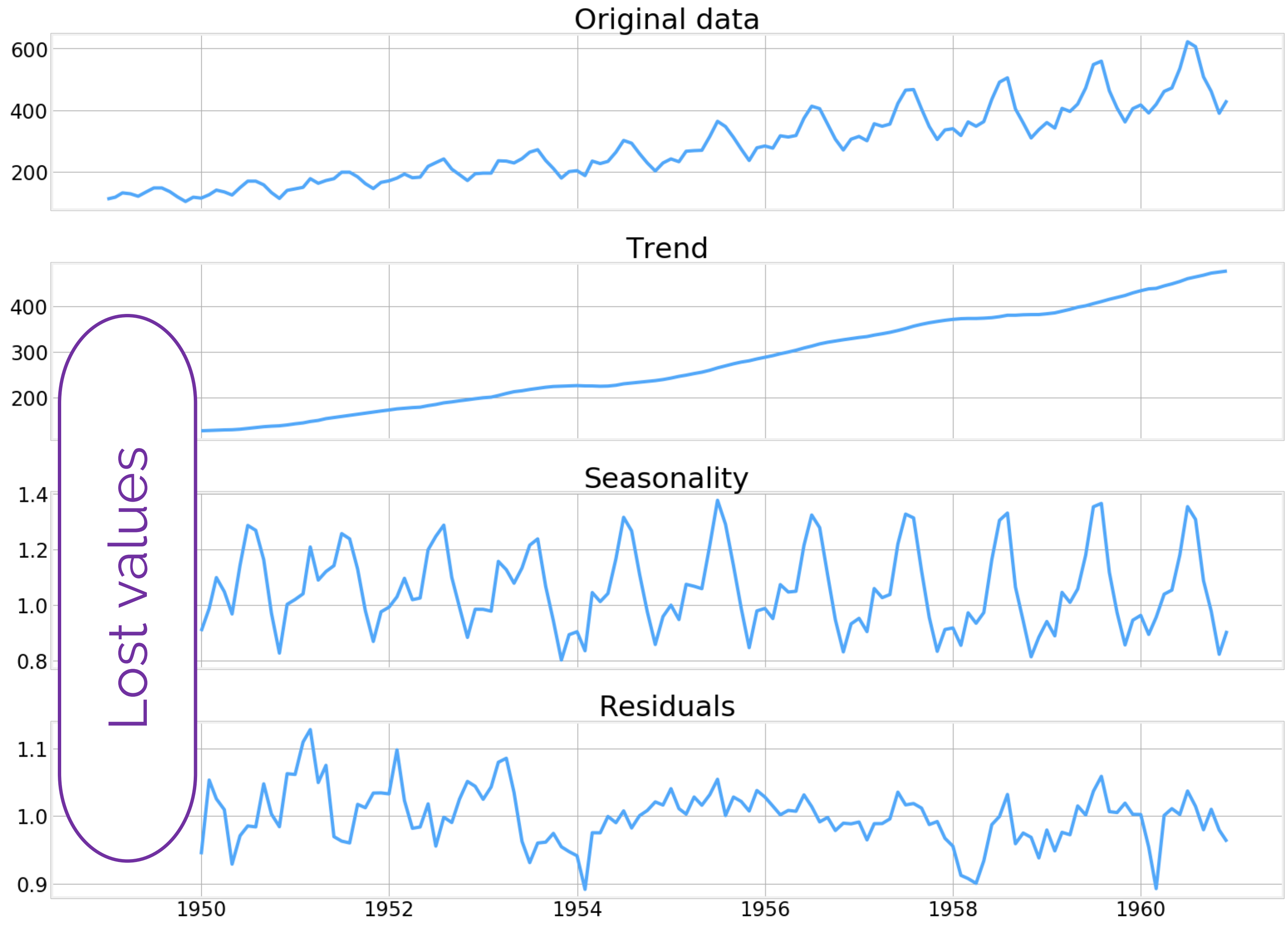

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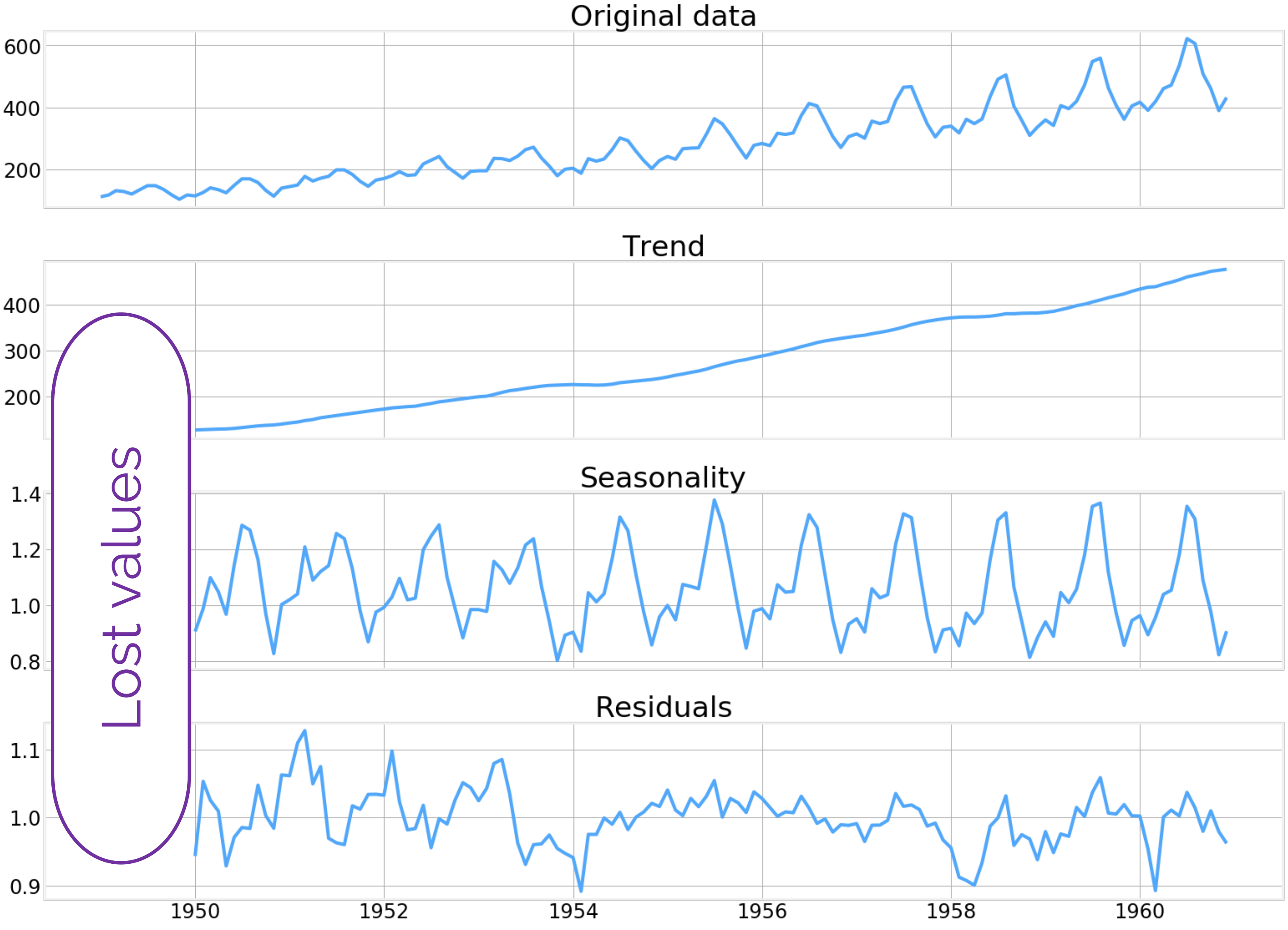
Lost values

Windowing

One common approach is to place all "lost values" at the beginning as it avoids "future leaking" when splitting the dataset



