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Perturbation-based thresholding search for packing equal circles and spheres

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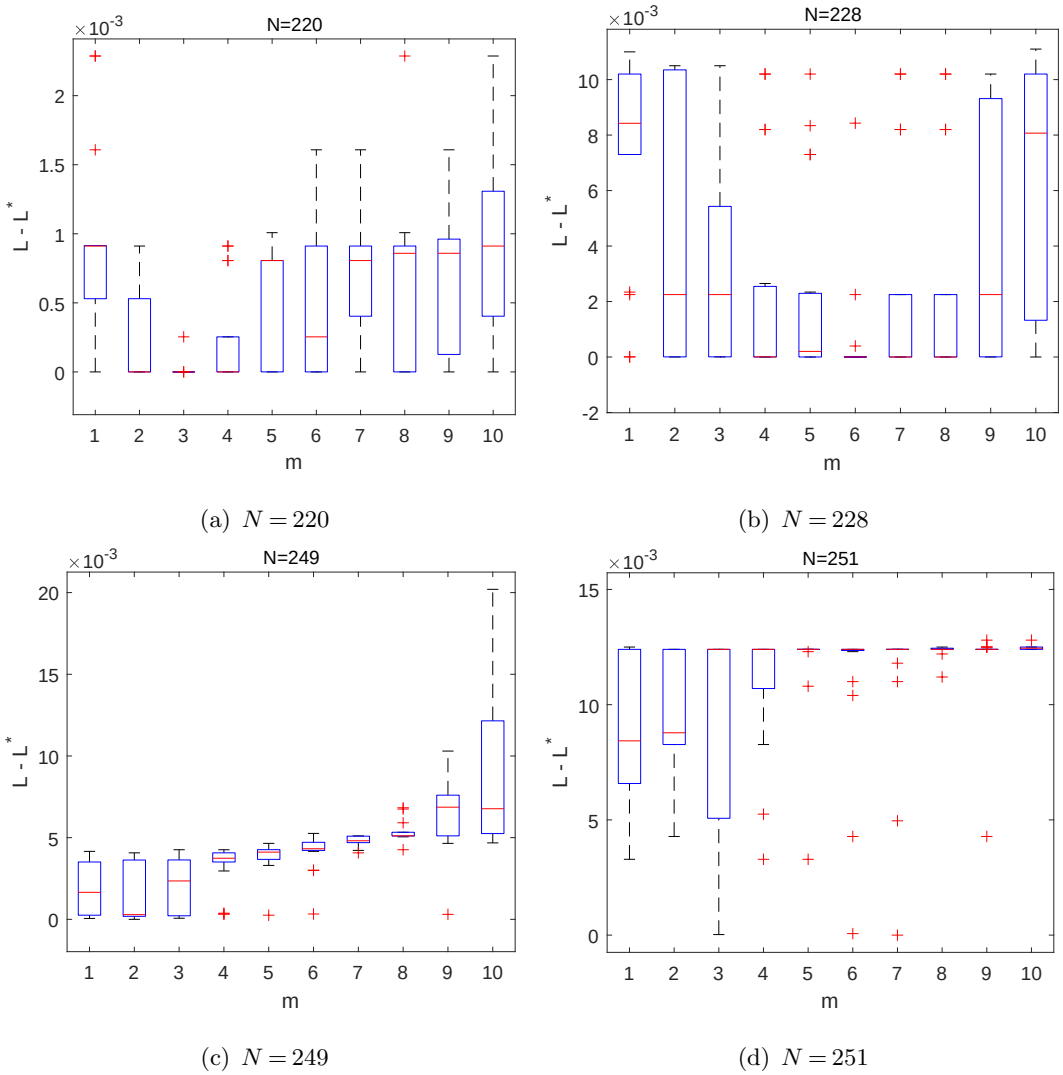
This online supplement provides the sensitivity analysis for two key parameters and the detailed computational results of the proposed PBTS algorithm on the studied PECS and PESC problems, and makes a comparison with the best known results in the literature.

Key words: Circle and sphere packing, constrained optimization, nonlinear non-convex optimization, global optimization, heuristics.

1. Sensitivity analysis of the parameters



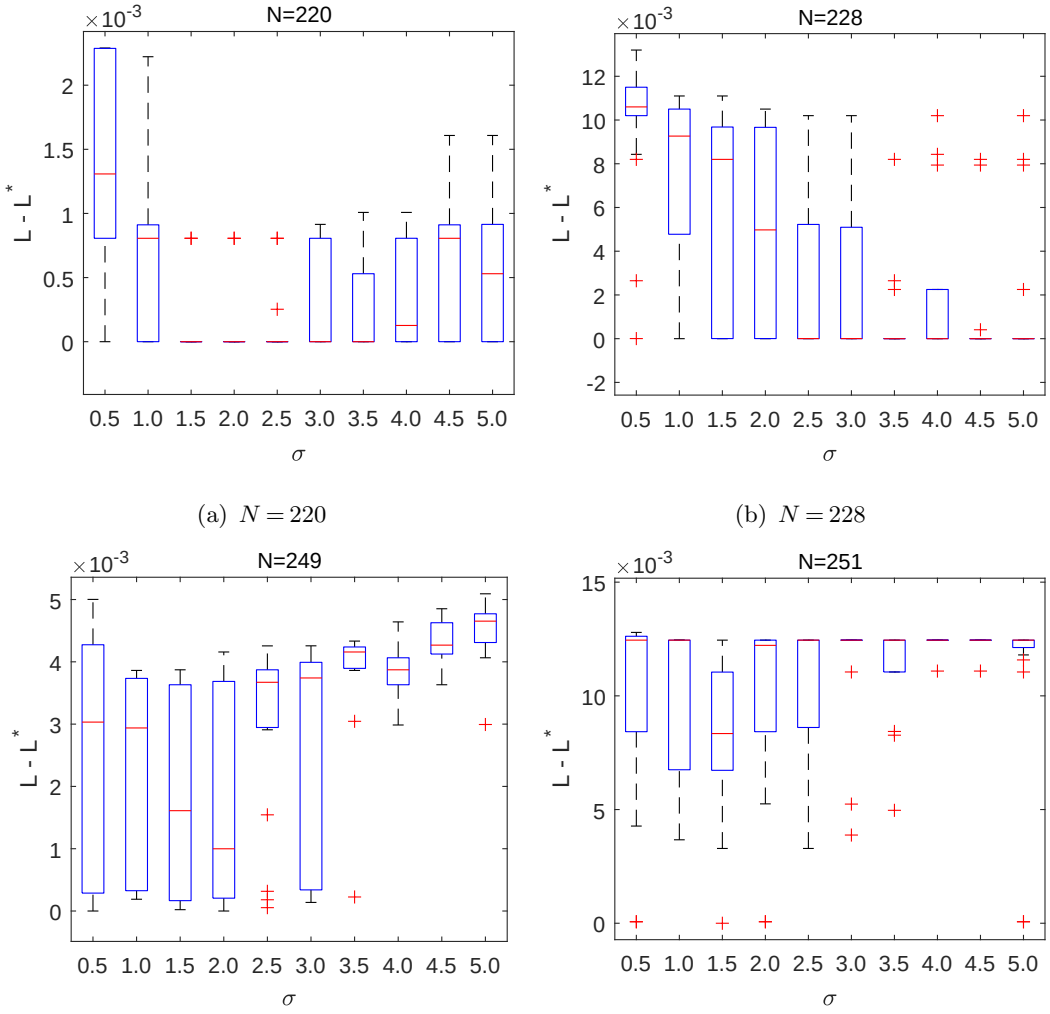
(a) $N = 220$ (b) $N = 228$

(c) $N = 249$ (d) $N = 251$

Figure 1 **Influence of parameter m on the algorithm for representative instances.**

The proposed algorithm employs several parameters. In this section, we focus on the two key parameters m and σ of the SRP operator, which is highly essential to the success of the algorithm. As previously noted, the parameter m represents the number of local descent moves after each small perturbation, and the parameter σ is proportional to the step size of local descent move which follows a direction opposite to the gradient $g(X)$ of objective function $E_L(X)$ at each step (see Algorithm 3 for more details)

First, to check the influence of parameter m, we create an algorithmic variant by disabling the URP operator and varying the value of parameter m in the interval $[1, 10]$. Then, we perform an experiment based on four representative PECS instances. For each tested



(a) $N = 220$ (b) $N = 228$

(c) $N = 249$ (d) $N = 251$

Figure 2 **Influence of parameter σ on the algorithm for representative instances.**

instance and each m value, the corresponding algorithm is executed 20 times and the results are shown in Fig. 1 with the box plots, where the X-axis represents the values of parameter m and the Y-axis indicates the gap between the result (L) of the current run and the best-known result (L^*).

Fig. 1 shows that the algorithm is sensitive to the setting of parameter m and the effectiveness of the parameter depends largely on the instances to be solved. For example, for $N = 220$, the setting $m = 3$ leads to the best performance. However, for $N = 228$, $m = 6$ is the best setting. For the remaining two instances $N = 249$ and $N = 251$, the setting $m \leq 3$ yields a satisfactory performance. Thus, it is difficult to find a single highly effective setting for the parameter m for all instances.

To check the influence of parameter σ, we create the second algorithmic variant by disabling the URP operator. In this experiment, we vary the value of parameter σ in the range of $\{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}$ and run the resulting algorithm 20 times for each instance and each tested parameter value. The experimental results are summarized by the box plots in Fig. 2.

