FitzHugh Nagumo Model of a Neuron

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1 Introduction

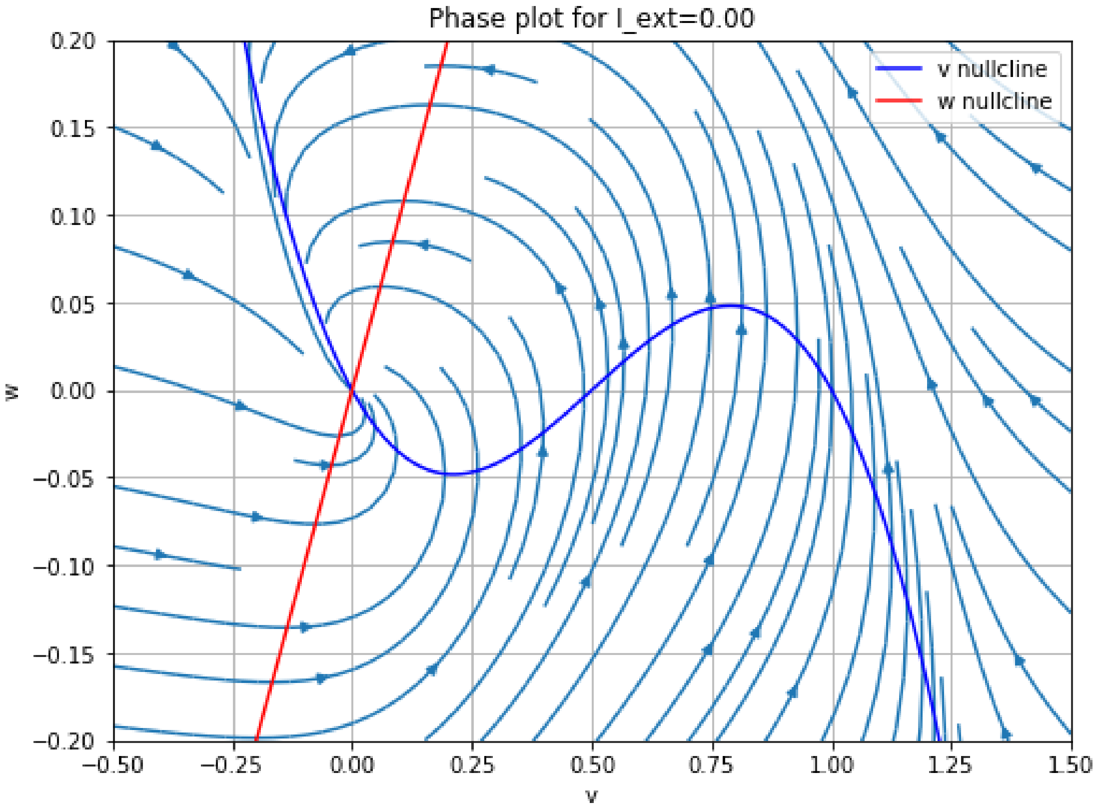
FitzHugh-Nagumo model is a two-variable neuron model, constructed by reducing the 4-variable Hodgkin Huxley model, by applying suitable assumptions. The FitzHugh-Nagumo Model is an example of a relaxation oscillator because, if the external stimulus I_{ext} exceeds a certain threshold value, the system will exhibit a characteristic excursion in phase space, before the variables v and w relax back to their rest values.This behaviour is typical for spike generations (a short, nonlinear elevation of membrane voltage v, diminished over time by a slower, linear recovery variable w) in a neuron after stimulation by an external input current.
We have the following two equations determining the system:
$\frac{dv}{dt} = f(v) - w + I_{\text{ext}}$
$\frac{dw}{dt} = bv - rw$
where, $f(v) = v(a - v)(v - 1)$

Here b and r are very small positive values.

2 Case 1:Excitability $(I_{\text{ext}} = 0)$

We set $I_{\text{ext}} = 0$.

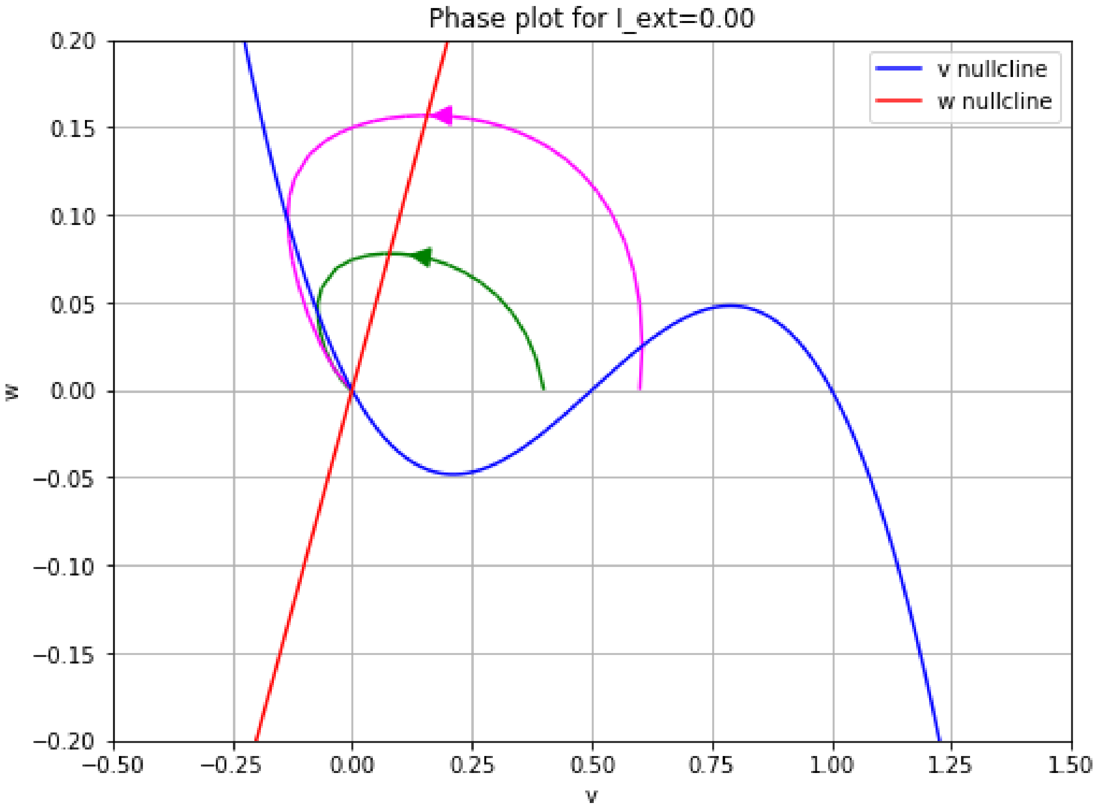
The plot below shows the phase trajectories as well as the null-clines



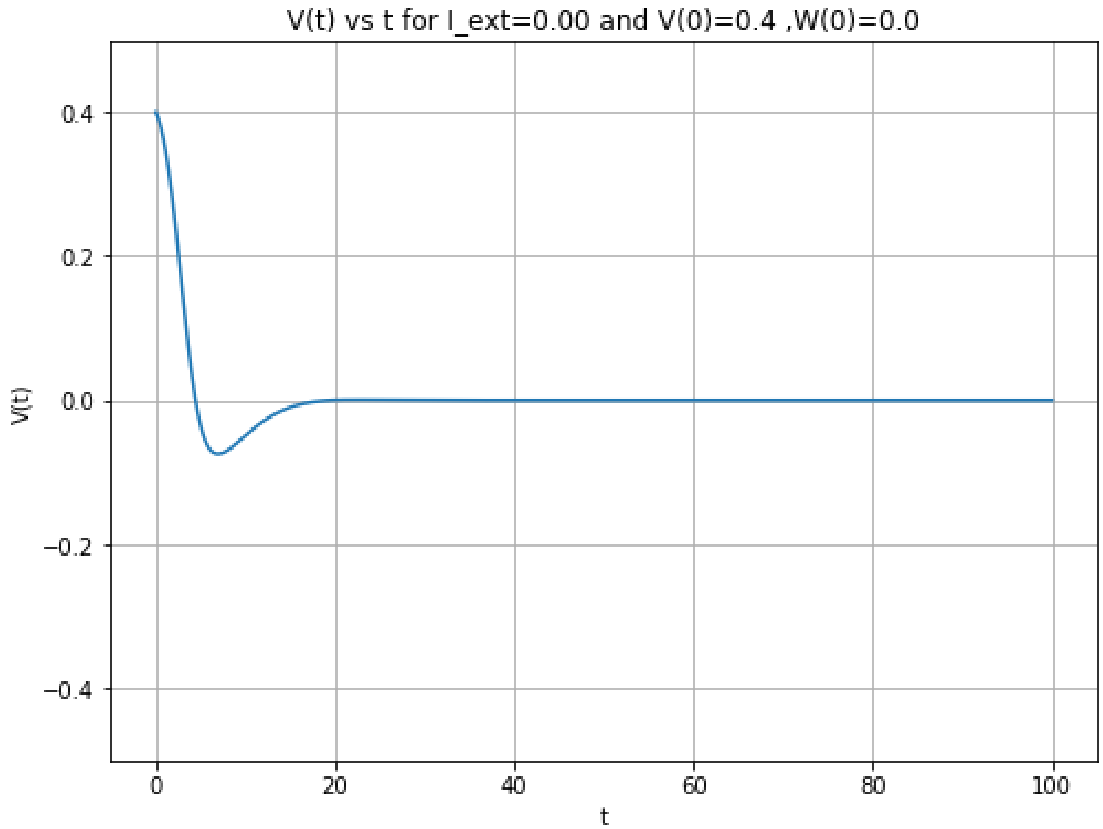
The plot below shows us the trajectories when:

- $v(0) = 0.4$: Green Trajectory
- $v(0) = 0.6$: Pink Trajectory

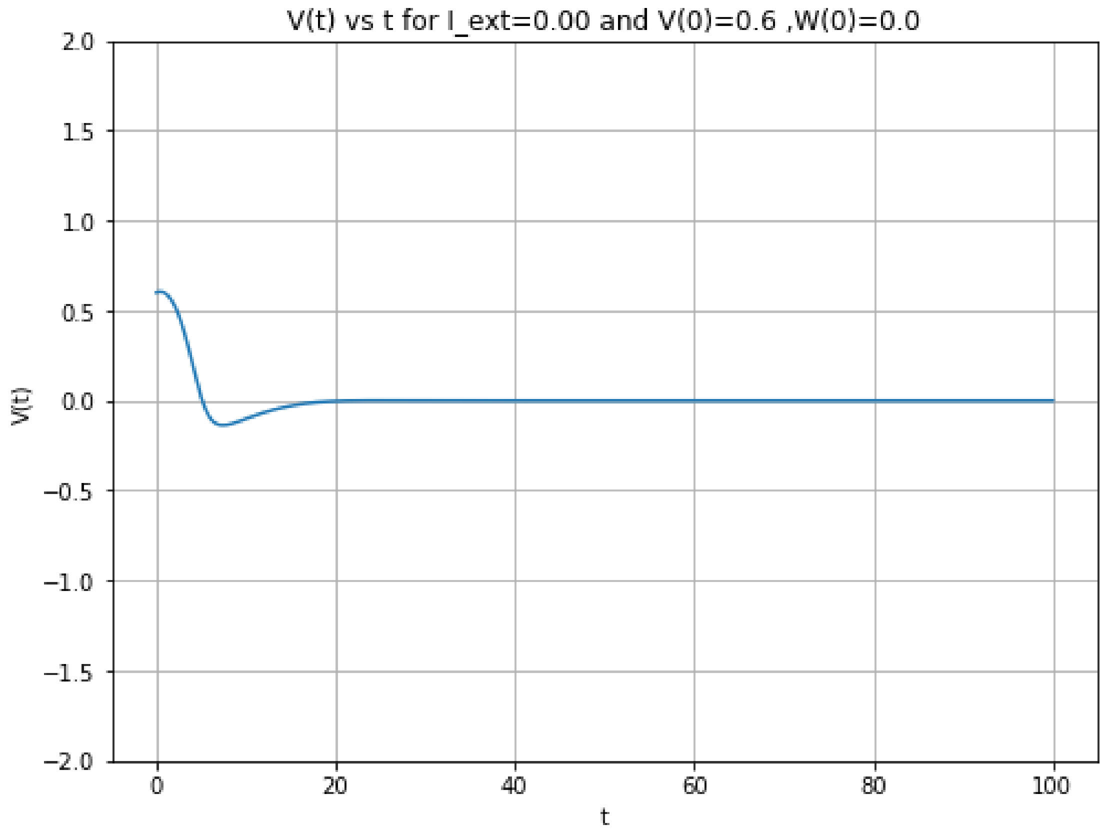
Both of the points end at the fixed point $(0, 0)$ which is stable.

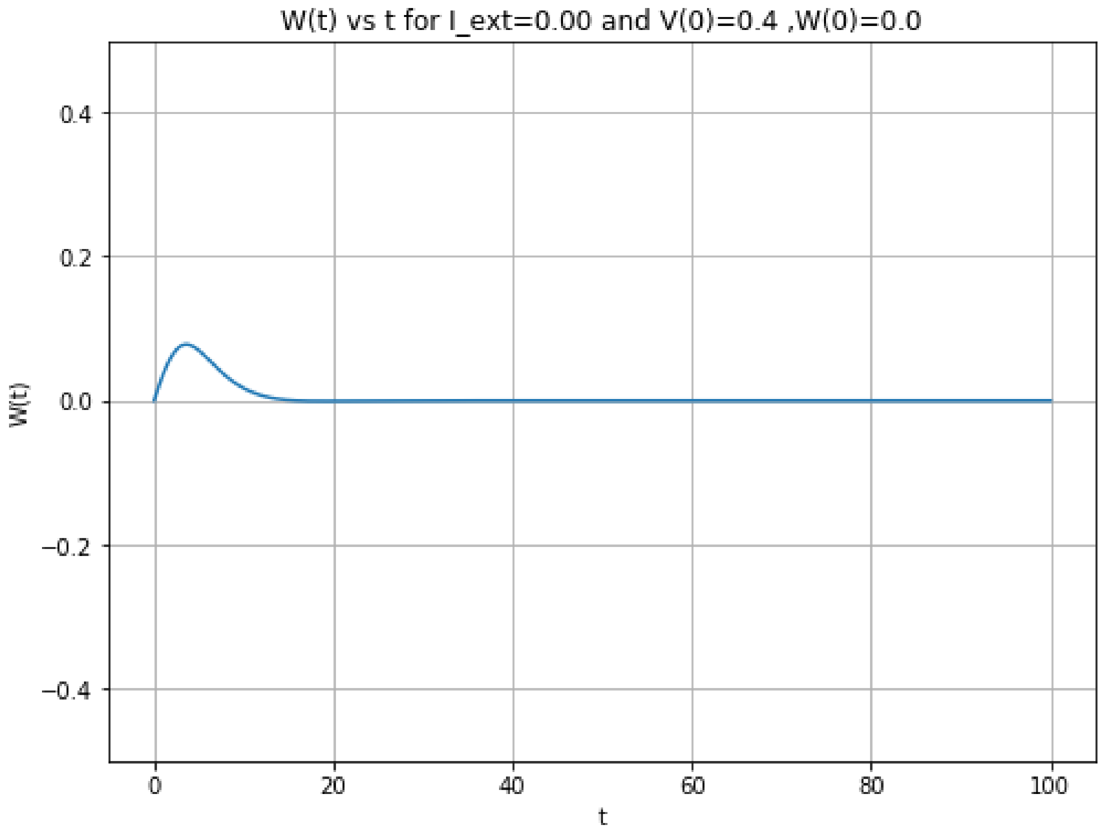


The plots below show the variation of $V(t)$ and $W(t)$ with time.
We have action potentials.

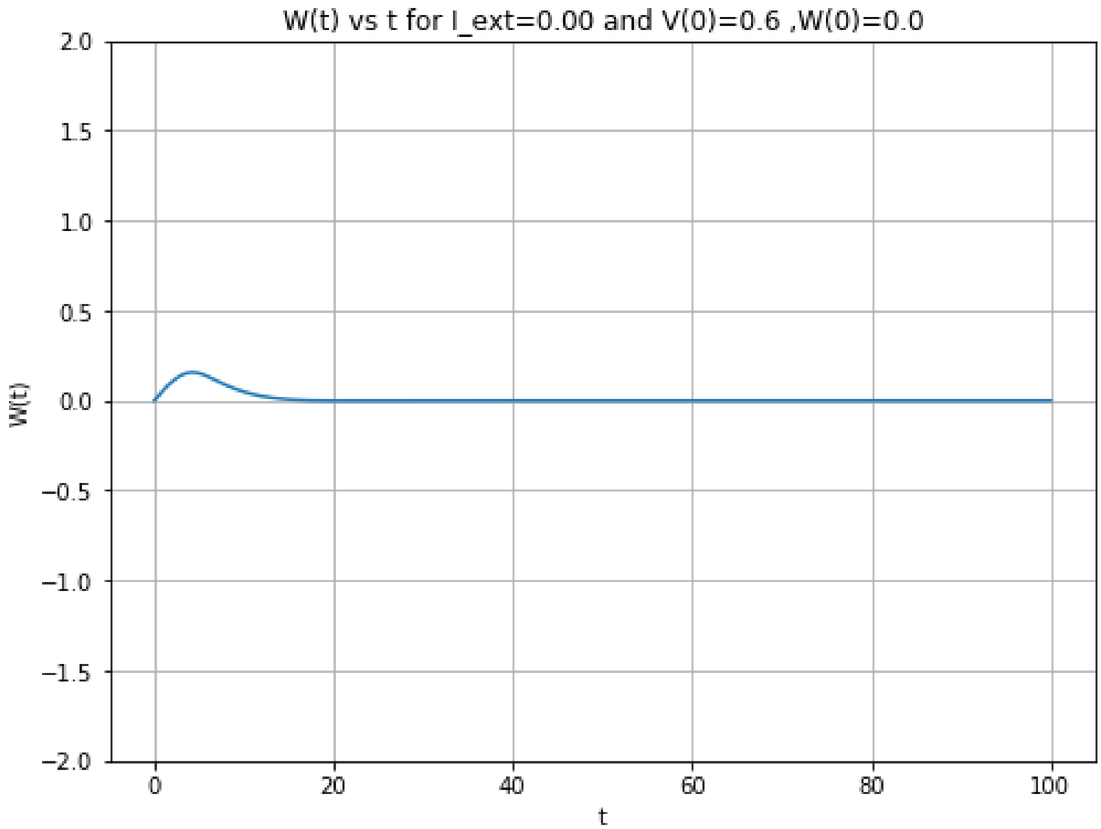


V(t) vs t for I_ext=0.00 and V(0)=0.4 ,W(0)=0.0
V(t)
t
0.4
0.2
0.0
−0.2
−0.4
0
20
40
60
80
100