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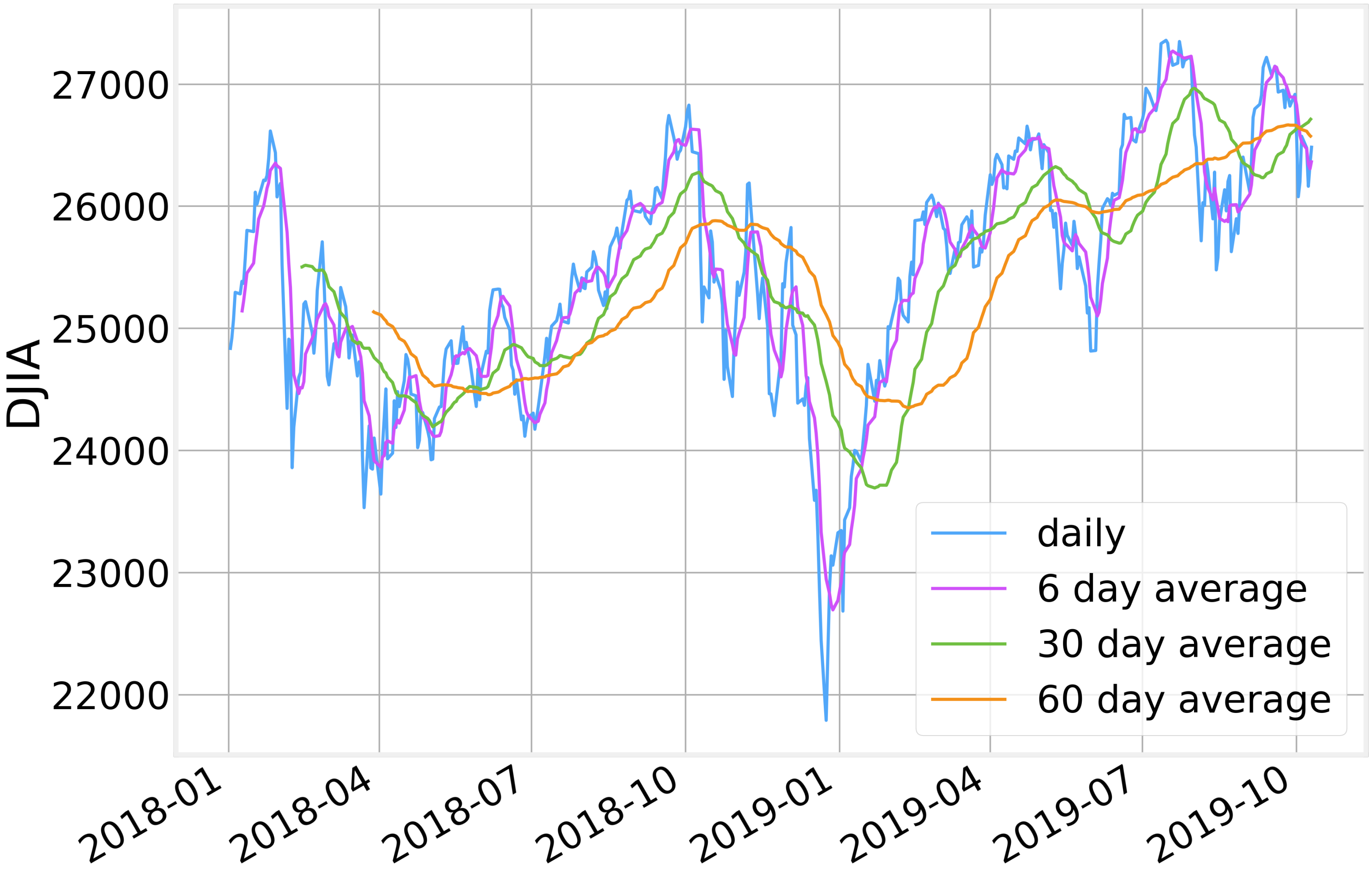
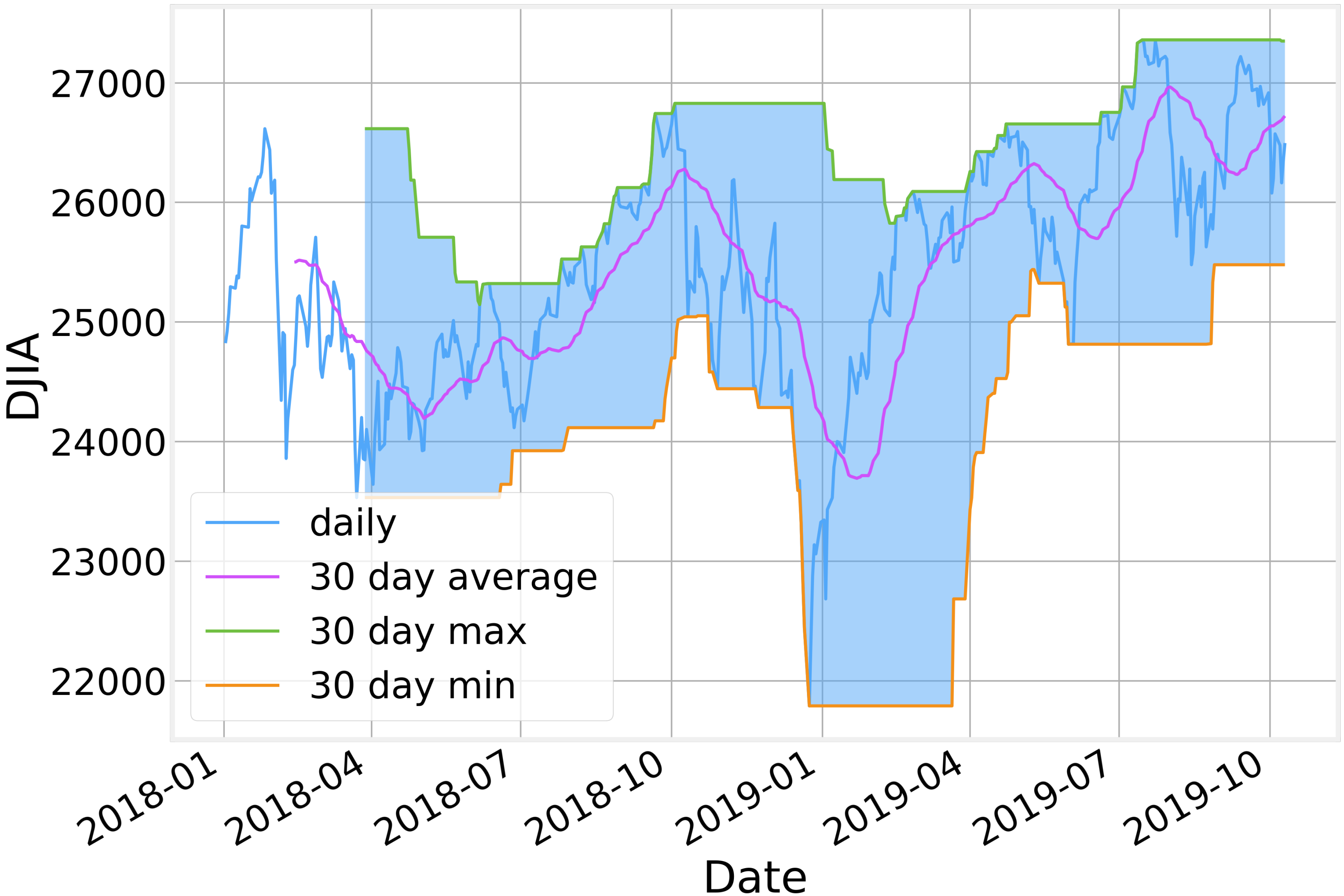
- Depending on the application we can choose to place the missing values in either or (or even both) extremes of the time interval