Similar to the previous experiment, Fig. 2 shows that the effectiveness of the parameter σ setting depends significantly on the instances to be solved and there does not exist a parameter setting which is best for all tested instances. For example, for the instance $N = 220$, the setting of $1.5 \leq \sigma \leq 2.5$ is very effective for the algorithm. However, for the instance $N = 228$, the setting $3.5 \leq \sigma \leq 5.0$ is very effective, but for the other instances, $\sigma = 1.5$ is preferable.

In sum, it is difficult to find settings for m and σ that work well over all instances. The default values $m = 5.0$ and $\sigma = 4.0$ prove to be reasonable globally, though better results can be obtained by fine-tuning these parameters according to the instances to be solved.

2. The detailed computational results and comparisons on PECS

Tables 1–8 summarize the computational results of the proposed PBTS algorithm on the PECS problem in the range of $2 \leq N \leq 400$. The first and second columns of the tables indicate the size of instances (N) and the best known results in the literature (denoted by L^*) in terms of the objective value, the results of the proposed PBTS algorithm are shown in columns 3–8, including the best objective value (L_{best}) over 20 independent runs, the average objective value (L_{avg}), the worst objective value (L_{worst}), the difference between L_{best} and L^*, the success rate (SR) of obtaining the best objective value (L_{best}), and the average running time in seconds for each run of the algorithm to obtain its final result ($time(s)$). In terms of L_{best}, L_{avg}, L_{worst}, the improved results are indicated in bold compared to the best known result L^*, and the worse results are indicated in italic. In addition, the last three rows of tables indicate the numbers of instances for which the proposed PBTS algorithm obtained a better, equal, and worse result compared to the best known result L^* in terms of L_{best}, L_{avg}, and L_{worst}.

Table 1 Computational results and comparison on the small instances with $2 \leq N \leq 50$.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best} - L^*$	SR	$time(s)$
2	3.414213562	3.414213562	3.414213562	3.414213562	0.0	20/20	0.00
3	3.931851653	3.931851653	3.931851653	3.931851653	0.0	20/20	0.00
4	4.000000000	4.000000000	4.000000000	4.000000000	0.0	20/20	0.03
5	4.828427125	4.828427125	4.828427125	4.828427125	0.0	20/20	0.14
6	5.328201177	5.328201177	5.328201177	5.328201177	0.0	20/20	0.00
7	5.732050808	5.732050808	5.732050808	5.732050808	0.0	20/20	0.00
8	5.863703305	5.863703305	5.863703305	5.863703305	0.0	20/20	0.15
9	6.000000000	6.000000000	6.000000000	6.000000000	0.0	20/20	0.13
10	6.747441523	6.747441523	6.747441523	6.747441523	0.0	20/20	0.01
11	7.022509503	7.022509503	7.022509503	7.022509503	0.0	20/20	0.10
12	7.144957554	7.144957554	7.144957554	7.144957554	0.0	20/20	0.23
13	7.463047829	7.463047829	7.463047829	7.463047829	0.0	20/20	0.45
14	7.732050808	7.732050808	7.732050808	7.732050808	0.0	20/20	0.37
15	7.863703305	7.863703305	7.863703305	7.863703305	0.0	20/20	0.42
16	8.000000000	8.000000000	8.000000000	8.000000000	0.0	20/20	0.21
17	8.532660347	8.532660348	8.532660348	8.532660348	0.0	20/20	0.57
18	8.656402355	8.656402355	8.656402355	8.656402355	0.0	20/20	0.58
19	8.907460939	8.907460939	8.907460939	8.907460939	0.0	20/20	1.03
20	8.978083353	8.978083353	8.978083353	8.978083353	0.0	20/20	0.57
21	9.358019959	9.358019959	9.358019959	9.358019959	0.0	20/20	1.01
22	9.463845091	9.463845091	9.463845091	9.463845091	0.0	20/20	1.11
23	9.727406610	9.727406610	9.727406610	9.727406610	0.0	20/20	1.09
24	9.863703305	9.863703305	9.863703305	9.863703305	0.0	20/20	0.93
25	10.000000000	10.000000000	10.000000000	10.000000000	0.0	20/20	0.58
26	10.377498204	10.377498204	10.377498204	10.377498204	0.0	20/20	1.69
27	10.479983040	10.479983040	10.479983040	10.479983040	0.0	20/20	1.73
28	10.675453694	10.675453694	10.675453694	10.675453694	0.0	20/20	2.43
29	10.815120018	10.815120018	10.815120018	10.815120018	0.0	20/20	1.65
30	10.908568331	10.908568331	10.908568331	10.908568331	0.0	20/20	1.17
31	11.193403352	11.193403352	11.193403352	11.193403352	0.0	20/20	3.57
32	11.381982427	11.381982427	11.381982427	11.381982427	0.0	20/20	2.00
33	11.463944032	11.463944033	11.463944033	11.463944033	0.0	20/20	2.15
34	11.727406610	11.727406610	11.727406610	11.727406610	0.0	20/20	2.20
35	11.863703305	11.863703305	11.863703305	11.863703305	0.0	20/20	1.69
36	12.000000000	12.000000000	12.000000000	12.000000000	0.0	20/20	1.27
37	12.181786397	12.181786397	12.181786397	12.181786397	0.0	20/20	58.31
38	12.238437644	12.238437644	12.238437644	12.238437644	0.0	20/20	4.26
39	12.289915109	12.289915109	12.289915109	12.289915109	0.0	20/20	2.76
40	12.628374926	12.628374927	12.628374927	12.628374927	0.0	20/20	6.26
41	12.746938453	12.746938453	12.746938453	12.746938453	0.0	20/20	8.64
42	12.853222145	12.853222146	12.853222146	12.853222146	0.0	20/20	2.38
43	13.099325141	13.099325141	13.099325141	13.099325141	0.0	20/20	6.50
44	13.195748126	13.195748126	13.195748126	13.195748126	0.0	20/20	23.17
45	13.381982427	13.381982427	13.381982427	13.381982427	0.0	20/20	3.56
46	13.463987888	13.463987888	13.463987888	13.463987888	0.0	20/20	4.53
47	13.677429883	13.677429883	13.677429883	13.677429883	0.0	20/20	15.94
48	13.805997054	13.805997054	13.805997054	13.805997054	0.0	20/20	16.19
49	13.948425087	13.948425087	13.948425087	13.948425087	0.0	20/20	16.09
50	14.010094916	14.010094916	14.010094916	14.010094916	0.0	20/20	23.93
#Better		0	0	0			
#Equal		49	49	49			
#Worst		0	0	0			

Table 2 Computational results and comparison on the small instances with $51 \leq N \leq 100$.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best} - L^*$	SR	$time(s)$
51	14.075954786	14.075954786	14.075954786	14.075954786	0.0	20/20	367
52	14.092904613	14.092904613	14.092904613	14.092904613	0.0	20/20	9
53	14.296487187	14.296487187	14.296487187	14.296487187	0.0	20/20	31
54	14.567588783	14.567588783	14.567588783	14.567588783	0.0	20/20	17
55	14.693919653	14.693919653	14.693919653	14.693919653	0.0	20/20	5
56	14.807664068	14.807664068	14.807664068	14.807664068	0.0	20/20	5
57	14.924287647	14.924287647	14.924287647	14.924287647	0.0	20/20	18
58	15.098217587	15.098217587	15.098217587	15.098217587	0.0	20/20	14
59	15.195837056	15.195837056	15.195837056	15.195837056	0.0	20/20	83
60	15.377420491	15.377420491	15.377420491	15.377420491	0.0	20/20	93
61	15.464012644	15.464012644	15.464012644	15.464012644	0.0	20/20	60
62	15.563673462	15.563673462	15.563673462	15.563673462	0.0	20/20	38
63	15.622186004	15.622186004	15.622186004	15.622186004	0.0	20/20	60
64	15.758209360	15.758209360	15.758209360	15.758209360	0.0	20/20	47
65	15.821794178	15.821794178	15.821794178	15.821794178	0.0	20/20	39
66	15.907796654	15.907796654	15.907796654	15.907796654	0.0	20/20	129
67	15.977649303	15.977649303	15.977649303	15.977649303	0.0	20/20	46
68	15.994861683	15.994861683	15.994861683	15.994861683	0.0	20/20	520
69	16.291003807	16.291003807	16.291003807	16.291003807	0.0	20/20	18
70	16.502550553	16.502550553	16.502550553	16.502550553	0.0	20/20	177
71	16.639895473	16.639895473	16.639895473	16.639895473	0.0	20/20	551
72	16.722127798	16.722127798	16.722127798	16.722127798	0.0	20/20	230
73	16.844644206	16.844644206	16.844644206	16.844644206	0.0	20/20	77
74	16.925520976	16.925520976	16.925520976	16.925520976	0.0	20/20	22
75	17.095614064	17.095614064	17.095614064	17.095614064	0.0	20/20	31
76	17.182566072	17.182566072	17.182566072	17.182566072	0.0	20/20	1947
77	17.285314432	17.285314432	17.285314432	17.285314432	0.0	20/20	118
78	17.330278639	17.330278639	17.330278639	17.330278639	0.0	20/20	39
79	17.388734506	17.388734506	17.388734506	17.388734507	0.0	20/20	82
80	17.430505055	17.430505055	17.430505055	17.430505055	0.0	20/20	151
81	17.583987817	17.583987817	17.583987817	17.583987817	0.0	20/20	84
82	17.695271678	17.695271678	17.695271678	17.695271678	0.0	20/20	95
83	17.815983792	17.815983792	17.815983792	17.815983792	0.0	20/20	29
84	17.902966174	17.902966174	17.902966174	17.902966174	0.0	20/20	317
85	17.959711485	17.959711485	17.959711485	17.959711485	0.0	20/20	145
86	17.994349965	17.994349965	17.994349965	17.994349965	0.0	20/20	800
87	18.283120057	18.283120057	18.283120057	18.283120057	0.0	20/20	22
88	18.380110530	18.380110530	18.380110530	18.380110530	0.0	20/20	376
89	18.536697919	18.536697919	18.536697919	18.536697919	0.0	20/20	681
90	18.604669056	18.604669056	18.604669056	18.604669056	0.0	20/20	80
91	18.692734847	18.692734847	18.692734847	18.692734847	0.0	20/20	220
92	18.755713984	18.755713984	18.755713984	18.755713984	0.0	20/20	1024
93	18.894150540	18.894150540	18.894150540	18.894150540	0.0	20/20	71
94	18.941057478	18.941057478	18.941057478	18.941057478	0.0	20/20	243
95	19.076554913	19.076554913	19.076554913	19.076554913	0.0	20/20	359
96	19.129447365	19.129447365	19.129447365	19.129447365	0.0	20/20	150
97	19.188408788	19.188408788	19.188408788	19.188408788	0.0	20/20	233
98	19.218577371	19.218577371	19.218577371	19.218577371	0.0	20/20	348
99	19.238684303	19.238684303	19.238684303	19.238684303	0.0	20/20	37
100	19.454847253	19.454847253	19.454847253	19.454847253	0.0	20/20	326
#Better		0	0	0			
#Equal		50	50	50			
#Worst		0	0	0			

