Heuristic Methods for Γ-Robust Mixed-Integer Linear Bilevel Problems

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1. Computational Results for Γ-Robust Knapsack Interdiction Problems (Min-Max Setting)

In the computational study, the following approaches are considered:

H-BKP: The heuristic in which we incorporate the bkpsolver (Weninger and Fukasawa 2023) for the solution of the deterministic interdiction problems.

H-IC: The heuristic in which we incorporate the branch-and-cut approach proposed by Fischetti et al. (2019) for the solution of the deterministic interdiction problems.

H-GI: The "Greedy Interdiction" heuristic proposed by DeNegre (2011), which has been adapted to account for a Γ-robust follower. Since the original method does not provide dual information, we further solve the so-called high-point relaxation (HPR) of the problem to obtain a valid lower bound.

E-MF: The exact single-leader multi-follower approach presented in our previous work in Beck et al. (2023). The code is publicly available at https://github.com/YasmineBeck/gamma-robust-knapsack-interdiction-solver.

Table 1. The number of instances (out of the 560 considered instances in the min-max setting with integer and continuous deviations, respectively) for which our presolve techniques have been applied.

	presolved	
Δf	sub-problems	variables
integer	560	500
continuous	440	520

Table 2. Statistics for the number of eliminated sub-problems and the number of fixed variables due to presolve (both in %) for the min-max setting with integer and continuous deviations.

Δf	presolved	min	1st quartile	median	3rd quartile	max
integer	sub-problems	40.91	63.84	73.21	80.00	88.00
	variables	0.00	3.48	5.00	6.04	8.00
continuous	sub-problems	0.00	3.13	8.89	21.08	44.00
	variables	0.00	3.48	5.00	6.67	9.09

Table 3. The number of instances for which a feasible point with finite gap is found ("feasible"; out of the 560 considered instances in the min-max setting with integer and continuous deviations, respectively) for the approaches H-BKP, H-IC, H-GI, and E-MF. Additionally, the number of instances solved to global optimality ("optimal"), along with the number of instances satisfying a sufficient optimality condition (Thm. 2 or Thm. 3 in the paper), is shown. For those instances with finite but non-zero gap ("open gap"), also the average gap ("average gap"; in %) is shown.

Δf		feasible	optimal	Thm. 2	Thm. 3	open gap	average gap
integer	H-BKP	560	555	340	340	5	0.12
	H-IC	517	513	277	315	4	0.14
	H-GI	560	4	–	–	556	100.00
	E-MF	560	526	–	–	34	6.08
continuous	H-BKP	560	554	359	359	6	0.08
	H-IC	481	476	266	309	5	0.10
	H-GI	560	4	–	–	556	100.00
	E-MF	560	524	–	–	36	7.03

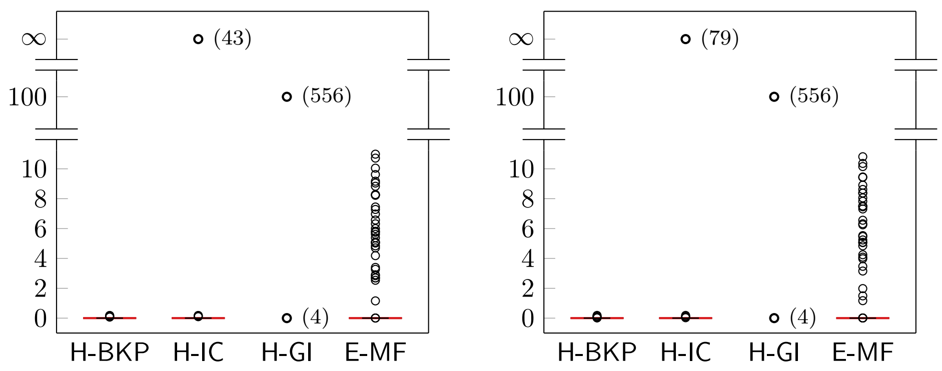

FIGURE 1. Box-plots of the optimality gaps (in %) for the approaches H-BKP, H-IC, H-GI, and E-MF in the min-max setting with integer (left) and continuous deviations (right).

