Simulation of Biological Neuronal Networks

Cortical networks: Background and simple models

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Outline

Biological background: Cortex structure

Some graph-theoretical concepts

Random networks

the balanced-random-network model (brunel, 2000)

Literature

Biological background: Cortex structure

- **Macroscopic structure:**
  - visible with your eyes
  - thousand/millions of cells forming a structure
  - e.g brain areas, long range connections
- **Microscopic structure:**
  - magnifying instruments are needed to recognize structure
  - depending on resolution: group of cells, single cell, molecules
  - e.g small neuronal network, short range connections

Microscopic structure: Cell types

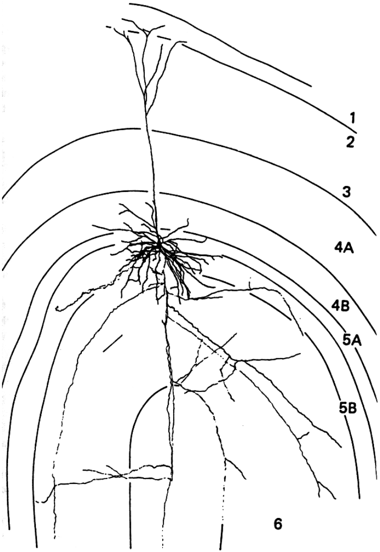
- cortical tissue is mainly composed of two cell types:
  - **neuroglia (glia)**:
    - important role in development of the brain
    - metabolic supportive role
    - control ionic composition of extracellular space
    - form myelin sheath around axons of neurons
    - do not take part in interaction between neurons on a millisecond scale (however, may play a role in slow modulations)
  - **nerve cells (neurons)**:
    - carry out information processing and storage in the brain

Microscopic structure: Neuron types

- pyramidal cell:
  - dendrites covered with spines
  - axon leaves cortex into white matter but has numerous branches close to cell body
  - axons makes excitatory synapses
  - cell receives input from inhibitory cells mainly at the soma and from excitatory cells at the basal and apical dendrite

Microscopic structure: Neuron types

Pyramidal cell



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- spiny stellate cell:
  - axon branches out within the cortex (rarely leaves the cortex)
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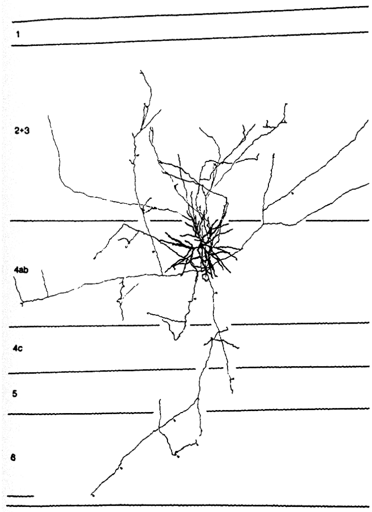
  - soma receives exclusively inhibitory and dendrites mostly excitatory inputs

Microscopic structure: Neuron types

Spiny stellate cell



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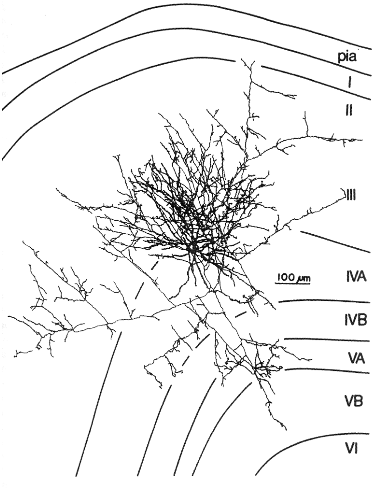
- smooth stellate cell (e.g. basket cell):
  - axon branches out only in the cortex
  - axons makes inhibitory synapses (GABA)
  - both the soma and the dendrites receive a mixture of excitatory and inhibitory inputs