V(t) vs t for I_ext=0.00 and V(0)=0.6 ,W(0)=0.0
V(t)
t
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
20
40
60
80
100



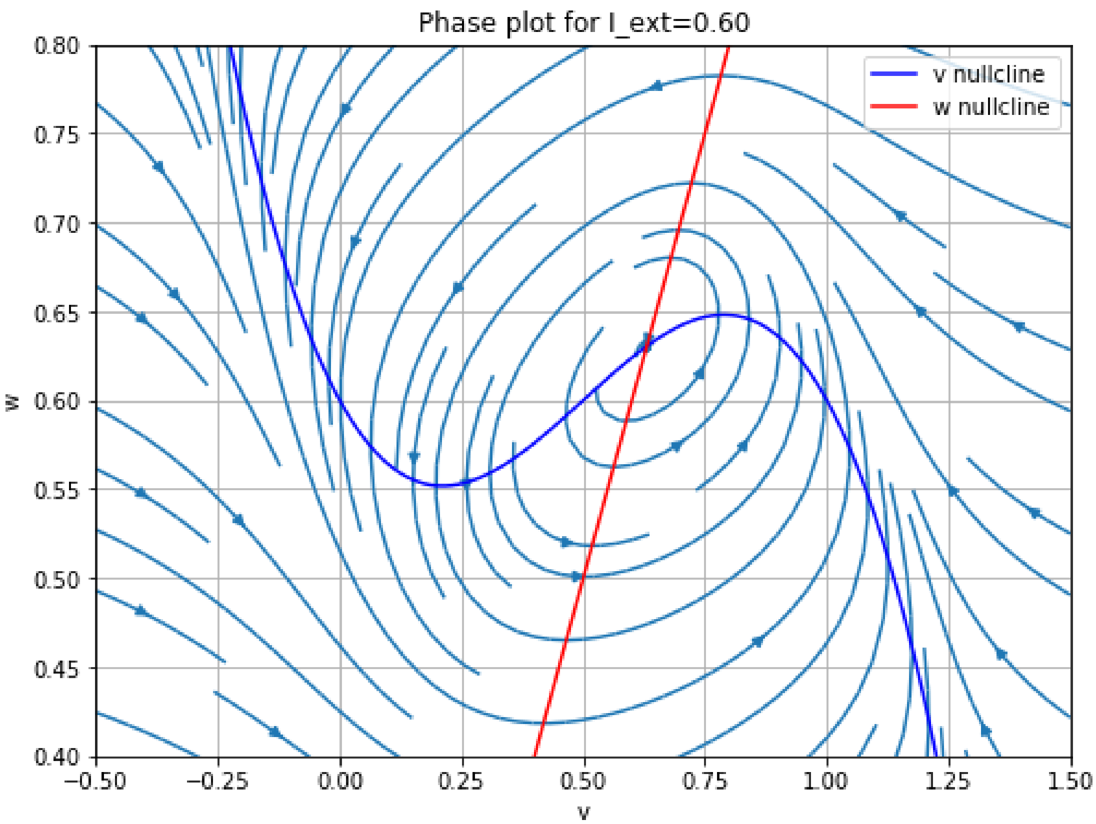
W(t) vs t for I_ext=0.00 and V(0)=0.4 ,W(0)=0.0
0.4
0.2
0.0
−0.2
−0.4
W(t)
0
20
40
60
80
100
t



3 Case 2:Limit Cycles($I_{\text{ext}} = 0.6$)

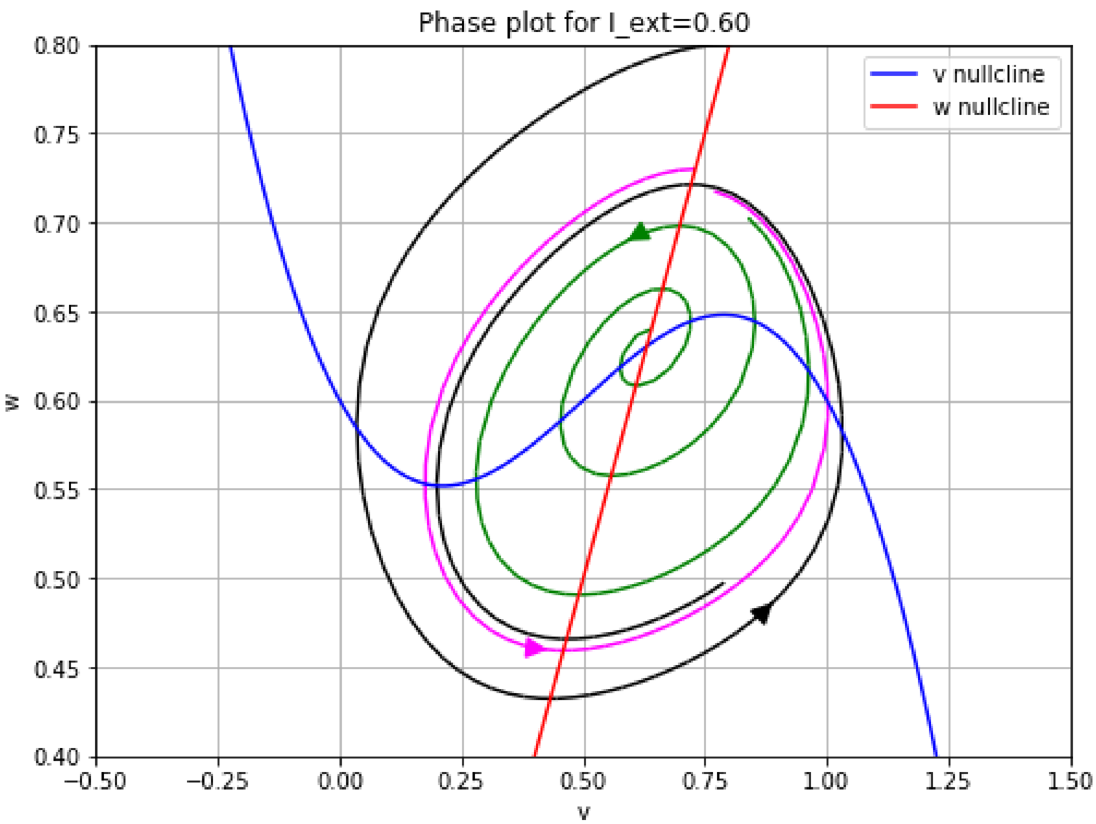
We set $I_{\text{ext}} = 0.6$.

The plot below shows the phase trajectories as well as the null-clines

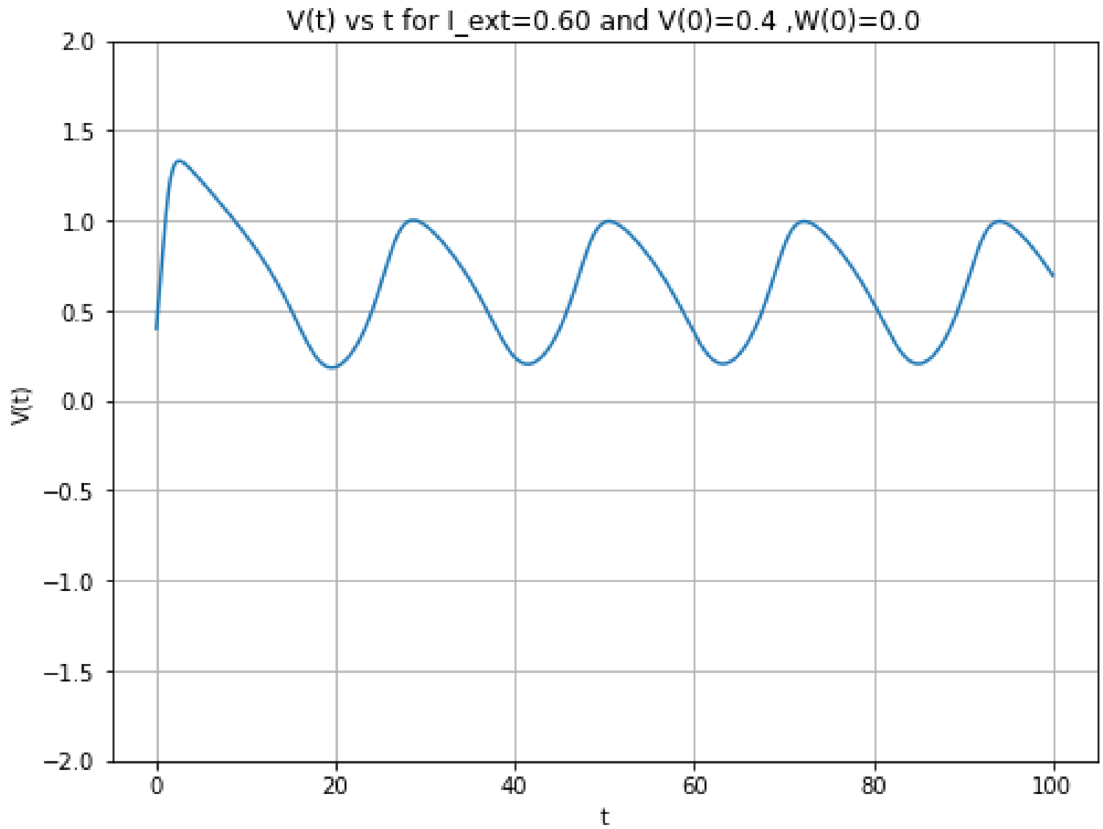


The below plot shows that:

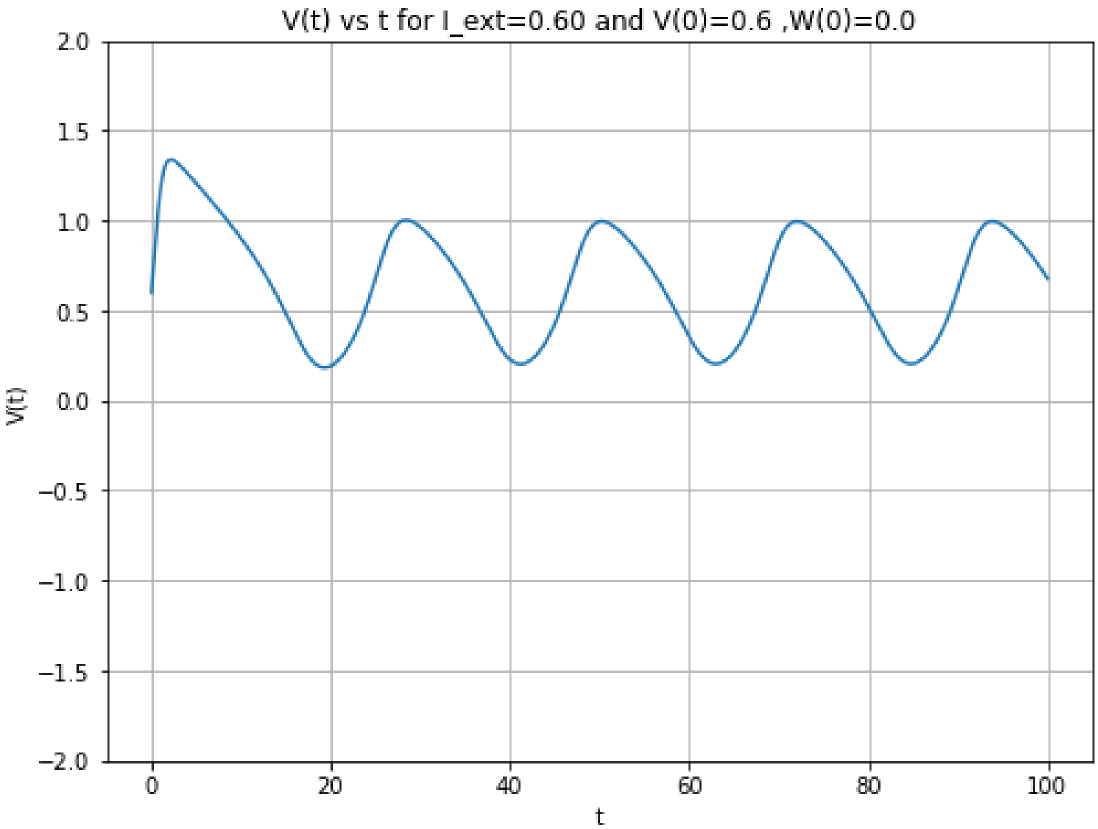
- A small perturbation from the fixed point is unstable as the trajectory(green path) moves towards the limit cycle (pink trajectory).
- The black trajectory is the one when the point is left from (0.8,0.8). This also moves towards the limit cycle proving that the limit cycle is indeed stable.

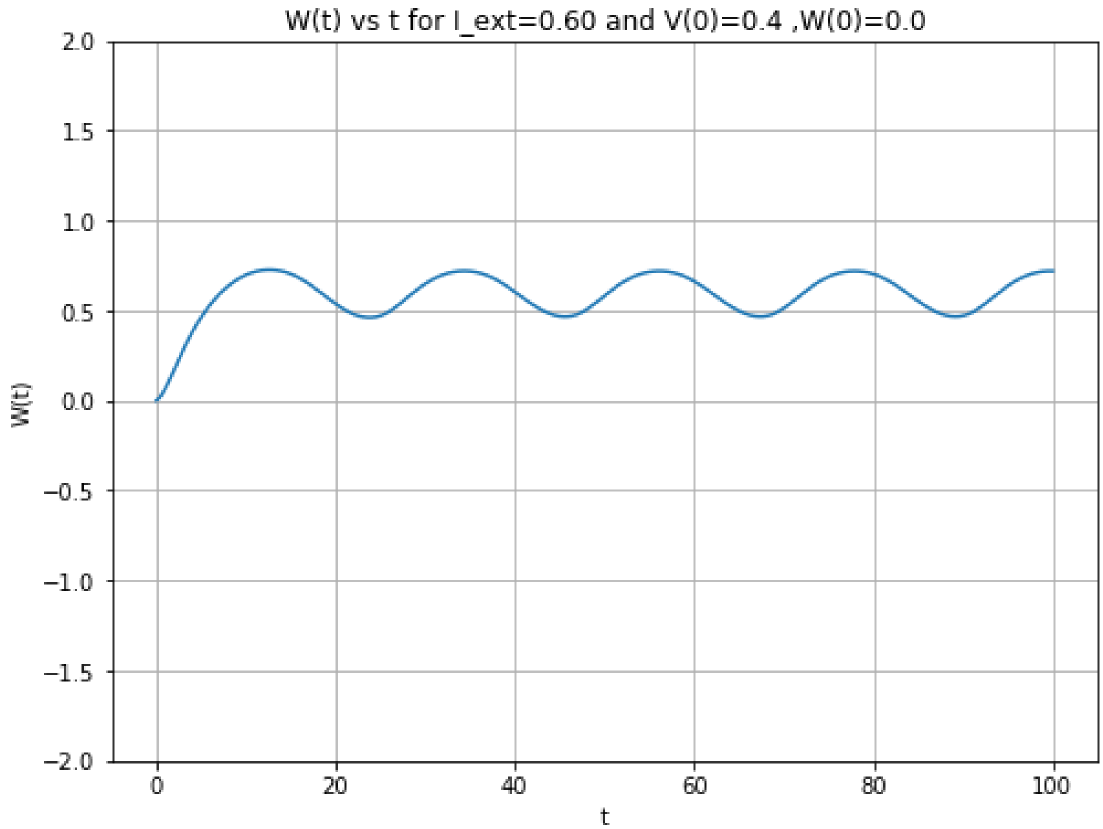


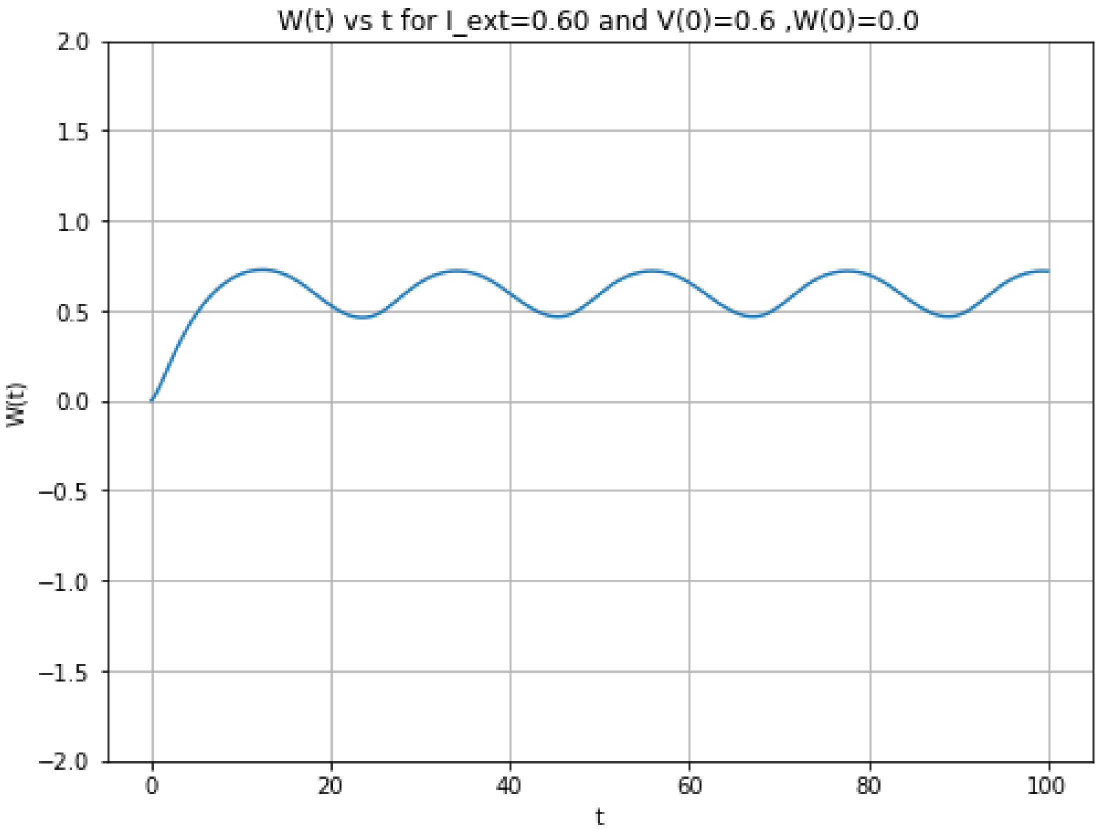
The plots below show the variation of $V(t)$ and $W(t)$ with time.
We have periodic firing.



