Simulation of Biological Neuronal Networks

Cortical networks: Background and simple models

Inst. of Neuroscience and Medicine (INM-6)
Computational and Systems Neuroscience
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Outline

Biological background: Cortex structure

Some graph-theoretical concepts

Random networks

The balanced-random-network model (Brunel, 2000)

Literature

Biological background: Cortex structure

- **macroscopic:**
  - visible with your eyes
  - thousands/millions of cells forming a structure
  - e.g. brain areas, long range connections
- **microscopic:**
  - magnifying instruments are needed to recognize structure
  - depending on resolution: group of cells, single cell, molecules
  - e.g. small neuronal network, short range connections

Microscopic structure of the Cortex: Cell types

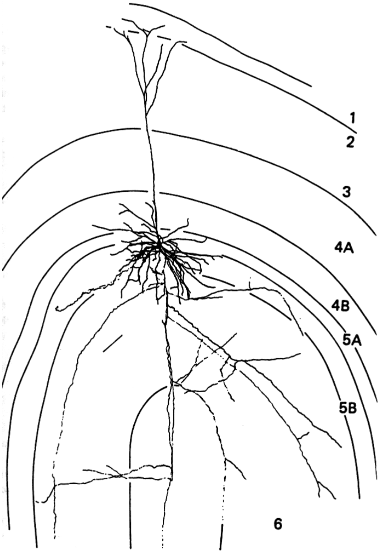
- cortical tissue is mainly composed of two cell types:
  - **neuroglia (glia)**:
    - important role in development of the brain
    - metabolic supportive role
    - control ionic composition of extracellular space
    - form myelin sheath around axons of neurons
    - do not take part in interaction between neurons on a millisecond scale (however, may play a role in slow modulations)
  - **nerve cells (neurons)**:
    - carry out information processing and storage in the brain

Microscopic structure of the Cortex: Neuron types

- pyramidal cell:
  - dendrites covered with spines
  - axon leaves cortex into white matter but has numerous branches close to cell body
  - axons makes excitatory synapses
  - cell receives input from inhibitory cells mainly at the soma and from excitatory cells at the basal and apical dendrite

Microscopic structure of the Cortex: Neuron types

Pyramidal cell



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- spiny stellate cell:
  - axon branches out within the cortex (rarely leaves the cortex)
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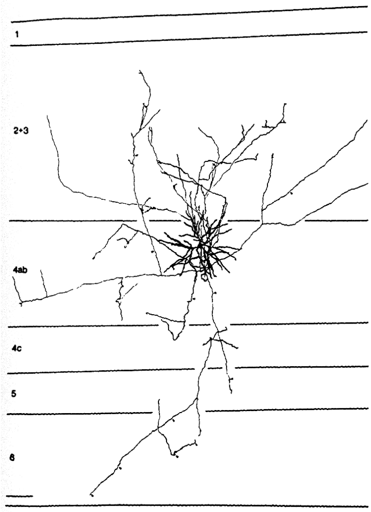
  - soma receives exclusively inhibitory and dendrites mostly excitatory inputs

Microscopic structure of the Cortex: Neuron types

Spiny stellate cell



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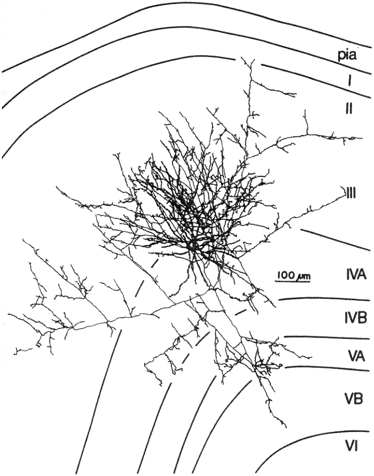
- smooth stellate cell (e.g. basket cell):
  - axon branches out only in the cortex
  - axons makes inhibitory synapses (GABA)
  - both the soma and the dendrites receive a mixture of excitatory and inhibitory inputs