Microscopic structure: Neuron types

Smooth stellate cell



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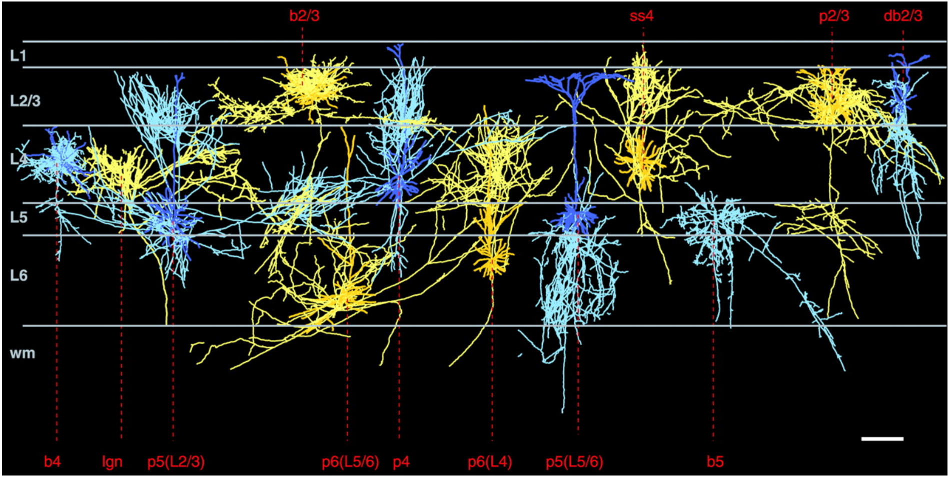
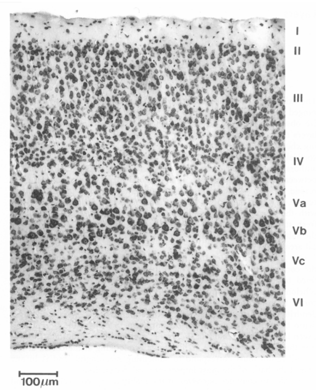
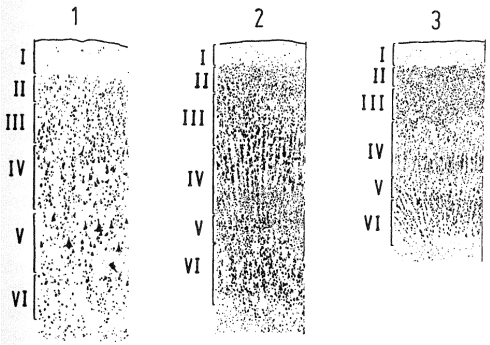


(Binzegger et al., 2004)

Microscopic structure: Cortex layers

- subdivision of neocortex into 6 (or 5) layers
  - layer I: very few neurons, only dendrites and mesh of horizontal axons
  - layer II/III: small pyramidal cells
  - layer IV: small and medium-size pyramidal cells and stellate cells in layer IVA, almost exclusively stellate cells in IVB
  - layer V: all cell types, very large pyramidal cells dominate
  - layer VI: small and medium-size pyramidal cells





primary motor area

primary sensory area

Cortex connectivity: Summary - some numbers

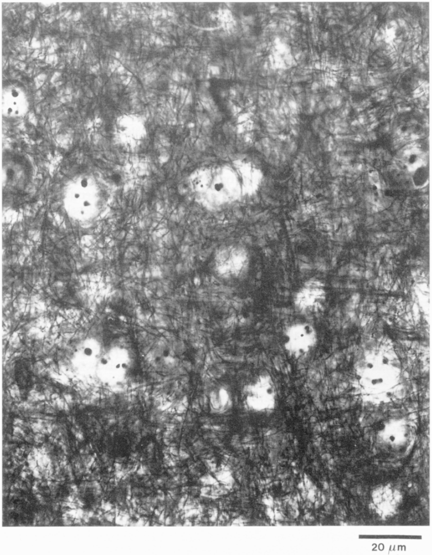


Table 1.5.1. *Densities of neurons in the cortex (thousands per cubic millimeter)*

A		B		C	
Animal	Density	Region	Density	Layer	Density
Mouse	142.5	Visual	106	I	20
Rat	105.0	Somatosensory	60	II	82
Guinea pig	52.5	Auditory	43	III	62
Rabbit	43.8	Motor	30	IV	67
Cat	30.8			V	61
Dog	24.5			VI	77
Monkey	21.5				
Human	10.5				
Elephant	6.9				
Whale	6.8				

