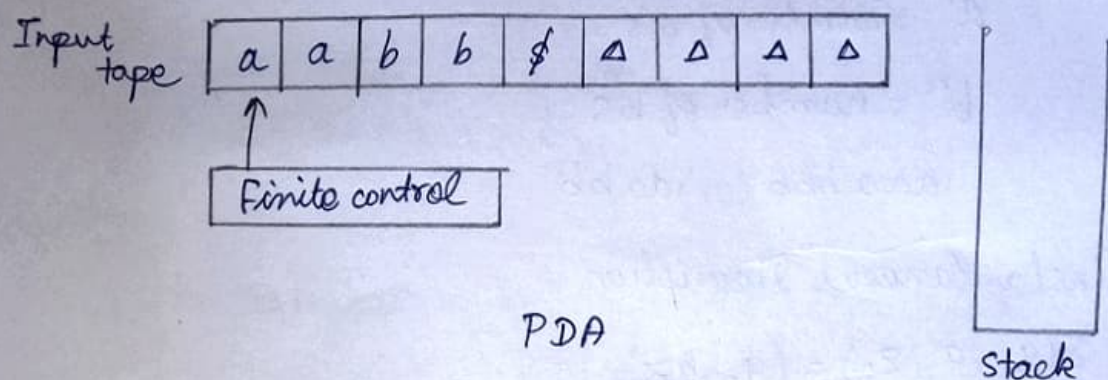


Pushdown Automata:-

Definition:

It have input tape, finite control and stack



PDA defined 7 components

$Q \rightarrow$ The finite set of states

$\Sigma \rightarrow$ Input set

$\Gamma \rightarrow$ stack alphabet

$q_0 \rightarrow$ Initial state

$z_0 \rightarrow$ start symbol, which is in Γ

$F \rightarrow$ Final states

$\delta \rightarrow$ Mapping function used to move from current state to next state

Design a PDA for accepting a language
 $\{L = a^n b^n / n \geq 1\}$

Given: $L = a^n b^n / n \geq 1$

Solution:

a^n = number of a's

b^n = number of b's

aaa bbb (or) aabb

Instantaneous Description

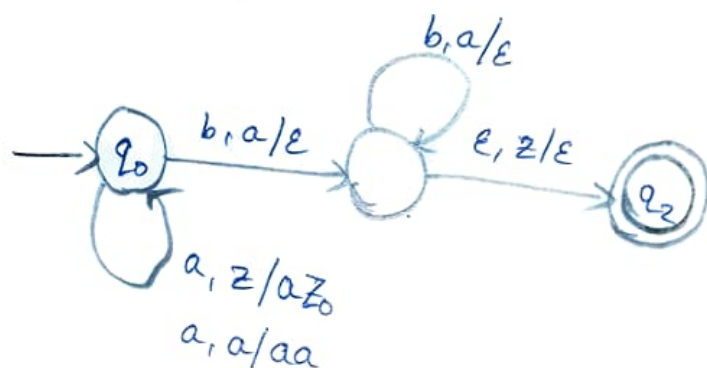
$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$$



CHOMSKY'S NORMAL FORM (CNF)

Non-terminal \rightarrow Non terminal \times non terminal

Non-terminal \rightarrow Terminal

Convert the following CFG into CNF

$$S \rightarrow a a a a S$$

$$S \rightarrow a a a a$$

Solution:

Non Terminals \rightarrow Nonterminal \times Non terminal

Non terminals \rightarrow Terminal

CFG given is

$$S \rightarrow a a a a S \quad \text{rule 1}$$

$$S \rightarrow a a a a \quad \text{rule 2}$$

$A \rightarrow a$ is in CNF

rule 1 and 2 becomes

$$A \rightarrow A A A A S$$

$$S \rightarrow A A A A$$

$S \rightarrow A \boxed{A A A A S}$ can be replaced by P_1

$$P_1 \rightarrow A A A S$$

$S \rightarrow A P_1$ is in CNF

The rule P_1 is not CNF, so convert it

$P_1 \rightarrow A \boxed{AAS}$ can be replaced by P_2

$P_1 \rightarrow AP_2$ which is in CNF

$P_2 \rightarrow AAS$ which is not in CNF, so let us
convert it to CNF

$P_2 \rightarrow A \boxed{AS}$ can be replaced by P_3

$P_2 \rightarrow AP_3$ where $P_3 \rightarrow AS$

$S \rightarrow AP_1$

$P_1 \rightarrow AP_2$

$P_2 \rightarrow AP_3$

$P_3 \rightarrow AS$

Now consider rule 2

$S \rightarrow AAAA$

If we break the rule 2 as

$$\begin{array}{cc} S \rightarrow & \boxed{AA} & \boxed{AA} \\ & \downarrow & \downarrow \\ & P_4 & P_5 \end{array}$$

The rule becomes

$S \rightarrow P_4 P_5$

is in CNF

But P_4 and P_1 indicates the same rule. So we can eliminate either of them

Rule 2 becomes

$$S \rightarrow P_4 P_4$$

Finally we can collectively show the CFG converted to CNF as

$$S \rightarrow AP_1$$

$$P_1 \rightarrow AP_2$$

$$P_2 \rightarrow AP_3$$

$$P_3 \rightarrow AS$$

$$S \rightarrow P_4 P_4$$

$$P_4 \rightarrow AA$$

$$A \rightarrow a$$

GREIBACH NORMAL FORM:

Non Terminal \rightarrow One terminal \times set of non terminal

$$NT \rightarrow TXNT$$

Convert the given CNF to CFG

$$S \rightarrow ABA$$

$$A \rightarrow aA/\epsilon$$

$$B \rightarrow bB/\epsilon$$

Solution:

Step 1:

There is ϵ -Production, so ^{we} will simplify the given grammar as follows.

$$S \rightarrow ABA|BA|AB|AA|A|B$$

$$A \rightarrow aA|a$$

$$B \rightarrow bB|b$$

Now eliminate unit productions

$$S \rightarrow A|B$$

So,

$$S \rightarrow ABA|BA|AB|AA|aA|a|bB|b$$

$$A \rightarrow aA|a$$

$$B \rightarrow bB|b$$

Step 2:

*. In above grammar the rules for A and B non-terminals are already in GNF.

*. In fact, some of the rules of S are also in GNF

x. He we will handle

$$S \rightarrow ABA \mid BA \mid AB \mid AA$$

By simply substituting the values of A and B here we need not have to bring grammar in ascending order of $A_i, A_j, A_k \dots$

Step 3: We will then get

$S \rightarrow ABA$	$S \rightarrow aABA \mid aBA$
$S \rightarrow BA$	$S \rightarrow bAB \mid bA$
$S \rightarrow AB$	$S \rightarrow aAB \mid aB$
$S \rightarrow AA$	$S \rightarrow aAA \mid aA$

Step 4:

Thus now all the rules are in GNF. To summarize

$$S \rightarrow aABA \mid aBA \mid bBA \mid bA \mid aAB \mid aB \mid aAA \mid aA$$

$$S \rightarrow aA \mid a \mid bB \mid b$$

$$A \rightarrow aA \mid a$$

$$B \rightarrow bB \mid b$$