Microscopic structure of the Cortex: Neuron types

Smooth stellate cell



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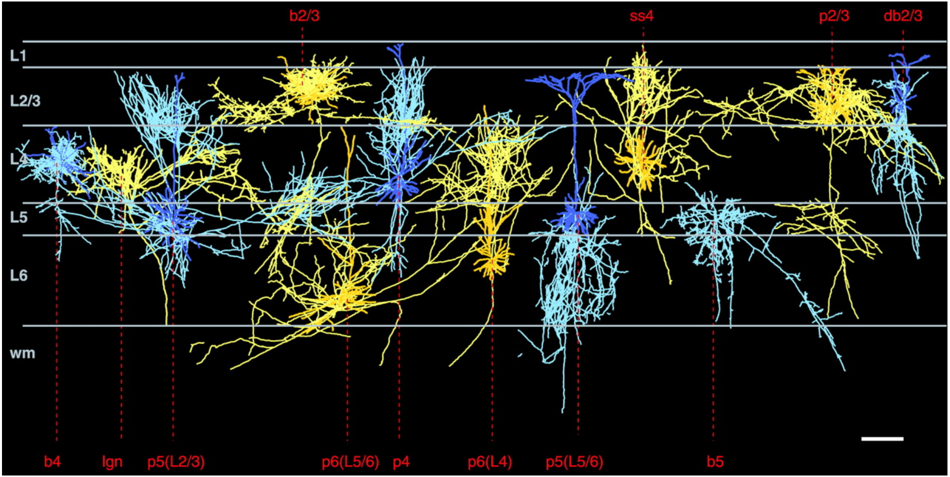
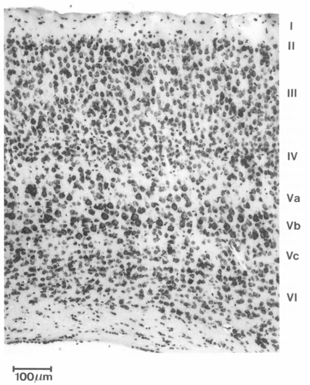
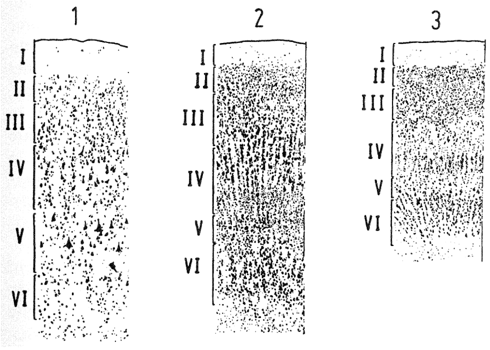


(Binzegger et al., 2004)

Microscopic structure of the Cortex: Cortex layers

- subdivision of neocortex into 6 (or 5) layers
  - layer I: very few neurons, only dendrites and mesh of horizontal axons
  - layer II/III: small pyramidal cells
  - layer IV: small and medium-size pyramidal cells and stellate cells in layer IVA, almost exclusively stellate cells in IVB
  - layer V: all cell types, very large pyramidal cells dominate
  - layer VI: small and medium-size pyramidal cells





primary motor area

primary sensory area

Cortex connectivity: Summary - some numbers

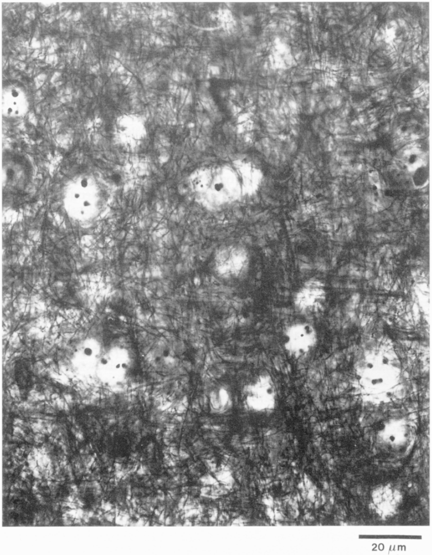


Table 1.5.1. *Densities of neurons in the cortex (thousands per cubic millimeter)*

A		B		C	
Animal	Density	Region	Density	Layer	Density
Mouse	142.5	Visual	106	I	20
Rat	105.0	Somatosensory	60	II	82
Guinea pig	52.5	Auditory	43	III	62
Rabbit	43.8	Motor	30	IV	67
Cat	30.8			V	61
Dog	24.5			VI	77
Monkey	21.5				
Human	10.5				
Elephant	6.9				
Whale	6.8				

Table 1.5.4. *Typical compositions of cortical tissues*

Variable	Value
Neuronal density	40,000/mm^3
Neuronal composition:	
Pyramidal	75%
Smooth stellate	15%
Spiny stellate	10%
Synaptic density	$8 \cdot 10^8$/mm^3
Axonal length density	3,200 m/mm^3
Dendritic length density	400 m/mm^3
Synapses per neuron	20,000
Inhibitory synapses per neuron	2,000
Excitatory synapses from remote sources per neuron	9,000
Excitatory synapses from local sources per neuron	9,000
Dendritic length per neuron	10 mm