Table 1.5.4. *Typical compositions of cortical tissues*

Variable	Value
Neuronal density	$40{,}000/mm^3$
Neuronal composition:	
Pyramidal	75%
Smooth stellate	15%
Spiny stellate	10%
Synaptic density	$8 \cdot 10^8/mm^3$
Axonal length density	$3{,}200\ m/mm^3$
Dendritic length density	$400\ m/mm^3$
Synapses per neuron	20,000
Inhibitory synapses per neuron	2,000
Excitatory synapses from remote sources per neuron	9,000
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Dendritic length per neuron	10 mm

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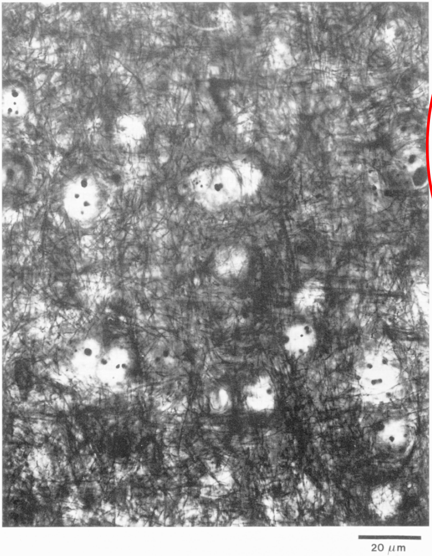

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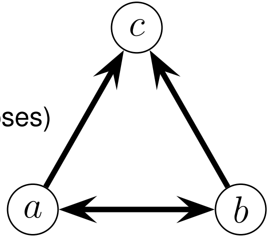
Some graph-theoretical concepts

- Graph: $G := (V, E)$

  V: set of 'vertices' (neurons)
  E: set of (ordered) pairs of vertices $\curvearrowright$ 'edges' (synapses)

  example:
  $V = \{a, b, c\}$, $E = \{\{a, b\}, \{b, a\}, \{b, c\}, \{a, c\}\}$



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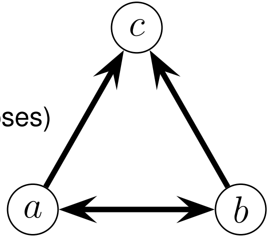

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- more convenient:

  adjacency matrix $A = \begin{array}{cc} \text{presynaptic} & \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} & \text{postsynaptic} \end{array}$ (columns=axons, rows=dendrites)

  with $A_{ij} = \begin{cases} 1 & \text{edge (synapse) } j \to i \text{ present} \\ 0 & \text{else} \end{cases}$

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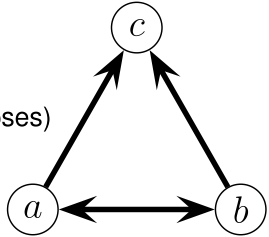

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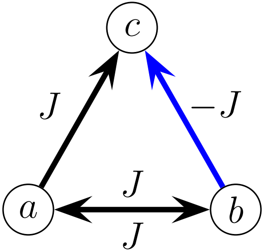
  weight matrix $W = \begin{pmatrix} 0 & J & 0 \\ J & 0 & 0 \\ J & -J & 0 \end{pmatrix}$

  (for weighted graphs)



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  example: linear firing-rate network model (Wilson-Cowan model)

$$\tau \frac{dy}{dt} = -y + Wy$$

with firing rates $y(t) = [y_1(t), \ldots, y_N(t)]^{\mathsf{T}}$ of neurons $1, \ldots, N$

$\curvearrowright$ stability of fixed point determined by eigenvalue spectrum of $W - 1$

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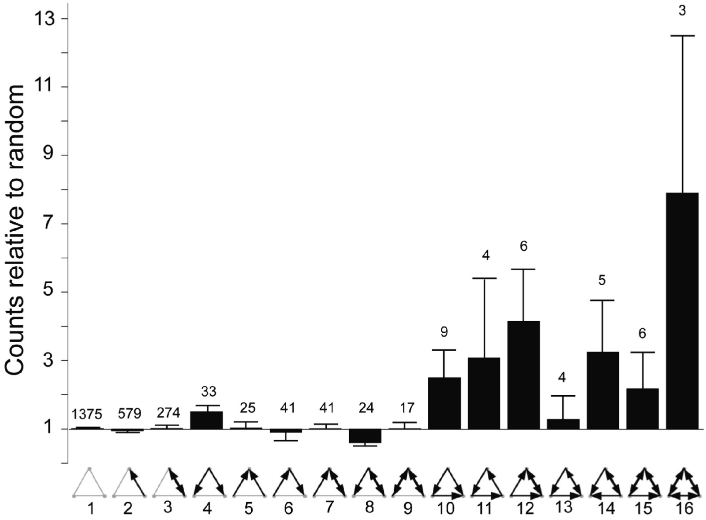