Running Values



Envelopes

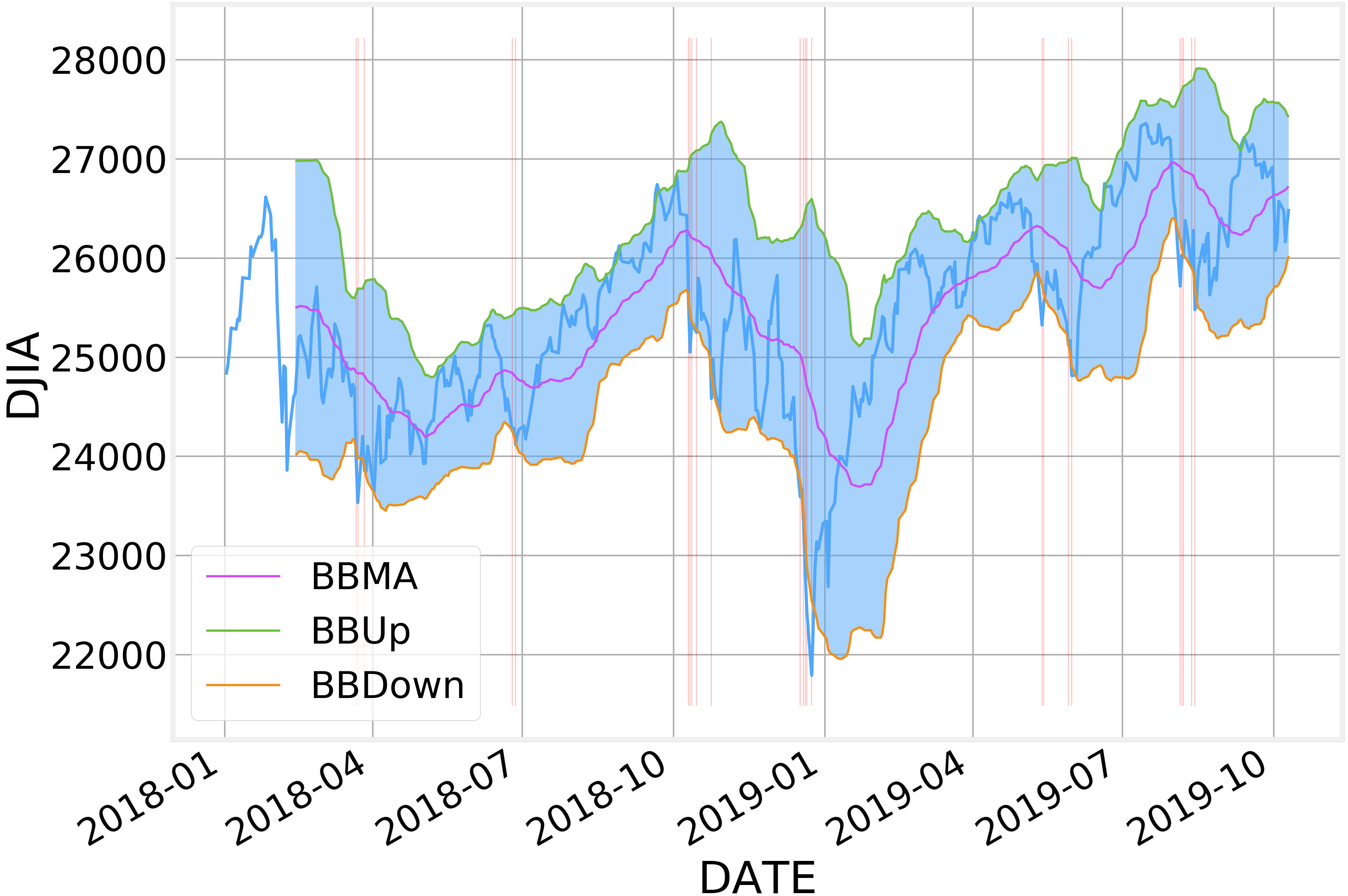


27000
26000
25000
24000
23000
22000
DJIA
2018-01
2018-04
2018-07
2018-10
2019-01
2019-04
2019-07
2019-10
Date
daily
30 day average
30 day max
30 day min

Bollinger Bands

- A common use for application for running values is the calculation of Bollinger Bands.
- Introduced by John Bollinger in the 1980s as a complement to more traditional time series technical analysis techniques.
- Bollinger Bands are defined by two components:
  - A $\boldsymbol{N}$ period moving average, μ_N
  - The area $\boldsymbol{K}$ standard deviations above and below the moving average $\mu_N \pm K\sigma_N$
- Both μ_N and σ_N are computed on a running window of size $\boldsymbol{N}$
- The values $\boldsymbol{N}$ and $\boldsymbol{K}$ are application specific. For stock trading, $\boldsymbol{N = 20}$ and $\boldsymbol{K = 2}$
- Whenever the time series steps out of the Bollinger Band that's a clear indication of a change in the temporal behavior.

Bollinger Bands



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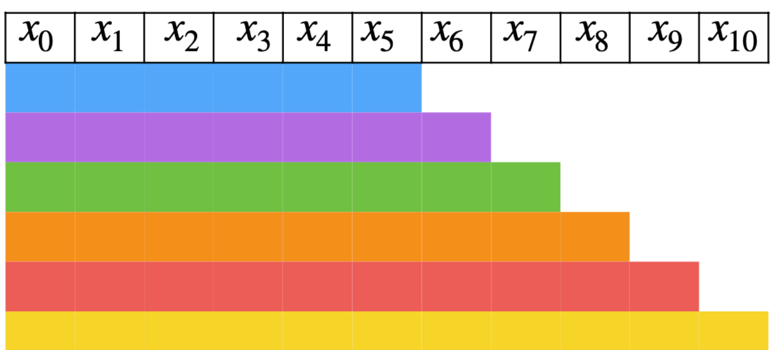

$$\begin{pmatrix} \alpha & & & & \\ \alpha(1-\alpha)^1 & \alpha & & & \\ \alpha(1-\alpha)^2 & \alpha(1-\alpha)^1 & \alpha & & \\ \vdots & \vdots & \vdots & \ddots & \\ \alpha(1-\alpha)^{n-1} & \alpha(1-\alpha)^{n-2} & \alpha(1-\alpha)^{n-3} & & \alpha \end{pmatrix}$$

