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Online Supplement for "An efficient optimization model and tabu search-based global optimization approach for continuous p-dispersion problem"

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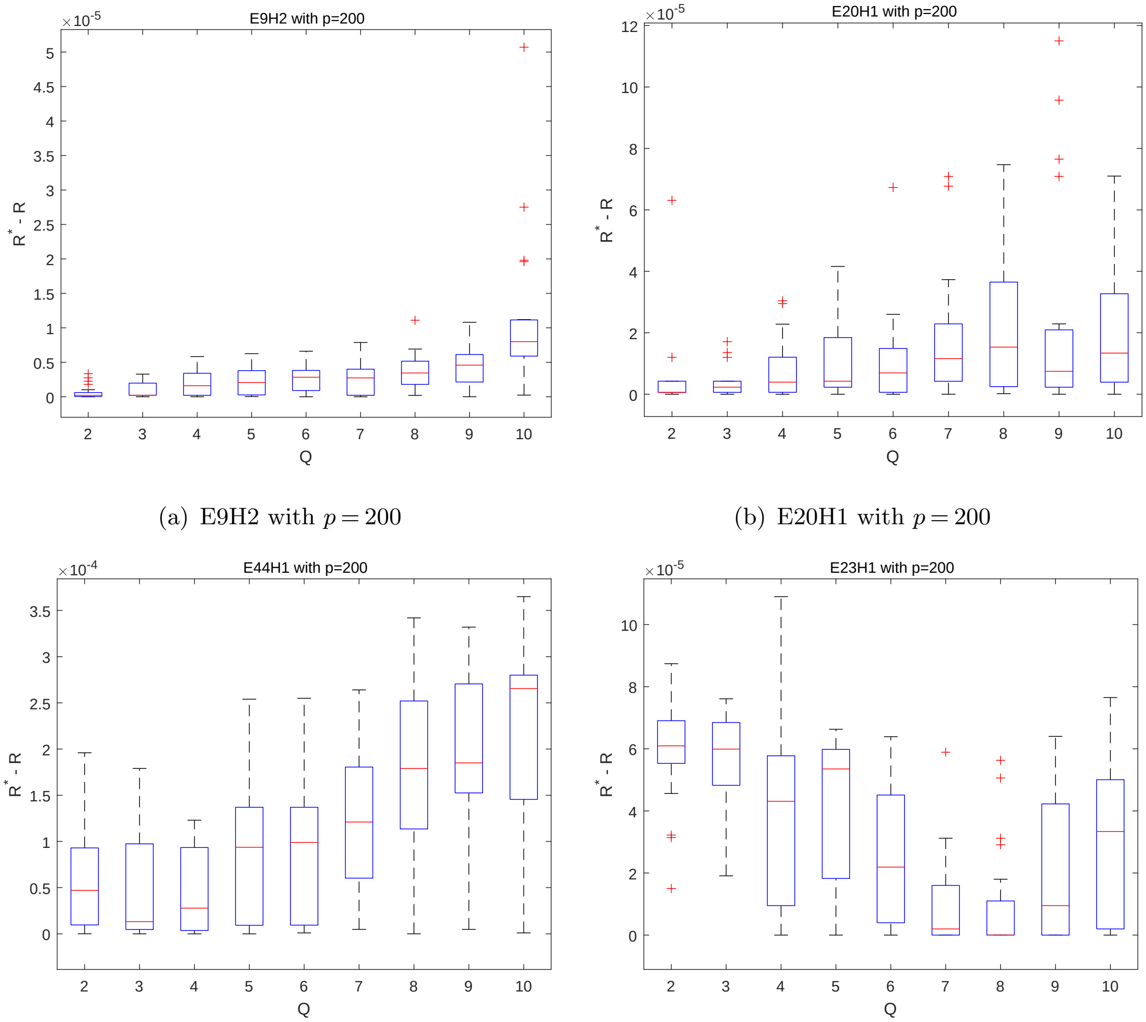
This online supplement provides a sensitivity analysis of several key parameters of the proposed TSGO algorithm and detailed computational results of the TSGO algorithm and its two main reference algorithms on benchmark instances.

Key words: Circle packing, continuous dispersion problem, global optimization, tabu search, nonlinear optimization.

1. Sensitivity analysis of the key parameters

This section presents a sensitivity analysis of the tabu tenure of the tabu search method and three key parameters, i.e., Q, θ_{max} and β_{max}.

1.1. Sensitivity analysis of parameter Q.



(a) E9H2 with $p = 200$

(b) E20H1 with $p = 200$

(c) E44H1 with $p = 200$

(d) E23H1 with $p = 200$

Figure 1 Influence of parameter Q on the performance of the TSGO algorithm for four representative instances.

Recall that Q is a parameter of the tabu search component of our TSGO algorithm, which determines the size of the neighborhood $N_{ins}(X)$ by setting $|N_{ins}(X)| = Q^2$. To show the influence of the neighborhood size determined by Q on the performance of TSGO algorithm and determine a suitable Q value, we performed an experiment based on 4 representative instances of the continuous p-dispersion problem with boundary constraints.

In this experiment, the TSGO algorithm was run 20 times for each instance and for each Q value in the range of $\{2, 3, \ldots, 10\}$, recording the objective values obtained. The experimental results are given in Fig. 1 using the box plots, where the X-axis indicates the values of the parameter Q and the Y-axis indicates the gap between the objective values (R, i.e., the radius of circles) and the best result obtained (R^*).

Fig. 1 shows that the performance of the algorithm is sensitive to the setting of Q and it is difficult to find a setting that works well on all instances. For 3 out of 4 instances (i.e., E9H2, E20H1, E44H1) the algorithms with $Q \leq 3$ performs better than $Q \geq 4$, which implies that the setting of $Q \leq 3$ is suitable for most tested instances. Thus, the default value of parameter Q is set to 3. However, for E23H1 the setting of $Q = 7$ or 8 produced a significantly better result than other settings, which means that the effectiveness of parameter Q depends also on the instances to be solved and that the algorithmic performance can be further improved by fine-tuning the value of Q according to the given instance.

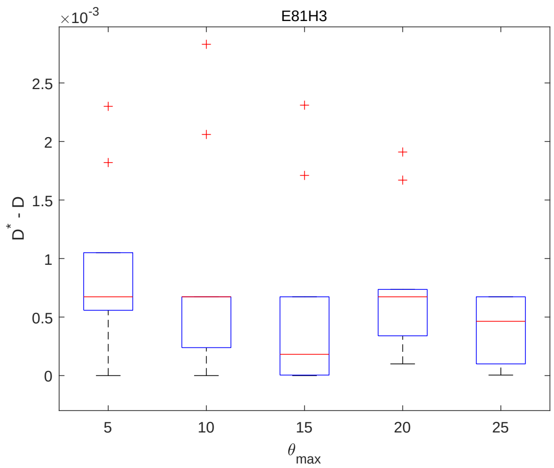
1.2. Sensitivity analysis of parameter θ_{max}.

Recall that θ_{max} is the parameter representing the search depth of the MBH method, which is one of the main components of TSGO, and a larger θ_{max} value means a more intensive search, and vice versa. To check the influence of θ_{max} on the performance of TSGO, we performed an experiment based on four representative instances of the point arrangement problem. In the experiment, the TSGO algorithm was performed 10 times for each instance and for each θ_{max} value in the range of $\{5, 10, 15, 20, 25\}$, and the results are given in Fig. 2 using the box plots, where the X-axis indicates the values of θ_{max} and the Y-axis indicates the gap between the objective values (D, i.e., the minimum distance between the dispersion points) and the best objective obtained (D^*).

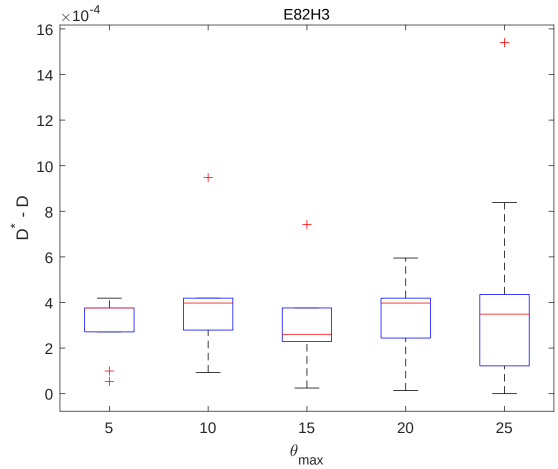
Fig. 2 shows that TSGO's performance depends on the setting of parameter θ_{max} and that a too large or too small θ_{max} value deteriorates its performance. Specifically, the setting of $\theta_{max} = 15$ leads to the best performance for 3 out of 4 instances among the tested settings. Moreover, $\theta_{max} = 15$ and $\theta_{max} = 20$ lead to similar performance for the remaining instance (i.e., E203H3). Thus, the default value of θ_{max} was set to 15 for the point arrangement problem in this work.

1.3. Sensitivity analysis of parameter β_{max}.

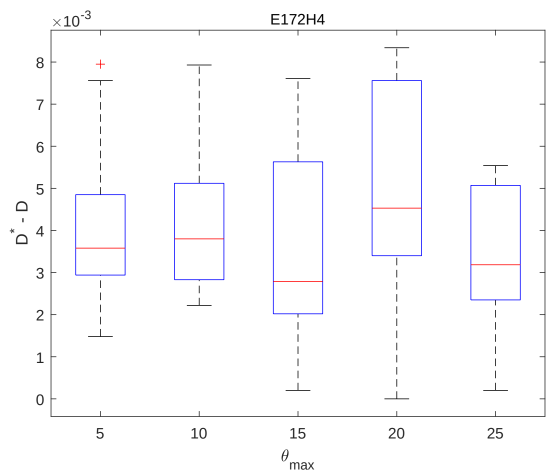
The parameter β_{max} represents the search depth of the tabu search method, and a larger β_{max} value favors a more intensive search for the proposed algorithm, and vice versa.



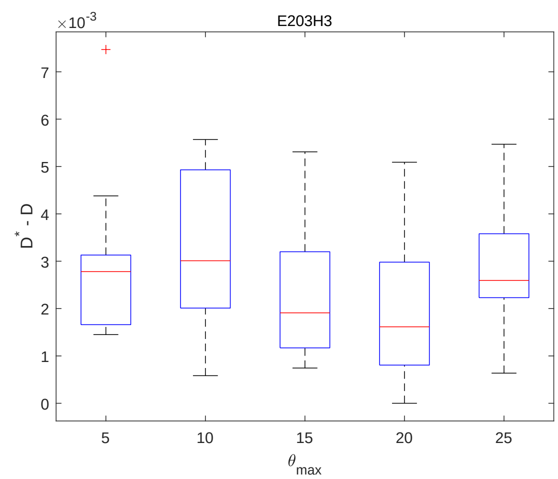
(a) E81H3 with $p = 100$



(b) E82H3 with $p = 100$



(c) E172H4 with $p = 100$

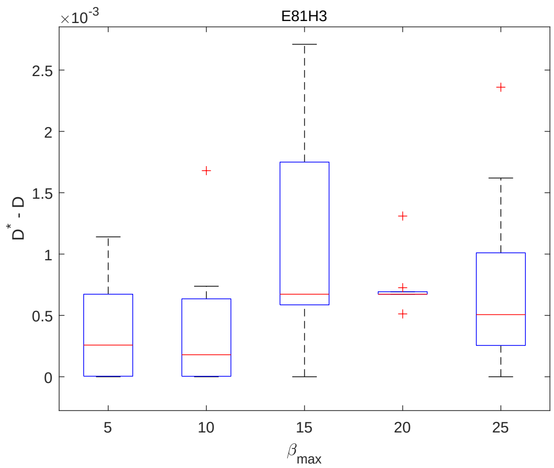


(d) E203H3 with $p = 100$

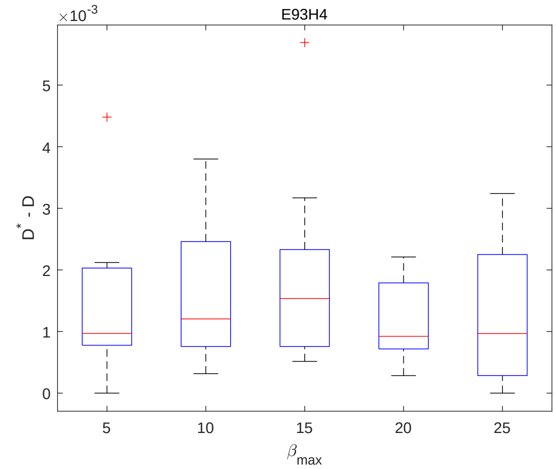
Figure 2 **Influence of parameter θ_{max} on the performance of the TSGO algorithm for four representative instances.**

To check its influence on the performance of TSGO, we conducted an experiment based on four representative instances of the point arrangement problem. In this experiment, TSGO was performed 10 times for each instance and for each β_{max} value in the range of $\{5, 10, 15, 20, 25\}$. The experiment results are summarized in Fig. 3 using the box plots, where the X-axis indicates the parameter values and the Y-axis indicates the gap between the objective values (D) and the best objective obtained (D^*).