Table 3 Computational results and comparison on the instances with $101 \leq N \leq 150$. In terms of L_{best}, L_{avg} and L_{worst}, the better results are indicated in bold compared to the best-known results L^* and the worse results are indicated in italic..

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best}-L^*$	SR	$time(s)$
101	19.577763885	19.577763885	19.577763885	19.577763885	0.0	20/20	2211
102	19.694956236	19.694956236	19.694956236	19.694956236	0.0	20/20	396
103	19.788462285	19.788462285	19.788462285	19.788462285	0.0	20/20	182
104	19.861449385	19.861449385	19.861449385	19.861449385	0.0	20/20	86
105	19.903290562	19.903290562	19.903290562	19.903290562	0.0	20/20	306
106	19.993921681	**19.993921677**	**19.993921678**	19.993921681	-3.74E-09	15/20	4279
107	20.199503688	20.199503688	20.199503688	*20.199503691*	0.0	19/20	2157
108	20.307240100	20.307240100	20.307240100	20.307240100	0.0	20/20	3193
109	20.383180784	20.383180784	*20.383180928*	*20.383181629*	0.0	16/20	5431
110	20.500428027	20.500428027	20.500428027	20.500428027	0.0	20/20	451
111	20.557546842	20.557546842	20.557546842	20.557546842	0.0	20/20	3349
112	20.641792165	20.641792165	20.641792165	20.641792165	0.0	20/20	317
113	20.722129539	20.722129539	20.722129539	20.722129539	0.0	20/20	128
114	20.760776357	20.760776357	20.760776357	20.760776357	0.0	20/20	327
115	20.870618895	20.870618895	20.870618895	20.870618895	0.0	20/20	203
116	20.931028475	20.931028475	20.931028475	20.931028475	0.0	20/20	293
117	20.989210316	20.989210316	20.989210316	20.989210316	0.0	20/20	85
118	21.020608262	21.020608262	21.020608262	21.020608262	0.0	20/20	113
119	21.032862363	21.032862363	21.032862363	21.032862363	0.0	20/20	52
120	21.039378281	21.039378281	21.039378281	21.039378281	0.0	20/20	54
121	21.325599420	21.325599420	21.325599420	21.325599420	0.0	20/20	1181
122	21.446943275	21.446943275	21.446943275	21.446943275	0.0	20/20	1465
123	21.575533833	21.575533833	21.575533833	21.575533833	0.0	20/20	102
124	21.692090591	21.692090591	21.692090591	21.692090591	0.0	20/20	1761
125	21.749373187	21.749373187	21.749373187	21.749373187	0.0	20/20	4440
126	21.820380580	21.820380580	21.820380580	21.820380580	0.0	20/20	1367
127	21.899151495	21.899151495	21.899151495	21.899151495	0.0	20/20	2150
128	21.990940091	21.990940091	21.990940091	21.990940091	0.0	20/20	161
129	22.123721974	22.123721974	*22.124138972*	*22.127317521*	0.0	3/20	7822
130	22.200928069	22.200928069	22.200928069	22.200928069	0.0	20/20	1055
131	22.278049843	22.278049843	22.278049843	22.278049843	0.0	20/20	1383
132	22.348181073	22.348181073	22.348181073	22.348181073	0.0	20/20	328
133	22.416678650	22.416678650	22.416678650	22.416678650	0.0	20/20	642
134	22.439514409	22.439514409	22.439514409	22.439514409	0.0	20/20	1569
135	22.506707939	22.506707939	22.506707939	22.506707939	0.0	20/20	1396
136	22.546235384	22.546235384	22.546235384	22.546235384	0.0	20/20	670
137	22.576918890	22.576918890	22.576918890	22.576918890	0.0	20/20	660
138	22.710685569	22.710685569	22.710685569	22.710685569	0.0	20/20	537
139	22.755332955	22.755332955	22.755332955	22.755332955	0.0	20/20	633
140	22.828528094	22.828528094	22.828528094	22.828528094	0.0	20/20	390
141	22.905063302	22.905063302	22.905063302	22.905063302	0.0	20/20	219
142	22.948078759	22.948078759	22.948078759	22.948078759	0.0	20/20	195
143	22.972392375	22.972392375	22.972392375	22.972392375	0.0	20/20	105
144	23.253767563	23.253767563	*23.253768119*	*23.253777647*	0.0	16/20	5451
145	23.325597250	23.325597250	23.325597250	23.325597250	0.0	20/20	1376
146	23.445951834	23.445951834	23.445951834	23.445951834	0.0	20/20	863
147	23.558795733	23.558795733	23.558795733	23.558795733	0.0	20/20	338
148	23.623784194	23.623784194	23.623784194	23.623784194	0.0	20/20	737
149	23.667595602	23.667595602	23.667595602	23.667595602	0.0	20/20	1645
150	23.727344772	23.727344772	23.727344772	23.727344772	0.0	20/20	1301
#Better		1	1	0			
#Equal		49	46	47			
#Worst		0	3	3			

Table 4 Computational results and comparison on the instances with $151 \leq N \leq 200$. In terms of L_{best}, L_{avg} and L_{worst}, the better results are indicated in bold compared to the best-known results L^* and the worse results are indicated in italic.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best}-L^*$	SR	$time(s)$
151	23.822514392	23.822514392	23.822514392	23.822514392	0.0	20/20	887
152	23.935009864	23.935009864	*23.935010133*	*23.935013342*	0.0	18/20	6642
153	24.010218767	**24.004609758**	**24.004609919**	**24.004610519**	-5.61E-03	13/20	7987
154	24.066270895	24.066270895	24.066270895	24.066270895	0.0	20/20	2447
155	24.132415945	24.132415945	24.132415945	24.132415945	0.0	20/20	1552
156	24.213445283	24.213445283	24.213445283	24.213445283	0.0	20/20	1941
157	24.259862394	**24.259862387**	*24.259862395*	*24.259862400*	-7.35E-09	4/20	3736
158	24.295613379	24.295613379	24.295613379	24.295613379	0.0	20/20	263
159	24.322506668	24.322506668	24.322506668	24.322506668	0.0	20/20	296
160	24.363743381	24.363743381	24.363743381	24.363743381	0.0	20/20	513
161	24.384089734	24.384089734	24.384089734	24.384089734	0.0	20/20	787
162	24.510337009	24.510337009	*24.510631478*	*24.516226394*	0.0	19/20	2810
163	24.581376428	24.581376428	24.581376428	24.581376428	0.0	20/20	322
164	24.676093960	24.676093960	24.676093960	24.676093960	0.0	20/20	974
165	24.766241302	24.766241302	24.766241302	24.766241302	0.0	20/20	232
166	24.852298048	24.852298048	24.852298048	24.852298048	0.0	20/20	391
167	24.907567166	24.907567166	24.907567166	24.907567166	0.0	20/20	140
168	24.936162965	24.936162965	24.936162965	*24.936162970*	0.0	19/20	920
169	25.122224029	25.122224029	25.122224029	25.122224029	0.0	20/20	189
170	25.252698973	**25.252697180**	**25.252697882**	*25.252699076*	-1.79E-06	5/20	6888
171	25.325602865	**25.325594655**	**25.325594752**	**25.325596586**	-8.21E-06	19/20	3909
172	25.417214968	**25.404460665**	*25.419025846*	*25.433205539*	-1.28E-02	4/20	8836
173	25.476711659	25.476711659	25.476711659	25.476711659	0.0	20/20	2930
174	25.523523830	25.523523830	*25.525171571*	*25.531750516*	0.0	10/20	9172
175	25.600918082	25.600918082	25.600918082	25.600918082	0.0	20/20	470
176	25.647066875	25.647066875	25.647066875	25.647066875	0.0	20/20	3718
177	25.733697786	25.733697786	25.733697786	25.733697786	0.0	20/20	4071
178	25.819402985	25.819402985	25.819402985	25.819402985	0.0	20/20	2648
179	25.851437558	25.851437558	*25.851437560*	*25.851437563*	0.0	12/20	3524
180	25.893244531	25.893244531	25.893244531	25.893244531	0.0	20/20	2570
181	25.985618408	**25.975260489**	**25.975260489**	**25.975260489**	-1.04E-02	20/20	2801
182	26.029681507	26.029681507	26.029681507	26.029681507	0.0	20/20	3048
183	26.072350968	26.072350968	26.072350968	26.072350968	0.0	20/20	891
184	26.122160481	26.122160481	*26.122187714*	*26.122705139*	0.0	19/20	3884
185	26.147038663	26.147038663	26.147038663	26.147038663	0.0	20/20	5949
186	26.154806378	26.154806378	*26.155005470*	*26.158747851*	0.0	13/20	8113
187	26.177280972	26.177280972	26.177280972	26.177280972	0.0	20/20	2124
188	26.185809225	26.185809225	26.185809225	26.185809225	0.0	20/20	1593
189	26.408530183	26.408530183	26.408530183	26.408530183	0.0	20/20	4320
190	26.528013597	26.528013597	26.528013597	26.528013597	0.0	20/20	569
191	26.635323947	26.635323947	26.635323947	26.635323947	0.0	20/20	490
192	26.706309251	26.706309251	26.706309251	*26.706309254*	0.0	19/20	4035
193	26.792374948	*26.792374956*	*26.792393690*	*26.792540178*	8.23E-09	2/20	10165
194	26.840126463	**26.836480274**	**26.837233280**	**26.838362788**	-3.65E-03	12/20	3250
195	26.872063371	26.872063371	26.872063371	26.872063371	0.0	20/20	574
196	26.992467225	26.992467225	26.992467225	26.992467225	0.0	20/20	414
197	27.121091211	27.121091211	27.121091211	27.121091211	0.0	20/20	4324
198	27.194835943	27.194835943	*27.194845501*	*27.194857182*	0.0	11/20	4335
199	27.272163202	27.272163202	*27.272163226*	*27.272163412*	0.0	11/20	6157
200	27.312853154	27.312853154	*27.323213491*	*27.327787437*	0.0	2/20	6734
#Better		7	5	4			
#Equal		42	33	31			
#Worst		1	12	15			