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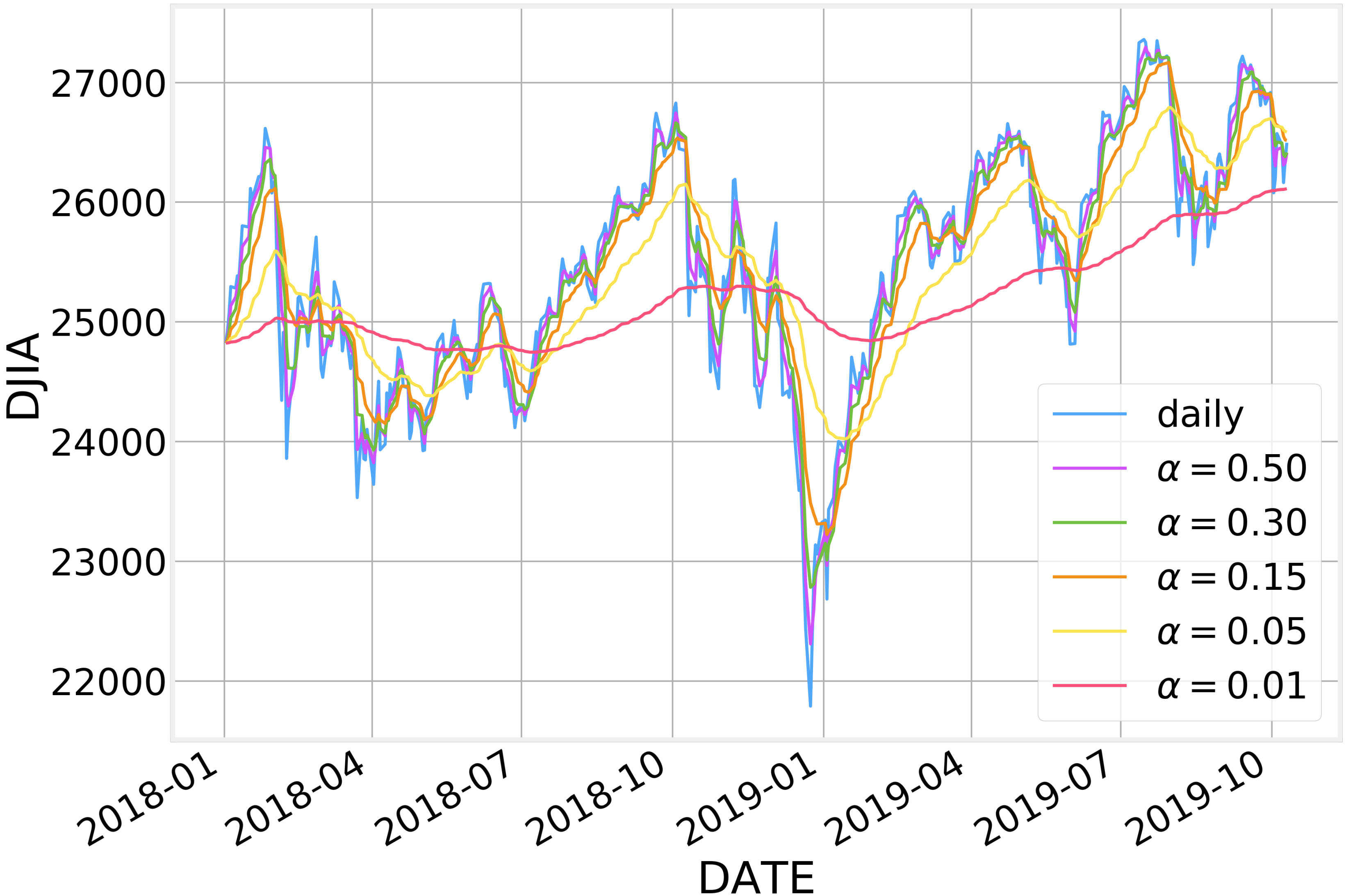

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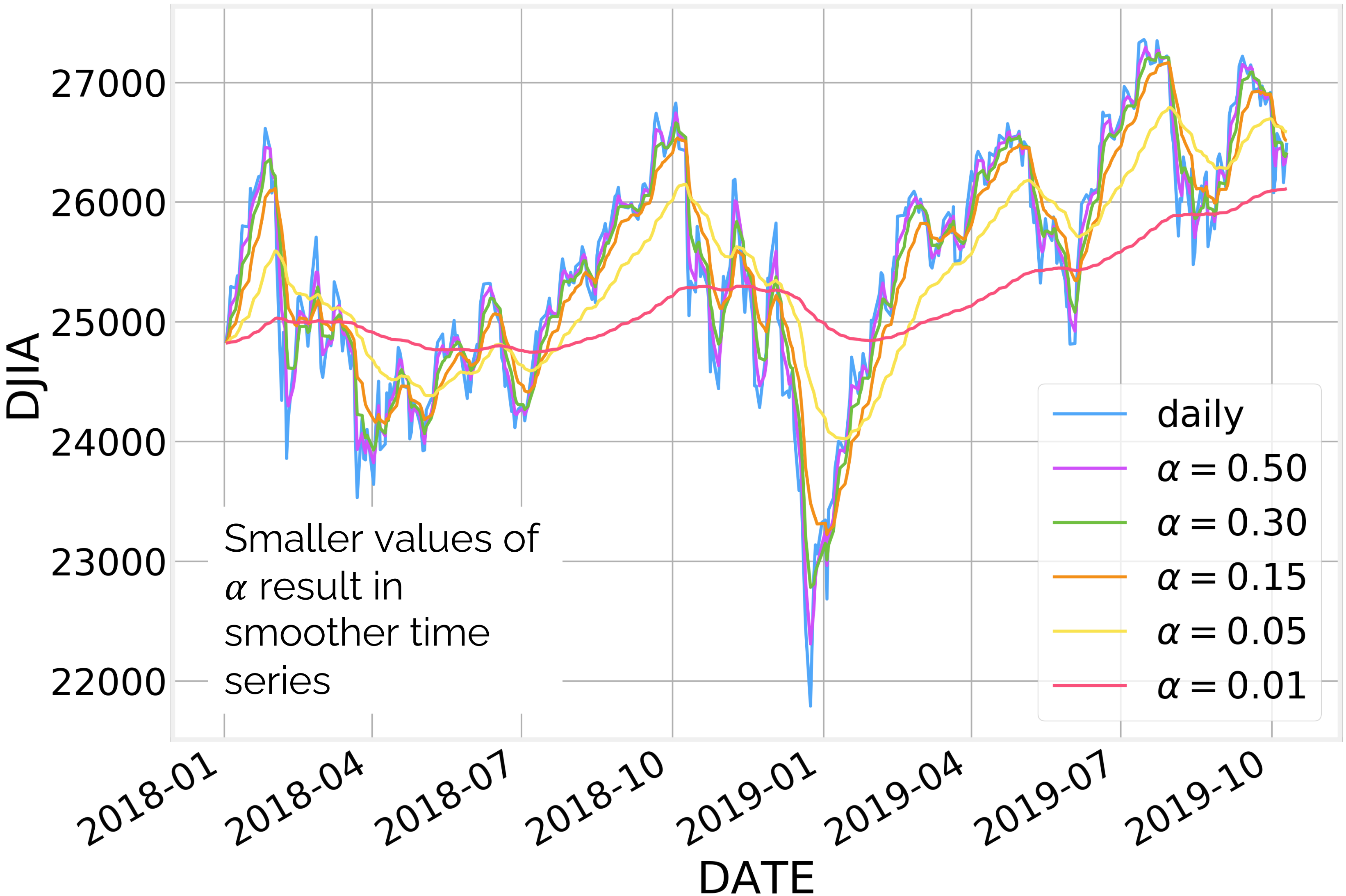

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- The value at time $t + 1$ is given by:

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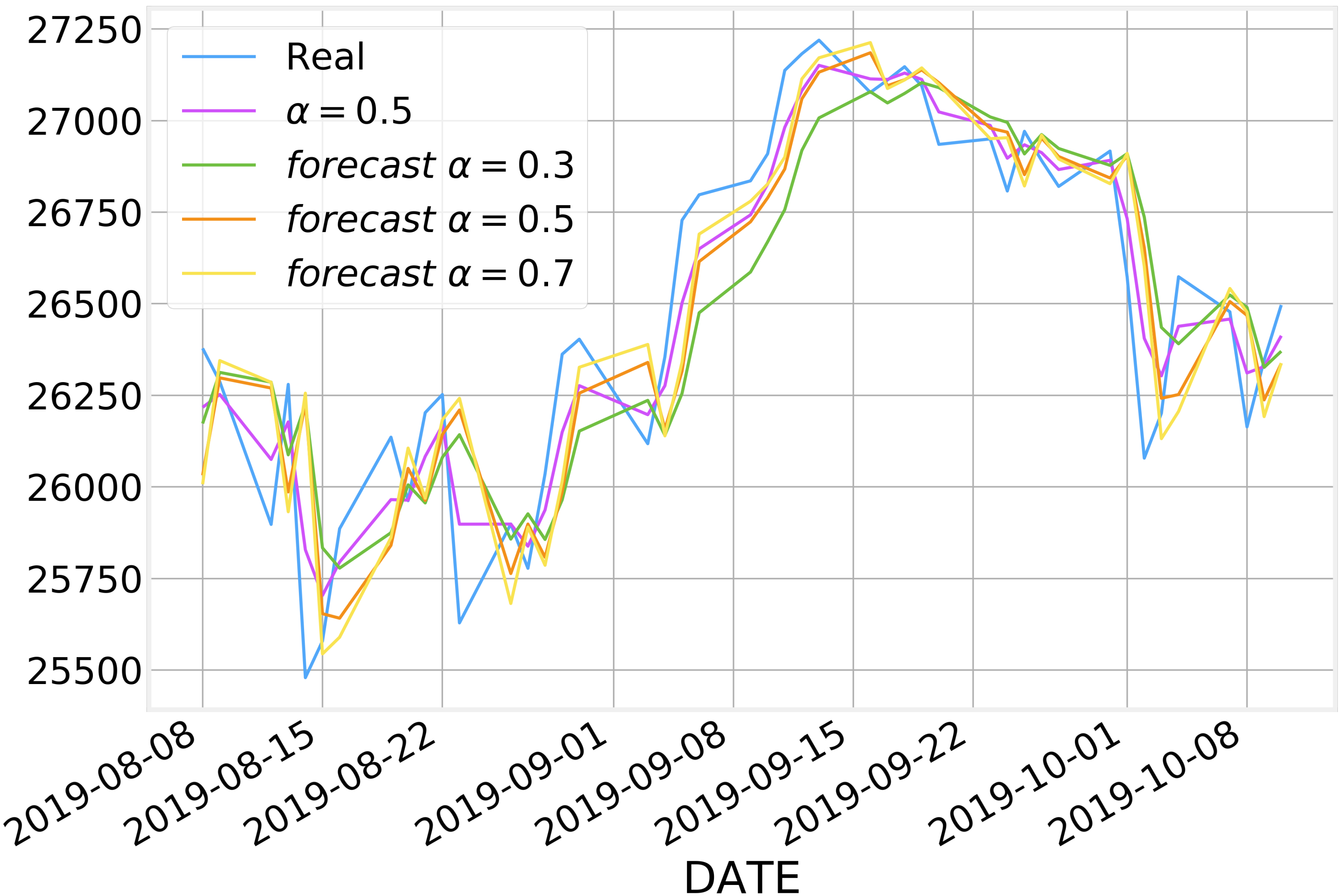
- Which we can consider to be a prediction on the value of x_{t+1}, based on the current value of z_t and some factor of out current error value $x_t - z_t$:

$$z_{t+1} = z_t + \alpha(x_t - z_t)$$

Forecasting



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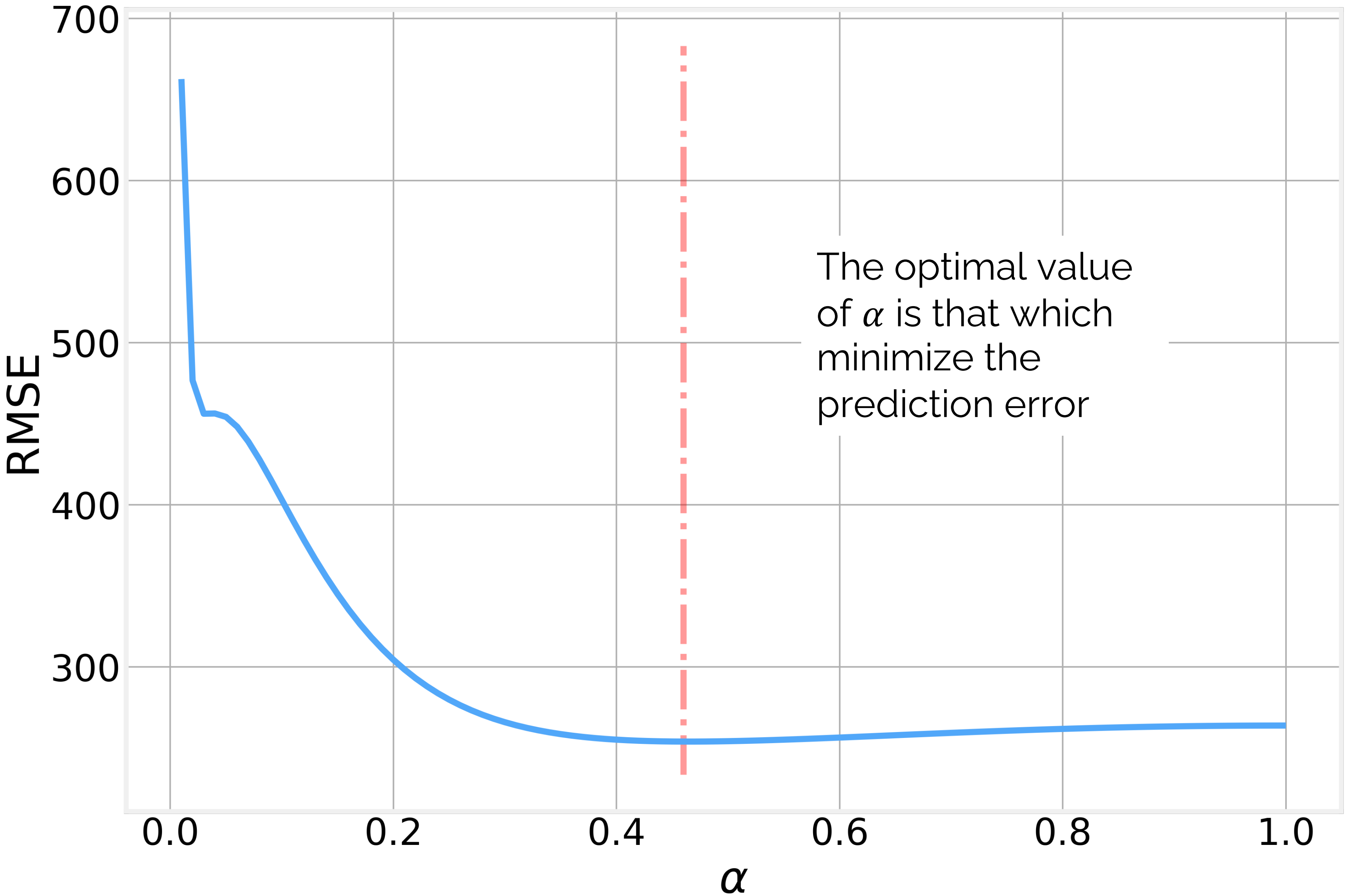

Fill Methods

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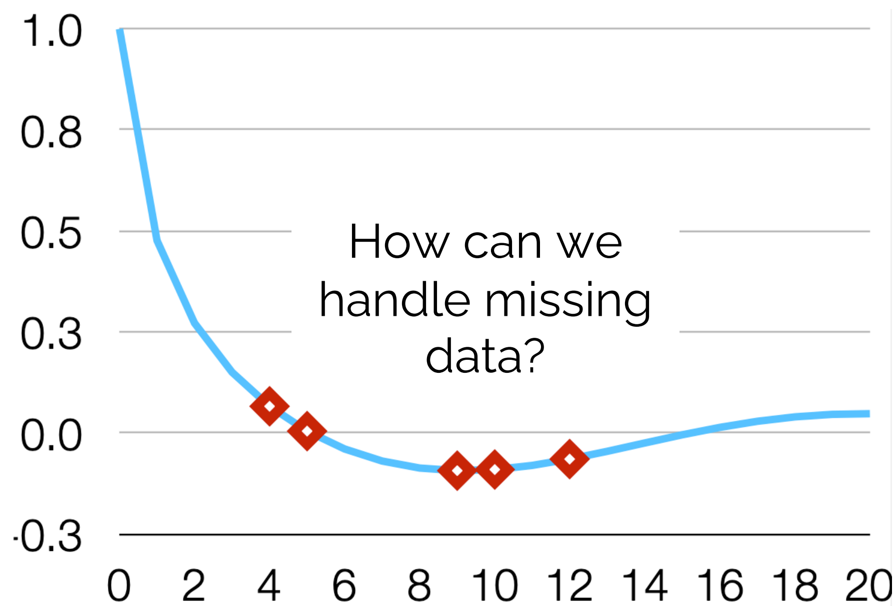