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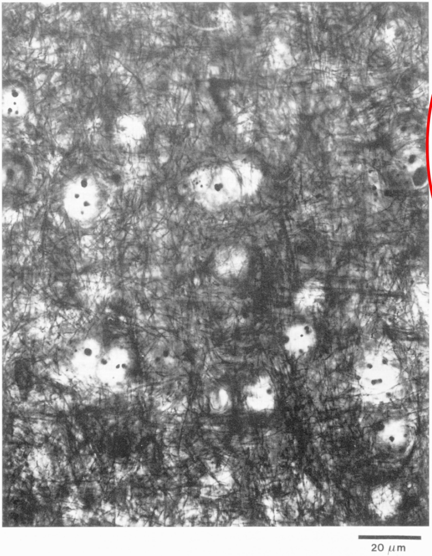

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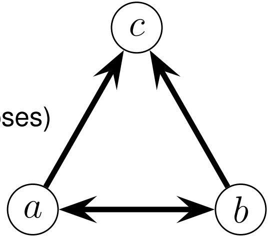
Some graph-theoretical concepts

- Graph: $G := (V, E)$

  V: set of 'vertices' (neurons)
  E: set of (ordered) pairs of vertices $\curvearrowright$ 'edges' (synapses)

  example:
  $V = \{a, b, c\}$, $E = \{\{a, b\}, \{b, a\}, \{b, c\}, \{a, c\}\}$



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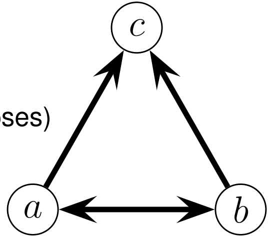

- more convenient:

  adjacency matrix $A = \begin{array}{cc} \text{presynaptic} & \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} & \text{postsynaptic} \end{array}$ (columns=axons, rows=dendrites)

  with $A_{ij} = \begin{cases} 1 & \text{edge (synapse) } j \to i \text{ present} \\ 0 & \text{else} \end{cases}$

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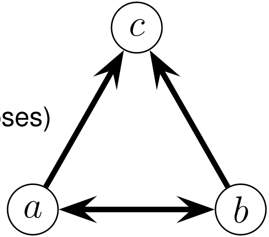

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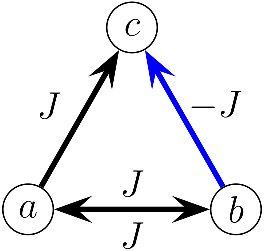
  weight matrix $W = \begin{pmatrix} 0 & J & 0 \\ J & 0 & 0 \\ J & -J & 0 \end{pmatrix}$

  (for weighted graphs)



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  example: linear firing-rate network model (Wilson-Cowan model)

$$\tau \frac{dy}{dt} = -y + Wy$$

  with firing rates $y(t) = [y_1(t), \ldots, y_N(t)]^{\mathsf{T}}$ of neurons $1, \ldots, N$

  $\curvearrowright$ stability of fixed point determined by eigenvalue spectrum of $W - 1$

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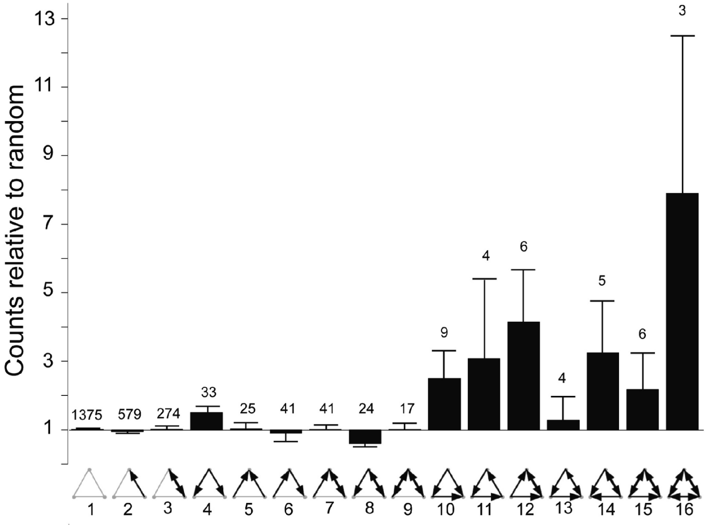

