

TIMESERIES: THE FINAL CHAPTER

11.9.2018

RECAP

- * power spectrum / psd
- * spectrogram
- * filtering
- * nyquist frequency

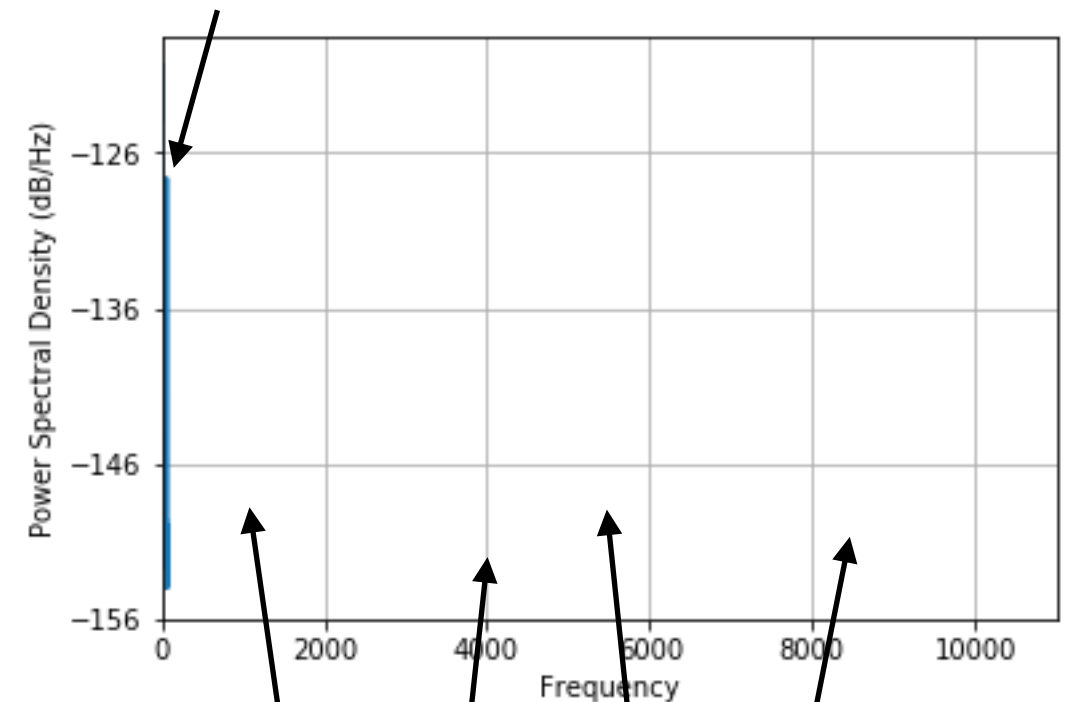
OVERSAMPLING

- * the EEG data from wednesday was originally collected at 22 kHz (22000 samples per second)
- * if we had used the original, the data file would have been 1.1 GB instead of 6.5 MB
- * and every analysis step that you ran would have taken at least 170x as long

OVERSAMPLING

- * would 22 kHz sampling be useful?
- * it would increase the Nyquist frequency from 64 Hz to 11000 Hz
- * but EEG can't see signals over ~100 Hz

all the
interesting
stuff



all the extras
you get from
22 kHz

OVERSAMPLING

- * the point: it's useful and good to *downsample* EEG signals from 22 kHz to 128 Hz
- * so how do we do that?