Table 5 Computational results and comparison on the instances with $201 \leq N \leq 250$. In terms of L_{best}, L_{avg} and L_{worst}, the better results are indicated in bold compared to the best-known results L^* and the worse results are indicated in italic.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best}-L^*$	SR	$time(s)$
201	27.345119023	27.345119023	27.345119023	27.345119023	0.0	20/20	6443
202	27.408554981	27.408554981	27.408554981	27.408554981	0.0	20/20	1609
203	27.490596149	27.490596149	27.490596149	27.490596149	0.0	20/20	913
204	27.523381977	27.523381977	27.523381977	27.523381977	0.0	20/20	4085
205	27.576876117	**27.567250117**	**27.567250139**	**27.567250555**	-9.63E-03	19/20	4326
206	27.649819060	27.649819060	27.649819060	27.649819060	0.0	20/20	556
207	27.677311798	**27.677311693**	**27.677311793**	27.677311798	-1.04E-07	1/20	7392
208	27.710218754	27.710218754	27.710218754	27.710218754	0.0	20/20	2195
209	27.816328561	27.816328561	27.816328561	27.816328561	0.0	20/20	2249
210	27.865431130	**27.865264940**	**27.865264940**	**27.865264940**	-1.66E-04	20/20	1298
211	27.895195152	27.895195152	27.895195152	27.895195152	0.0	20/20	752
212	27.943373578	27.943373578	27.943373578	27.943373578	0.0	20/20	1709
213	27.967301678	27.967301678	27.967301678	27.967301678	0.0	20/20	1349
214	27.980730351	27.980730351	27.980730351	27.980730351	0.0	20/20	1287
215	27.999469469	**27.999457017**	**27.999461572**	**27.999466531**	-1.25E-05	1/20	16056
216	27.999894488	**27.999890345**	**27.999890351**	**27.999890448**	-4.14E-06	19/20	9867
217	28.186892582	28.186892582	*28.187008295*	*28.187223191*	0.0	13/20	7949
218	28.396275192	**28.396274204**	**28.396274204**	**28.396274204**	-9.88E-07	20/20	1977
219	28.505470621	**28.503438714**	**28.503438715**	**28.503438717**	-2.03E-03	14/20	11992
220	28.581964018	**28.579623308**	**28.579955723**	**28.581843660**	-2.34E-03	10/20	14190
221	28.650217350	**28.647970386**	**28.647970386**	**28.647970386**	-2.25E-03	20/20	4808
222	28.688667811	28.688667811	*28.689163526*	*28.697683795*	0.0	18/20	8698
223	28.746373096	**28.744925481**	**28.744925481**	**28.744925481**	-1.45E-03	20/20	6158
224	28.791478737	**28.791012002**	**28.791012002**	**28.791012002**	-4.67E-04	20/20	460
225	28.892529959	**28.872701621**	**28.872701621**	**28.872701621**	-1.98E-02	20/20	581
226	28.980195790	**28.977237925**	**28.977981653**	*28.980217880*	-2.96E-03	1/20	15892
227	29.047402657	**29.043187997**	**29.043208869**	**29.043532621**	-4.21E-03	12/20	18009
228	29.113455127	29.113455127	*29.114686490*	*29.123606101*	0.0	10/20	15884
229	29.166477858	**29.142413853**	**29.142413853**	**29.142413853**	-2.41E-02	20/20	2486
230	29.186674755	**29.186139367**	**29.186139367**	**29.186139367**	-5.35E-04	20/20	9779
231	29.248534290	29.248534290	29.248534290	29.248534290	0.0	20/20	3620
232	29.331629096	**29.331615175**	**29.331615175**	**29.331615175**	-1.39E-05	20/20	6843
233	29.372019605	**29.371417294**	**29.371417294**	**29.371417294**	-6.02E-04	20/20	3202
234	29.399297204	29.399297204	29.399297204	29.399297204	0.0	20/20	4746
235	29.412549407	**29.412498235**	**29.412498235**	**29.412498235**	-5.12E-05	20/20	1144
236	29.483100577	29.483100577	29.483100577	29.483100577	0.0	20/20	1697
237	29.498377024	**29.498316339**	**29.498316371**	**29.498316409**	-6.07E-05	11/20	4747
238	29.528432420	29.528432420	29.528432420	29.528432420	0.0	20/20	11688
239	29.620823282	29.620823282	29.620823282	29.620823282	0.0	20/20	3583
240	29.666279530	**29.666279350**	**29.666279350**	**29.666279350**	-1.80E-07	20/20	5577
241	29.693078678	29.693078678	29.693078678	29.693078678	0.0	20/20	1147
242	29.752947872	29.752947872	*29.752947873*	*29.752947875*	0.0	13/20	7630
243	29.860683998	29.860683998	*29.860684000*	*29.860684011*	0.0	17/20	3213
244	29.914182807	**29.912487768**	**29.912487768**	**29.912487773**	-1.70E-03	19/20	9015
245	29.942673375	29.942673375	*29.942690758*	*29.942694523*	0.0	1/20	9001
246	29.968323001	**29.967083637**	**29.967084244**	**29.967094660**	-1.24E-03	2/20	9466
247	29.979731873	29.979731873	*29.984857292*	*29.999879920*	0.0	7/20	15410
248	30.183466092	30.183466092	30.183466092	30.183466092	0.0	20/20	2265
249	30.335969818	**30.330357801**	**30.334288216**	**30.335152142**	-5.61E-03	1/20	10783
250	30.417031057	**30.413810109**	**30.415458027**	*30.417095278*	-3.22E-03	3/20	15749
#Better		25	25	22			
#Equal		25	18	19			
#Worst		0	7	9			