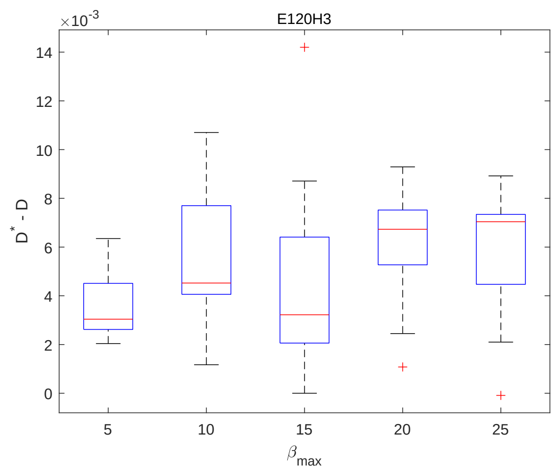
One observes from Fig. 3 that TSGO is sensitive to the setting of parameter β_{max} and that the best setting of β_{max} depends on the instance to be solved. For example, for E81H3 with $p = 100$ $\beta_{max} = 10$ results in the best result among the five settings, but for E93H4



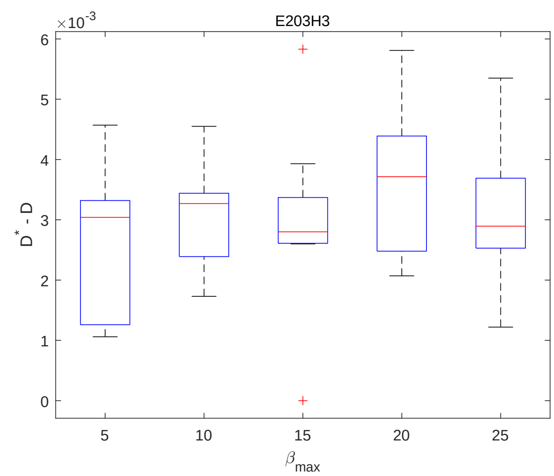
(a) E81H3 with $p = 100$



(b) E93H4 with $p = 100$



(c) E120H3 with $p = 100$



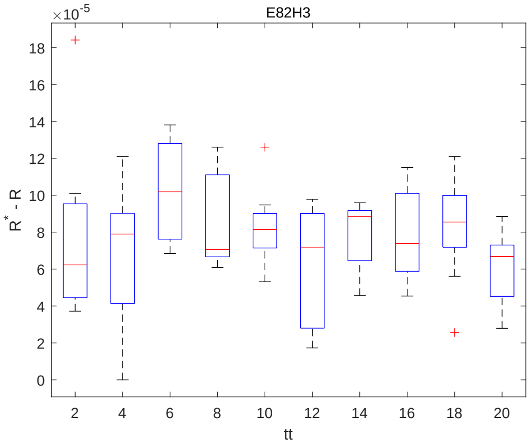
(d) E203H3 with $p = 100$

Figure 3 **Influence of parameter β_{max} on the performance of the TSGO algorithm for four representative instances.**

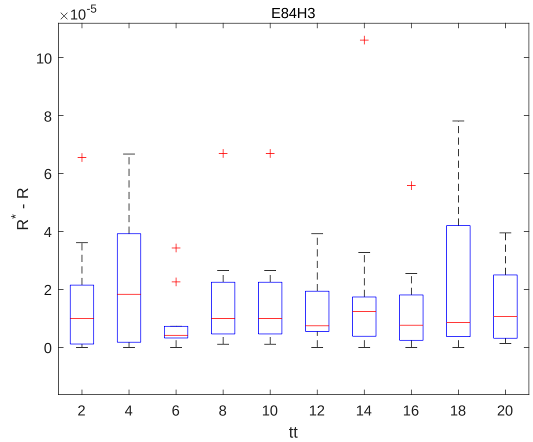
the setting of $\beta_{max} = 20$ results in the best result. On the other hand, a small β_{max} value like $\beta_{max} = 5$ is able to yield the desired result for most instances tested. Thus, the default value of β_{max} was set to 5 for the point arrangement problem in this work.

1.4. Sensitivity analysis of tabu tenure.

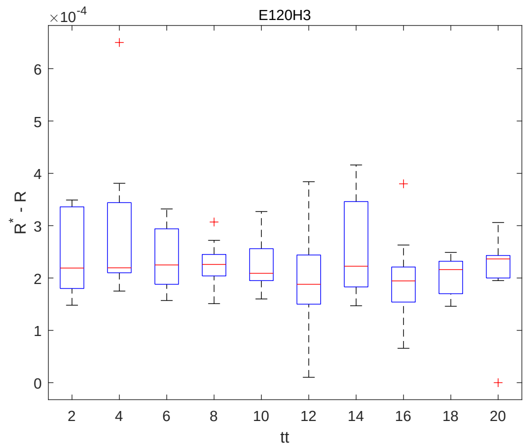
To evaluate the influence of the tabu tenure tt on the performance of the algorithm, we performed an experiment based on six representative instances of the equal circle packing problem, where the TSGO algorithm was performed 10 times for each instance and for each tabu tenure tt in the range of $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$. The results are summarized in Fig.4 using the box plots, where the X-axis indicates the value of tabu tenure tt and the



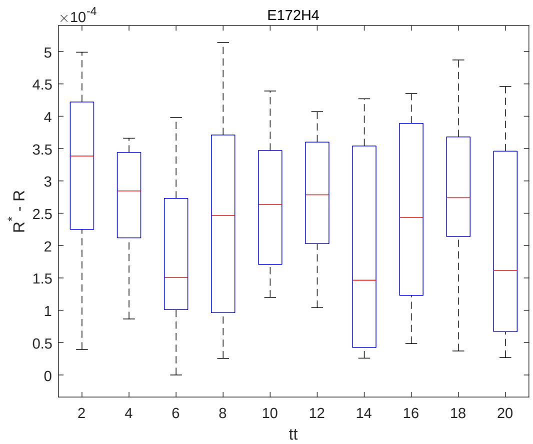
(a) E82H3 with $p = 200$



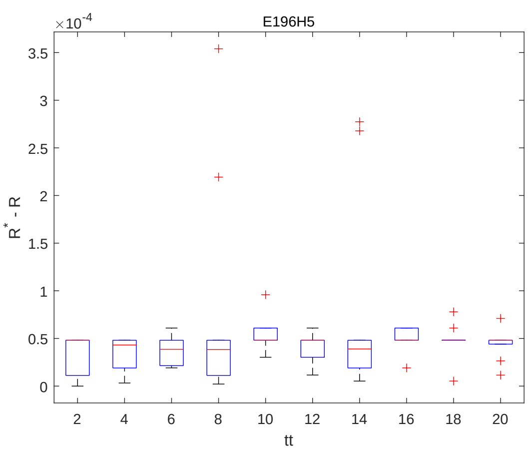
(b) E84H3 with $p = 200$



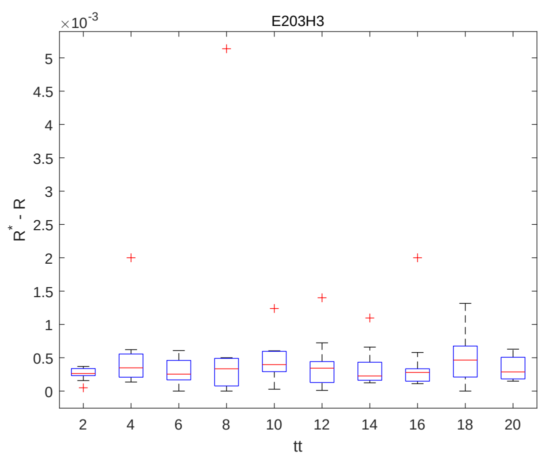
(c) E120H3 with $p = 200$



(d) E172H4 with $p = 200$



(e) E196H5 with $p = 200$



(f) E203H3 with $p = 200$

Figure 4 Influence of tabu tenure tt on the performance of TSGO algorithm for six representative instances.

Y-axis indicates the gap between the objective values (R, i.e., the radius of circles) and the best objective obtained (R^*).

Fig.4 shows that the best setting of the tabu tenure tt depends on the instance to be solved and it is difficult to find a general tt value for all instances. Based on the observation that the TSGO algorithm performs well for most tt values in the interval $[6, 10]$ and for most tested instances, we empirically set $tt = 5 + rand(5)$, where $rand(5)$ is a random number between 0 and 5.

2. Detailed computational results on the continuous p-dispersion problem with boundary constraints

Table 1 Computational results of the TSGO algorithm on 60 simple instances with boundary constraints.