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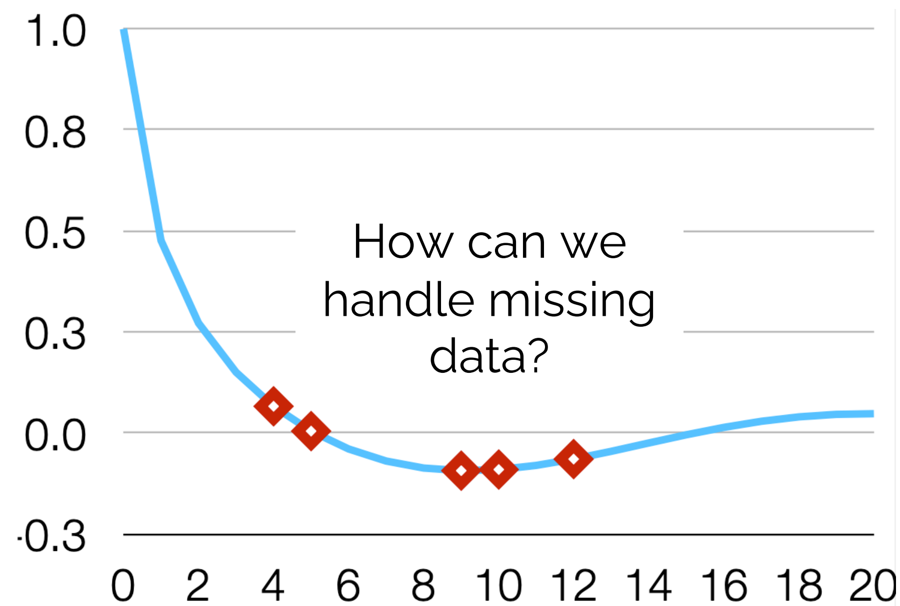

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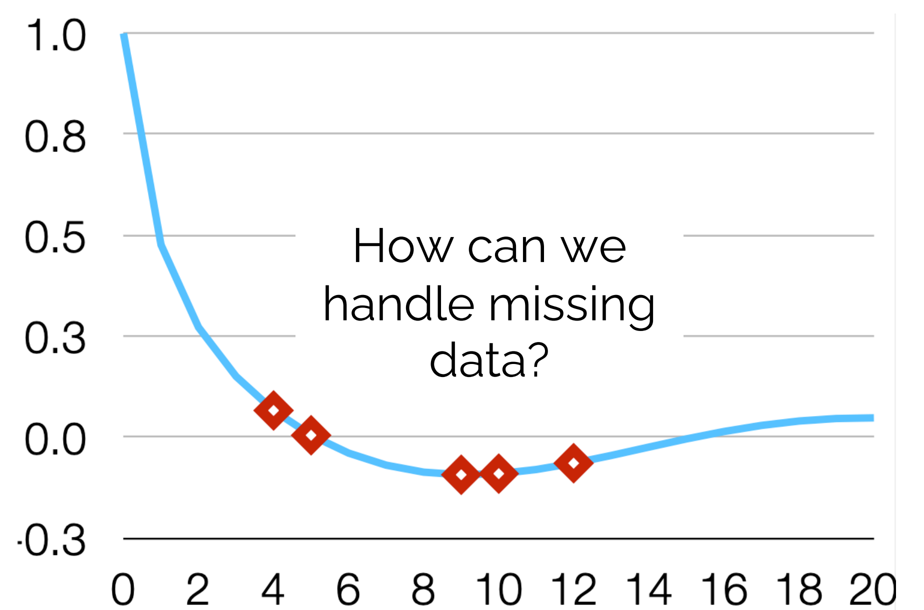

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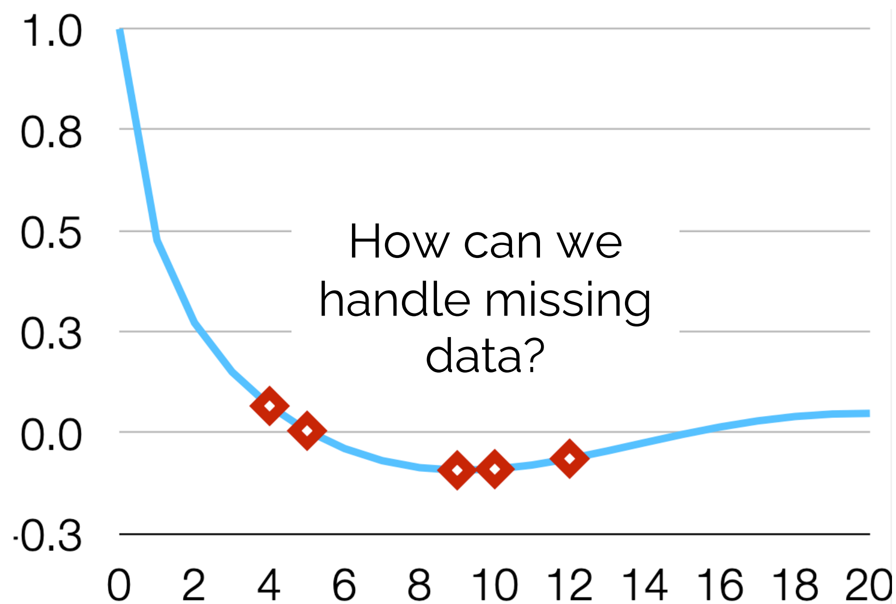

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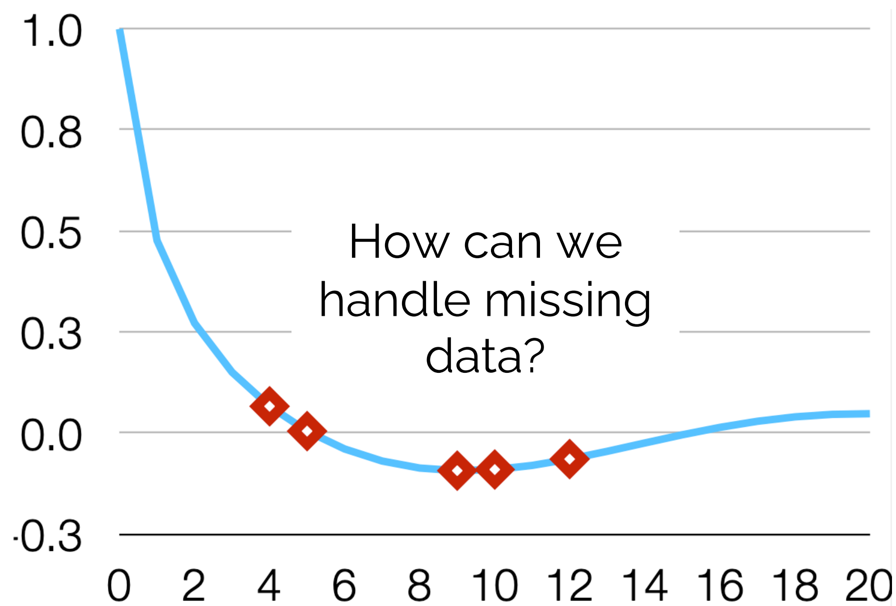

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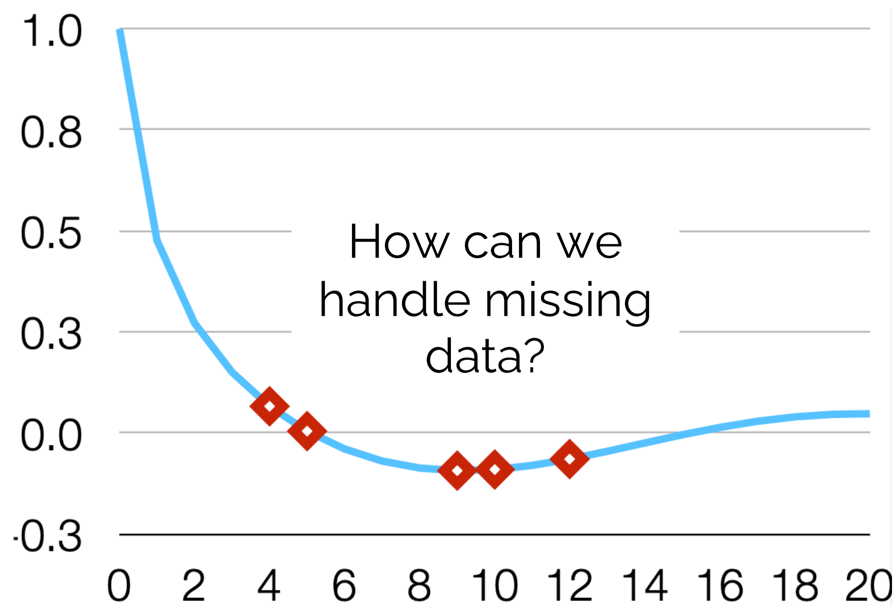