Song et al. (2005), Highly Nonrandom Features of Synaptic Connectivity in Local Cortical Circuits

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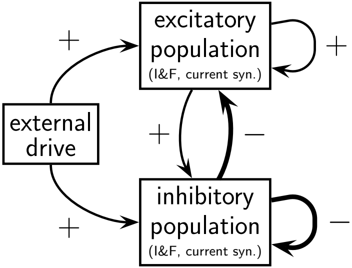
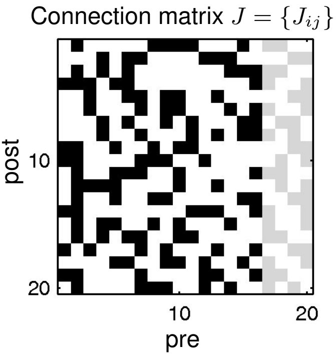
  - ...and infinitely many more

the balanced-random-network model (brunel, 2000)





- N_E excitatory, N_I inhibitory neurons, $N = N_\mathsf{E} + N_\mathsf{I} \sim 10^{4\ldots5}$, N_E:N_I=4:1
- random Dale-conform connectivity with fixed in-degrees $K_{\mathsf{E/I}} = N_{\mathsf{E/I}}/10$
- free parameters: g (ratio of inhibitory to excitatory synaptic weight), $\frac{\nu_\text{ext}}{\nu_\text{thr}}$ (ratio of external rate to threshold rate)

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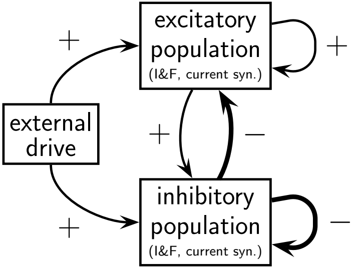

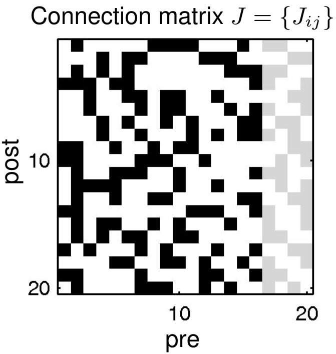

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- LIF dynamics with current synapses
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Activity regimes

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↷ stationary and oscillatory states

SR:	synchronous regular
AI:	asynchronous irregular
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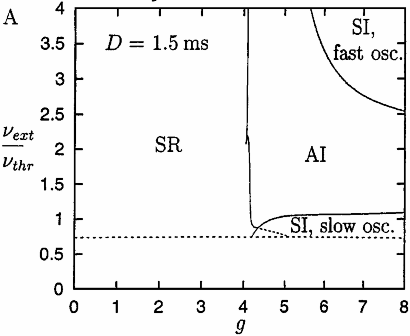
theory



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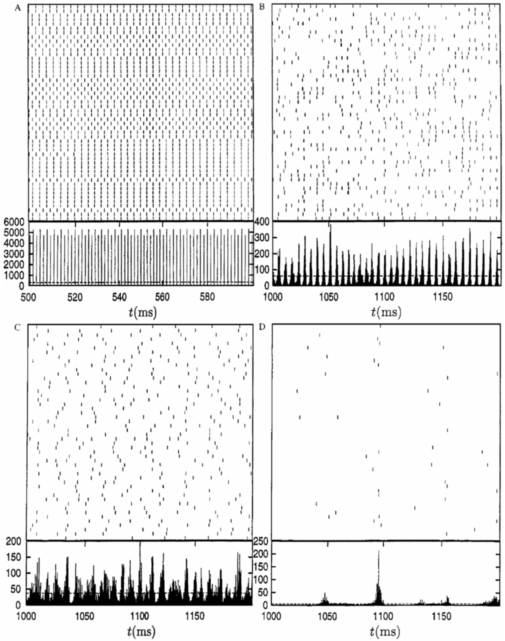