Instance				TSGO (This work)					
Container	$\|E\|$	$\|H\|$	p	R_{best}	R_{avg}	R_{worst}	SR	σ	$time(s)$
E4H0a	4	0	50	0.0713771039	0.0713771039	0.0713771039	10/10	0.00	9.3
E4H0b	4	0	50	0.2721671717	0.2721671717	0.2721671717	10/10	0.00	5.5
E6H0	6	0	50	0.1226935182	0.1226935182	0.1226935182	10/10	0.00	2.4
E9H2	9	2	50	0.3689738179	0.3689738179	0.3689738179	10/10	0.00	19.5
E11H0	11	0	50	0.4368343737	0.4368343737	0.4368343737	10/10	0.00	11.7
E12H0a	12	0	50	0.6225798476	0.6225798476	0.6225798476	10/10	0.00	9.8
E12H0b	12	0	50	0.4620855791	0.4620855791	0.4620855791	10/10	0.00	42.6
E12H2	12	2	50	0.4448612715	0.4448612715	0.4448612715	10/10	0.00	15.7
E18H0	18	0	50	0.7877071509	0.7877071509	0.7877071509	10/10	0.00	13.3
E20H1	20	1	50	0.4137431060	0.4137431060	0.4137431060	10/10	0.00	29.4
E20H2	20	2	50	0.6402224988	0.6402224988	0.6402224988	10/10	0.00	19.4
E23H1	23	1	50	0.4005485124	0.4005485124	0.4005485124	10/10	0.00	10.6
E27H1	27	1	50	0.4844567507	0.4844567507	0.4844567507	10/10	0.00	8.8
E44H1	44	1	50	0.5345742337	0.5345742337	0.5345742337	10/10	0.00	122.4
E59H1	59	1	50	0.0199947494	0.0199947494	0.0199947494	10/10	0.00	111.8
E4H0a	4	0	100	0.0514010718	0.0514010718	0.0514010718	10/10	0.00	31.6
E4H0b	4	0	100	0.1978021937	0.1978021937	0.1978021937	10/10	0.00	110.5
E6H0	6	0	100	0.0882825264	0.0882825264	0.0882825264	10/10	0.00	77.1
E9H2	9	2	100	0.2801620820	0.2801620820	0.2801620820	10/10	0.00	107.8
E11H0	11	0	100	0.3181468932	0.3181468932	0.3181468932	10/10	0.00	142.4
E12H0a	12	0	100	0.4517241097	0.4517241097	0.4517241097	10/10	0.00	565.2
E12H0b	12	0	100	0.3361308135	0.3361308135	0.3361308135	10/10	0.00	351.4
E12H2	12	2	100	0.3172501921	0.3172501921	0.3172501921	10/10	0.00	252.1
E18H0	18	0	100	0.5761713874	0.5761713874	0.5761713874	10/10	0.00	400.1
E20H1	20	1	100	0.2989547054	0.2989547054	0.2989547054	10/10	0.00	1120.8
E20H2	20	2	100	0.4733830176	0.4733830176	0.4733830176	10/10	0.00	175.2
E23H1	23	1	100	0.2896330498	0.2896316545	0.2896216366	5/10	3.35E-06	2162.0
E27H1	27	1	100	0.3541756870	0.3541756870	0.3541756870	10/10	0.00	362.5
E44H1	44	1	100	0.3907223295	0.3907223295	0.3907223295	10/10	0.00	565.0
E59H1	59	1	100	0.0150349633	0.0150349588	0.0150349406	8/10	9.09E-09	1541.7
E4H0a	4	0	150	0.0421454577	0.0421454454	0.0421453344	9/10	3.70E-08	2062.1
E4H0b	4	0	150	0.1636890708	0.1636890708	0.1636890708	10/10	0.00	805.5
E6H0	6	0	150	0.0725873025	0.0725873025	0.0725873025	10/10	0.00	921.9
E9H2	9	2	150	0.2333481422	0.2333481422	0.2333481422	10/10	0.00	2878.0
E11H0	11	0	150	0.2628105627	0.2628104872	0.2628102971	7/10	1.15E-07	5247.1
E12H0a	12	0	150	0.3730487798	0.3730484422	0.3730454043	9/10	1.01E-06	4207.5
E12H0b	12	0	150	0.2783387066	0.2783387066	0.2783387066	10/10	0.00	1504.2
E12H2	12	2	150	0.2636360626	0.2636360626	0.2636360626	10/10	0.00	1512.0
E18H0	18	0	150	0.4800573753	0.4800534988	0.4800437063	1/10	5.02E-06	4397.4
E20H1	20	1	150	0.2508905275	0.2508892317	0.2508853392	2/10	1.68E-06	4907.4
E20H2	20	2	150	0.3929380796	0.3929380796	0.3929380796	10/10	0.00	1000.7
E23H1	23	1	150	0.2425858370	0.2425856154	0.2425853361	2/10	1.79E-07	5166.3
E27H1	27	1	150	0.2930756336	0.2930447613	0.2929186640	7/10	5.24E-05	4199.4
E44H1	44	1	150	0.3253697234	0.3253657613	0.3253339089	2/10	1.06E-05	4159.2
E59H1	59	1	150	0.0126021266	0.0125992932	0.0125955719	1/10	1.58E-06	6371.8
E4H0a	4	0	200	0.0366127989	0.0366083653	0.0365930121	3/10	7.69E-06	8538.4
E4H0b	4	0	200	0.1429882126	0.1429882126	0.1429882126	10/10	0.00	4186.0
E6H0	6	0	200	0.0636129856	0.0636129856	0.0636129856	10/10	0.00	7224.2
E9H2	9	2	200	0.2051780567	0.2051759042	0.2051720545	1/10	1.85E-06	8318.8
E11H0	11	0	200	0.2300479293	0.2300479293	0.2300479293	10/10	0.00	1821.3
E12H0a	12	0	200	0.3303513838	0.3303506670	0.3303497622	5/10	7.63E-07	4106.0
E12H0b	12	0	200	0.2397150046	0.2397147555	0.2397145332	3/10	1.81E-07	7345.3
E12H2	12	2	200	0.2334870331	0.2334801734	0.2334692402	1/10	7.44E-06	10673.5
E18H0	18	0	200	0.4191458163	0.4190839637	0.4189449983	2/10	6.27E-05	12150.4
E20H1	20	1	200	0.2172791061	0.2172676640	0.2172289079	1/10	1.42E-05	10461.0
E20H2	20	2	200	0.3447056183	0.3446855508	0.3446820377	1/10	6.79E-06	9201.7
E23H1	23	1	200	0.2102037845	0.2101661510	0.2101431359	1/10	1.99E-05	11331.5
E27H1	27	1	200	0.2595897715	0.2595486576	0.2595101870	1/10	3.16E-05	9821.7
E44H1	44	1	200	0.2816788437	0.2815233736	0.2814098733	1/10	7.34E-05	12057.6
E59H1	59	1	200	0.0111178733	0.0111156356	0.0111114885	1/10	1.72E-06	10219.1

Table 2 Computational results of the TSGO algorithm on 45 complicated instances with boundary constraints.

Instance				TSGO (This work)					
Container	$\|E\|$	$\|H\|$	p	R_{best}	R_{avg}	R_{worst}	SR	σ	$time(s)$
E101H2	101	2	100	0.5131508522	0.5131508522	0.5131508522	10/10	0.00	1397.0
E101H3	101	3	100	0.7238846144	0.7238775278	0.7238753902	2/10	3.65E-06	2268.4
E106H3	106	3	100	0.6039167191	0.6039165094	0.6039146219	9/10	6.29E-07	1982.0
E106H5	106	5	100	0.5332531100	0.5331975141	0.5331581027	1/10	2.55E-05	2813.1
E107H3	107	3	100	1.1937852368	1.1937852368	1.1937852368	10/10	0.00	690.6
E120H3	120	3	100	0.8773789961	0.8773533279	0.8773503983	1/10	8.56E-06	1495.2
E172H4	172	4	100	0.4001953141	0.4001905629	0.4001790263	5/10	6.44E-06	2856.6
E193H1	193	1	100	0.8969499567	0.8966769264	0.8963631269	1/10	1.95E-04	2435.9
E196H5	196	5	100	0.4756223520	0.4756214961	0.4756137936	9/10	2.57E-06	1817.9
E203H3	203	3	100	0.2964789245	0.2957012517	0.2950784493	1/10	3.95E-04	3460.3
E60H1	60	1	100	0.1389364842	0.1389224366	0.1389060989	3/10	1.20E-05	2364.9
E81H3	81	3	100	0.4596304590	0.4596270833	0.4596199620	5/10	3.63E-06	2565.6
E82H3	82	3	100	0.4239299929	0.4239299929	0.4239299929	10/10	0.00	1585.3
E84H3	84	3	100	0.4320481250	0.4320481250	0.4320481250	10/10	0.00	1163.4
E93H4	93	4	100	0.5936444451	0.5936093706	0.5935254279	1/10	4.51E-05	2503.9
E101H2	101	2	150	0.4246301877	0.4245517063	0.4244364627	1/10	6.42E-05	5233.0
E101H3	101	3	150	0.6022981927	0.6022931886	0.6022903843	1/10	2.51E-06	5311.9
E106H3	106	3	150	0.4996758005	0.4995933394	0.4994616507	1/10	7.46E-05	6205.4
E106H5	106	5	150	0.4444091109	0.4443309623	0.4442520370	1/10	4.18E-05	5681.8
E107H3	107	3	150	0.9872881106	0.9872064963	0.9870158687	1/10	8.05E-05	5813.1
E120H3	120	3	150	0.7306445570	0.7306432422	0.7306324040	8/10	3.62E-06	5624.6
E172H4	172	4	150	0.3332646035	0.3331663193	0.3330925105	1/10	4.18E-05	5907.3
E193H1	193	1	150	0.7409137632	0.7404794296	0.7389815314	1/10	5.26E-04	6926.1
E196H5	196	5	150	0.3997206613	0.3995365485	0.3988107216	1/10	2.65E-04	6546.5
E203H3	203	3	150	0.2512178335	0.2500754648	0.2483444155	1/10	9.41E-04	6987.4
E60H1	60	1	150	0.1151505241	0.1151269139	0.1151106888	1/10	1.08E-05	5677.9
E81H3	81	3	150	0.3802766660	0.3802496299	0.3801959565	1/10	2.89E-05	5473.0
E82H3	82	3	150	0.3490791105	0.3490722172	0.3490514816	2/10	9.80E-06	5087.5
E84H3	84	3	150	0.3579871420	0.3579385074	0.3577198430	3/10	8.18E-05	5645.2
E93H4	93	4	150	0.4948698733	0.4948247350	0.4947154867	1/10	4.19E-05	6173.2
E101H2	101	2	200	0.3708266880	0.3708085633	0.3708000494	2/10	1.01E-05	9235.6
E101H3	101	3	200	0.5271473970	0.5270959759	0.5268497538	1/10	9.64E-05	11894.7
E106H3	106	3	200	0.4384167531	0.4383871870	0.4383279802	1/10	2.87E-05	11821.2
E106H5	106	5	200	0.3910559351	0.3910204812	0.3909799396	1/10	2.56E-05	11785.9
E107H3	107	3	200	0.8601603145	0.8600083142	0.8597126848	1/10	1.69E-04	11456.7
E120H3	120	3	200	0.6352456666	0.6351595315	0.6350429648	1/10	5.28E-05	12337.9
E172H4	172	4	200	0.2938264340	0.2935365465	0.2933340360	1/10	1.54E-04	12277.3
E193H1	193	1	200	0.6497271655	0.6495499522	0.6494284771	1/10	1.19E-04	10798.2
E196H5	196	5	200	0.3532967101	0.3532569447	0.3531768106	1/10	3.09E-05	10726.4
E203H3	203	3	200	0.2233159947	0.2232194868	0.2231246243	1/10	5.38E-05	12856.2
E60H1	60	1	200	0.1002674230	0.1002478734	0.1002322688	1/10	1.02E-05	8220.7
E81H3	81	3	200	0.3322827528	0.3321969424	0.3321376262	1/10	4.86E-05	11181.5
E82H3	82	3	200	0.3045495631	0.3044844759	0.3044037274	1/10	4.26E-05	9352.3
E84H3	84	3	200	0.3144135487	0.3143854408	0.3143467468	1/10	2.24E-05	12855.1
E93H4	93	4	200	0.4323742595	0.4322075414	0.4318942208	1/10	1.55E-04	12831.8

This section reports the detailed computational results of the TSGO, BH* and MBH* algorithms for the continuous p-dispersion problem with the boundary constraints (i.e., the equal circle packing problem). Tables 1–2 give the detailed computational results of the TSGO respectively for two sets of instances, where the first set contains 60 relatively simple instances and the second set contains 45 complicated instances. Tables 3–4 give the results of the BH* algorithm, and Tables 5–6 give the results of the MBH* algorithm. The first three columns of the tables respectively indicate the names of container, the

Table 3 Computational results of the BH* algorithm on 60 simple instances with boundary constraints.