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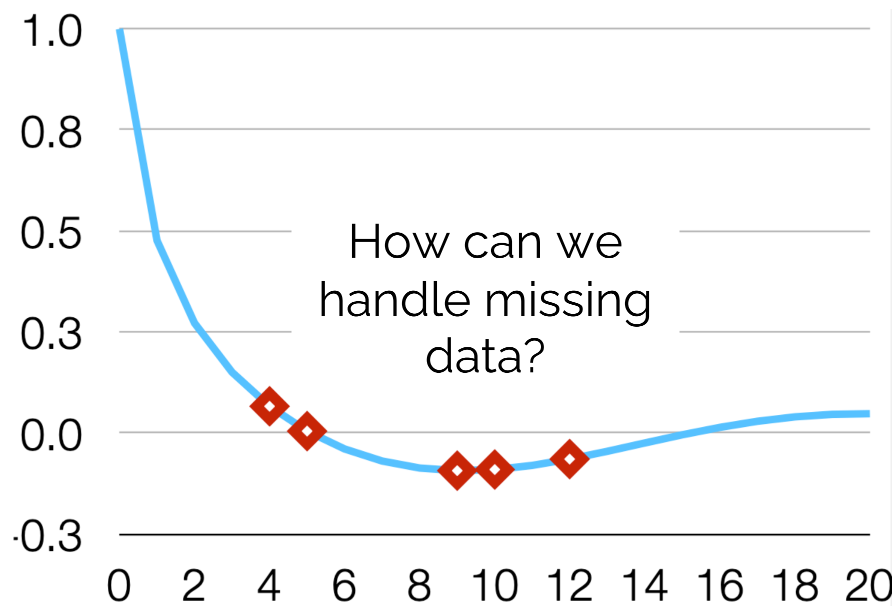

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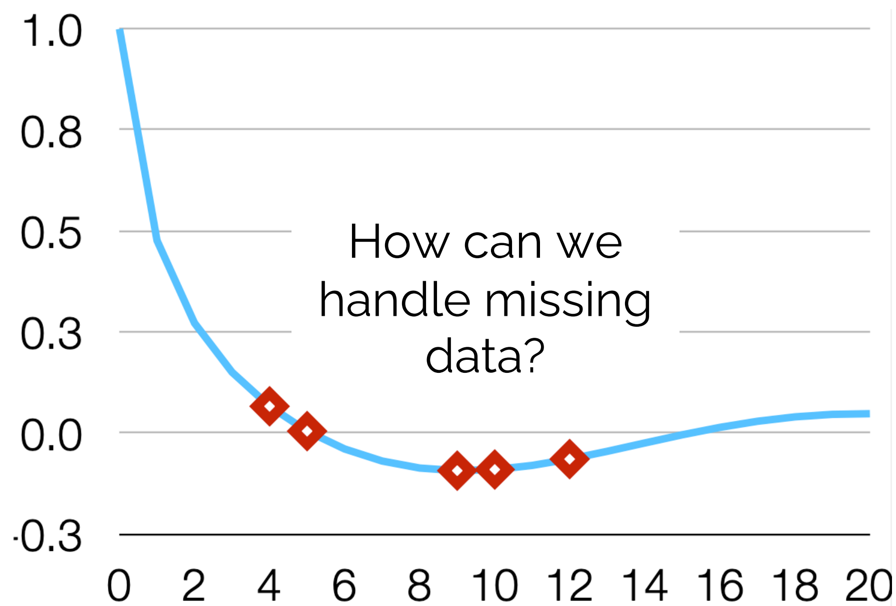

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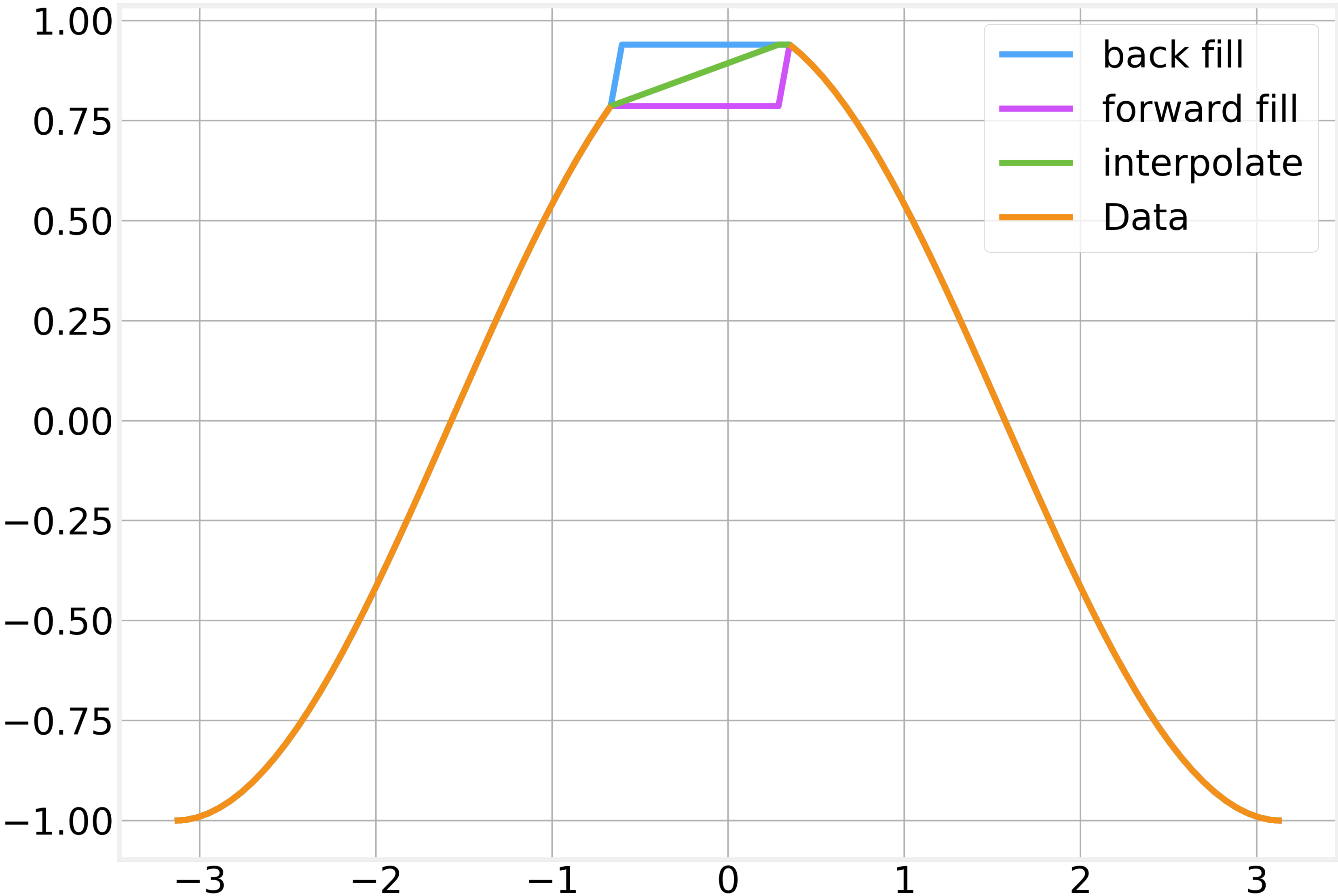