V(t) vs t for I_ext=0.60 and V(0)=0.4 ,W(0)=0.0
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
V(t)
0
20
40
60
80
100
t


V(t) vs t for I_ext=0.60 and V(0)=0.6 ,W(0)=0.0
V(t)
t
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
20
40
60
80
100

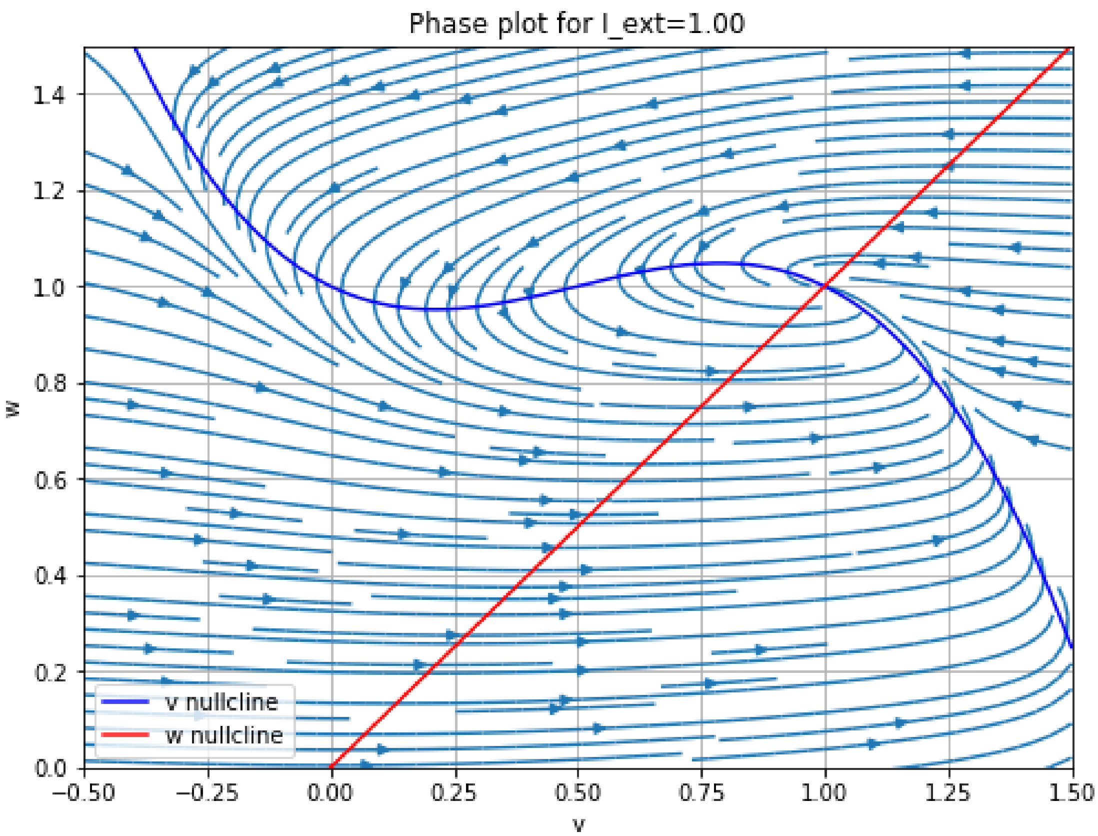

W(t) vs t for I_ext=0.60 and V(0)=0.4 ,W(0)=0.0
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
W(t)
0
20
40
60
80
100
t



4 Case 3:Depolarization ($I_{\text{ext}} = 1$)

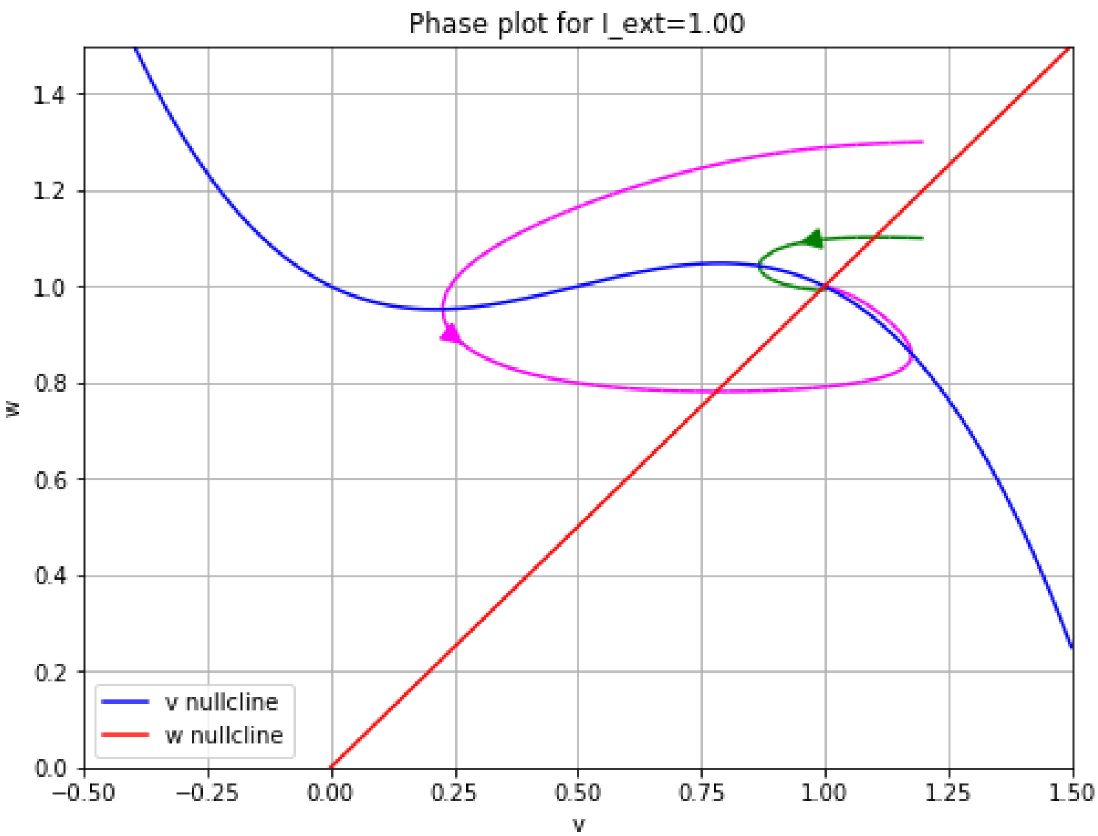
We set $I_{\text{ext}} = 1$.