Instance				Basin-hopping					
Container	$\|E\|$	$\|H\|$	p	R_{best}	R_{avg}	R_{worst}	SR	σ	$time(s)$
E4H0a	4	0	50	0.0713771039	0.0713771039	0.0713771039	10/10	0.00	5.3
E4H0b	4	0	50	0.2721671717	0.2721671717	0.2721671717	10/10	0.00	9.4
E6H0	6	0	50	0.1226935182	0.1226935182	0.1226935182	10/10	0.00	2.9
E9H2	9	2	50	0.3686157489	0.3634125564	0.3572806998	1/10	2.87E-03	27.1
E11H0	11	0	50	0.4368343737	0.4357490250	0.4337620103	1/10	1.02E-03	40.7
E12H0a	12	0	50	0.6225798476	0.6225642581	0.6225027784	5/10	2.43E-05	29.4
E12H0b	12	0	50	0.4620855791	0.4619205142	0.4618509975	2/10	8.77E-05	29.4
E12H2	12	2	50	0.4448612715	0.4448604939	0.4448597163	1/10	3.48E-07	13.0
E18H0	18	0	50	0.7877071509	0.7877071509	0.7877071509	10/10	0.00	14.0
E20H1	20	1	50	0.4137431060	0.4137379287	0.4137333919	4/10	4.31E-06	18.3
E20H2	20	2	50	0.6402224988	0.6402224988	0.6402224988	10/10	0.00	20.2
E23H1	23	1	50	0.4005485124	0.4005485124	0.4005485124	10/10	0.00	9.0
E27H1	27	1	50	0.4844567507	0.4843452715	0.4840050329	7/10	1.81E-04	23.8
E44H1	44	1	50	0.5345742337	0.5344076418	0.5342642960	1/10	1.14E-04	36.2
E59H1	59	1	50	0.0199320353	0.0198443767	0.0197552772	1/10	3.95E-05	31.0
E4H0a	4	0	100	0.0514010718	0.0514010668	0.0514010216	9/10	1.51E-08	125.1
E4H0b	4	0	100	0.1978021937	0.1978021937	0.1978021937	10/10	0.00	72.1
E6H0	6	0	100	0.0882825264	0.0882825264	0.0882825264	10/10	0.00	161.3
E9H2	9	2	100	0.2801620820	0.2801620820	0.2801620820	10/10	0.00	788.2
E11H0	11	0	100	0.3181468932	0.3181468837	0.3181468775	4/10	7.70E-09	130.2
E12H0a	12	0	100	0.4517241097	0.4517241097	0.4517241097	10/10	0.00	934.5
E12H0b	12	0	100	0.3361308135	0.3361308135	0.3361308135	10/10	0.00	85.4
E12H2	12	2	100	0.3172501921	0.3172493170	0.3172472749	7/10	1.34E-06	1551.5
E18H0	18	0	100	0.5761713874	0.5761713874	0.5761713874	10/10	0.00	964.0
E20H1	20	1	100	0.2989547054	0.2989526815	0.2989506483	5/10	2.02E-06	1710.7
E20H2	20	2	100	0.4733830176	0.4733823640	0.4733801150	4/10	8.23E-07	708.5
E23H1	23	1	100	0.2896330498	0.2896299959	0.2896216366	4/10	4.48E-06	1791.0
E27H1	27	1	100	0.3541756870	0.3541756870	0.3541756870	10/10	0.00	216.2
E44H1	44	1	100	0.3907223295	0.3907223295	0.3907223295	10/10	0.00	1224.5
E59H1	59	1	100	0.0150349633	0.0150334227	0.0150292503	2/10	2.30E-06	2278.6
E4H0a	4	0	150	0.0421454577	0.0421453491	0.0421453344	1/10	3.69E-08	916.6
E4H0b	4	0	150	0.1636890708	0.1636890465	0.1636890405	2/10	1.21E-08	1177.0
E6H0	6	0	150	0.0725873025	0.0725872801	0.0725870787	9/10	6.72E-08	2162.5
E9H2	9	2	150	0.2333481422	0.2333137881	0.2332328075	2/10	3.81E-05	3948.0
E11H0	11	0	150	0.2628105627	0.2628082575	0.2627995664	3/10	3.67E-06	3421.3
E12H0a	12	0	150	0.3730487798	0.3730279634	0.3729689162	4/10	2.90E-05	3376.0
E12H0b	12	0	150	0.2783387066	0.2783385307	0.2783382260	4/10	2.02E-07	633.7
E12H2	12	2	150	0.2636360626	0.2636357700	0.2636338518	3/10	6.67E-07	2423.6
E18H0	18	0	150	0.4800548333	0.4800409940	0.4800160364	1/10	1.07E-05	4243.1
E20H1	20	1	150	0.2508850879	0.2508718500	0.2508473809	1/10	1.25E-05	3255.3
E20H2	20	2	150	0.3929380796	0.3929376328	0.3929355544	5/10	7.30E-07	2080.1
E23H1	23	1	150	0.2425848102	0.2425745664	0.2425624936	1/10	7.22E-06	3129.5
E27H1	27	1	150	0.2930400468	0.2929020446	0.2928217394	1/10	6.80E-05	2688.1
E44H1	44	1	150	0.3253694621	0.3253445564	0.3253234180	1/10	2.05E-05	3134.0
E59H1	59	1	150	0.0126020291	0.0125991512	0.0125946799	1/10	1.95E-06	4814.0
E4H0a	4	0	200	0.0366127743	0.0366045964	0.0365929910	1/10	9.47E-06	7416.6
E4H0b	4	0	200	0.1429882126	0.1429867411	0.1429851295	2/10	1.24E-06	7355.0
E6H0	6	0	200	0.0636129856	0.0636129785	0.0636129592	7/10	1.09E-08	4370.0
E9H2	9	2	200	0.2051749126	0.2051581717	0.2051371586	1/10	1.18E-05	7671.1
E11H0	11	0	200	0.2300479293	0.2300475234	0.2300462181	7/10	6.35E-07	6865.2
E12H0a	12	0	200	0.3303497674	0.3303312676	0.3303044960	1/10	1.62E-05	10572.9
E12H0b	12	0	200	0.2397150105	0.2397144563	0.2397141176	1/10	2.51E-07	1550.0
E12H2	12	2	200	0.2334882838	0.2334860073	0.2334799229	1/10	2.15E-06	6242.4
E18H0	18	0	200	0.4190034532	0.4188462333	0.4186487620	1/10	1.05E-04	9323.8
E20H1	20	1	200	0.2172626068	0.2172200262	0.2171481115	1/10	4.02E-05	5994.6
E20H2	20	2	200	0.3446792546	0.3446675117	0.3446425598	1/10	1.04E-05	7744.7
E23H1	23	1	200	0.2101428378	0.2101305311	0.2101131861	1/10	1.02E-05	5201.6
E27H1	27	1	200	0.2595897715	0.2595276865	0.2593957750	1/10	5.92E-05	6017.7
E44H1	44	1	200	0.2813674161	0.2812891330	0.2811962338	1/10	5.41E-05	8308.3
E59H1	59	1	200	0.0111175198	0.0111155130	0.0111133976	1/10	1.32E-06	10174.0

Table 4 Computational results of the BH* algorithm on 45 complicated instances with boundary constraints.

Instance				Basin-hopping					
Container	$\|E\|$	$\|H\|$	p	R_{best}	R_{avg}	R_{worst}	SR	σ	$time(s)$
E101H2	101	2	100	0.5131508522	0.5131462111	0.5131387318	1/10	3.47E-06	1608.6
E101H3	101	3	100	0.7236466019	0.7233832936	0.7231338721	1/10	1.52E-04	2331.5
E106H3	106	3	100	0.6039146219	0.6037780899	0.6033798239	2/10	1.44E-04	2398.2
E106H5	106	5	100	0.5322579826	0.5308443285	0.5296996479	1/10	7.64E-04	2148.6
E107H3	107	3	100	1.1937852368	1.1937852368	1.1937852368	10/10	0.00	1042.3
E120H3	120	3	100	0.8773504855	0.8772410287	0.8768077425	4/10	2.17E-04	1791.1
E172H4	172	4	100	0.4001953141	0.4001810712	0.4001767418	1/10	5.19E-06	1477.4
E193H1	193	1	100	0.8960463038	0.8953314693	0.8941600123	1/10	4.98E-04	1892.2
E196H5	196	5	100	0.4727866982	0.4709323222	0.4705322320	1/10	8.08E-04	1672.7
E203H3	203	3	100	0.2954793213	0.2932913997	0.2920748608	1/10	1.10E-03	1477.2
E60H1	60	1	100	0.1389364842	0.1389127305	0.1388932970	1/10	1.16E-05	1934.4
E81H3	81	3	100	0.4596181712	0.4595817333	0.4595293464	1/10	2.85E-05	2180.0
E82H3	82	3	100	0.4239063629	0.4238394299	0.4237861413	1/10	3.33E-05	2403.8
E84H3	84	3	100	0.4320481250	0.4320481250	0.4320481250	10/10	0.00	1026.5
E93H4	93	4	100	0.5936319873	0.5934827880	0.5931926460	1/10	1.33E-04	2473.0
E101H2	101	2	150	0.4246039525	0.4245003932	0.4244055027	1/10	7.00E-05	2861.5
E101H3	101	3	150	0.6022903843	0.6022903843	0.6022903843	10/10	0.00	3008.7
E106H3	106	3	150	0.4994823977	0.4994604604	0.4994208894	2/10	1.61E-05	5303.1
E106H5	106	5	150	0.4443063213	0.4441837084	0.4439883475	1/10	9.66E-05	4432.4
E107H3	107	3	150	0.9872584433	0.9871599470	0.9870806370	1/10	7.38E-05	4702.7
E120H3	120	3	150	0.7306445570	0.7306208580	0.7305410419	1/10	4.00E-05	4428.5
E172H4	172	4	150	0.3331491441	0.3330541489	0.3328827019	1/10	8.96E-05	4090.3
E193H1	193	1	150	0.7408977499	0.7406671323	0.7405158861	1/10	1.35E-04	5038.0
E196H5	196	5	150	0.3973150431	0.3955168408	0.3942261920	1/10	1.10E-03	4484.6
E203H3	203	3	150	0.2501434557	0.2495857093	0.2483444155	1/10	5.47E-04	3406.5
E60H1	60	1	150	0.1151603319	0.1151416702	0.1151321382	1/10	9.08E-06	3730.0
E81H3	81	3	150	0.3801972735	0.3801341086	0.3800693193	1/10	3.25E-05	3938.4
E82H3	82	3	150	0.3490790239	0.3490590187	0.3490261268	1/10	2.36E-05	2994.9
E84H3	84	3	150	0.3579755298	0.3578167382	0.3576707261	1/10	1.13E-04	5167.9
E93H4	93	4	150	0.4948046173	0.4947339911	0.4946839366	1/10	3.93E-05	4609.4
E101H2	101	2	200	0.3707706478	0.3706979030	0.3706596653	1/10	3.69E-05	10336.5
E101H3	101	3	200	0.5271335274	0.5268565459	0.5266142702	1/10	1.93E-04	9959.2
E106H3	106	3	200	0.4384135384	0.4383338932	0.4381814371	1/10	7.91E-05	11577.4
E106H5	106	5	200	0.3909836192	0.3909032381	0.3907981109	1/10	6.78E-05	10181.3
E107H3	107	3	200	0.8600500229	0.8598515845	0.8597461859	1/10	8.64E-05	10511.7
E120H3	120	3	200	0.6352015578	0.6349999088	0.6348730430	1/10	9.62E-05	9001.6
E172H4	172	4	200	0.2937985244	0.2936556747	0.2935399283	1/10	9.97E-05	10240.2
E193H1	193	1	200	0.6495304786	0.6493698892	0.6492518575	1/10	9.08E-05	9288.5
E196H5	196	5	200	0.3528640237	0.3526929504	0.3523534104	1/10	1.48E-04	9264.2
E203H3	203	3	200	0.2218022505	0.2214404497	0.2211647640	1/10	1.55E-04	9203.5
E60H1	60	1	200	0.1002412092	0.1002173527	0.1002054011	1/10	1.11E-05	9291.7
E81H3	81	3	200	0.3321541887	0.3320943976	0.3319899467	1/10	4.92E-05	6834.4
E82H3	82	3	200	0.3045767624	0.3044992578	0.3044429049	1/10	3.96E-05	9037.4
E84H3	84	3	200	0.3144020791	0.3143300047	0.3141287446	1/10	8.86E-05	11028.9
E93H4	93	4	200	0.4321912739	0.4320001139	0.4317981134	1/10	1.11E-04	9814.6

numbers of edges of polygonal container and the numbers of holes in the region, and the fourth column gives the size of problem (p). The results of the corresponding algorithm are given in columns 5–10, including the best objective value (R_{best}) over 10 independent runs, the average objective value (R_{avg}), the worst objective value (R_{worst}), the success rate (SR) of obtaining the best objective value R_{best}, the standard deviation of objective values obtained (σ), and the average running time in seconds to obtain its final result ($time(s)$) for each run of the algorithm. In addition, Fig. 5 gives the geometrical configurations of best solutions found in this work for 9 representative instances.

Table 5 Computational results of the MBH* algorithm on 60 simple instances with boundary constraints.