Table 6 Computational results and comparison on the instances with $251 \leq N \leq 300$. In terms of L_{best}, L_{avg} and L_{worst}, the better results are indicated in bold compared to the best-known results L^* and the worse results are indicated in italic.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best} - R^*$	SR	$time(s)$
251	30.514477048	**30.488615243**	**30.500267826**	**30.501062819**	-2.59E-02	1/20	4567
252	30.556359106	**30.554594854**	**30.554599337**	**30.554684510**	-1.76E-03	19/20	9991
253	30.591417781	30.591417781	*30.591476303*	*30.591983157*	0.0	17/20	13067
254	30.637242017	**30.629122311**	**30.630238212**	**30.634701830**	-8.12E-03	16/20	18066
255	30.688409827	**30.688395200**	**30.688395919**	**30.688408914**	-1.46E-05	18/20	10648
256	30.724745895	**30.724745891**	30.724745895	*30.724745902*	-3.62E-09	9/20	2599
257	30.824174685	**30.818433601**	**30.818440411**	**30.818484908**	-5.74E-03	1/20	14376
258	30.891238918	30.891238918	*30.891238920*	*30.891238924*	0.0	13/20	11157
259	30.971015228	**30.959405025**	**30.961895185**	**30.962546517**	-1.16E-02	4/20	14057
260	31.002117303	**30.976264045**	**30.976264090**	**30.976264127**	-2.59E-02	1/20	9077
261	31.021231115	**31.020513868**	**31.020836629**	31.021231115	-7.17E-04	11/20	10018
262	31.061690871	31.061690871	31.061690871	31.061690871	0.0	20/20	3547
263	31.135387935	**31.135221398**	**31.135221400**	**31.135221432**	-1.67E-04	19/20	9598
264	31.200792357	**31.198320781**	**31.198326135**	**31.198424318**	-2.47E-03	13/20	16570
265	31.217512841	**31.217512377**	**31.217512378**	**31.217512379**	-4.64E-07	20/20	8818
266	31.255856470	**31.252989666**	**31.252989666**	**31.252989666**	-2.87E-03	20/20	4041
267	31.262745851	**31.262008812**	**31.262008812**	**31.262008812**	-7.37E-04	20/20	6908
268	31.304614023	**31.290274210**	**31.290274210**	**31.290274210**	-1.43E-02	20/20	5666
269	31.314762420	**31.314762419**	31.314762420	31.314762420	-3.57E-10	19/20	2042
270	31.331235580	31.331235580	31.331235580	31.331235580	0.0	20/20	2886
271	31.437717984	**31.437713367**	**31.437715579**	31.437717984	-4.62E-06	8/20	14734
272	31.553295627	**31.553030915**	**31.553149780**	**31.553230504**	-2.65E-04	1/20	12750
273	31.605207996	**31.605028606**	**31.605029727**	**31.605046999**	-1.79E-04	12/20	11563
274	31.674215412	31.674215412	31.674215412	31.674215412	0.0	20/20	619
275	31.760669189	**31.727696959**	**31.738119037**	**31.742626469**	-3.30E-02	1/20	19092
276	31.835204513	31.835204513	31.835204513	31.835204513	0.0	20/20	9265
277	31.876485279	**31.875605745**	**31.875605878**	**31.875606279**	-8.80E-04	15/20	10373
278	31.904139803	**31.896621548**	**31.896985326**	**31.903897095**	-7.52E-03	19/20	5544
279	31.930082787	31.930082787	*31.930229633*	*31.930837449*	0.0	8/20	13074
280	31.978915453	31.978915453	*31.978959697*	*31.979037555*	0.0	8/20	11871
281	32.180781578	32.180781578	32.180781578	32.180781578	0.0	20/20	624
282	32.271825992	**32.247367828**	**32.247369679**	**32.247402924**	-2.45E-02	5/20	14743
283	32.349428155	**32.328946505**	**32.336018811**	**32.337443225**	-2.05E-02	1/20	14098
284	32.371743196	**32.371671534**	*32.375020635*	*32.396526683*	-7.17E-05	1/20	21863
285	32.442795717	**32.440898585**	**32.440923378**	**32.441245131**	-1.90E-03	10/20	18883
286	32.453648756	**32.453626290**	**32.453626290**	**32.453626290**	-2.25E-05	20/20	7931
287	32.502565870	32.502565870	*32.509629320*	*32.515411025*	0.0	9/20	16060
288	32.572406278	**32.571998609**	**32.571998621**	**32.571998729**	-4.08E-04	18/20	5641
289	32.607735870	32.607735870	*32.607761797*	*32.608254401*	0.0	19/20	10889
290	32.658281934	**32.619667676**	**32.619669609**	**32.619672285**	-3.86E-02	11/20	19575
291	32.737055178	**32.713223771**	**32.713223771**	**32.713223771**	-2.38E-02	20/20	10660
292	32.784245264	**32.771377288**	**32.771377288**	**32.771377288**	-1.29E-02	20/20	3335
293	32.803781063	**32.803747327**	**32.803747329**	**32.803747333**	-3.37E-05	9/20	5671
294	32.827175126	32.827175126	32.827175126	32.827175126	0.0	20/20	5856
295	32.847220793	32.847220793	32.847220793	32.847220793	0.0	20/20	13273
296	32.939976201	**32.939965541**	**32.939965541**	**32.939965541**	-1.07E-05	20/20	9668
297	32.991738983	**32.990170865**	**32.990170865**	**32.990170865**	-1.57E-03	20/20	5128
298	33.024730665	33.024730665	33.024730665	33.024730665	0.0	20/20	4681
299	33.058743039	**33.058648838**	**33.058648838**	**33.058648838**	-9.42E-05	20/20	6103
300	33.091154939	33.091154939	33.091154939	33.091154939	0.0	20/20	7146
#Better		35	32	30			
#Equal		15	11	12			
#Worst		0	7	8			

Table 7 Computational results and comparison on the instances with $301 \leq N \leq 350$. In terms of L_{best}, L_{avg} and L_{worst}, the better results are indicated in bold compared to the best-known results L^* and the worse results are indicated in italic.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best}-L^*$	SR	$time(s)$
301	33.107664708	**33.107188751**	**33.107577871**	**33.107618293**	-4.76E-04	1/20	27273
302	33.115483620	**33.114674076**	**33.114674076**	**33.114674076**	-8.10E-04	20/20	3946
303	33.126591528	33.126591528	*33.126597403*	*33.126709027*	0.0	19/20	11029
304	33.132402078	33.132402078	33.132402078	33.132402078	0.0	20/20	1555
305	33.369469591	**33.365641905**	**33.365683242**	**33.366039406**	-3.83E-03	15/20	24675
306	33.472130383	**33.433775371**	**33.433776475**	**33.433779787**	-3.84E-02	15/20	21022
307	33.533342804	**33.528160031**	**33.529035141**	**33.530159038**	-5.18E-03	1/20	29231
308	33.592291592	**33.581624679**	**33.581624744**	**33.581624864**	-1.07E-02	13/20	15652
309	33.662242538	**33.653059584**	**33.653059584**	**33.653059584**	-9.18E-03	20/20	7198
310	33.708969061	**33.706874743**	**33.706874978**	**33.706877034**	-2.09E-03	17/20	16353
311	33.766968464	**33.763821954**	**33.763822813**	**33.763823099**	-3.15E-03	2/20	13722
312	33.810134916	**33.800477552**	**33.800522662**	**33.800560717**	-9.66E-03	5/20	18280
313	33.819660112	**33.819647499**	**33.819647513**	**33.819647536**	-1.26E-05	1/20	7532
314	33.863166811	**33.863128095**	**33.863135821**	**33.863136895**	-3.87E-05	1/20	15440
315	33.977361251	**33.977355640**	**33.977357605**	33.977361251	-5.61E-06	7/20	14728
316	34.106569996	**34.066488590**	**34.069965233**	**34.081965925**	-4.01E-02	6/20	23702
317	34.157653052	**34.139244391**	**34.146102257**	**34.150979337**	-1.84E-02	1/20	17202
318	34.230632899	**34.212087493**	**34.213468635**	**34.214221930**	-1.85E-02	1/20	26754
319	34.278012052	**34.244701317**	**34.258088313**	**34.268965251**	-3.33E-02	1/20	26101
320	34.301272474	**34.296258944**	**34.296315949**	**34.297399020**	-5.01E-03	15/20	22515
321	34.330158112	**34.304146089**	**34.304146929**	**34.304147004**	-2.60E-02	1/20	18378
322	34.384694645	**34.363743605**	**34.371430235**	**34.384693716**	-2.10E-02	11/20	20772
323	34.453936080	**34.443942400**	**34.443942404**	**34.443942411**	-9.99E-03	13/20	16237
324	34.469293461	**34.469293437**	**34.469293437**	**34.469293437**	-2.45E-08	20/20	10581
325	34.497557560	**34.495918525**	**34.495918525**	**34.495918538**	-1.64E-03	19/20	13277
326	34.538708641	**34.530949661**	**34.530949661**	**34.530949661**	-7.76E-03	20/20	6945
327	34.591603459	**34.591579616**	**34.591579666**	**34.591579868**	-2.38E-05	16/20	11477
328	34.627862893	**34.627258700**	**34.627258700**	**34.627258700**	-6.04E-04	20/20	7720
329	34.640006924	34.640006924	34.640006924	34.640006924	0.0	20/20	4178
330	34.665648167	**34.665385656**	**34.665385656**	**34.665385656**	-2.63E-04	20/20	12718
331	34.757395176	**34.757317756**	**34.757317756**	**34.757317756**	-7.74E-05	20/20	3312
332	34.790927642	**34.790437779**	**34.790437779**	**34.790437779**	-4.90E-04	20/20	14915
333	34.824467247	**34.824467058**	**34.824467058**	**34.824467058**	-1.90E-07	20/20	5802
334	34.848927176	**34.848252392**	**34.848252392**	**34.848252392**	-6.75E-04	20/20	4862
335	34.888861313	**34.888813154**	**34.888813154**	**34.888813154**	-4.82E-05	20/20	4900
336	34.923285419	**34.923052382**	**34.923052382**	**34.923052382**	-2.33E-04	20/20	3792
337	34.942152251	34.942152251	34.942152251	34.942152251	0.0	20/20	2810
338	34.974622430	**34.974609760**	**34.974609760**	**34.974609760**	-1.27E-05	20/20	3198
339	34.987295963	**34.987292522**	**34.987292522**	**34.987292522**	-3.44E-06	20/20	1670
340	34.992358965	34.992358965	34.992358965	34.992358965	0.0	20/20	858
341	35.292676076	**35.276436460**	**35.276569349**	**35.276989927**	-1.62E-02	1/20	17677
342	35.353034567	**35.349992599**	**35.349992814**	**35.349993029**	-3.04E-03	10/20	16610
343	35.444701701	**35.424528912**	**35.424580372**	**35.424634999**	-2.02E-02	1/20	23820
344	35.501634171	**35.494954267**	**35.494955153**	**35.494971987**	-6.68E-03	19/20	16019
345	35.565061674	**35.557861904**	**35.557861904**	**35.557861904**	-7.20E-03	20/20	5325
346	35.620800461	**35.615716868**	**35.615853648**	**35.617898150**	-5.08E-03	4/20	23135
347	35.658729029	**35.653297131**	**35.653325991**	**35.653575336**	-5.43E-03	6/20	20300
348	35.685807919	**35.685495033**	**35.685495952**	**35.685512355**	-3.13E-04	10/20	21069
349	35.719505469	**35.718924303**	**35.718924310**	**35.718924372**	-5.81E-04	14/20	11232
350	35.771493660	**35.771461077**	**35.771461079**	**35.771461086**	-3.26E-05	15/20	4976
#Better		45	45	44			
#Equal		5	4	5			
#Worst		0	1	1			

