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Tables and Figures from Average case sub-quadratic exact and heuristic procedures for the traveling salesman 2-OPT neighborhood

Paolo Vidoni and Giuseppe Lancia

Table 1 Average number of moves evaluated for finding the best move on a random tour. Results for uniform instances.

n	CE	$\mathcal{A}_g$	$\mathcal{A}_L$	$\mathcal{H}(\delta_n)$	$\frac{\mathcal{A}_L}{\mathcal{A}_g}$
2,000	1,999,000	3,080	3,813	7,230	1.23
4,000	7,998,000	6,101	7,738	14,469	1.26
6,000	17,997,000	9,284	11,741	21,614	1.26
8,000	31,996,000	12,066	15,403	28,943	1.27
10,000	49,995,000	15,489	19,709	36,187	1.27
12,000	71,994,000	18,698	23,754	43,471	1.27
14,000	97,993,000	21,306	27,324	50,625	1.28
16,000	127,992,000	25,172	32,097	57,914	1.27
18,000	161,991,000	27,432	35,331	64,754	1.28
20,000	199,990,000	31,454	39,898	71,726	1.26
22,000	241,989,000	34,993	44,170	79,325	1.26
24,000	287,988,000	36,664	47,107	86,796	1.28

Table 2 Average number of moves evaluated for finding the best move on a random tour. Results for Euclidean instances.

n	CE	$\mathcal{A}_g$	$\mathcal{A}_L$	$\mathcal{H}(\delta_n)$	$\frac{\mathcal{A}_L}{\mathcal{A}_g}$
2,000	1,999,000	140.1	189.8	2,135.9	1.35
4,000	7,998,000	125.2	159.7	1,881.7	1.27
6,000	17,997,000	128.2	165.2	1,741.2	1.28
8,000	31,996,000	120.5	153.8	1,658.5	1.27
10,000	49,995,000	116.9	149.7	1,629.7	1.28
12,000	71,994,000	116.8	149,4	1,582.3	1.27
14,000	97,993,000	120.3	151.2	1,543.9	1.25
16,000	127,992,000	114.5	148.1	1,552.5	1.29
18,000	161,991,000	109.9	139.3	1,523.1	1,26
20,000	199,990,000	121.9	158.1	1,504.0	1,29
22,000	241,989,000	115.2	147.3	1,506.5	1.27
24,000	287,988,000	113.3	146.8	1,503.8	1.29

Table 3 Average number of moves evaluated for finding the best move on a random tour. Results for TSPLIB instances.

name	n	CE	$\mathcal{A}_g$	$\mathcal{A}_L$	$\frac{\mathcal{A}_g}{\mathcal{A}_L}$
`euc2d/rl5915`	5,915	17,490,655	233.8	290.5	1.24
`euc2d/rl5934`	5,934	17,603,211	147.9	179.2	1.21
`ceil2d/pla7397`	7,397	27,354,106	92.1	119.7	1.29
`euc2d/rl11849`	11,849	70,193,476	146.3	157.0	1.07
`euc2d/usa13509`	13,509	91,239,786	127.4	165.2	1.29
`euc2d/brd14051`	14,051	98,708,275	241.6	282.5	1.17
`euc2d/d15112`	15,112	114,178,716	185.7	279.6	1.50
`euc2d/d18512`	18,512	171,337,816	282.7	367.9	1.30
`ceil2d/pla33810`	33,810	571,541,145	219.4	289.9	1.32