Instance				MBH					
Container	$\|E\|$	$\|H\|$	p	R_{best}	R_{avg}	R_{worst}	SR	σ	$time(s)$
E4H0a	4	0	50	0.0713771039	0.0713771039	0.0713771039	10/10	0.00	4.0
E4H0b	4	0	50	0.2721671717	0.2721671717	0.2721671717	10/10	0.00	5.4
E6H0	6	0	50	0.1226935182	0.1226935182	0.1226935182	10/10	0.00	4.0
E9H2	9	2	50	0.3656662863	0.3616720252	0.3566548400	1/10	2.61E-03	33.0
E11H0	11	0	50	0.4368343737	0.4365780965	0.4357715132	4/10	3.38E-04	30.4
E12H0a	12	0	50	0.6225798476	0.6225763213	0.6225445849	9/10	1.06E-05	31.7
E12H0b	12	0	50	0.4620855791	0.4618933105	0.4618376288	1/10	7.26E-05	24.3
E12H2	12	2	50	0.4448612715	0.4448608827	0.4448604939	5/10	3.89E-07	23.3
E18H0	18	0	50	0.7877071509	0.7877071509	0.7877071509	10/10	0.00	13.3
E20H1	20	1	50	0.4137431060	0.4137363551	0.4137333919	2/10	3.50E-06	16.9
E20H2	20	2	50	0.6402224988	0.6401546638	0.6395449671	8/10	2.03E-04	20.8
E23H1	23	1	50	0.4005485124	0.4005485124	0.4005485124	10/10	0.00	7.4
E27H1	27	1	50	0.4840050329	0.4840050329	0.4840050329	10/10	0.00	15.6
E44H1	44	1	50	0.5345665428	0.5342859578	0.5336099632	1/10	2.53E-04	36.3
E59H1	59	1	50	0.0198678555	0.0198127987	0.0197552311	1/10	4.44E-05	27.6
E4H0a	4	0	100	0.0514010718	0.0514010718	0.0514010718	10/10	0.00	231.4
E4H0b	4	0	100	0.1978021937	0.1978021937	0.1978021937	10/10	0.00	185.6
E6H0	6	0	100	0.0882825264	0.0882825264	0.0882825264	10/10	0.00	92.0
E9H2	9	2	100	0.2801620820	0.2798302518	0.2794630634	3/10	2.46E-04	2941.1
E11H0	11	0	100	0.3181468932	0.3181468853	0.3181468775	5/10	7.86E-09	276.7
E12H0a	12	0	100	0.4517241097	0.4517241097	0.4517241097	10/10	0.00	1200.6
E12H0b	12	0	100	0.3361308135	0.3361308135	0.3361308135	10/10	0.00	142.0
E12H2	12	2	100	0.3172501921	0.3172501921	0.3172501921	10/10	0.00	776.9
E18H0	18	0	100	0.5761713874	0.5761713874	0.5761713874	10/10	0.00	613.8
E20H1	20	1	100	0.2989547054	0.2989547054	0.2989547054	10/10	0.00	1544.6
E20H2	20	2	100	0.4733830079	0.4733827279	0.4733822983	6/10	3.51E-07	260.8
E23H1	23	1	100	0.2896330498	0.2896324594	0.2896271455	9/10	1.77E-06	1667.7
E27H1	27	1	100	0.3541756870	0.3541756870	0.3541756870	10/10	0.00	335.6
E44H1	44	1	100	0.3907223295	0.3907223295	0.3907223295	10/10	0.00	1066.9
E59H1	59	1	100	0.0150349633	0.0150349542	0.0150349406	6/10	1.11E-08	2077.0
E4H0a	4	0	150	0.0421454577	0.0421453491	0.0421453344	1/10	3.69E-08	1682.5
E4H0b	4	0	150	0.1636890405	0.1636890405	0.1636890405	10/10	0.00	1699.7
E6H0	6	0	150	0.0725873025	0.0725873025	0.0725873025	10/10	0.00	2177.9
E9H2	9	2	150	0.2333481422	0.2332790355	0.2331292927	3/10	7.44E-05	5324.0
E11H0	11	0	150	0.2628105627	0.2627997871	0.2627777167	1/10	1.32E-05	5268.9
E12H0a	12	0	150	0.3730487798	0.3730429057	0.3730115372	7/10	1.18E-05	4578.3
E12H0b	12	0	150	0.2783387066	0.2783384399	0.2783381591	2/10	1.90E-07	2081.5
E12H2	12	2	150	0.2636360626	0.2636228471	0.2635727378	3/10	2.14E-05	4563.1
E18H0	18	0	150	0.4800569423	0.4800544890	0.4800446523	1/10	3.51E-06	3928.7
E20H1	20	1	150	0.2508904348	0.2508899440	0.2508875776	2/10	8.04E-07	3335.0
E20H2	20	2	150	0.3929380796	0.3929379824	0.3929375939	8/10	1.94E-07	2313.4
E23H1	23	1	150	0.2425853555	0.2425802538	0.2425499672	1/10	1.05E-05	4884.8
E27H1	27	1	150	0.2930778541	0.2929866807	0.2928857460	1/10	7.59E-05	5448.3
E44H1	44	1	150	0.3253697234	0.3253641926	0.3253337514	1/10	1.04E-05	4020.1
E59H1	59	1	150	0.0126004282	0.0125977296	0.0125942582	1/10	2.10E-06	4983.6
E4H0a	4	0	200	0.0366127127	0.0365977879	0.0365837617	1/10	9.78E-06	9329.6
E4H0b	4	0	200	0.1429882126	0.1429880516	0.1429879687	3/10	1.06E-07	7036.3
E6H0	6	0	200	0.0636129856	0.0636129789	0.0636129632	7/10	1.03E-08	7340.8
E9H2	9	2	200	0.2051734607	0.2050920008	0.2050184327	1/10	5.92E-05	9089.7
E11H0	11	0	200	0.2300479269	0.2300465204	0.2300354350	8/10	3.73E-06	10117.1
E12H0a	12	0	200	0.3303513835	0.3303323760	0.3302731613	1/10	2.88E-05	13302.0
E12H0b	12	0	200	0.2397149858	0.2397146114	0.2397142110	1/10	2.28E-07	2692.9
E12H2	12	2	200	0.2334869495	0.2334026236	0.2333424972	1/10	5.24E-05	9666.0
E18H0	18	0	200	0.4191417996	0.4190123862	0.4188161775	1/10	1.02E-04	11804.2
E20H1	20	1	200	0.2172697806	0.2172338832	0.2171686562	1/10	2.89E-05	13046.0
E20H2	20	2	200	0.3447052767	0.3446854835	0.3446778800	1/10	7.39E-06	9545.9
E23H1	23	1	200	0.2102077787	0.2101637466	0.2101163560	1/10	2.99E-05	11021.2
E27H1	27	1	200	0.2595897715	0.2595432008	0.2594498405	5/10	4.95E-05	10223.3
E44H1	44	1	200	0.2814511662	0.2814024292	0.2813687533	1/10	2.66E-05	10452.3
E59H1	59	1	200	0.0111161222	0.0111136582	0.0111100607	1/10	1.76E-06	10668.5

Table 6 Computational results of the MBH* algorithm on 45 complicated instances with boundary constraints.

Instance				MBH					
Container	$\|E\|$	$\|H\|$	p	R_{best}	R_{avg}	R_{worst}	SR	σ	$time(s)$
E101H2	101	2	100	0.5131508522	0.5131375011	0.5130322996	3/10	3.51E-05	2676.4
E101H3	101	3	100	0.7238754116	0.7237614097	0.7235120559	1/10	1.54E-04	2611.4
E106H3	106	3	100	0.6039167191	0.6038959925	0.6037136472	7/10	6.08E-05	2397.7
E106H5	106	5	100	0.5328329107	0.5323526847	0.5312126077	1/10	5.06E-04	2869.1
E107H3	107	3	100	1.1937852368	1.1937852368	1.1937852368	10/10	0.00	604.6
E120H3	120	3	100	0.8773504855	0.8772947645	0.8768077425	6/10	1.62E-04	1857.0
E172H4	172	4	100	0.3970925656	0.3954351442	0.3916029705	1/10	1.78E-03	2578.8
E193H1	193	1	100	0.8970410179	0.8964794893	0.8961972146	1/10	2.77E-04	2572.5
E196H5	196	5	100	0.4652219431	0.4595169179	0.4543347868	1/10	3.38E-03	1804.4
E203H3	203	3	100	0.2892741421	0.2862528842	0.2818039493	1/10	2.37E-03	3476.9
E60H1	60	1	100	0.1389366502	0.1389155735	0.1389001085	1/10	1.30E-05	2501.5
E81H3	81	3	100	0.4596304590	0.4596304590	0.4596304590	10/10	0.00	2083.2
E82H3	82	3	100	0.4239299929	0.4239298328	0.4239291923	8/10	3.20E-07	1403.5
E84H3	84	3	100	0.4320481250	0.4320481250	0.4320481250	10/10	0.00	1145.2
E93H4	93	4	100	0.5936386605	0.5936305684	0.5936202818	1/10	4.78E-06	2722.0
E101H2	101	2	150	0.4246094078	0.4244980399	0.4244080511	1/10	6.59E-05	5945.0
E101H3	101	3	150	0.6022967719	0.6022689505	0.6020687695	2/10	6.68E-05	5440.3
E106H3	106	3	150	0.4996271982	0.4992973653	0.4983357367	1/10	3.45E-04	6155.2
E106H5	106	5	150	0.4444220001	0.4442980251	0.4442403628	1/10	6.28E-05	6090.1
E107H3	107	3	150	0.9872061372	0.9871103817	0.9869386607	1/10	7.77E-05	5692.9
E120H3	120	3	150	0.7306445568	0.7303939124	0.7299580764	2/10	2.72E-04	5945.4
E172H4	172	4	150	0.3331649258	0.3331410341	0.3330710559	2/10	2.55E-05	5954.0
E193H1	193	1	150	0.7409843995	0.7405549328	0.7400004318	1/10	2.59E-04	6112.8
E196H5	196	5	150	0.3971790864	0.3959672532	0.3929897609	1/10	1.29E-03	6055.5
E203H3	203	3	150	0.2483444155	0.2473432994	0.2449480525	3/10	1.08E-03	5544.1
E60H1	60	1	150	0.1151655673	0.1151512904	0.1151400557	1/10	6.82E-06	4639.9
E81H3	81	3	150	0.3802772848	0.3802365212	0.3801608921	1/10	3.94E-05	4912.3
E82H3	82	3	150	0.3490480371	0.3490358746	0.3490324492	2/10	6.08E-06	4886.6
E84H3	84	3	150	0.3579715432	0.3578566666	0.3577256969	1/10	9.31E-05	5453.8
E93H4	93	4	150	0.4949205191	0.4948543190	0.4947966163	1/10	4.22E-05	6143.9
E101H2	101	2	200	0.3708004504	0.3706963168	0.3705426420	1/10	9.21E-05	12444.9
E101H3	101	3	200	0.5268009361	0.5265542452	0.5262630474	1/10	1.91E-04	11027.1
E106H3	106	3	200	0.4384076393	0.4382993999	0.4380720423	1/10	9.62E-05	12067.6
E106H5	106	5	200	0.3909721826	0.3908447285	0.3907593101	1/10	6.44E-05	13431.8
E107H3	107	3	200	0.8601675879	0.8600649578	0.8597594956	1/10	1.32E-04	11941.6
E120H3	120	3	200	0.6352180420	0.6351366779	0.6349540662	1/10	7.72E-05	11122.8
E172H4	172	4	200	0.2938115968	0.2935779286	0.2933022486	1/10	1.77E-04	12560.0
E193H1	193	1	200	0.6496346640	0.6494441862	0.6493282527	1/10	7.93E-05	11664.1
E196H5	196	5	200	0.3532782171	0.3529051796	0.3524081717	1/10	2.09E-04	12676.7
E203H3	203	3	200	0.2230280767	0.2218444831	0.2213785287	1/10	5.53E-04	11169.2
E60H1	60	1	200	0.1002605523	0.1002302451	0.1002059940	1/10	1.44E-05	11051.1
E81H3	81	3	200	0.3322738793	0.3321386094	0.3320064877	1/10	7.22E-05	12811.5
E82H3	82	3	200	0.3045651214	0.3044837034	0.3044114861	1/10	4.78E-05	11140.6
E84H3	84	3	200	0.3143943149	0.3142707651	0.3141422594	1/10	8.67E-05	11265.4
E93H4	93	4	200	0.4321330929	0.4318109785	0.4315437465	1/10	1.82E-04	12904.3

3. Detailed computational results on the continuous p-dispersion problem without boundary constraint

This section reports the detailed computational results of TSGO, BH* and MBH* for the continuous p-dispersion problem without boundary constraint (i.e., the point arrangement problem). The results of the TSGO algorithm are summarized in Tables 7–8 respectively for two sets of benchmark instances, where the symbols are the same as in the previous tables. The results of the BH* algorithm are summarized in Tables 9–10, and the results of the MBH* algorithm are summarized in Tables 11–12. In addition, Fig. 6 gives the geometrical configurations of best solutions found in this work for 9 representative instances.