Table 8 Computational results and comparison on the instances with $351 \leq N \leq 400$. In terms of L_{best}, L_{avg} and L_{worst}, the better results are indicated in bold compared to the best-known results L^* and the worse results are indicated in italic.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best}-L^*$	SR	$time(s)$
351	35.793678470	**35.793675756**	*35.797816186*	*35.827566117*	-2.71E-06	7/20	28861
352	35.952656162	**35.915743200**	**35.917088505**	**35.929190777**	-3.69E-02	6/20	21812
353	35.995094545	**35.984799658**	**35.987168600**	**35.990703540**	-1.03E-02	1/20	21339
354	36.006531662	**36.006336175**	*36.013929005*	*36.022136454*	-1.95E-04	1/20	31358
355	36.098028799	**36.075573954**	**36.077536051**	**36.081465518**	-2.25E-02	1/20	26490
356	36.125768096	**36.098038505**	**36.098297471**	**36.098362212**	-2.77E-02	4/20	23129
357	36.144514642	**36.143900174**	**36.143913849**	**36.144167141**	-6.14E-04	15/20	24475
358	36.157541679	36.157541679	36.157541679	36.157541679	0.0	20/20	17020
359	36.245440882	**36.214080980**	**36.235661527**	*36.245965821*	-3.14E-02	4/20	25236
360	36.313349883	**36.296057544**	**36.296125346**	**36.296768738**	-1.73E-02	3/20	31953
361	36.329623084	**36.316021050**	**36.316821430**	*36.332027037*	-1.36E-02	1/20	16856
362	36.351806981	**36.351199205**	**36.351199205**	**36.351199205**	-6.08E-04	20/20	20609
363	36.365172174	36.365172174	*36.365178312*	*36.365294946*	-5.17E-10	19/20	24732
364	36.393238790	**36.367921257**	**36.367921391**	**36.367923784**	-2.53E-02	18/20	23451
365	36.440462302	**36.430903871**	**36.432088152**	**36.434285757**	-9.56E-03	6/20	24676
366	36.449293679	**36.448985622**	**36.448985622**	**36.448985622**	-3.08E-04	20/20	12385
367	36.461059367	**36.461059364**	36.461059367	*36.461059376*	-2.68E-09	4/20	13044
368	36.476301347	36.476301347	*36.479495528*	*36.540184954*	0.0	19/20	13775
369	36.575190355	36.575190355	*36.575193739*	*36.575229653*	0.0	10/20	24203
370	36.617447623	**36.616904172**	**36.616904172**	**36.616904172**	-5.43E-04	20/20	10886
371	36.630690132	**36.630690061**	**36.630690061**	**36.630690061**	-7.12E-08	20/20	12644
372	36.694638066	36.694638066	36.694638066	36.694638066	0.0	20/20	5963
373	36.783094613	36.783094613	36.783094613	36.783094613	0.0	20/20	5907
374	36.860238563	**36.860209804**	**36.860209804**	**36.860209804**	-2.88E-05	20/20	3205
375	36.905797738	**36.905763282**	**36.905763282**	**36.905763282**	-3.45E-05	20/20	4332
376	36.934713162	**36.933889065**	**36.933889066**	**36.933889075**	-8.24E-04	16/20	12615
377	36.956938729	**36.951443202**	**36.951961114**	**36.953958755**	-5.50E-03	5/20	21894
378	36.964726408	**36.964387688**	**36.964396666**	**36.964423602**	-3.39E-04	15/20	15730
379	37.168995378	**37.163358097**	**37.163404719**	**37.164180855**	-5.64E-03	8/20	21796
380	37.268203266	**37.254728252**	**37.255051015**	**37.255422785**	-1.35E-02	3/20	30854
381	37.328685368	**37.317946628**	**37.318049661**	**37.319009325**	-1.07E-02	1/20	27013
382	37.399833611	**37.367815912**	**37.367868868**	**37.368525344**	-3.20E-02	6/20	27115
383	37.453237457	**37.427104603**	**37.430305466**	**37.439045676**	-2.61E-02	1/20	28439
384	37.509762344	**37.480145181**	**37.481772844**	**37.485982558**	-2.96E-02	1/20	21122
385	37.524483179	**37.520288824**	**37.523293997**	*37.529436580*	-4.19E-03	1/20	20164
386	37.555106698	**37.553644453**	**37.553644454**	**37.553644456**	-1.46E-03	11/20	7067
387	37.596693465	**37.561988045**	**37.561988045**	**37.561988045**	-3.47E-02	20/20	19677
388	37.634842769	**37.626421391**	**37.626421391**	**37.626421391**	-8.42E-03	20/20	2130
389	37.669261935	**37.668211363**	**37.668211363**	**37.668211363**	-1.05E-03	20/20	2288
390	37.697840561	**37.696589756**	**37.696589756**	**37.696589756**	-1.25E-03	20/20	13990
391	37.786012401	**37.769753050**	**37.769763033**	**37.769777333**	-1.63E-02	5/20	29084
392	37.861151301	**37.836566990**	**37.837784516**	**37.839320024**	-2.46E-02	1/20	34452
393	37.919186895	**37.899488243**	**37.899704475**	**37.900021701**	-1.97E-02	3/20	31459
394	37.930436797	**37.930272137**	**37.930278074**	**37.930310285**	-1.65E-04	4/20	24346
395	37.962314535	**37.960263101**	**37.960273664**	**37.960474373**	-2.05E-03	19/20	14013
396	37.975202348	37.975202348	*37.975627191*	*37.983699109*	0.0	18/20	25958
397	38.023361885	**38.023245990**	**38.023250121**	**38.023252875**	-1.16E-04	8/20	23505
398	38.082889973	**38.053496853**	**38.066853125**	**38.082490840**	-2.94E-02	4/20	11965
399	38.148780581	**38.128423455**	**38.128423455**	**38.128423455**	-2.04E-02	20/20	7370
400	38.164523000	**38.164286993**	**38.164286993**	**38.164286993**	-2.36E-04	20/20	9305
#Better		43	40	37			
#Equal		7	4	3			
#Worst		0	6	10			

3. The detailed computational results and comparisons on PESC

The computational results of the proposed PBTS algorithm on the PESC problem are summarized in Tables 9–12. The first and second columns of the tables indicate the size of instances (N) and the best known results in the literature (denoted by L^*) in terms of the objective value, the results of the proposed PBTS algorithm are shown in columns 3–8, including the best objective value (L_{best}) over 10 independent runs, the average objective value (L_{avg}), the worst objective value (L_{worst}), the difference between L_{best} and L^*, the success rate (SR) of obtaining the best objective value (L_{best}), and the average running time in seconds for each run of the algorithm to obtain its final result ($time(s)$). In terms of L_{best}, L_{avg}, L_{worst}, the improved results are indicated in bold compared to the best known result L^*, and the worse results are indicated in italic. In addition, the last three rows of tables indicate the numbers of instances for which the proposed PBTS algorithm obtained a better, equal, and worse result compared to the best known result L^* in terms of L_{best}, L_{avg}, and L_{worst}.

Table 9 Computational results and comparison on the PESC instances with $2 \leq N \leq 50$. In terms of L_{best}, L_{avg} and L_{worst}, the better results are indicated in bold compared to the best-known results L^* and the worse results are indicated in italic.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best}-L^*$	SR	$time(s)$
2	3.154700538	3.154700538	3.154700538	3.154700538	0.0	10/10	0.00
3	3.414213562	3.414213562	3.414213562	3.414213562	0.0	10/10	0.00
4	3.414213562	3.414213562	3.414213562	3.414213562	0.0	10/10	0.00
5	3.788854382	3.788854382	3.788854382	3.788854382	0.0	10/10	0.00
6	3.885618083	3.885618083	3.885618083	3.885618083	0.0	10/10	0.02
7	3.997822724	3.997822724	3.997822724	3.997822724	0.0	10/10	0.19
8	4.000000000	4.000000000	4.000000000	4.000000000	0.0	10/10	0.10
9	4.309401077	4.309401077	4.309401077	4.309401077	0.0	10/10	0.05
10	4.666666667	4.666666667	4.666666667	4.666666667	0.0	10/10	0.03
11	4.816439741	4.816439741	4.816439741	4.816439741	0.0	10/10	0.20
12	4.828427024	*4.828427124*	*4.828427051*	*4.828427053*	1.00E-07	1/10	434.11
13	4.828427125	4.828427125	4.828427125	4.828427125	0.0	10/10	0.14
14	4.828427125	4.828427125	4.828427125	4.828427125	0.0	10/10	0.34
15	5.200000000	5.200000000	5.200000000	5.200000000	0.0	10/10	1.37
16	5.296700829	5.296700829	5.296700829	5.296700829	0.0	10/10	0.75
17	5.299831646	5.299831646	5.299831646	5.299831646	0.0	10/10	0.86
18	5.328201177	5.328201177	5.328201177	5.328201177	0.0	10/10	0.43
19	5.458953217	5.458953217	5.458953217	5.458953217	0.0	10/10	0.82
20	5.605154958	5.605154958	5.605154958	5.605154958	0.0	10/10	1.30
21	5.642734410	5.642734410	5.642734410	5.642734410	0.0	10/10	22.86
22	5.771236166	5.771236166	5.771236166	5.771236166	0.0	10/10	2.10
23	5.820153191	5.820153191	5.820153191	5.820153191	0.0	10/10	2.54
24	5.863703305	5.863703305	5.863703305	5.863703305	0.0	10/10	1.97
25	5.959330822	5.959330822	5.959330822	5.959330822	0.0	10/10	4.66
26	5.991426247	5.991426247	5.991426247	5.991426247	0.0	10/10	6.73
27	6.000000000	6.000000000	6.000000000	6.000000000	0.0	10/10	1.58
28	6.242547728	6.242547728	6.242547728	6.242547728	0.0	10/10	125.52
29	6.242640687	6.242640687	6.242640687	6.242640687	0.0	10/10	5.20
30	6.242640687	6.242640687	6.242640687	6.242640687	0.0	10/10	2.08
31	6.242640687	6.242640687	6.242640687	6.242640687	0.0	10/10	1.53
32	6.242640687	6.242640687	6.242640687	6.242640687	0.0	10/10	2.92
33	6.468078047	6.468078047	6.468078047	6.468078047	0.0	10/10	6.04
34	6.573883166	6.573883166	6.573883166	6.573883166	0.0	10/10	4.30
35	6.593325909	6.593325909	6.593325909	6.593325909	0.0	10/10	60.83
36	6.694418120	6.694418120	6.694418120	6.694418120	0.0	10/10	1005.93
37	6.708634483	6.708634483	6.708634483	6.708634483	0.0	10/10	69.67
38	6.709663574	6.709663574	6.709663574	6.709663574	0.0	10/10	7.63
39	6.774270117	6.774270117	6.774270117	6.774270117	0.0	10/10	4.22
40	6.800000000	6.800000000	6.800000000	6.800000000	0.0	10/10	3.46
41	6.904271251	6.904271251	6.904271251	6.904271251	0.0	10/10	11.51
42	6.990786312	6.990786312	6.990786312	6.990786312	0.0	10/10	775.96
43	7.059538630	7.059538630	7.059538630	7.059538630	0.0	10/10	22.34
44	7.099216689	7.099216689	7.099216689	7.099216689	0.0	10/10	11.28
45	7.110760321	7.110760321	7.110760321	7.110760321	0.0	10/10	56.08
46	7.130283920	7.130283920	7.130283920	7.130283920	0.0	10/10	11.06
47	7.144956748	7.144956748	*7.144956836*	*7.144957035*	0.0	1/10	233.25
48	7.144957554	7.144957554	7.144957554	7.144957554	0.0	10/10	4.87
49	7.329940291	7.329940291	7.329940291	7.329940291	0.0	10/10	92.87
50	7.355401307	7.355401307	7.355401307	7.355401307	0.0	10/10	71.24
#Better		0	0	0			
#Equal		48	47	47			
#Worst		1	2	2			

