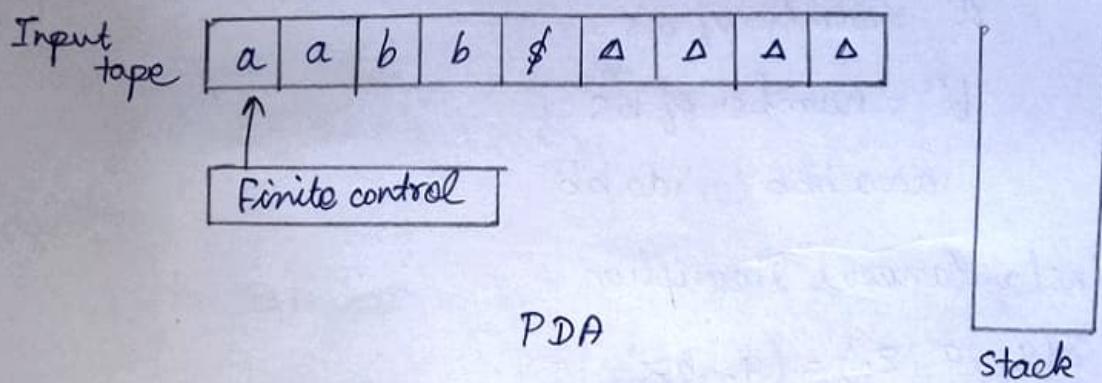


## Push down Automata:

Definition:

It have input tape, finite control and stack



PDA defined 7 components

$Q$  → The finite set of states

$\Sigma$  → Input set

$\Gamma$  → stack alphabet

$q_0$  → Initial state

$z_0$  → start symbol, which is in  $\Gamma$

$F$  → Final states

$\delta$  → Mapping function used to map from current state to next state

Design a PDA for accepting a language  
 $\{L = a^n b^n / n \geq 1\}$

Given:

$$L = a^n b^n / n \geq 1$$

Solution:

$a^n$  = number of a's

$b^n$  = number of b's

aaa bbb (or) aabb

Instantaneous Description

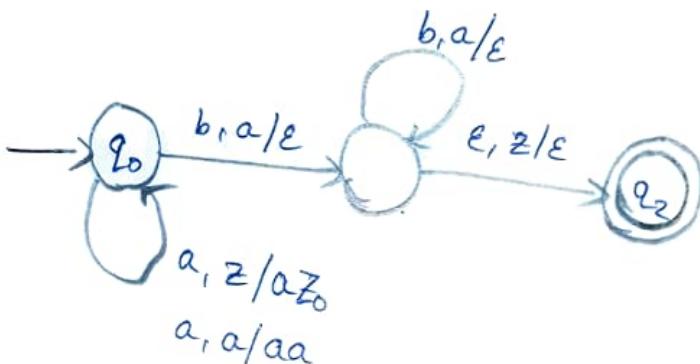
$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, a, z_0) = (q_2, \epsilon)$$



## CHOMSKY'S NORMAL FORM (CNF)

Non-terminal  $\rightarrow$  Non terminal  $\times$  non terminal

Non-terminal  $\rightarrow$  Terminal

Convert the following CFG into CNF

$$S \rightarrow aaaaS$$

$$S \rightarrow aaaa$$

Solution:

Non Terminals  $\rightarrow$  Nonterminal  $\times$  non terminal

Non terminals  $\rightarrow$  Terminal

CFG given is

$$S \rightarrow aaaaS \quad \text{rule 1}$$

$$S \rightarrow aaaa \quad \text{rule 2}$$

$A \rightarrow a$  is in CNF

rule 1 and 2 becomes

$$A \rightarrow AAAAS$$

$$S \rightarrow AAAA$$

$S \rightarrow A \boxed{AAAAS}$  can be replaced by  $P_1$

$$P_1 \rightarrow AAAAS$$

$$S \rightarrow AP_1 \text{ is in CNF}$$

The rule  $P_1$  is not CNF, so convert it

$P_1 \rightarrow A \boxed{AAS}$  can be replaced by  $P_2$

$P_1 \rightarrow AP_2$  which is in CNF

$P_2 \rightarrow AAS$  which is not in CNF, so let us convert it to CNF

$P_2 \rightarrow A \boxed{AS}$  can be replaced by  $P_3$

$P_2 \rightarrow AP_3$  where  $P_3 \rightarrow AS$

$S \rightarrow AP_1$

$P_1 \rightarrow AP_2$

$P_2 \rightarrow AP_3$

$P_3 \rightarrow AS$

Now consider rule 2

$S \rightarrow AAAA$

If we break the rule & as

$$S \rightarrow \boxed{AA} \quad \boxed{AA}$$

$\downarrow$                      $\downarrow$   
 $P_4$                      $P_5$

The rule becomes

$S \rightarrow P_4 P_5$

is in CNF

But  $P_1$  and  $P_2$  indicates the same rule. So we can eliminate either of them.

Rule 2 becomes

$$S \rightarrow P_4 P_4$$

Finally we can collectively show the CFG converted to CNF as

$$S \rightarrow AP_1$$

$$P_1 \rightarrow AP_2$$

$$P_2 \rightarrow AP_3$$

$$P_3 \rightarrow AS$$

$$S \rightarrow P_4 P_4$$

$$P_4 \rightarrow AA$$

$$A \rightarrow a$$

### GREIBACH NORMAL FORM:

Non Terminal  $\rightarrow$  One terminal  $\times$  set of non terminal

$$NT \rightarrow T \times NT$$

Convert the given CNF to CFG

$$S \rightarrow ABA$$

$$A \rightarrow aA / \epsilon$$

$$B \rightarrow bB / \epsilon$$

Solution:

Step 1:

There is  $\epsilon$ -Production, so we will simplify the given grammar as follows.

$$S \rightarrow ABA | BA | AB | AA | A | B$$

$$A \rightarrow aA | a$$

$$B \rightarrow bB | b$$

Now eliminate unit productions

$$S \rightarrow A | B$$

so,

$$S \rightarrow ABA | BA | AB | AA | aA | a | bB | b$$

$$A \rightarrow aA | a$$

$$B \rightarrow bB | b$$

Step 2:

\* In above grammar the rules for A and B non-terminals are already in GNF.

\* In fact, some of the rules of S are also in GNF

x. Here we will handle

$$S \rightarrow ABA | BA | AB | AA$$

By simply substituting the values of A and B  
here we need not have to bring grammar in  
ascending order of  $A_i, A_j, A_k \dots$ .

Step 3: We will then get

$$S \rightarrow ABA$$

$$S \rightarrow BA$$

$$S \rightarrow PB$$

$$S \rightarrow AA$$

$$S \rightarrow aABA | ABA$$

$$S \rightarrow bAB | bA$$

$$S \rightarrow aAB | aB$$

$$S \rightarrow aAA | aB$$

Step 4:

Thus now all the rules are in GNF. To summarize

$$S \rightarrow aABA | ABA | bBA | BA | aAB | AB | aAA | aA$$

$$S \rightarrow aA | a | bB | b$$

$$A \rightarrow aA | a$$

$$B \rightarrow bB | b$$