Table 7 Computational results of the TSGO algorithm on 30 simple instances without boundary constraint.

Instance				TSGO (This work)					
Container	$\|E\|$	$\|H\|$	p	D_{best}	D_{avg}	D_{worst}	SR	σ	$time(s)$
E4H0a	4	0	50	0.1664988509	0.1664988509	0.1664988509	10/10	0.00	180.5
E4H0b	4	0	50	0.7232436230	0.7232436230	0.7232436230	10/10	0.00	739.5
E6H0	6	0	50	0.2933786207	0.2933780098	0.2933779419	1/10	1.02E-07	19.1
E9H2	9	2	50	1.1252199453	1.1252199453	1.1252199453	10/10	0.00	901.6
E11H0	11	0	50	1.1330657353	1.1330657353	1.1330657353	10/10	0.00	1013.1
E12H0a	12	0	50	1.6364303754	1.6364303754	1.6364303754	10/10	0.00	616.7
E12H0b	12	0	50	1.1512460802	1.1512460802	1.1512460802	10/10	0.00	1116.5
E12H2	12	2	50	1.2090410970	1.2090410970	1.2090410970	10/10	0.00	779.9
E18H0	18	0	50	2.2664436736	2.2664436736	2.2664436736	10/10	0.00	1068.9
E20H1	20	1	50	1.1494074146	1.1492486621	1.1478238181	7/10	2.37E-04	1488.1
E20H2	20	2	50	1.7642062877	1.7642062866	1.7642062831	10/10	0.00	460.2
E23H1	23	1	50	1.0945028103	1.0944875530	1.0943526665	7/10	2.25E-05	2122.1
E27H1	27	1	50	1.3363883975	1.3363490513	1.3359949361	9/10	5.90E-05	1020.2
E44H1	44	1	50	1.4231184848	1.4231165211	1.4231119392	7/10	1.50E-06	1865.9
E59H1	59	1	50	0.0625365968	0.0625045164	0.0624105491	1/10	1.78E-05	1706.7
E4H0a	4	0	100	0.1145682248	0.1145670435	0.1145631122	2/10	8.87E-07	5158.0
E4H0b	4	0	100	0.4826373801	0.4826359562	0.4826303355	5/10	1.03E-06	4267.0
E6H0	6	0	100	0.1998574281	0.1998568131	0.1998543437	8/10	6.17E-07	5872.6
E9H2	9	2	100	0.7511278062	0.7505626930	0.7496910108	1/10	2.22E-04	8439.7
E11H0	11	0	100	0.7656953424	0.7656381539	0.7655095383	1/10	2.59E-05	5744.0
E12H0a	12	0	100	1.1078034265	1.1077925638	1.1076947990	9/10	1.63E-05	8483.8
E12H0b	12	0	100	0.7755043306	0.7754471255	0.7754149644	2/10	1.88E-05	3320.2
E12H2	12	2	100	0.8146928310	0.8145986367	0.8142894851	3/10	7.75E-05	6330.5
E18H0	18	0	100	1.4928956213	1.4920205838	1.4906975757	1/10	3.49E-04	8936.5
E20H1	20	1	100	0.7640895485	0.7638340934	0.7634178672	1/10	9.61E-05	8420.7
E20H2	20	2	100	1.1842695666	1.1842567749	1.1842401334	2/10	5.13E-06	6673.3
E23H1	23	1	100	0.7211499167	0.7207608891	0.7202295349	2/10	1.74E-04	7346.7
E27H1	27	1	100	0.9088386700	0.9087282509	0.9079030060	4/10	1.38E-04	7385.1
E44H1	44	1	100	0.9656074713	0.9649732229	0.9635173694	1/10	3.46E-04	10796.5
E59H1	59	1	100	0.0424207994	0.0423973095	0.0423371213	1/10	1.33E-05	7539.1

Table 8 Computational results of the TSGO algorithm on 45 complicated instances without boundary constraint.

Instance				TSGO (This work)					
Container	$\|E\|$	$\|H\|$	p	D_{best}	D_{avg}	D_{worst}	SR	σ	$time(s)$
E101H2	101	2	50	1.9296611433	1.9290832147	1.9260122914	6/10	5.64E-04	2716.6
E101H3	101	3	50	2.7597012147	2.7583284264	2.7500455113	8/10	1.51E-03	1522.4
E106H3	106	3	50	2.2785596163	2.2757095844	2.2735971160	1/10	9.59E-04	2635.2
E106H5	106	5	50	2.1503429086	2.1495595896	2.1482859621	4/10	3.47E-04	2788.9
E107H3	107	3	50	4.3776888646	4.3764749744	4.3676150769	7/10	1.49E-03	2774.0
E120H3	120	3	50	3.3711157887	3.3707419523	3.3688833673	5/10	3.22E-04	2084.6
E172H4	172	4	50	1.6083403888	1.6079020027	1.6062504085	5/10	3.43E-04	2573.1
E193H1	193	1	50	3.4931672197	3.4849529684	3.4732117898	1/10	2.68E-03	1422.9
E196H5	196	5	50	2.1525660024	2.1420151416	2.1257351904	1/10	4.21E-03	3887.8
E203H3	203	3	50	1.2963445060	1.2950615485	1.2913780638	3/10	7.32E-04	2812.4
E60H1	60	1	50	0.5509805177	0.5509709816	0.5509397844	6/10	7.91E-06	1875.4
E81H3	81	3	50	1.6902021702	1.6900946298	1.6898437021	7/10	8.21E-05	2245.0
E82H3	82	3	50	1.5477286761	1.5464949834	1.5433672155	1/10	7.17E-04	2686.8
E84H3	84	3	50	1.6205855793	1.6205050592	1.6201584144	5/10	7.57E-05	2096.3
E93H4	93	4	50	2.4096068833	2.4057504187	2.3995648727	2/10	1.73E-03	2549.6
E101H2	101	2	100	1.2833642210	1.2822230393	1.2811424026	1/10	3.34E-04	10678.4
E101H3	101	3	100	1.8527605603	1.8496615212	1.8473991377	1/10	9.04E-04	10525.8
E106H3	106	3	100	1.5397160821	1.5382859065	1.5367187289	1/10	4.45E-04	10505.0
E106H5	106	5	100	1.4471430022	1.4460133622	1.4435874200	1/10	6.64E-04	9858.7
E107H3	107	3	100	2.9423829506	2.9397676542	2.9356166933	1/10	1.17E-03	9959.0
E120H3	120	3	100	2.2581001199	2.2535581665	2.2504824507	1/10	1.11E-03	11658.0
E172H4	172	4	100	1.0759548576	1.0736695957	1.0710510662	1/10	8.86E-04	12042.6
E193H1	193	1	100	2.3343111553	2.3287152398	2.3192866300	2/10	2.53E-03	7972.2
E196H5	196	5	100	1.4209052194	1.4128738037	1.4083978659	1/10	1.62E-03	12567.9
E203H3	203	3	100	0.8557972355	0.8544934583	0.8525183466	1/10	4.62E-04	7296.4
E60H1	60	1	100	0.3582993202	0.3580180655	0.3577563662	1/10	8.46E-05	7573.0
E81H3	81	3	100	1.1485321215	1.1479214956	1.1469500706	1/10	2.45E-04	4950.3
E82H3	82	3	100	1.0474228732	1.0471543246	1.0470037558	1/10	6.41E-05	8110.2
E84H3	84	3	100	1.0871845694	1.0863459158	1.0841220576	1/10	4.42E-04	10003.2
E93H4	93	4	100	1.5937588256	1.5930971364	1.5900537773	1/10	5.18E-04	12201.4
E101H2	101	2	150	1.0175701224	1.0163457379	1.0153537781	1/10	3.85E-04	24165.8
E101H3	101	3	150	1.4758619447	1.4748651219	1.4739416150	1/10	3.29E-04	19676.8
E106H3	106	3	150	1.2316898249	1.2296099299	1.2267879837	1/10	7.42E-04	20253.7
E106H5	106	5	150	1.1389649453	1.1373346074	1.1361706108	1/10	4.24E-04	14919.8
E107H3	107	3	150	2.3574562564	2.3532942920	2.3477191885	1/10	1.51E-03	23193.7
E120H3	120	3	150	1.8007023797	1.7959811597	1.7913526206	1/10	1.25E-03	24401.7
E172H4	172	4	150	0.8516701412	0.8510752628	0.8504897298	1/10	1.76E-04	22075.5
E193H1	193	1	150	1.8630694963	1.8562745973	1.8529781995	1/10	1.81E-03	25078.5
E196H5	196	5	150	1.1096389040	1.1045592005	1.0996482711	1/10	1.47E-03	26994.1
E203H3	203	3	150	0.6828531990	0.6802848226	0.6787715690	1/10	5.62E-04	12261.9
E60H1	60	1	150	0.2829748122	0.2827246206	0.2825256834	1/10	5.87E-05	23385.7
E81H3	81	3	150	0.9163973051	0.9149487359	0.9134319129	1/10	3.98E-04	22917.4
E82H3	82	3	150	0.8321274553	0.8314888335	0.8307940763	1/10	1.76E-04	22437.6
E84H3	84	3	150	0.8629057017	0.8620988622	0.8609983138	1/10	3.14E-04	21476.5
E93H4	93	4	150	1.2604568525	1.2595900730	1.2577228575	1/10	4.25E-04	19305.2

Table 9 Computational results of the BH* algorithm on 30 simple instances without boundary constraint.

Instance				Basin-hopping					
Container	$\|E\|$	$\|H\|$	p	D_{best}	D_{avg}	D_{worst}	SR	σ	$time(s)$
E4H0a	4	0	50	0.1664988509	0.1664988509	0.1664988509	10/10	0.00	235.0
E4H0b	4	0	50	0.7232436230	0.7232436230	0.7232436230	10/10	0.00	1409.8
E6H0	6	0	50	0.2933779419	0.2933779419	0.2933779419	10/10	0.00	36.4
E9H2	9	2	50	1.1252199453	1.1250991089	1.1241573407	8/10	1.58E-04	1762.4
E11H0	11	0	50	1.1330657353	1.1330162295	1.1328182064	8/10	4.95E-05	1060.2
E12H0a	12	0	50	1.6364303754	1.6364303754	1.6364303754	10/10	0.00	1467.9
E12H0b	12	0	50	1.1512460802	1.1512460802	1.1512460802	10/10	0.00	244.7
E12H2	12	2	50	1.2090410970	1.2089809516	1.2089357034	4/10	2.47E-05	1317.2
E18H0	18	0	50	2.2664436736	2.2664388856	2.2664277138	7/10	3.66E-06	880.0
E20H1	20	1	50	1.1494074146	1.1487617666	1.1474782420	6/10	4.03E-04	999.7
E20H2	20	2	50	1.7642062858	1.7640056517	1.7638036026	5/10	1.00E-04	1178.0
E23H1	23	1	50	1.0945028103	1.0944768737	1.0943314661	7/10	2.75E-05	771.0
E27H1	44	1	50	1.3363883975	1.3353178683	1.3341503446	2/10	3.76E-04	1600.3
E44H1	27	1	50	1.4231184848	1.4229226152	1.4226789226	2/10	7.20E-05	1711.4
E59H1	59	1	50	0.0625113933	0.0624453409	0.0623820199	1/10	2.30E-05	2104.1
E4H0a	4	0	100	0.1145681001	0.1145662184	0.1145628786	2/10	8.41E-07	5491.1
E4H0b	4	0	100	0.4826373801	0.4826356082	0.4826285040	7/10	1.58E-06	5426.4
E6H0	6	0	100	0.1998574337	0.1998560311	0.1998543373	4/10	7.06E-07	4811.3
E9H2	9	2	100	0.7494661035	0.7478403599	0.7460060656	1/10	5.88E-04	8130.4
E11H0	11	0	100	0.7655396868	0.7652491178	0.7648781012	1/10	1.19E-04	5013.9
E12H0a	12	0	100	1.1078034265	1.1077982675	1.1077640795	7/10	5.86E-06	5830.0
E12H0b	12	0	100	0.7755043144	0.7754464078	0.7751726491	2/10	4.76E-05	5436.0
E12H2	12	2	100	0.8146928310	0.8143454056	0.8138911318	2/10	1.61E-04	5445.7
E18H0	18	0	100	1.4913313807	1.4902461911	1.4881580646	1/10	5.01E-04	4922.2
E20H1	20	1	100	0.7639769482	0.7636734678	0.7630778453	1/10	1.33E-04	4749.2
E20H2	20	2	100	1.1842551290	1.1829491272	1.1806416366	1/10	6.61E-04	6246.5
E23H1	23	1	100	0.7209507423	0.7196800678	0.7186048229	1/10	3.35E-04	7233.2
E27H1	27	1	100	0.9087618011	0.9063990417	0.9049753033	1/10	5.31E-04	7206.3
E44H1	44	1	100	0.9655365849	0.9633779411	0.9626444341	1/10	4.36E-04	10230.6
E59H1	59	1	100	0.0424090996	0.0423971176	0.0423592941	1/10	8.59E-06	5309.8

Table 10 Computational results of the BH* algorithm on 45 complicated instances without boundary constraint.