Table 10 Computational results and comparison on the PESC instances with $51 \leq N \leq 100$. In terms of L_{best}, L_{avg} and L_{worst}, the better results are indicated in bold compared to the best-known results L^* and the worse results are indicated in italic.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best}-L^*$	SR	$time(s)$
51	7.406141350	7.406141350	7.406141350	7.406141350	0.0	10/10	1469
52	7.472722440	7.472722440	7.472722440	7.472722440	0.0	10/10	218
53	7.505769764	7.505769764	7.505769764	7.505769764	0.0	10/10	56
54	7.564111736	7.564111736	7.564111736	7.564111736	0.0	10/10	3463
55	7.620773667	*7.620882328*	*7.628059544*	*7.636212661*	1.09E-04	1/10	7165
56	7.653389321	*7.653432823*	*7.653432823*	*7.653432823*	4.35E-05	10/10	5372
57	7.655669707	7.655669707	*7.655716903*	*7.655758979*	0.0	1/10	9899
58	7.656841094	7.656841094	*7.656853184*	*7.656854184*	0.0	1/10	3670
59	7.656854249	7.656854249	7.656854249	7.656854249	0.0	10/10	148
60	7.656854249	7.656854249	7.656854249	7.656854249	0.0	10/10	77
61	7.656854249	7.656854249	7.656854249	7.656854249	0.0	10/10	22
62	7.656854249	7.656854249	7.656854249	7.656854249	0.0	10/10	24
63	7.656854249	7.656854249	7.656854249	7.656854249	0.0	10/10	48
64	7.889947715	7.889947715	7.889947715	7.889947715	0.0	10/10	2047
65	7.925020971	7.925020971	*7.925022749*	*7.925038736*	0.0	8/10	6318
66	7.952940940	7.952940940	*7.953029851*	*7.953163218*	0.0	6/10	3036
67	8.033348821	8.033348821	8.033348821	8.033348821	0.0	10/10	162
68	8.101386823	8.101386823	*8.101390107*	*8.101391235*	0.0	1/10	6413
69	8.120834459	8.120834459	8.120834459	8.120834459	0.0	10/10	2275
70	8.127588230	8.127588230	*8.127588231*	*8.127588234*	0.0	7/10	6590
71	8.128132184	8.128132184	8.128132184	8.128132184	0.0	10/10	325
72	8.128258770	8.128258770	8.128258770	8.128258770	0.0	10/10	141
73	8.211019328	8.211019328	8.211019328	8.211019328	0.0	10/10	122
74	8.241389611	8.241389611	*8.241389617*	*8.241389619*	0.0	1/10	3023
75	8.242565702	*8.242565746*	*8.242565933*	*8.242566059*	4.40E-08	1/10	4648
76	8.320605897	8.320605897	8.320605897	8.320605897	0.0	10/10	108
77	8.399470764	8.399470764	*8.399470767*	*8.399470793*	0.0	9/10	4472
78	8.435541485	8.435541485	8.435541485	8.435541485	0.0	10/10	3995
79	8.473570565	8.473570565	8.473570565	8.473570565	0.0	10/10	1100
80	8.489819691	8.489819691	8.489819691	8.489819691	0.0	10/10	1782
81	8.539868132	8.539868132	*8.540155680*	*8.541305874*	0.0	8/10	4485
82	8.559217076	8.559217076	8.559217076	*8.559217077*	0.0	7/10	5757
83	8.599549559	8.599549559	8.599549559	8.599549559	0.0	10/10	7053
84	8.599663291	8.599663291	8.599663291	8.599663291	0.0	10/10	135
85	8.599663291	8.599663291	8.599663291	8.599663291	0.0	10/10	188
86	8.599663291	8.599663291	8.599663291	8.599663291	0.0	10/10	633
87	8.599663291	8.599663291	8.599663291	8.599663291	0.0	10/10	277
88	8.656402355	8.656402355	8.656402355	8.656402355	0.0	10/10	33
89	8.789682205	*8.789686708*	*8.789686708*	*8.789686708*	4.50E-06	10/10	2581
90	8.827500619	8.827500619	8.827500619	8.827500619	0.0	10/10	1113
91	8.863214510	8.863214510	8.863214510	8.863214510	0.0	10/10	245
92	8.888778080	8.888778080	*8.888865375*	*8.889526593*	0.0	6/10	7170
93	8.898548794	8.898548794	8.898548794	8.898548794	0.0	10/10	1377
94	8.923821830	**8.923821803**	**8.923821803**	**8.923821803**	-2.75E-08	10/10	968
95	8.947693827	*8.947729596*	*8.948764749*	*8.949115497*	3.58E-05	1/10	7885
96	8.956098398	**8.956012987**	**8.956036903**	8.956098398	-8.54E-05	1/10	8414
97	8.964667081	*8.964667091*	*8.964667227*	*8.964667510*	9.58E-09	1/10	7530
98	8.967082193	8.967082193	*8.967082195*	*8.967082212*	0.0	5/10	5251
99	8.977671508	8.977671508	*8.977671528*	*8.977671576*	0.0	1/10	2951
100	8.978083353	8.978083353	8.978083353	8.978083353	0.0	10/10	63
#Better		2	2	1			
#Equal		42	30	30			
#Worst		6	18	19			