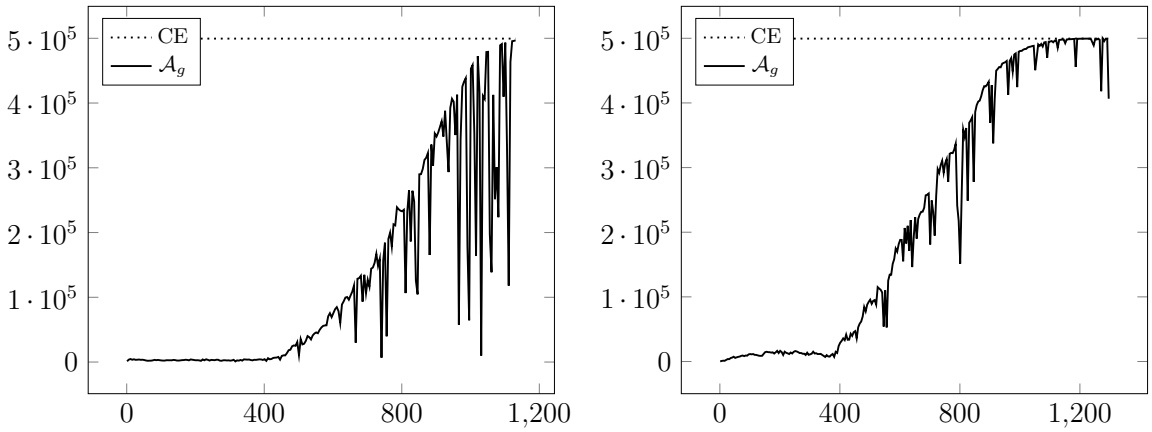

Figure 5 **Number of moves evaluated per LS step.** $n = 1000$**. Left, UNI instance. Right EUC instance.**

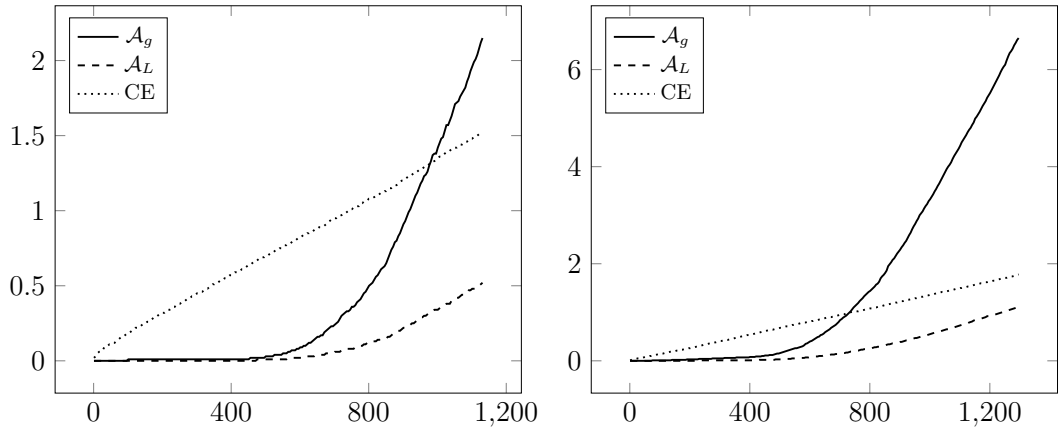

Figure 6 **Cumulative time (sec) up to LS step.** $n = 1000$**. Left, UNI instance. Right EUC instance.**

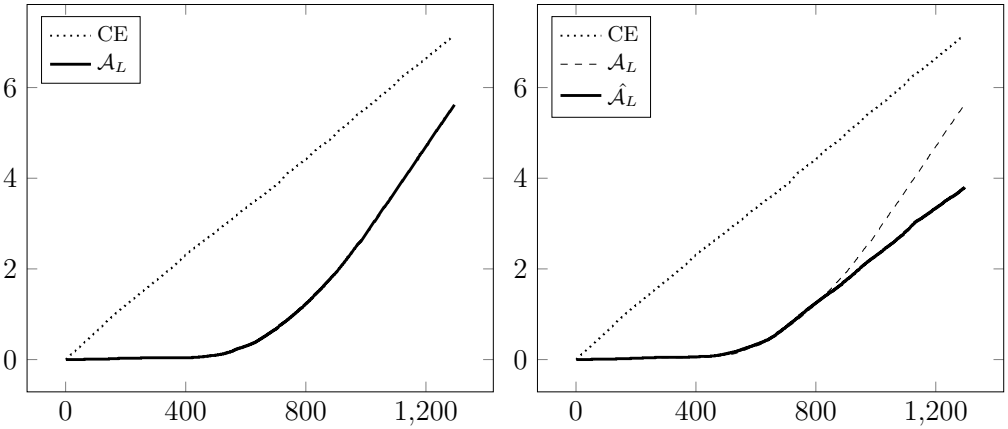

Figure 7 **Cumulative time (sec) up to LS step. TSPLIB instance pr1002,** $n = 1002$**. Left,** $\mathcal{A}_L$ **vs CE. Right,** $\hat{\mathcal{A}}_L$ **vs CE.**