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  - Going from weekly to daily frequency requires specifying how to allocate the values for each day of the week

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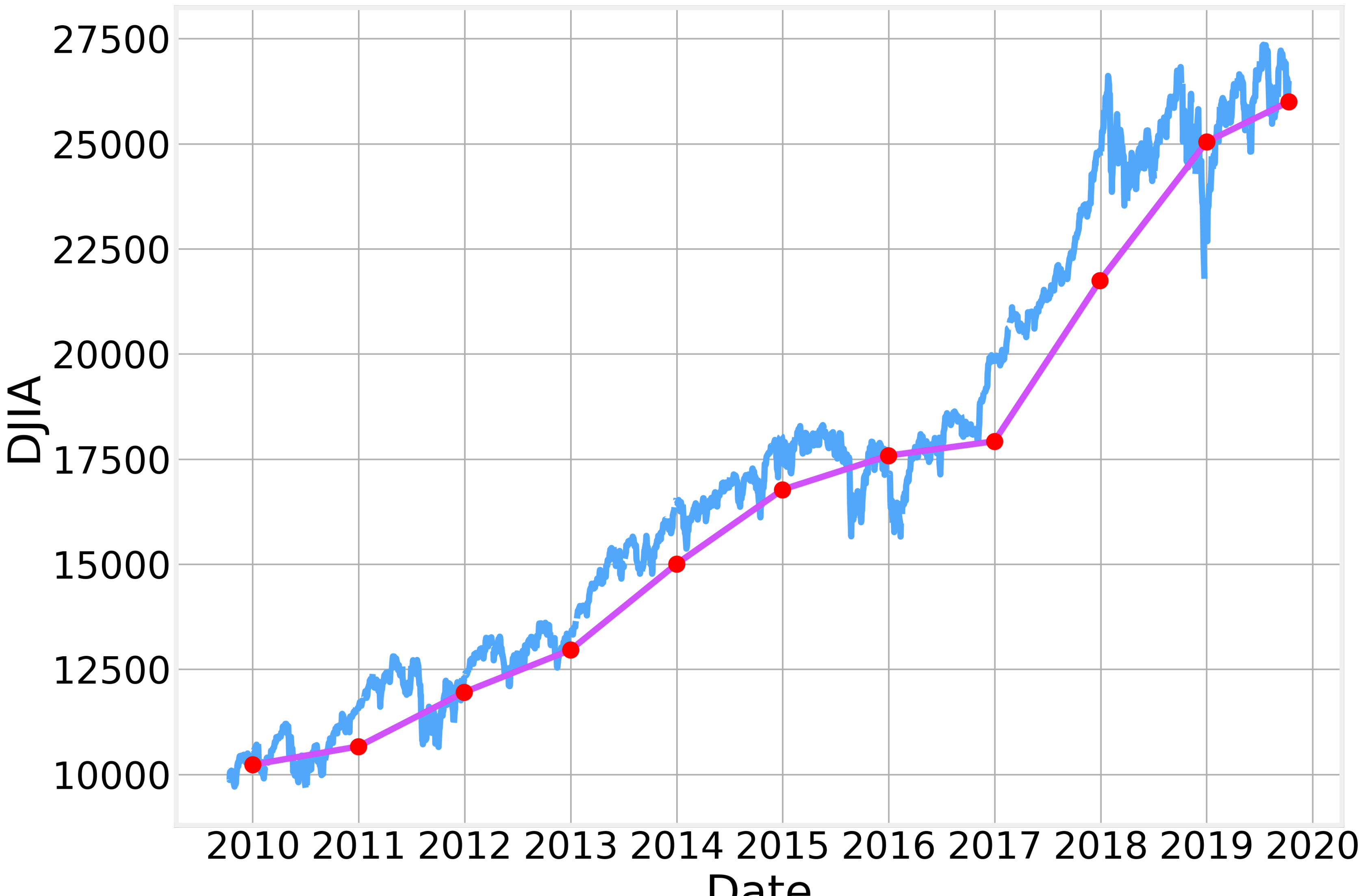

27500
25000
22500
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Date

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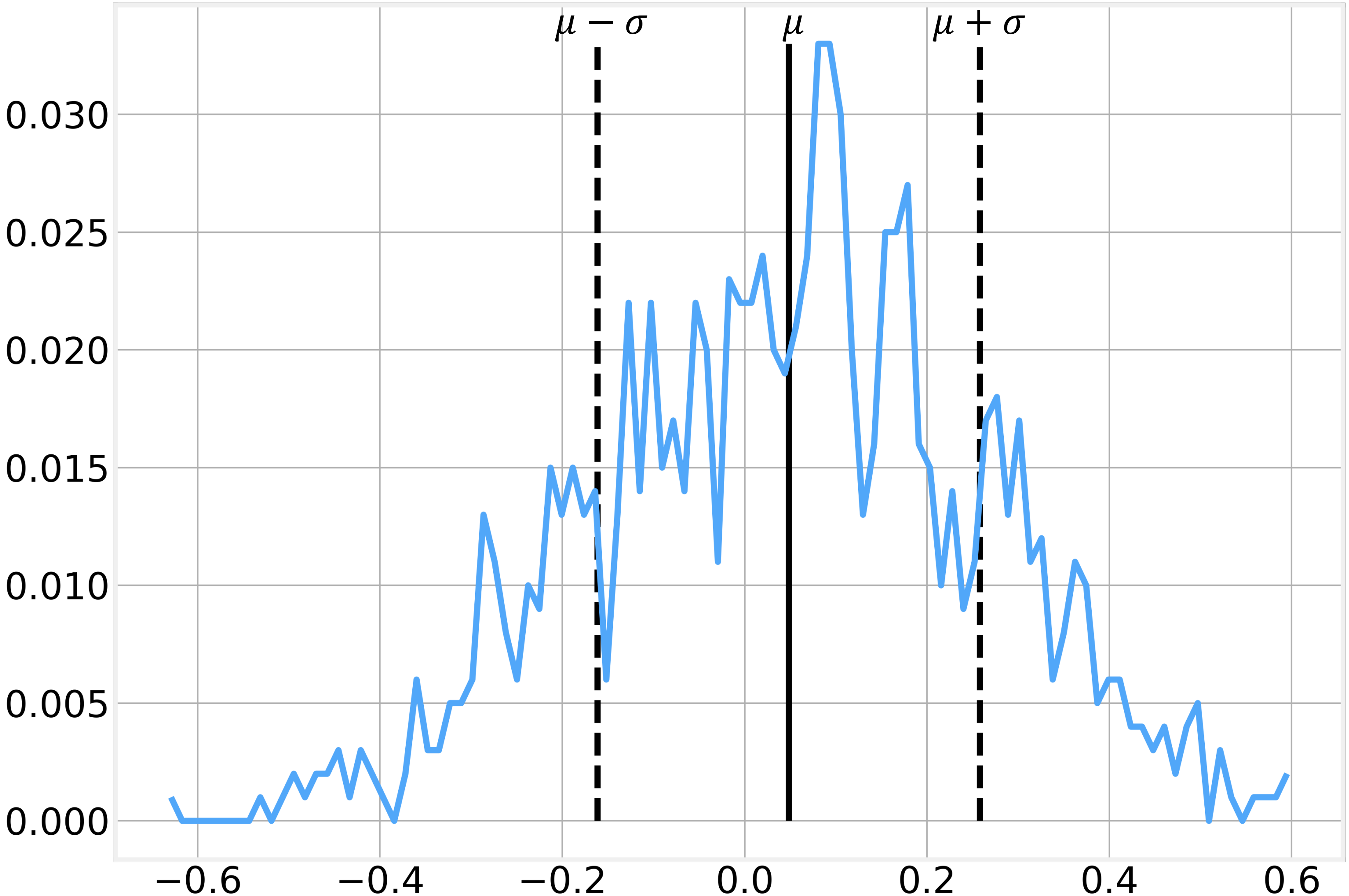

Code – Transformations