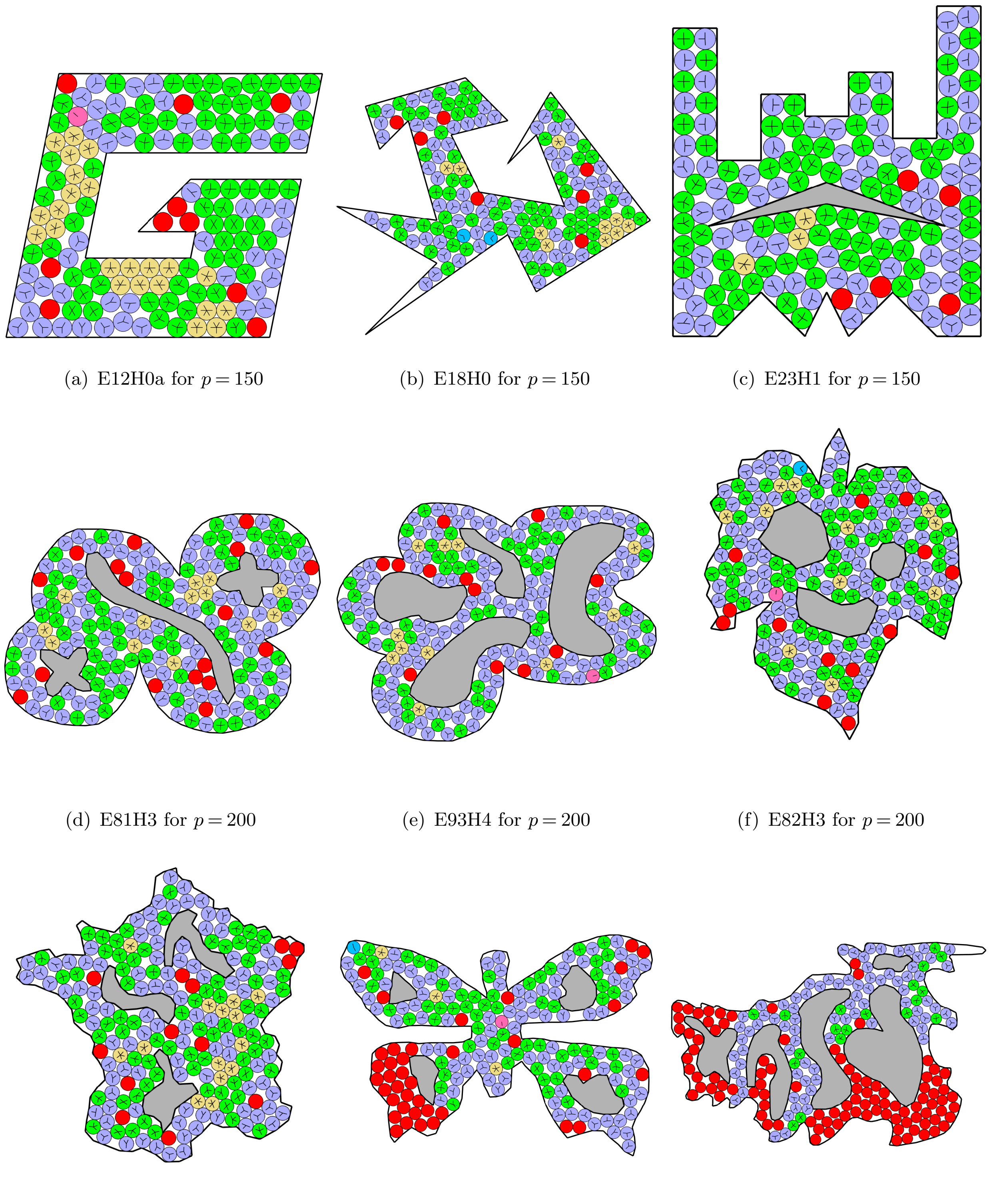
Instance				Basin-hopping					
Container	$\|E\|$	$\|H\|$	p	D_{best}	D_{avg}	D_{worst}	SR	σ	$time(s)$
E101H2	101	2	50	1.9294792183	1.7373305150	0.9582699256	1/10	1.66E-01	3576.2
E101H3	101	3	50	2.7597012147	2.7579227475	2.7542749209	6/10	1.10E-03	1911.0
E106H3	106	3	50	2.2760303651	2.2721291472	2.2656263048	1/10	1.77E-03	1953.8
E106H5	106	5	50	2.1492581495	2.1474806367	2.1443049302	2/10	9.70E-04	2444.3
E107H3	107	3	50	4.3776888646	4.3744222239	4.3665051231	1/10	1.99E-03	2741.0
E120H3	120	3	50	3.3713160251	3.3694128674	3.3688833673	1/10	4.61E-04	2240.0
E172H4	172	4	50	1.6086033153	1.6073993775	1.6058290113	1/10	4.27E-04	2078.2
E193H1	193	1	50	3.4885688530	3.4809669506	3.4737344936	1/10	1.92E-03	1918.0
E196H5	196	5	50	2.1462437211	2.1393500463	2.1258826241	1/10	3.13E-03	2780.3
E203H3	203	3	50	1.2947250241	1.2907711108	1.2829250186	2/10	1.69E-03	2705.9
E60H1	60	1	50	0.5509805177	0.5509455932	0.5508558364	1/10	1.89E-05	1672.9
E81H3	81	3	50	1.6902021702	1.6895559247	1.6879125148	3/10	3.77E-04	2550.2
E82H3	82	3	50	1.5476779639	1.5460645514	1.5432248699	1/10	8.69E-04	2304.9
E84H3	84	3	50	1.6205855793	1.6202744664	1.6199020060	1/10	9.71E-05	2591.9
E93H4	93	4	50	2.4057626982	2.4038957388	2.3985438152	4/10	1.45E-03	2567.8
E101H2	101	2	100	1.2840210583	1.2816485778	1.2806627457	1/10	4.69E-04	10487.4
E101H3	101	3	100	1.8474917504	1.8454387326	1.8427915595	1/10	6.82E-04	9669.7
E106H3	106	3	100	1.5368444673	1.5336969076	1.5314968823	1/10	7.95E-04	7813.6
E106H5	106	5	100	1.4471249445	1.4448386089	1.4406016680	1/10	1.07E-03	8291.4
E107H3	107	3	100	2.9391747678	2.9367746053	2.9334389332	1/10	1.02E-03	7076.3
E120H3	120	3	100	2.2566551682	2.2527411251	2.2471432621	1/10	1.62E-03	8783.9
E172H4	172	4	100	1.0758381251	1.0727718180	1.0704585715	1/10	7.98E-04	10689.5
E193H1	193	1	100	2.3254402936	2.3205631956	2.3149865899	1/10	1.47E-03	9249.4
E196H5	196	5	100	1.4098783673	1.4065769102	1.4037875223	1/10	8.40E-04	10447.4
E203H3	203	3	100	0.8522060778	0.8507183234	0.8489127720	1/10	5.47E-04	5676.0
E60H1	60	1	100	0.3582299427	0.3577676480	0.3571829765	1/10	1.42E-04	7899.8
E81H3	81	3	100	1.1482805892	1.1472277187	1.1441207090	1/10	6.45E-04	3039.9
E82H3	82	3	100	1.0473915944	1.0469480201	1.0458872171	1/10	1.85E-04	2793.5
E84H3	84	3	100	1.0869889031	1.0853519460	1.0841252819	1/10	4.57E-04	8166.1
E93H4	93	4	100	1.5934017314	1.5916298296	1.5899418976	1/10	5.19E-04	7782.2
E101H2	101	2	150	1.0166314840	1.0127636545	0.9996473509	1/10	2.28E-03	22759.0
E101H3	101	3	150	1.4749399669	1.4733512716	1.4693714428	1/10	1.06E-03	7479.9
E106H3	106	3	150	1.2301928882	1.2272249098	1.2247890921	1/10	9.08E-04	10787.9
E106H5	106	5	150	1.1379054440	1.1370513922	1.1363740161	1/10	2.74E-04	12795.7
E107H3	107	3	150	2.3545859328	2.3507354789	2.3482549453	1/10	1.14E-03	19836.0
E120H3	120	3	150	1.7957563567	1.7927535594	1.7881833987	1/10	1.22E-03	18717.6
E172H4	172	4	150	0.8524978245	0.8508764111	0.8487578335	1/10	6.11E-04	21729.9
E193H1	193	1	150	1.8619806126	1.8540394316	1.8453921476	1/10	2.69E-03	23883.0
E196H5	196	5	150	1.1085450824	1.1015314163	1.0944318454	1/10	2.24E-03	22917.5
E203H3	203	3	150	0.6798294129	0.6776296256	0.6765371099	1/10	4.46E-04	11821.8
E60H1	60	1	150	0.2827053641	0.2824164724	0.2822220188	1/10	7.09E-05	14629.7
E81H3	81	3	150	0.9151500861	0.9131879749	0.9121142852	1/10	5.03E-04	15451.9
E82H3	82	3	150	0.8314206381	0.8310173646	0.8303719556	1/10	1.64E-04	16126.1
E84H3	84	3	150	0.8630536076	0.8620981076	0.8612198615	1/10	3.29E-04	17433.8
E93H4	93	4	150	1.2604775170	1.2587040149	1.2561608210	1/10	5.68E-04	13075.9

Table 11 Computational results of the MBH* algorithm on 30 relatively simple instances without boundary constraint.