The plot below shows the phase trajectories as well as the null-clines

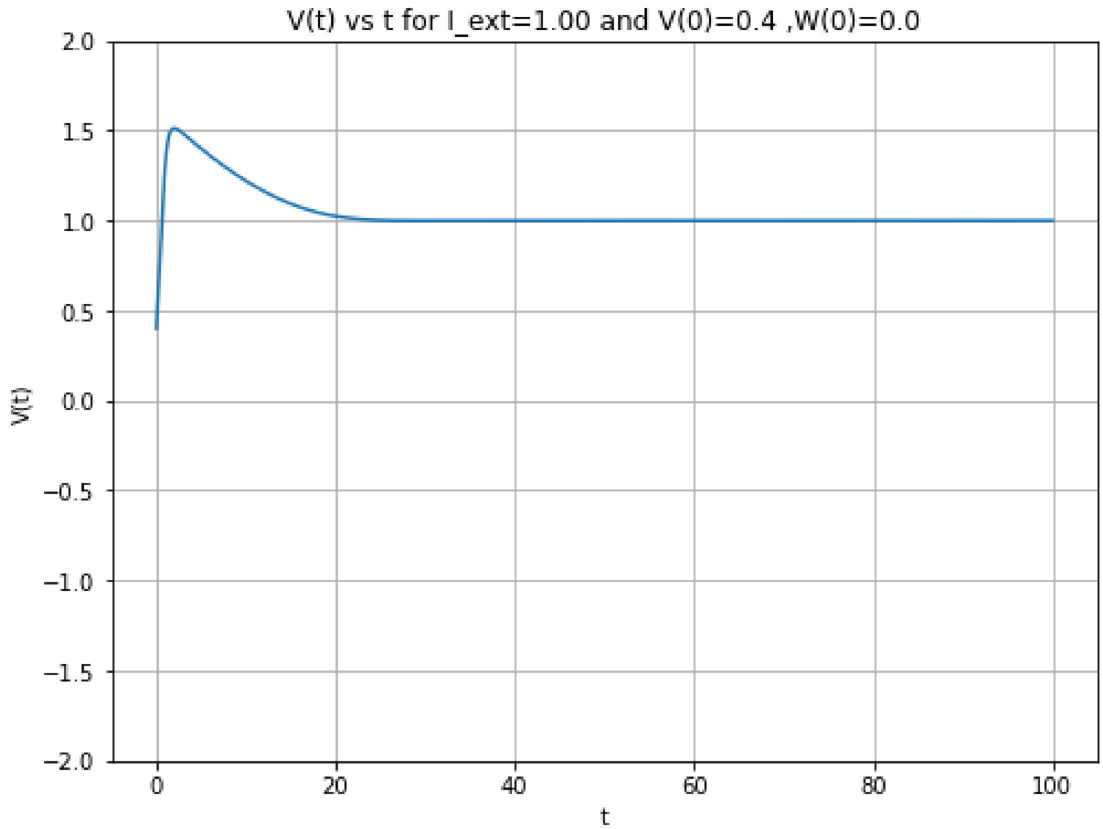


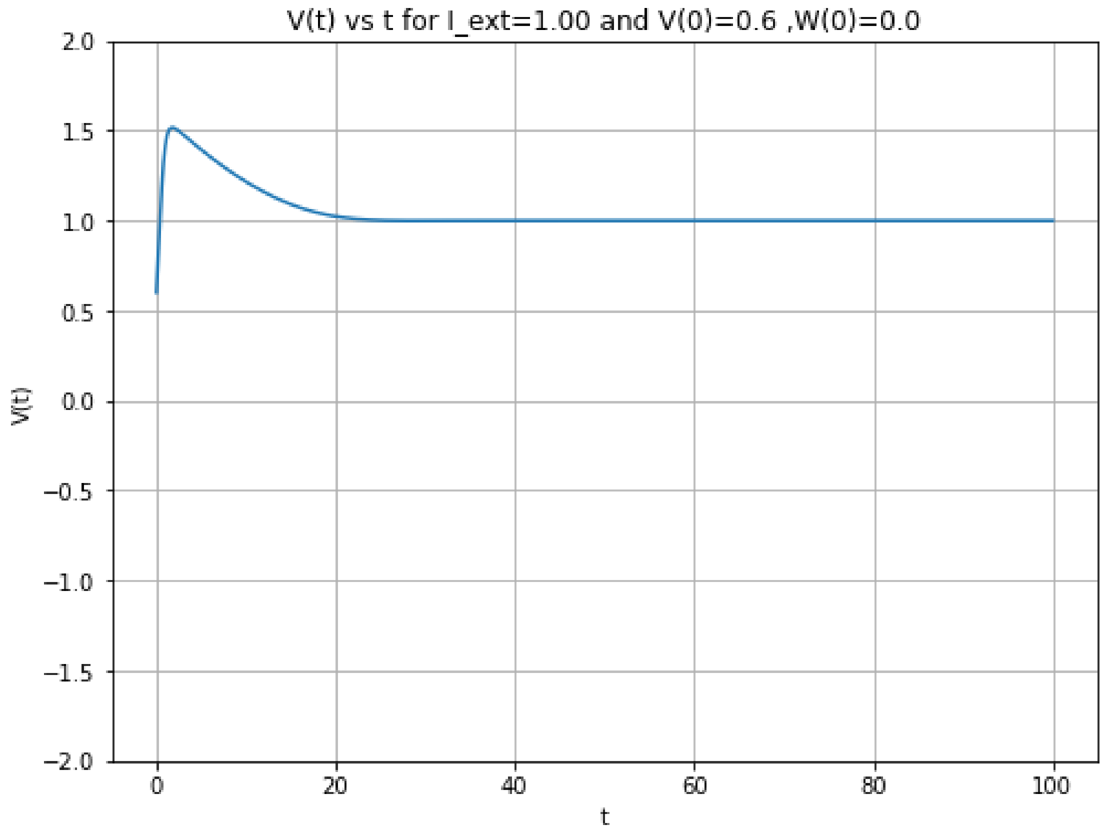
The below plot shows that:

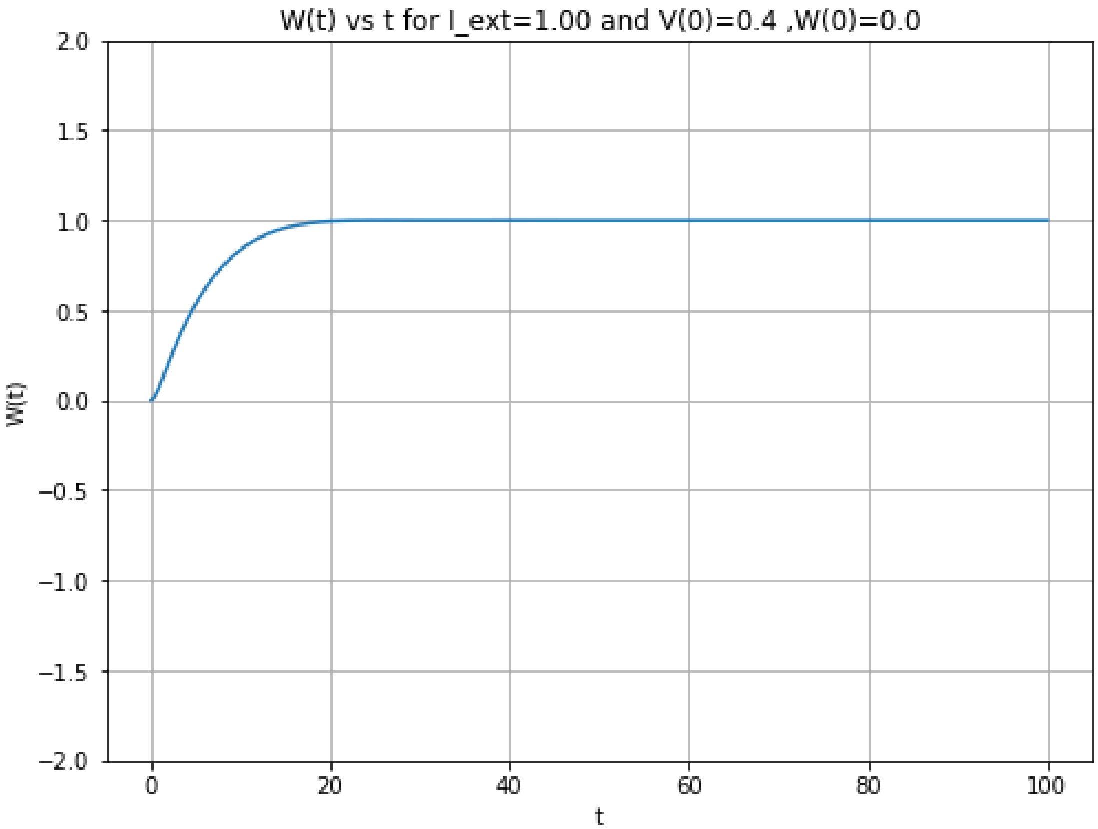
- A small perturbation from the fixed point is stable as after a small perturbation from the fixed point the point moves to the fixed point(green and pink trajectories)

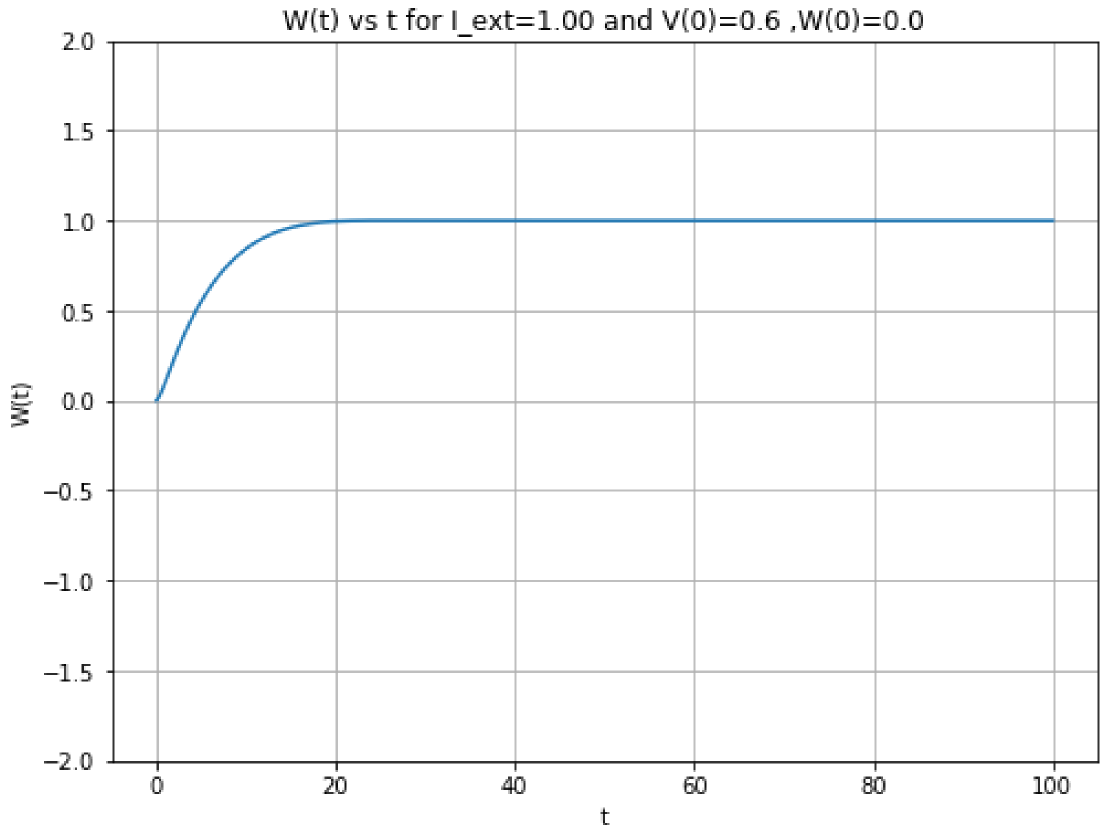


The plots below show the variation of $V(t)$ and $W(t)$ with time.
We have depolarization in the output.