Song et al. (2005), Highly Nonrandom Features of Synaptic Connectivity in Local Cortical Circuits

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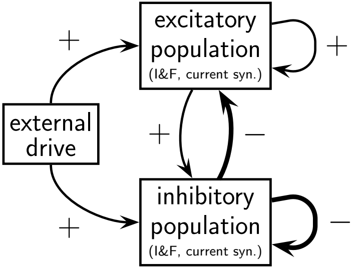
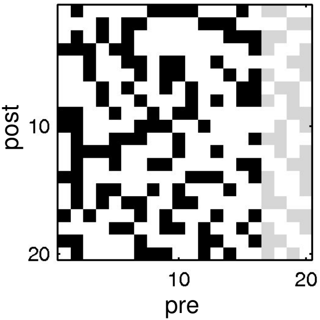
  - ... and infinitely many more

The balanced-random-network model (Brunel, 2000)

Simple model of a local cortex volume ($\sim 1\text{mm}^3$, $\sim 10^{4\ldots5}$ neurons)



Connection matrix $J = \{J_{ij}\}$



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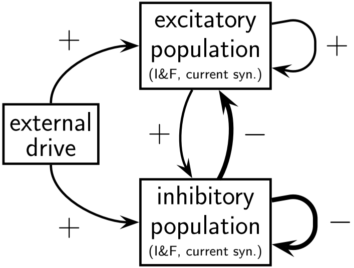

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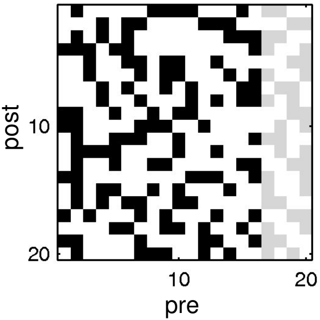

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- LIF dynamics with current synapses

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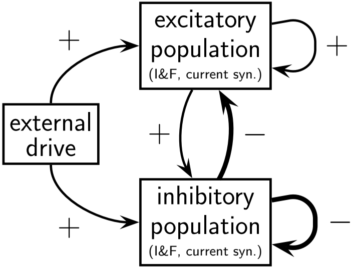

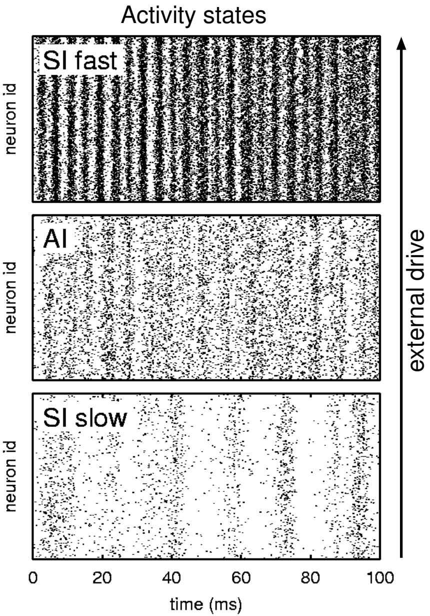

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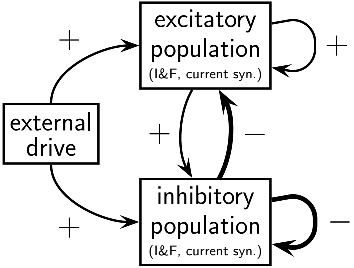
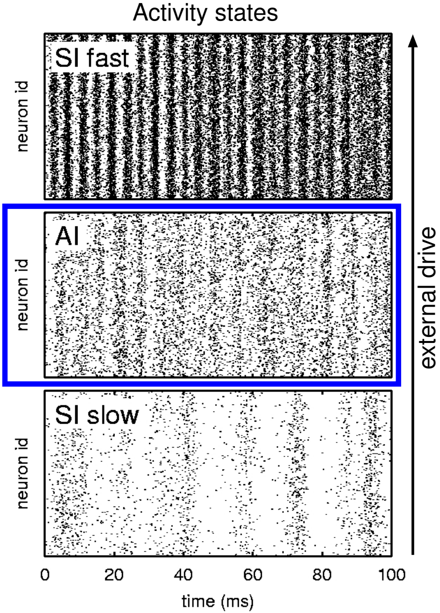