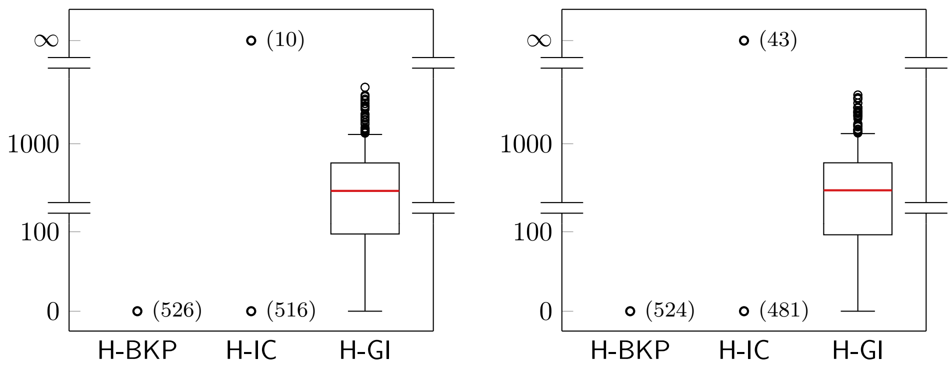

FIGURE 2. Box-plots of the ex-post optimality gaps (in %) for the approaches H-BKP, H-IC, and H-GI for the 526 instances with integer deviations (left) and the 524 instances with continuous deviations (right) that have been solved to global optimality by E-MF. Values above 100 % are shown on a log-scaled y-axis.

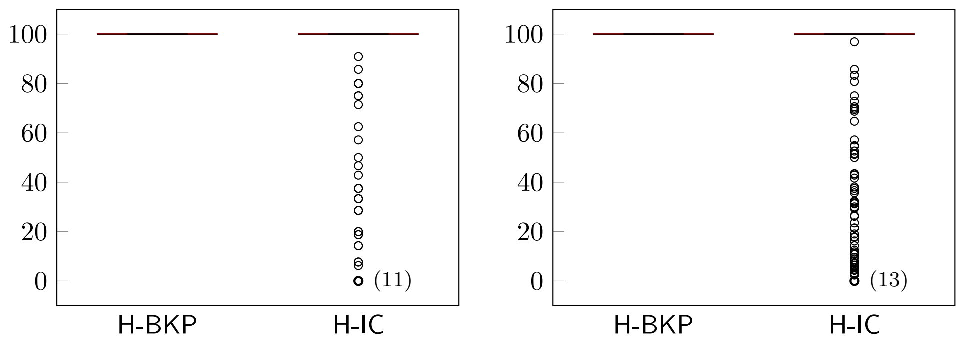

FIGURE 3. Box-plots of the percentages of solved sub-problems (out of the total number of sub-problems to be solved) for the approaches H-BKP and H-IC in the min-max setting with integer deviations (left) and continuous deviations (right).