V(t) vs t for I_ext=1.00 and V(0)=0.4 ,W(0)=0.0
V(t)
t
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
20
40
60
80
100


V(t) vs t for I_ext=1.00 and V(0)=0.6 ,W(0)=0.0
V(t)
t
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
20
40
60
80
100


W(t) vs t for I_ext=1.00 and V(0)=0.4 ,W(0)=0.0
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
W(t)
0
20
40
60
80
100
t

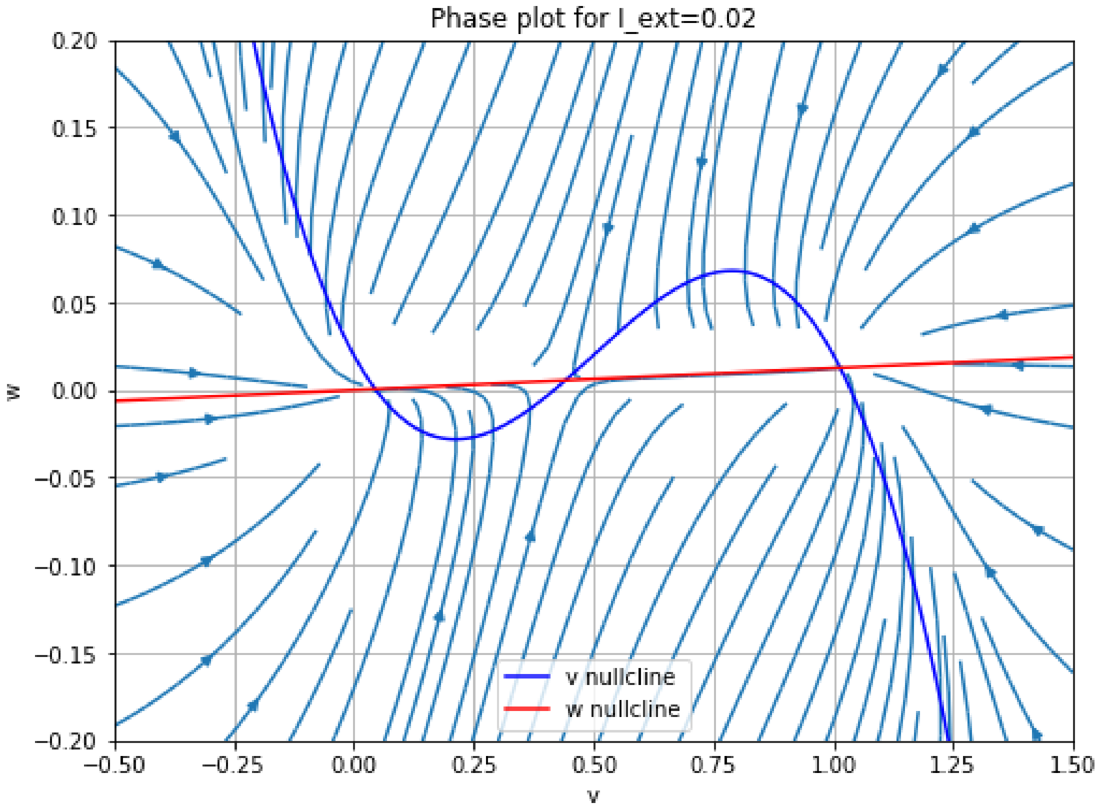


5 Case 4:Bistability (a=0.5,b=0.01,r=0.8)

We set $I_{\text{ext}} = 0.02$.

The plot below shows the phase trajectories as well as the null-clines.

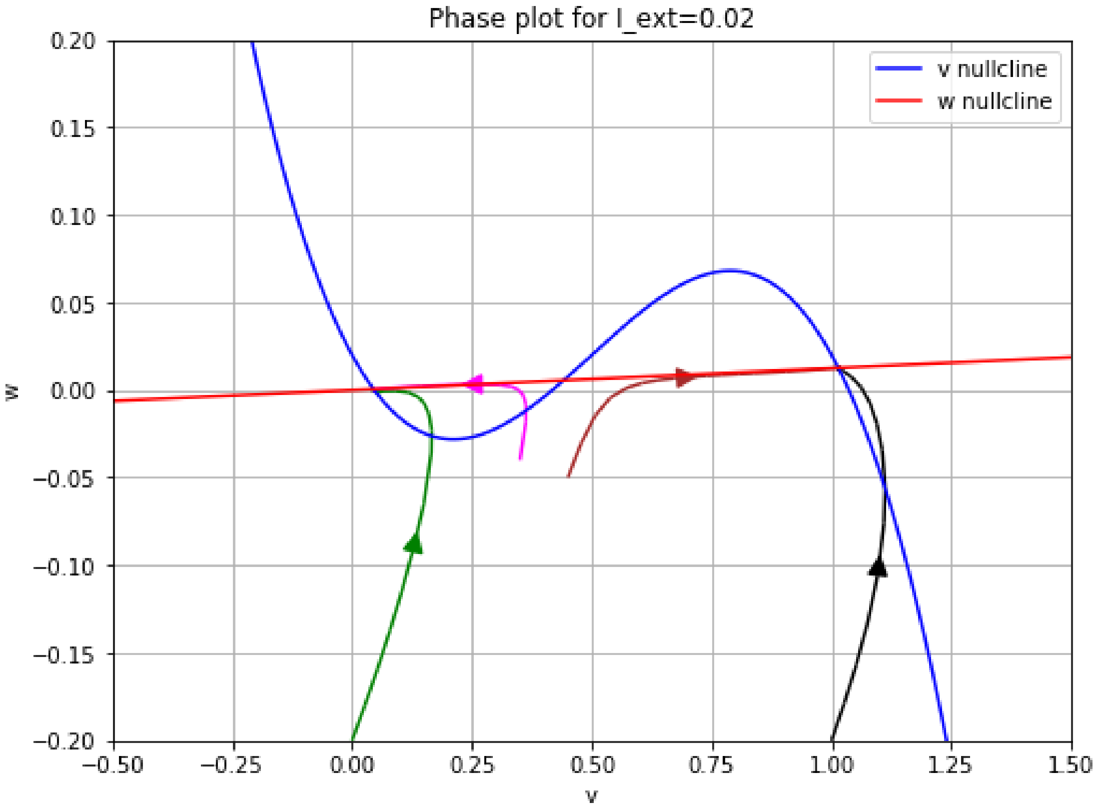
The w-nullcline cuts the v-nullcline three times.