Table 11 Computational results and comparison on the PESC instances with $101 \leq N \leq 150$. In terms of L_{best}, L_{avg} and L_{worst}, the better results are indicated in bold compared to the best-known results L^* and the worse results are indicated in italic.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best}-L^*$	SR	$time(s)$
101	9.071067812	**9.071067798**	**9.071067810**	9.071067812	-1.43E-08	1/10	11220
102	9.071067812	**9.071067752**	**9.071067806**	9.071067812	-6.03E-08	1/10	3715
103	9.071067812	9.071067812	9.071067812	9.071067812	0.0	10/10	3740
104	9.071067812	9.071067812	9.071067812	9.071067812	0.0	10/10	2251
105	9.071067812	9.071067812	9.071067812	9.071067812	0.0	10/10	743
106	9.071067812	9.071067812	9.071067812	9.071067812	0.0	10/10	269
107	9.071067812	9.071067812	9.071067812	9.071067812	0.0	10/10	215
108	9.071067812	9.071067812	9.071067812	9.071067812	0.0	10/10	216
109	9.256067536	**9.256055096**	**9.256055514**	**9.256056491**	-1.24E-05	7/10	11143
110	9.315177255	**9.315142168**	*9.315212581*	*9.315221869*	-3.51E-05	1/10	14974
111	9.350891923	**9.350777107**	**9.350777108**	**9.350777109**	-1.15E-04	6/10	11277
112	9.381202299	**9.381195998**	**9.381200412**	*9.381202321*	-6.30E-06	3/10	9137
113	9.432624730	**9.432624729**	**9.432624729**	9.432624730	-3.86E-10	8/10	8677
114	9.466230073	**9.466077280**	**9.466083267**	**9.466089398**	-1.53E-04	1/10	21491
115	9.511189528	**9.506114111**	**9.506203107**	**9.506583834**	-5.08E-03	1/10	26933
116	9.521926074	**9.521920535**	*9.521937937*	*9.522003637*	-5.54E-06	2/10	27387
117	9.536291586	**9.535542146**	**9.535575768**	**9.535619417**	-7.49E-04	1/10	24729
118	9.539688701	**9.539355204**	**9.539357756**	**9.539379920**	-3.33E-04	1/10	21276
119	9.541327956	**9.541325287**	*9.541328387*	*9.541335327*	-2.67E-06	1/10	22175
120	9.542056732	**9.542033251**	**9.542036993**	**9.542048033**	-2.35E-05	5/10	21320
121	9.542472333	9.542472333	9.542472333	9.542472333	0.0	10/10	657
122	9.542472333	9.542472333	9.542472333	9.542472333	0.0	10/10	7150
123	9.647335796	**9.647281695**	**9.647281696**	**9.647281696**	-5.41E-05	3/10	10293
124	9.655733621	**9.655730130**	**9.655730134**	**9.655730143**	-3.49E-06	2/10	10017
125	9.656804408	**9.656804109**	*9.656807388*	*9.656836792*	-2.99E-07	1/10	17708
126	9.656854249	9.656854249	9.656854249	9.656854249	0.0	10/10	2578
127	9.734886581	**9.734698832**	**9.734698832**	**9.734698832**	-1.88E-04	10/10	459
128	9.790670672	9.790670672	9.790670672	9.790670672	0.0	10/10	625
129	9.831897328	**9.831895609**	**9.831896468**	9.831897328	-1.72E-06	5/10	15246
130	9.853471206	*9.853907242*	*9.854004373*	*9.854015165*	4.36E-04	1/10	11370
131	9.877737172	**9.877735748**	**9.877735748**	**9.877735748**	-1.42E-06	10/10	2329
132	9.900529520	**9.900168047**	**9.900168047**	**9.900168047**	-3.61E-04	10/10	2876
133	9.915905083	9.915905083	9.915905083	9.915905083	0.0	10/10	403
134	9.969788469	**9.968398864**	**9.968645712**	**9.969623732**	-1.39E-03	1/10	28954
135	9.997248155	**9.992767099**	*9.997381737*	*9.998641055*	-4.48E-03	1/10	26822
136	10.005773959	**10.002501384**	**10.003742524**	**10.004988547**	-3.27E-03	1/10	26944
137	10.009904202	**10.009483099**	**10.009680230**	**10.009875386**	-4.21E-04	1/10	18545
138	10.013103535	**10.013103499**	*10.013144985*	*10.013518360*	-3.59E-08	9/10	12086
139	10.013875394	**10.013871054**	**10.013875013**	*10.013876785*	-4.34E-06	1/10	7933
140	10.013876853	10.013876853	10.013876853	10.013876853	0.0	10/10	429
141	10.013876853	10.013876853	10.013876853	10.013876853	0.0	10/10	275
142	10.013876853	10.013876853	10.013876853	10.013876853	0.0	10/10	236
143	10.013876853	10.013876853	10.013876853	10.013876853	0.0	10/10	181
144	10.013876853	10.013876853	10.013876853	10.013876853	0.0	10/10	182
145	10.182224825	**10.182224821**	**10.182224821**	**10.182224821**	-4.49E-09	10/10	480
146	10.234614828	**10.234607999**	**10.234607999**	**10.234607999**	-6.83E-06	10/10	13881
147	10.261562986	**10.261561623**	**10.261561623**	**10.261561623**	-1.36E-06	10/10	11644
148	10.300703719	**10.297021071**	**10.297030385**	**10.297096775**	-3.68E-03	7/10	23757
149	10.326031998	**10.323831269**	**10.325884477**	*10.327169850*	-2.20E-03	1/10	28434
150	10.341280996	**10.333464149**	**10.340354502**	*10.341368987*	-7.82E-03	1/10	27849
#Better		33	27	19			
#Equal		16	16	20			
#Worst		1	7	11			

Table 12 Computational results and comparison on the PESC instances with $151 \leq N \leq 200$. In terms of L_{best}, L_{avg} and L_{worst}, the better results are indicated in bold compared to the best-known results L^* and the worse results are indicated in italic.

		PBTS (this work)					
N	L^*	L_{best}	L_{avg}	L_{worst}	$L_{best}-L^*$	SR	$time(s)$
151	10.365253238	**10.358713606**	**10.358713607**	**10.358713607**	-6.54E-03	10/10	16650
152	10.374597071	**10.372956934**	**10.373321274**	**10.373920178**	-1.64E-03	1/10	19768
153	10.379012294	10.379012294	10.379012294	10.379012294	0.0	10/10	5015
154	10.397742319	**10.392876250**	**10.392876250**	**10.392876250**	-4.87E-03	10/10	9584
155	10.432530776	**10.420260141**	**10.427699561**	**10.429561486**	-1.23E-02	2/10	28985
156	10.441716576	10.441716576	*10.442288619*	*10.443779819*	0.0	2/10	28905
157	10.450379879	**10.450379451**	**10.450379701**	**10.450379809**	-4.28E-07	1/10	12180
158	10.466444211	**10.450379879**	**10.450379879**	**10.450379879**	-1.61E-02	10/10	3258
159	10.468298279	**10.468298215**	**10.468298216**	**10.468298219**	-6.40E-08	1/10	12943
160	10.475182230	**10.475182106**	*10.475654767*	*10.479908715*	-1.24E-07	9/10	11790
161	10.479983040	**10.479983036**	**10.479983038**	10.479983040	-4.25E-09	1/10	5665
162	10.479983040	10.479983040	10.479983040	10.479983040	0.0	10/10	167
163	10.485281374	10.485281374	10.485281374	10.485281374	0.0	10/10	14199
164	10.485281374	10.485281374	*10.488511331*	*10.517580945*	0.0	9/10	12534
165	10.485281374	10.485281374	10.485281374	10.485281374	0.0	10/10	8833
166	10.485281374	10.485281374	*10.495814352*	*10.590611153*	0.0	9/10	10697
167	10.485281374	10.485281374	10.485281374	10.485281374	0.0	10/10	12509
168	10.485281374	10.485281374	10.485281374	10.485281374	0.0	10/10	11233
169	10.485281374	10.485281374	10.485281374	10.485281374	0.0	10/10	7305
170	10.485281374	10.485281374	10.485281374	10.485281374	0.0	10/10	8954
171	10.485281374	10.485281374	10.485281374	10.485281374	0.0	10/10	10832
172	10.485281374	10.485281374	10.485281374	10.485281374	0.0	10/10	8865
173	10.691905399	*10.691937617*	*10.692023721*	*10.692457981*	3.22E-05	1/10	24389
174	10.731753582	**10.723893469**	**10.724306791**	**10.724699074**	-7.86E-03	1/10	27666
175	10.747079787	**10.746999788**	**10.747032429**	*10.747110657*	-8.00E-05	1/10	30256
176	10.769091939	**10.769085289**	*10.769092376*	*10.769104934*	-6.65E-06	1/10	23716
177	10.797319352	**10.795952217**	**10.796247861**	**10.797210032**	-1.37E-03	1/10	28788
178	10.874840404	**10.843408746**	**10.843412293**	**10.843416634**	-3.14E-02	1/10	19977
179	10.906678902	**10.878347102**	**10.878379424**	**10.878410485**	-2.83E-02	1/10	28452
180	10.908568331	**10.907954805**	**10.908062110**	**10.908161133**	-6.14E-04	1/10	27000
181	10.934189362	**10.925071993**	**10.927841431**	**10.929516821**	-9.12E-03	1/10	24037
182	10.945963366	**10.925072004**	**10.933672289**	**10.945683459**	-2.09E-02	4/10	23550
183	10.948204573	**10.943111098**	**10.946483741**	*10.948731302*	-5.09E-03	1/10	32062
184	10.949401271	**10.949347370**	*10.949680571*	*10.950586218*	-5.39E-05	1/10	28785
185	10.954519536	**10.953766994**	**10.954361610**	*10.955192297*	-7.53E-04	1/10	31908
186	10.955528631	**10.954992621**	**10.955149270**	**10.955303610**	-5.36E-04	1/10	31233
187	10.956001897	**10.955754270**	**10.955805068**	**10.955862655**	-2.48E-04	1/10	28741
188	10.956685895	10.956685895	10.956685895	10.956685895	0.0	10/10	3123
189	10.956685895	10.956685895	10.956685895	10.956685895	0.0	10/10	3674
190	10.956685895	10.956685895	10.956685895	10.956685895	0.0	10/10	7131
191	11.064604714	**11.050914327**	**11.050927060**	**11.050944819**	-1.37E-02	1/10	18574
192	11.071024807	**11.068578718**	**11.069427605**	**11.070888037**	-2.45E-03	1/10	32518
193	11.071067812	**11.070025179**	**11.070608305**	**11.070954343**	-1.04E-03	1/10	31844
194	11.071067812	**11.070959098**	**11.071024624**	**11.071067631**	-1.09E-04	1/10	22539
195	11.071067812	11.071067812	11.071067812	11.071067812	0.0	10/10	682
196	11.071067812	11.071067812	11.071067812	11.071067812	0.0	10/10	335
197	11.183482546	**11.148837915**	**11.148837917**	**11.148837919**	-3.46E-02	4/10	15727
198	11.216034385	**11.210995781**	**11.211004188**	**11.211027779**	-5.04E-03	3/10	12052
199	11.249615229	**11.225541804**	**11.235037146**	**11.237981142**	-2.41E-02	1/10	28035
200	11.271851411	**11.249879531**	**11.253167051**	**11.264665663**	-2.20E-02	1/10	33317
#Better		31	28	24			
#Equal		18	15	16			
#Worst		1	7	10			