Table 4 Running times (sec) and move evaluations for best-improvement LS convergences with CE, $\mathcal{A}_L$ and $\hat{\mathcal{A}}_L$. Averages over 10 runs per instance.

type	n	steps	$\bar{m}(CE)$	$\bar{t}(CE)$	$\bar{m}(\mathcal{A}_L)$	$\bar{t}(\mathcal{A}_L)$	$\bar{m}(\hat{\mathcal{A}}_L)$	$\bar{t}(\hat{\mathcal{A}}_L)$	speed-up
UNI	500	553	124,750	0.72	36,058	0.37	48,456	0.34	2.11×
	1,000	1,147	499,500	6.56	138,478	2.97	194,364	2.74	2.39×
	1,500	1,767	1,124,250	23.67	295,899	10.33	449,733	10.19	2.32×
	2,000	2,437	1,999,000	59.89	555,098	30.12	835,071	27.65	2.15×
	2,500	3,108	3,123,750	129.53	875,875	67.07	1,335,617	60.13	2.16×
	3,000	3,695	4,498,500	238.62	1,194,544	117.06	1,889,970	110.45	2.16×
EUC	500	588	124,750	0.93	58,295	0.59	61,651	0.54	1.72×
	1,000	1,290	499,500	7.15	227,535	5.34	253,496	4.07	1.75×
	1,500	1,972	1,124,250	26.07	495,043	20.08	565,779	14.99	1.73×
	2,000	2,681	1,999,000	73.46	856,554	53.54	1,004,952	37.82	1.94×
	2,500	3,399	3,123,750	141.72	1,324,269	111.10	1,576,564	77.69	1.82×
	3,000	4,156	4,498,500	278.88	1,904,449	218.87	2,298,937	149.34	1.86×
TSPLIB	rat575	1,340	165,025	1.54	78,442	1.38	81,287	0.98	1.57×
	pr1002	2,605	501,501	12.26	227,041	9.93	252,224	6.17	1.98×
	u1432	3,427	1,024,596	33.66	436,540	28.54	472,799	18.15	1.85×
	u2152	5,651	2,314,476	136.52	935,776	124.01	1,138,229	75.91	1.79×
	pr2392	6,576	2,859,636	198.14	1,203,348	167.72	1,444,297	103.85	1.90×
	pcb3038	8,174	4,613,203	496.96	2,007,906	417.04	2,299,242	255.49	1.94×

Table 5 First- and Best-improvement LS starting at a random tour. Averages over multiple runs per instance.

		F-I CE				F-I $\mathcal{A}_L$				B-I $\hat{\mathcal{A}}_L$			
type	n	f^*	$\bar{f}$	$\bar{t}$	steps	f^*	$\bar{f}$	$\bar{t}$	steps	f^*	$\bar{f}$	$\bar{t}$	steps
U	500	6.02	6.66	0.02	2365	**6.01**	6.50	0.06	1252	6.13	**6.28**	0.26	561
	1000	**8.29**	8.85	0.12	5670	8.32	8.72	0.63	2790	8.44	**8.63**	2.88	1160
	1500	10.25	10.72	0.36	9242	**10.03**	**10.43**	2.18	4382	10.41	10.53	10.02	1766
	2000	11.66	12.35	0.65	12753	11.84	12.04	5.85	6046	**11.55**	**11.81**	25.58	2437
	2500	13.22	13.62	1.28	17317	13.06	13.40	12.82	7711	**12.74**	**12.98**	54.05	3070
E	500	18.29	18.70	0.02	3661	**17.60**	**18.04**	0.14	1690	17.85	18.33	0.59	599
	1000	25.54	26.15	0.11	9208	**24.69**	**25.08**	1.07	3803	25.44	25.62	4.34	1280
	1500	31.43	31.97	0.22	16057	**30.19**	**30.57**	3.51	6098	30.98	31.10	19.91	1991
	2000	35.94	36.70	0.44	23348	**34.63**	**35.06**	8.33	8582	35.89	36.00	45.91	2710
	2500	40.28	41.18	0.51	29583	**39.24**	**39.54**	17.27	11058	40.15	40.38	88.51	3404
T	rat575	7519	7658	0.02	4405	**7334**	**7491**	0.08	1801	7463	7573	0.73	670
	d657	53142	54852	0.03	5183	**51703**	**52866**	0.40	2432	53267	53806	1.35	820
	pr1002	286207	293431	0.06	9283	**275177**	**277871**	1.03	3848	281382	284240	4.38	1288
	u1060	247260	253046	0.05	11052	**237925**	**240977**	1.41	4109	242578	247453	5.40	1373
	rl1323	293319	309967	0.09	13169	**282121**	**288530**	1.82	5595	296168	302311	10.36	1842
	u1432	172482	175573	0.11	15239	**167760**	**169218**	0.79	5355	170097	171333	12.49	1722
	u2152	73788	76083	0.30	25016	**71363**	**72857**	6.72	8979	72852	73922	57.86	2840
	pr2392	421939	435466	0.44	29542	**406889**	**412659**	15.19	10463	421003	423614	75.38	3275
H	Tnm511	**8945401**	9119258	0.01	3098	8958219	**8998822**	0.06	1764	8983466	9073819	0.44	714
	Tnm1021	18570469	18803382	0.04	7029	**18547346**	**18597745**	0.49	3965	18613850	18714185	3.84	1542
	Tnm1501	27621647	28001323	0.09	13174	**27601361**	**27685473**	1.67	6152	27654368	27761642	13.89	2312
	Tnm2011	37292079	37880141	0.18	18568	**37245854**	**37285407**	5.30	8630	37335996	37699400	38.67	3208
	Tnm2521	46925076	47590156	0.33	23635	**46888936**	**46924132**	10.86	10971	46956008	47259950	79.60	4016