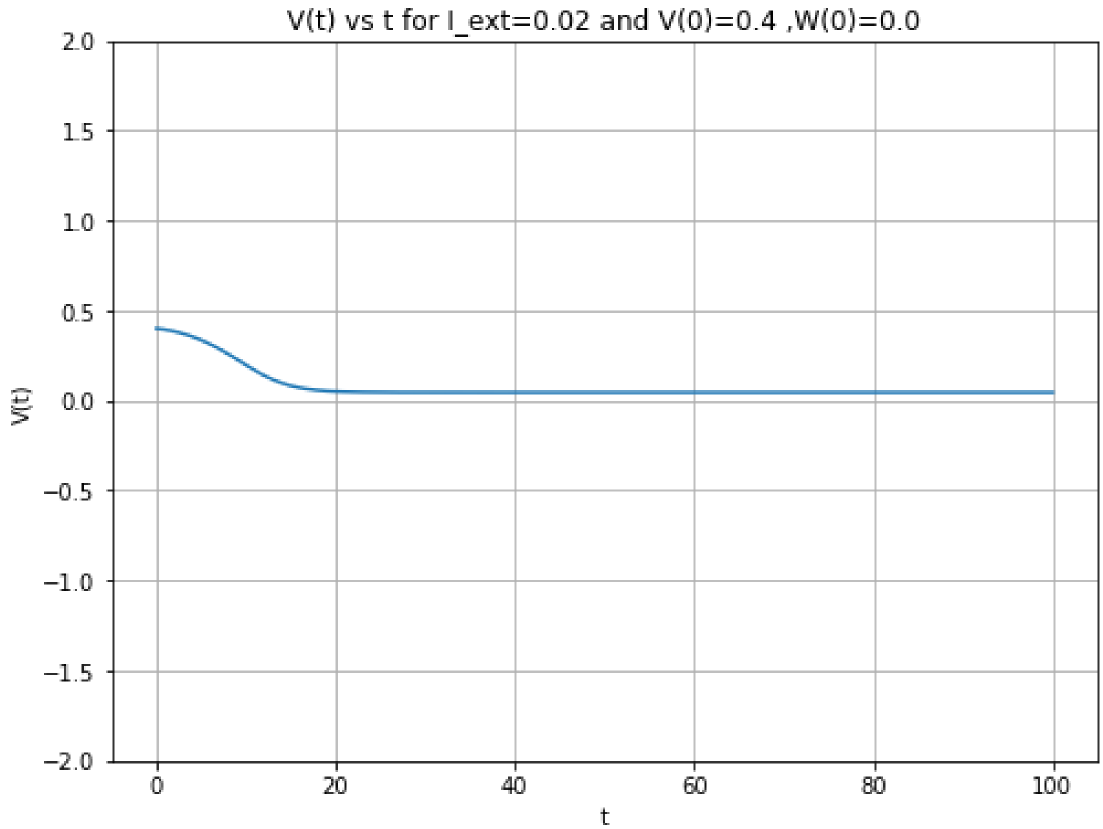


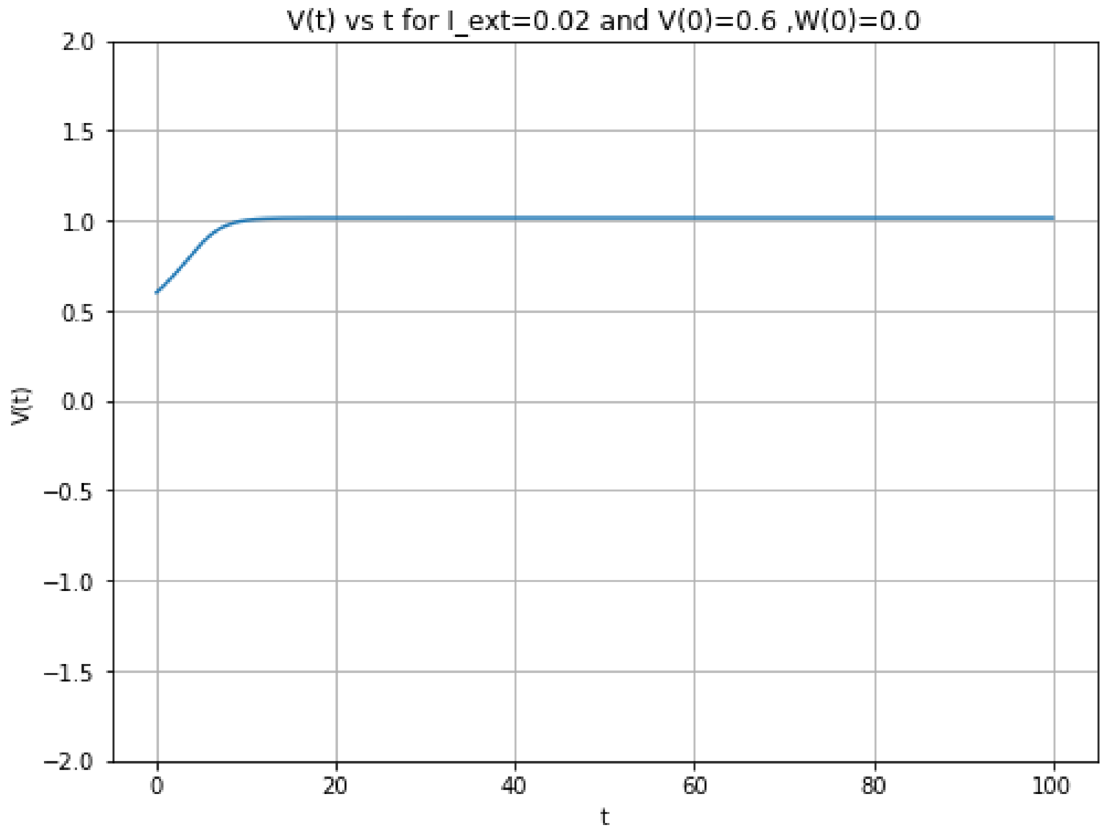
Let the points of intersection of the nullclines be p_1, p_2, p_3 from left to right.

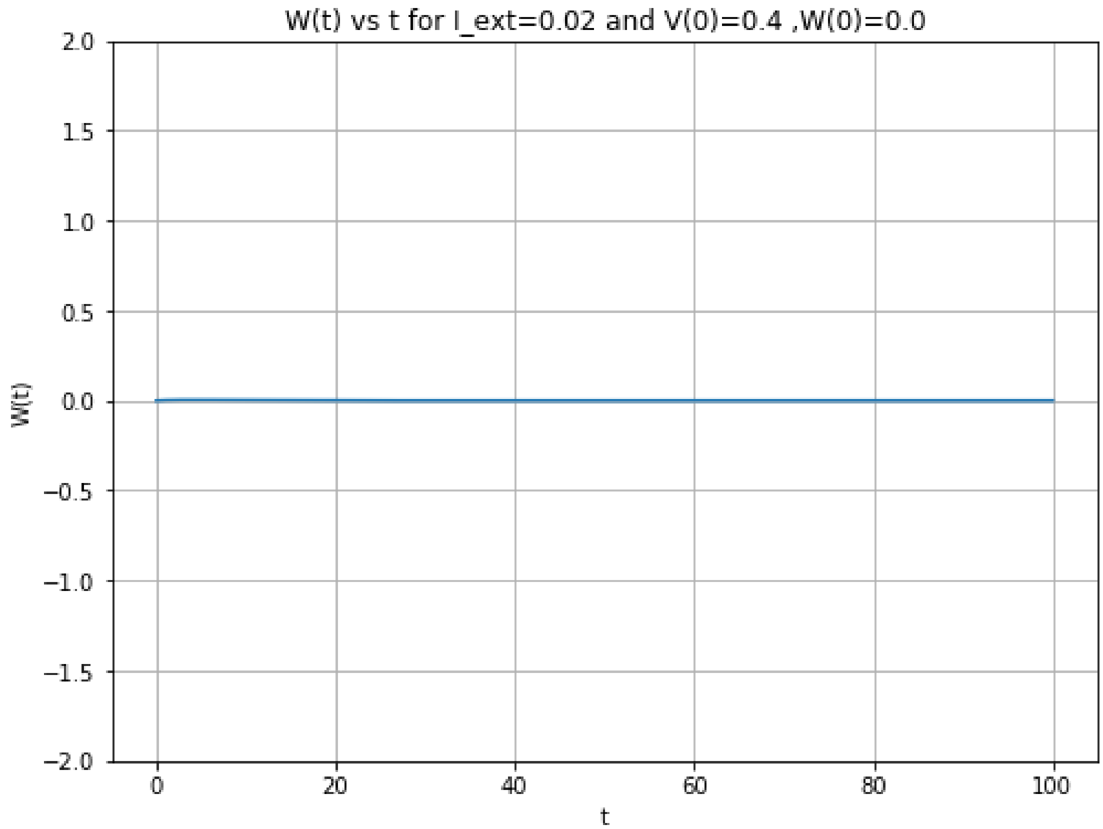
- We can see that p_1 is stable as any small perturbation around it brings it back to that point(green trajectory).
- We can see that p_2 is unstable as any small perturbation around it brings it to p_1 or p_3.(pink and brown trajectories)
- We can see that p_3 is stable as any small perturbation around it brings it back to that point.(black trajectory)

  Therefore there are two stable points and one unstable point.

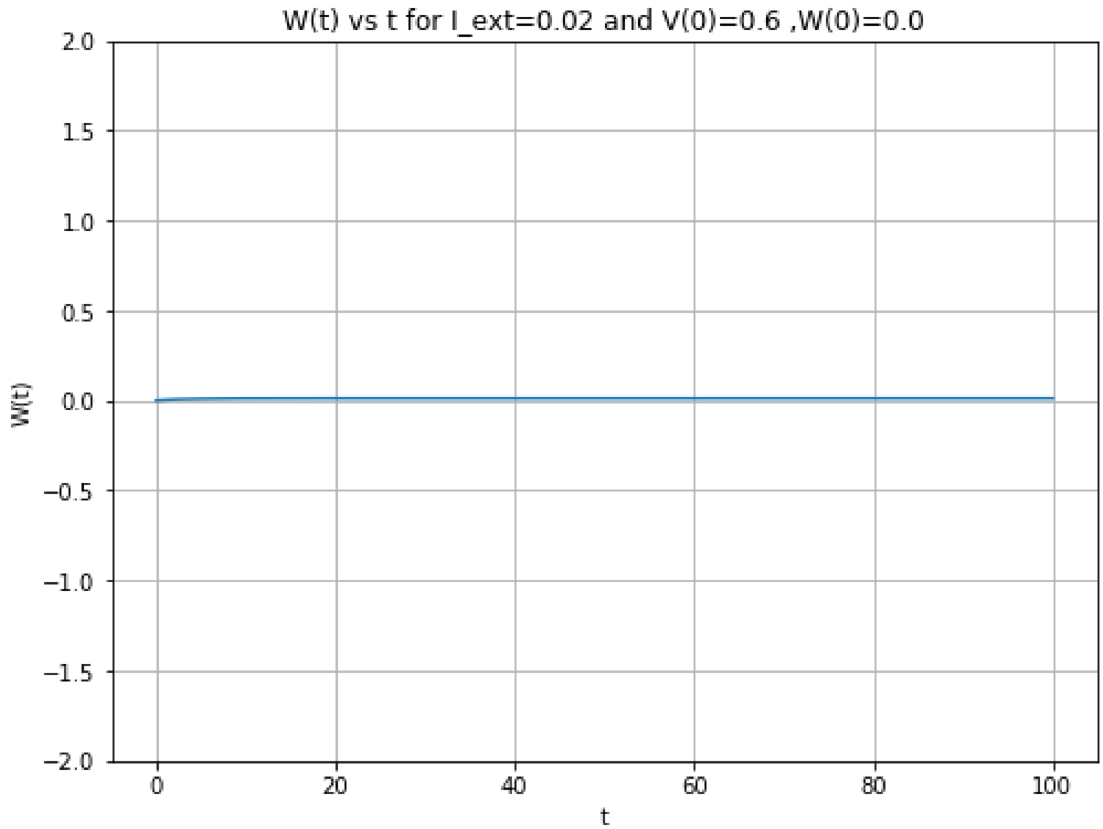

Phase plot for I_ext=0.02
v nullcline
w nullcline
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
w
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
1.25
1.50
v


V(t) vs t for I_ext=0.02 and V(0)=0.4 ,W(0)=0.0
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
V(t)
0
20
40
60
80
100
t


V(t) vs t for I_ext=0.02 and V(0)=0.6 ,W(0)=0.0
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
V(t)
0
20
40
60
80
100
t



W(t) vs t for I_ext=0.02 and V(0)=0.4 ,W(0)=0.0
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
W(t)
0
20
40
60
80
100
t


W(t) vs t for I_ext=0.02 and V(0)=0.6 ,W(0)=0.0
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
W(t)
0
20
40
60
80
100
t