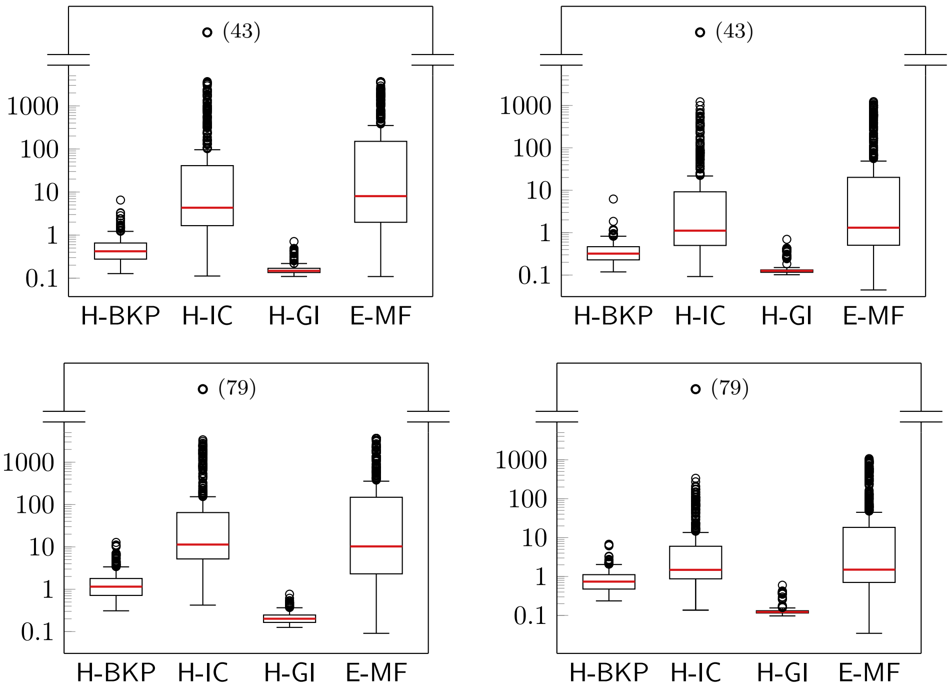

FIGURE 4. Box-plots of the sequential (left) and the idealized parallel runtimes (right) for the approaches H-BKP, H-IC, H-GI, and E-MF in the min-max setting with integer deviations (top) and continuous deviations (bottom). Sequential and idealized parallel runtimes (in s) are depicted on a log-scaled y-axis.

2. Computational Results for Generalized Γ-Robust Knapsack Interdiction Problems (General Bilevel Setting)

In the computational study, the following approaches are considered:

H: The heuristic in which we incorporate a problem-tailored branch-and-cut approach for the solution of the deterministic bilevel problems. The method is based on H-IC, which we have adapted accordingly to account for the more general setting. We abbreviate the heuristic in the sequential and the idealized parallel setting by H-seq and H-ideal, respectively.

H-OS: The ONE-SHOT heuristic presented in Fischetti et al. (2018). The original method computes a bilevel-feasible point and an upper bound for the optimal objective function value, but no lower bound is provided. To have at least some basis for evaluating the quality of the obtained solutions, we thus compute a valid lower bound by solving the HPR of the original bilevel problem.

H-IT: The ITERATE heuristic presented in Fischetti et al. (2018). As before, we solve the HPR of the original bilevel problem to obtain a valid lower bound.

E: An exact branch-and-cut approach tailored to the mixed-integer linear reformulation of the considered generalized knapsack interdiction instances.

TABLE 4. The number of instances for which a feasible point with finite gap is found ("feasible"; out of the 560 considered instances in the general bilevel setting with integer and continuous deviations, respectively) for the approaches H, H-OS, H-IT, and E. Additionally, the number of instances solved to global optimality ("optimal"), along with the number of instances satisfying the sufficient optimality condition in Thm. 5 in the paper, is shown. For those instances with finite but non-zero gap ("open gap"), also the average gap ("average gap"; in %) is shown.

Δf		feasible	optimal	Thm. 5	open gap	average gap
integer	H	249	186	70	63	1.94
	H-OS	560	0	–	560	100.00
	H-IT	560	0	–	560	100.00
	E	480	236	–	244	23.05
continuous	H	217	172	58	45	2.10
	H-OS	560	0	–	560	100.00
	H-IT	560	0	–	560	100.00
	E	474	230	–	244	22.01

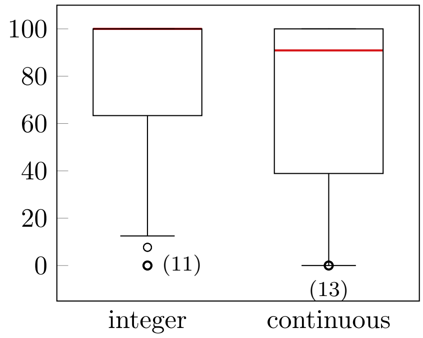

FIGURE 5. Box-plots of the percentages of solved sub-problems (out of the total number of sub-problems to be solved) for H in the general bilevel setting with integer and continuous deviations.