Table 6 First- and Best-improvement LS starting at a NN tour. Averages over multiple runs per instance.

		F-I CE				F-I $\mathcal{A}_L$				B-I $\hat{\mathcal{A}}_L$			
type	n	f^*	$\bar{f}$	$\bar{t}$	steps	f^*	$\bar{f}$	$\bar{t}$	steps	f^*	$\bar{f}$	$\bar{t}$	steps
U	1000	3.70	4.01	0.04	104	**3.56**	3.85	0.03	97	3.57	**3.69**	0.15	53
	2000	4.24	4.48	0.21	157	4.06	4.30	0.23	152	**3.94**	**4.02**	0.88	91
	3000	4.43	4.69	0.56	198	4.30	4.51	0.73	182	**3.99**	**4.17**	2.63	108
	4000	4.71	4.91	1.19	236	4.50	4.66	1.15	219	**4.30**	**4.38**	5.48	126
	5000	4.83	5.01	1.80	267	4.61	4.79	2.10	247	**4.41**	**4.46**	9.45	144
E	1000	24.71	24.99	0.04	269	**24.28**	**24.46**	0.22	208	24.44	24.51	1.08	152
	2000	34.76	35.07	0.16	576	33.97	34.14	2.06	458	**33.93**	**34.09**	10.11	322
	3000	42.52	42.99	0.43	858	**41.82**	**42.00**	7.73	652	41.85	42.02	40.22	463
	4000	48.90	49.30	0.95	1098	**47.87**	**48.08**	20.49	850	47.99	**48.08**	87.33	588
	5000	54.85	55.20	1.39	1327	**53.45**	53.76	40.43	1046	53.63	**53.75**	188.86	732
T	u2319	245487	247069	0.16	487	243417	244955	0.22	396	**240007**	**241229**	11.61	282
	pr2392	403571	411427	0.20	598	**396172**	400396	2.27	470	397387	**399683**	13.26	310
	pcb3038	148651	149989	0.40	815	145590	146534	4.97	633	**145541**	**145796**	35.59	422
	fl3795	30087	31003	0.63	864	29483	30009	4.95	622	**29463**	**29854**	55.36	453
	rl5915	605735	613195	2.07	792	**591490**	**595127**	11.35	585	594073	597075	104.51	406
H	Tnm1021	18655658	18745272	0.01	17	**18635276**	18716006	0.02	21	18645515	**18676812**	0.08	12
	Tnm2011	37365235	37462906	0.04	14	**37361409**	37460712	0.05	15	37368017	**37429604**	0.21	13
	Tnm3001	**56079824**	56174525	0.15	19	56125606	56192125	0.17	19	56096925	**56122793**	0.89	14
	Tnm4021	75361046	75486418	0.30	14	75387526	75473227	0.32	15	**75320675**	**75437195**	1.25	11
	Tnm5011	**94106391**	94259213	0.54	13	94168303	94262754	0.43	9	94192902	**94229562**	1.34	8