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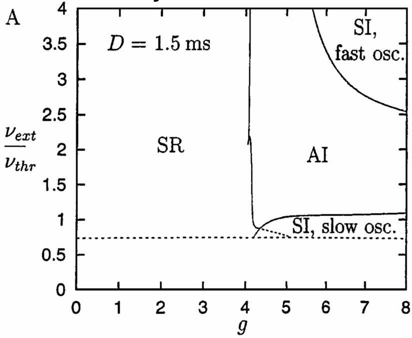
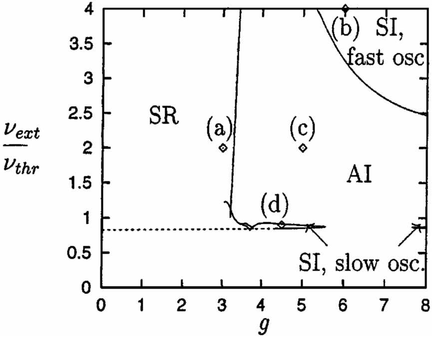


simulation



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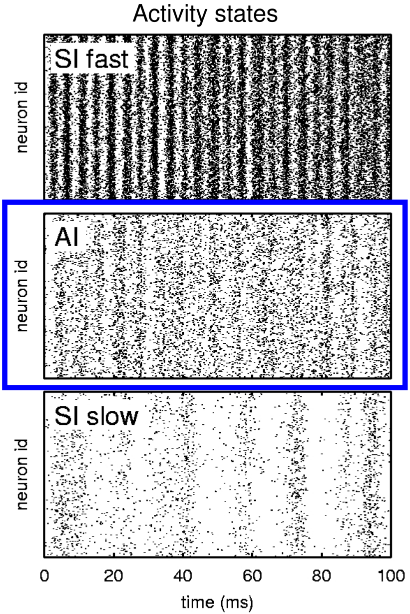

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  - large membrane potential fluctuations
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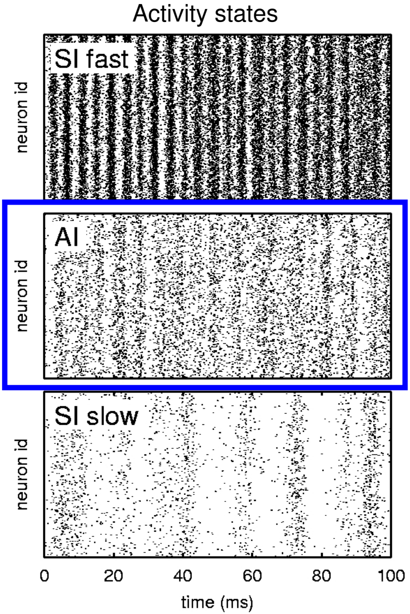

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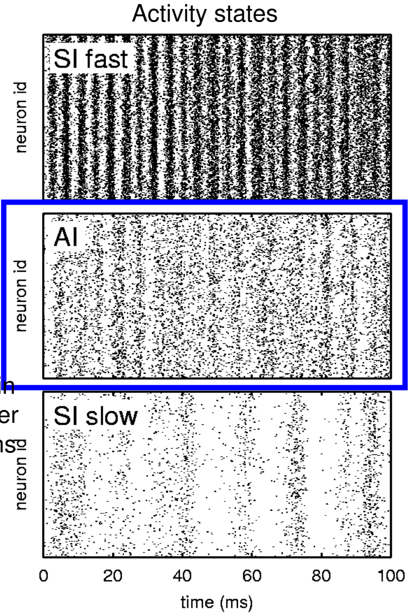

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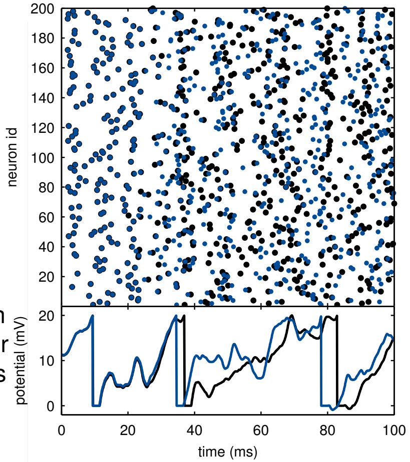
- chaotic spiking (high sensitivity)

Chaos in random networks



Simulation of two identical networks ($N \sim 10000$), slight perturbation of initial membrane potential of one neuron ($\Delta V = 0.1$ mV)

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