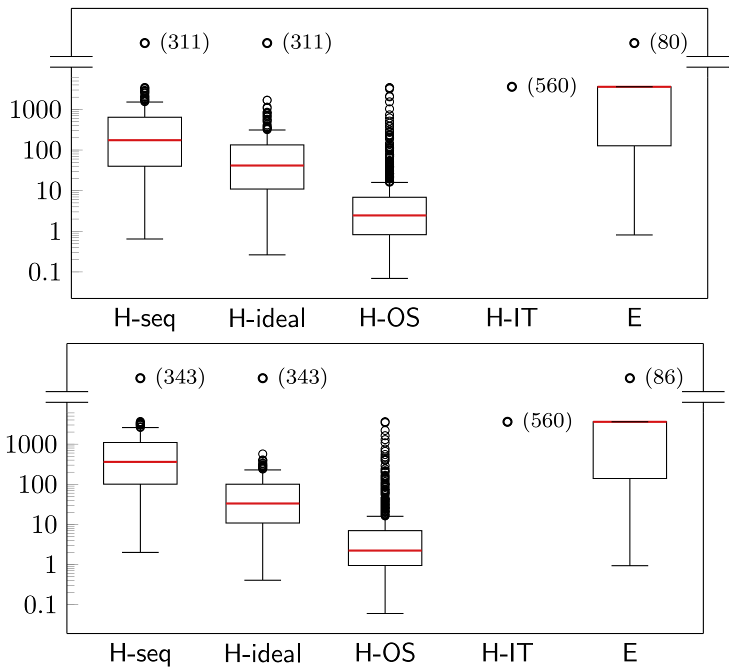

FIGURE 6. Box-plots of the runtimes for the approaches in the general bilevel setting with integer (top) and continuous deviations (bottom). Runtimes (in s) are depicted on a log-scaled y-axis.

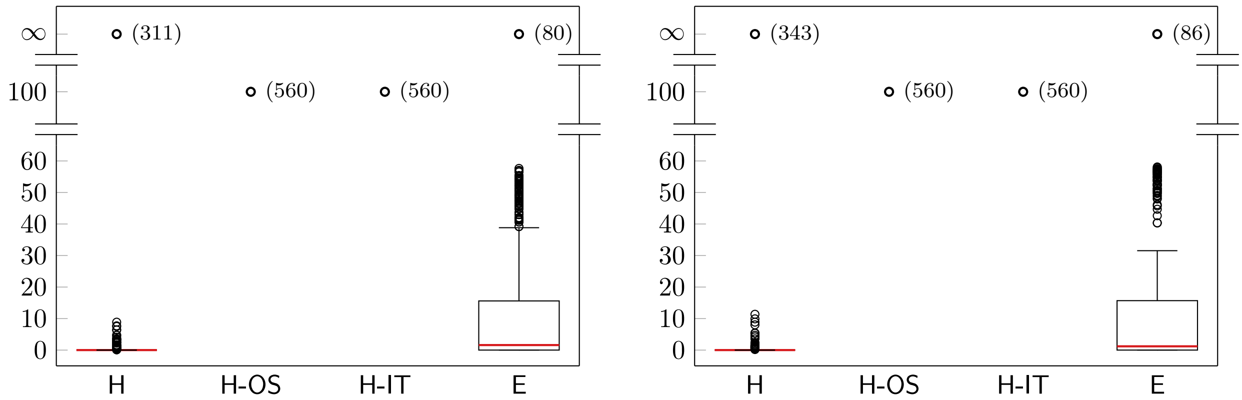

FIGURE 7. Box-plots of the optimality gaps (in %) for the approaches in the general bilevel setting with integer (top) and continuous deviations (bottom).

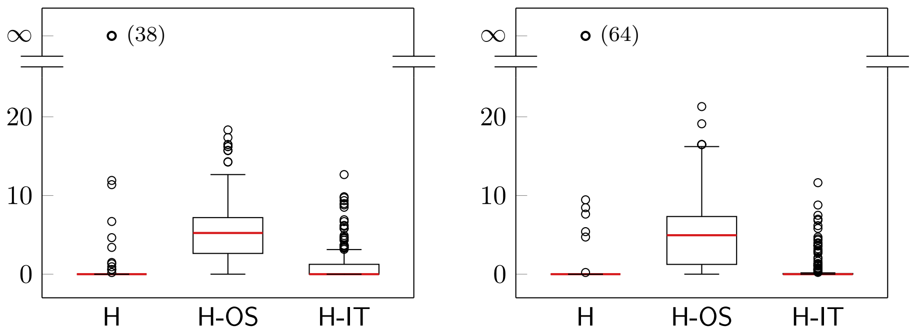

FIGURE 8. Box-plots of the ex-post optimality gaps (in %) for the approaches H, H-OS, and H-IT for the 236 instances with integer (left) and the 230 instances with continuous deviations (right) that have been solved to global optimality by E.

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