- in-vivo like activity
  - large membrane potential fluctuations
  - low firing rates
  - irregular spiking
- global oscillatory modes in various frequency bands



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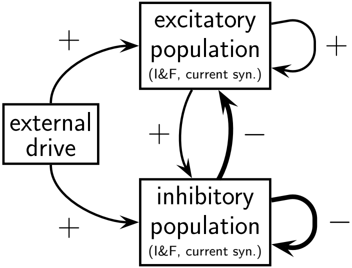
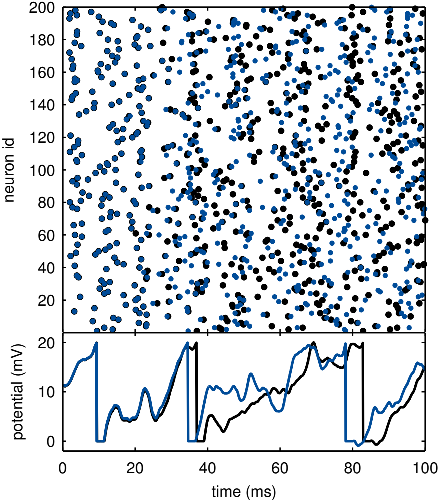


- in-vivo like activity
  - large membrane potential fluctuations
  - low firing rates
  - irregular spiking
- global oscillatory modes in various frequency bands
- chaotic spiking (high sensitivity)

Chaos in random networks



Simulation of two identical networks ($N \sim 10000$), slight perturbation of initial membrane potential of one neuron ($\Delta V = 0.1$ mV)

Activity regimes

- analysis of network dynamics in diffusion approximation (small synaptic weights, high firing rates, small correlations) (see Brunel, 2000)

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AI:	asynchronous irregular
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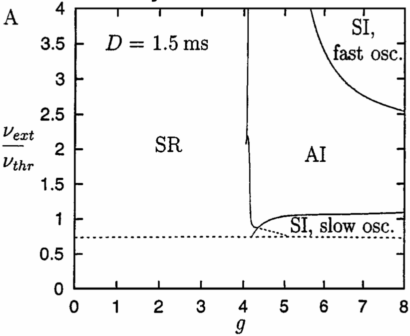
theory



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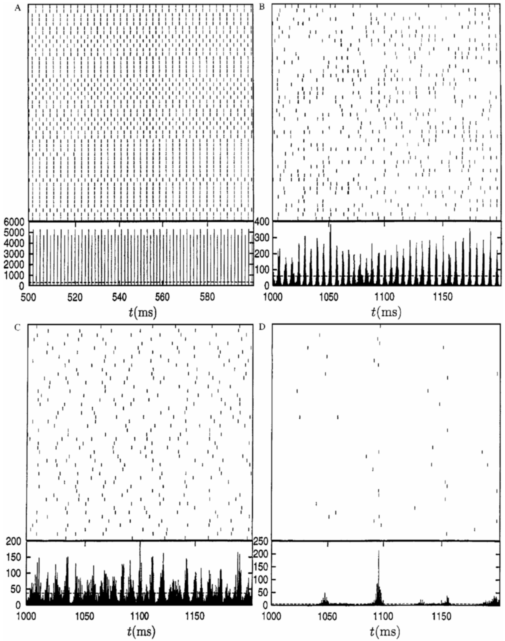

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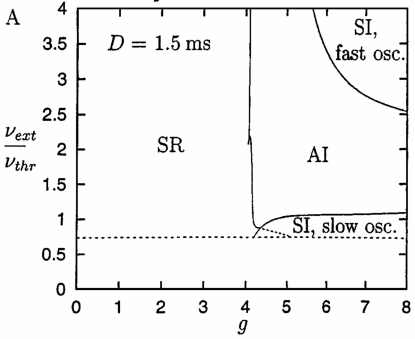
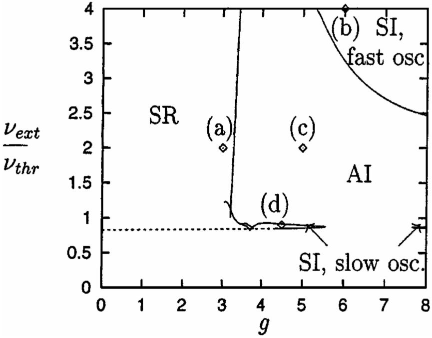


simulation



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