Instance				Basin-hopping					
Container	$\|E\|$	$\|H\|$	p	D_{best}	D_{avg}	D_{worst}	SR	σ	$time(s)$
E4H0a	4	0	50	0.1664988509	0.1664988509	0.1664988509	10/10	0.00	491.2
E4H0b	4	0	50	0.7232436230	0.7232436230	0.7232436230	10/10	0.00	662.8
E6H0	6	0	50	0.2933779419	0.2933779419	0.2933779419	10/10	0.00	49.5
E9H2	9	2	50	1.1252199453	1.1252199453	1.1252199453	10/10	0.00	841.0
E11H0	11	0	50	1.1330657353	1.1330657353	1.1330657353	10/10	0.00	967.4
E12H0a	12	0	50	1.6364303754	1.6364303754	1.6364303754	10/10	0.00	666.3
E12H0b	12	0	50	1.1512460802	1.1512460802	1.1512460802	10/10	0.00	488.7
E12H2	12	2	50	1.2090410970	1.2090410970	1.2090410970	10/10	0.00	1373.8
E18H0	18	0	50	2.2664436736	2.2664436736	2.2664436736	10/10	0.00	532.7
E20H1	20	1	50	1.1494074146	1.1483431535	1.1475611405	2/10	2.83E-04	1862.2
E20H2	20	2	50	1.7642062920	1.7642062866	1.7642062834	8/10	1.21E-08	381.2
E23H1	23	1	50	1.0945028103	1.0943600422	1.0932145191	8/10	1.92E-04	1985.0
E27H1	27	1	50	1.3363883975	1.3363490513	1.3359949361	9/10	5.90E-05	1938.3
E44H1	44	1	50	1.4231184848	1.4230690380	1.4229607184	3/10	3.49E-05	1990.1
E59H1	59	1	50	0.0625365968	0.0624835536	0.0624554522	1/10	1.61E-05	1866.9
E4H0a	4	0	100	0.1145682248	0.1145671956	0.1145640931	2/10	6.61E-07	6166.4
E4H0b	4	0	100	0.4826373801	0.4826179302	0.4825356355	4/10	1.75E-05	7812.4
E6H0	6	0	100	0.1998574216	0.1997843134	0.1991362773	5/10	1.08E-04	8935.3
E9H2	9	2	100	0.7511238475	0.7502858249	0.7492433888	1/10	2.57E-04	11835.5
E11H0	11	0	100	0.7656897992	0.7655114064	0.7649914495	1/10	9.52E-05	6899.5
E12H0a	12	0	100	1.1078034265	1.1077723402	1.1075502010	6/10	3.75E-05	9443.9
E12H0b	12	0	100	0.7755043105	0.7754653422	0.7754149644	3/10	1.78E-05	7972.2
E12H2	12	2	100	0.8146928235	0.8146115427	0.8142894851	5/10	7.22E-05	5162.5
E18H0	18	0	100	1.4935027529	1.4928152147	1.4904918591	1/10	4.46E-04	9056.9
E20H1	59	1	100	0.7638994850	0.7636264998	0.7630640975	1/10	1.18E-04	6597.8
E20H2	20	1	100	1.1842672680	1.1842389685	1.1841346694	1/10	1.80E-05	5169.2
E23H1	20	2	100	0.7211250210	0.7205693410	0.7200443848	1/10	1.66E-04	8966.1
E27H1	23	1	100	0.9088386862	0.9077689522	0.9067437064	1/10	4.04E-04	8322.5
E44H1	44	1	100	0.9655319844	0.9643480822	0.9628066562	1/10	4.84E-04	10325.3
E59H1	27	1	100	0.0424419006	0.0423749032	0.0421698750	1/10	3.72E-05	6288.4

Table 12 Computational results of the MBH* algorithm on 45 complicated instances without boundary constraint.

Instance				Basin-hopping					
Container	$\|E\|$	$\|H\|$	p	D_{best}	D_{avg}	D_{worst}	SR	σ	$time(s)$
E101H2	101	2	50	1.9296611433	1.9293080575	1.9279676174	5/10	2.57E-04	3019.1
E101H3	101	3	50	2.7597012147	2.7528616370	2.7451484943	3/10	2.86E-03	1982.8
E106H3	106	3	50	2.2785596163	2.2753498323	2.2712160749	1/10	1.16E-03	2144.3
E106H5	106	5	50	2.1503429086	2.1495254107	2.1490434210	3/10	2.72E-04	2619.3
E107H3	107	3	50	4.3776888646	4.3763245600	4.3650783757	8/10	1.88E-03	1951.4
E120H3	120	3	50	3.3713160251	3.3707797719	3.3690613268	1/10	3.04E-04	2321.8
E172H4	172	4	50	1.6083403888	1.6080920152	1.6078112251	4/10	1.19E-04	2604.7
E193H1	193	1	50	3.4930481925	3.4828879612	3.4690733136	1/10	3.40E-03	2139.2
E196H5	196	5	50	2.1458081483	2.1359675846	2.1126245719	1/10	4.32E-03	3044.0
E203H3	203	3	50	1.2963445060	1.2944729939	1.2881415582	4/10	1.26E-03	2854.5
E60H1	60	1	50	0.5509805177	0.5508626165	0.5506094089	2/10	6.40E-05	2174.6
E81H3	81	3	50	1.6902021702	1.6900255476	1.6895113479	6/10	1.17E-04	1541.5
E82H3	82	3	50	1.5477194735	1.5455161767	1.5429605308	1/10	9.53E-04	2494.9
E84H3	84	3	50	1.6205855793	1.6202793679	1.6200263765	2/10	1.01E-04	2228.8
E93H4	93	4	50	2.4087452204	2.4047430949	2.3975061928	1/10	1.49E-03	2178.3
E101H2	101	2	100	1.2839958467	1.2822682646	1.2803151948	1/10	4.82E-04	9696.0
E101H3	101	3	100	1.8517629108	1.8493839200	1.8468545093	1/10	8.89E-04	9336.4
E106H3	106	3	100	1.5389609812	1.5373017679	1.5355478145	1/10	5.62E-04	8820.2
E106H5	106	5	100	1.4471430022	1.4449916914	1.4422214241	1/10	9.16E-04	9055.9
E107H3	107	3	100	2.9422207363	2.9394661810	2.9367264776	1/10	8.60E-04	8926.1
E120H3	120	3	100	2.2585254097	2.2548931409	2.2511126934	1/10	1.23E-03	12440.1
E172H4	172	4	100	1.0767384436	1.0749063077	1.0727082268	1/10	5.74E-04	9578.5
E193H1	193	1	100	2.3343111553	2.3295235036	2.3214644932	2/10	1.97E-03	7944.6
E196H5	196	5	100	1.4138861609	1.4103438073	1.4065984035	1/10	1.23E-03	10583.1
E203H3	203	3	100	0.8564731696	0.8539301996	0.8521302869	1/10	6.78E-04	5641.0
E60H1	60	1	100	0.3582466332	0.3578719449	0.3572269441	1/10	1.39E-04	11181.3
E81H3	81	3	100	1.1485277749	1.1471834164	1.1455728502	1/10	4.88E-04	6745.9
E82H3	82	3	100	1.0474433895	1.0470108082	1.0464577206	2/10	1.59E-04	5507.8
E84H3	84	3	100	1.0872842260	1.0860224488	1.0830364389	1/10	5.83E-04	9933.0
E93H4	93	4	100	1.5939701632	1.5925545584	1.5911251813	1/10	5.50E-04	11966.2
E101H2	101	2	150	1.0155772115	1.0143834682	1.0128444937	1/10	4.04E-04	21128.3
E101H3	101	3	150	1.4759039711	1.4737548666	1.4681973948	1/10	1.03E-03	16911.7
E106H3	106	3	150	1.2307360530	1.2276956567	1.2250162675	1/10	7.08E-04	15194.5
E106H5	106	5	150	1.1372298361	1.1364212055	1.1347835054	1/10	3.69E-04	19205.7
E107H3	107	3	150	2.3577371747	2.3531119086	2.3472667062	1/10	1.61E-03	21671.7
E120H3	120	3	150	1.7975153377	1.7955887570	1.7924087476	1/10	7.07E-04	20227.9
E172H4	172	4	150	0.8525962904	0.8509029887	0.8494290019	1/10	4.90E-04	17591.8
E193H1	193	1	150	1.8620687888	1.8561542253	1.8499537490	1/10	2.10E-03	20907.5
E196H5	196	5	150	1.1096023844	1.1007478968	1.0851449447	1/10	3.46E-03	21476.2
E203H3	203	3	150	0.6801848429	0.6786975696	0.6774633897	1/10	4.19E-04	19141.6
E60H1	60	1	150	0.2828959343	0.2826403084	0.2823869775	1/10	6.12E-05	22588.3
E81H3	81	3	150	0.9155216189	0.9141138269	0.9134880776	1/10	2.96E-04	21073.8
E82H3	82	3	150	0.8324378837	0.8314429821	0.8305306328	1/10	2.79E-04	18772.3
E84H3	84	3	150	0.8629312866	0.8618367930	0.8607773850	1/10	3.75E-04	23571.1
E93H4	93	4	150	1.2601949020	1.2588156611	1.2567688378	1/10	5.40E-04	22484.4

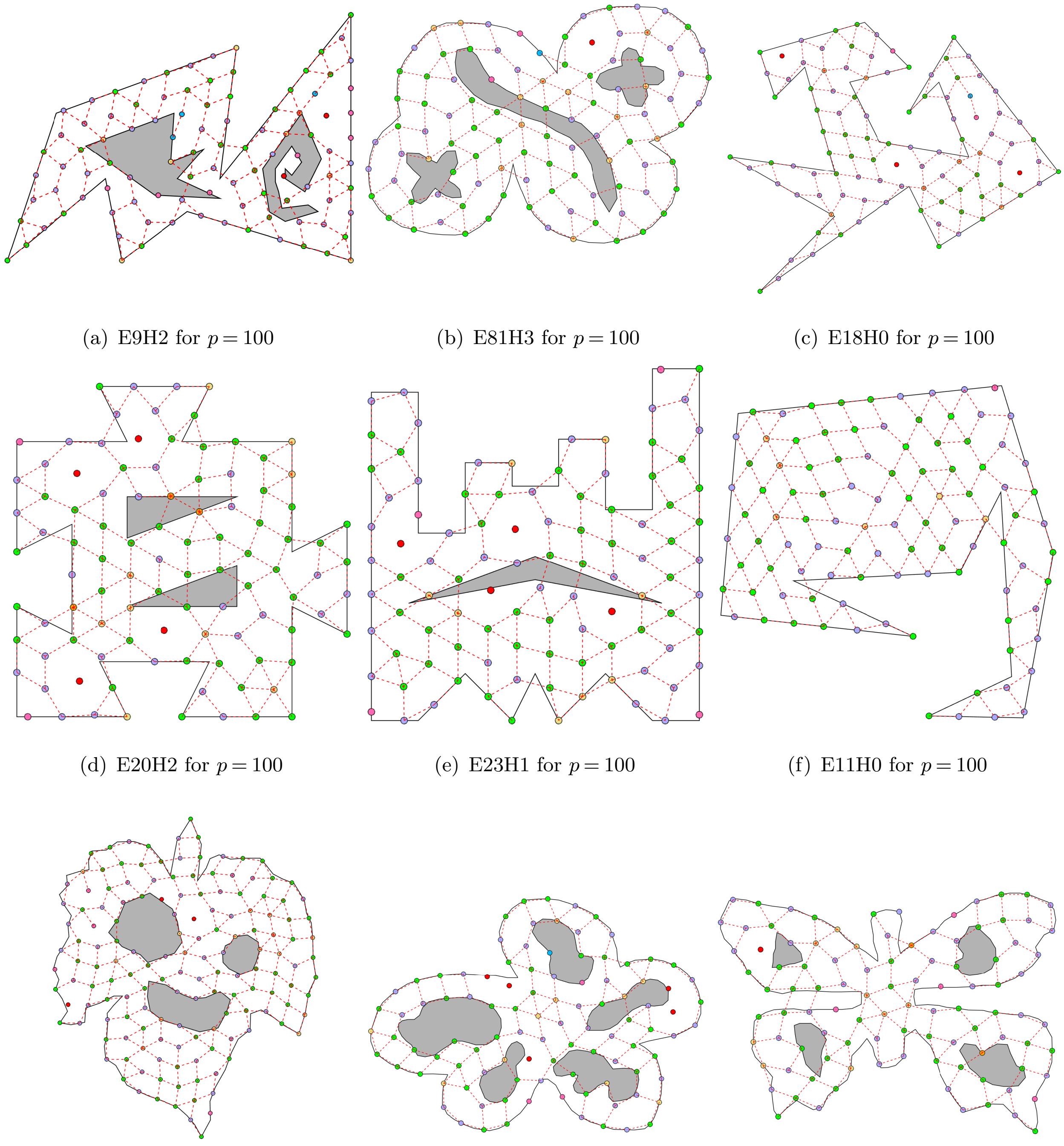


(a) E12H0a for $p = 150$ (b) E18H0 for $p = 150$ (c) E23H1 for $p = 150$

(d) E81H3 for $p = 200$ (e) E93H4 for $p = 200$ (f) E82H3 for $p = 200$

(g) E107H3 for $p = 200$ (h) E172H4 for $p = 200$ (i) E196H5 for $p = 200$

Figure 5 Best configurations found in this work for other representative instances, where the circles are colored according to their number of neighbors.



(a) E9H2 for $p=100$ (b) E81H3 for $p=100$ (c) E18H0 for $p=100$

(d) E20H2 for $p=100$ (e) E23H1 for $p=100$ (f) E11H0 for $p=100$

(g) E82H3 for $p=100$ (h) E106H5 for $p=150$ (i) E172H4 for $p=100$

Figure 6 **Best configurations found in this work for other representative instances, where two points are connected by a dotted line if the distance between them equals the minimum distance found D_{best}.**