SUBSAMPLING

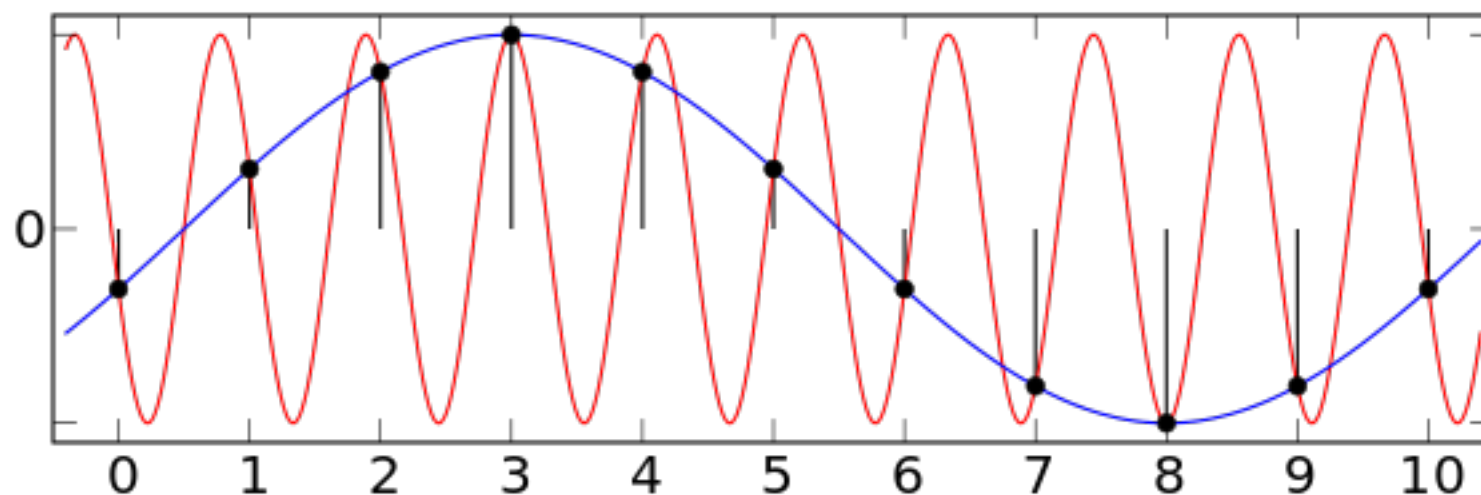
- * suppose (for simplicity) we have a 20 kHz signal and want to downsample it to 2 kHz

SUBSAMPLING

- * one idea: just take every 10th sample!
- * (this is called **subsampling**)
- * taking every 10th sample is ***LITERALLY THE WORST IDEA***
- * (let's see an example)

SUBSAMPLING

- * the weird thing about subsampling is that, instead of removing high frequencies, it *turns them into low frequencies*
- * this is called **aliasing**



ALIASING IN IMAGES

Original Image



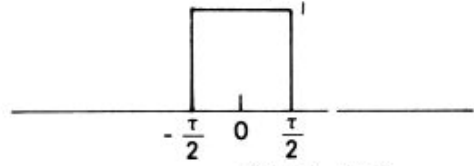
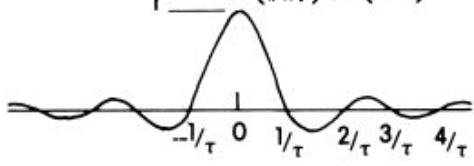
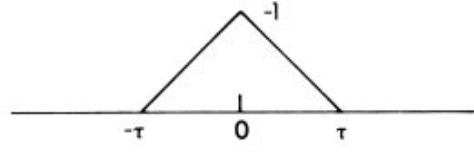
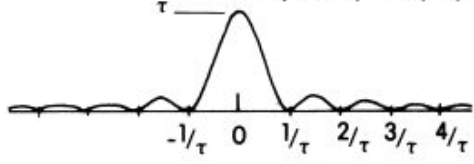
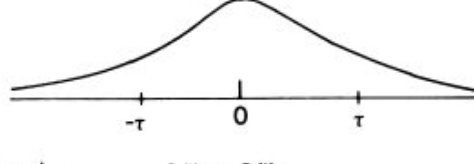
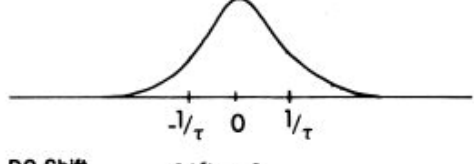
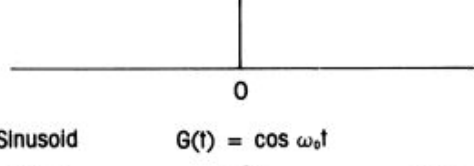
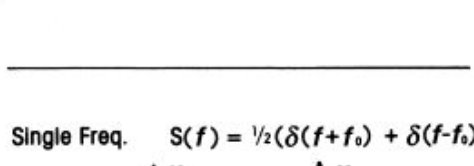
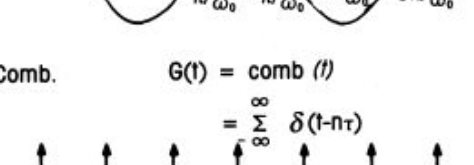
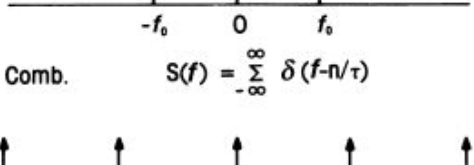


Subsampled



high frequency
pattern (bricks) is
aliased to low
frequency “moiré
pattern”

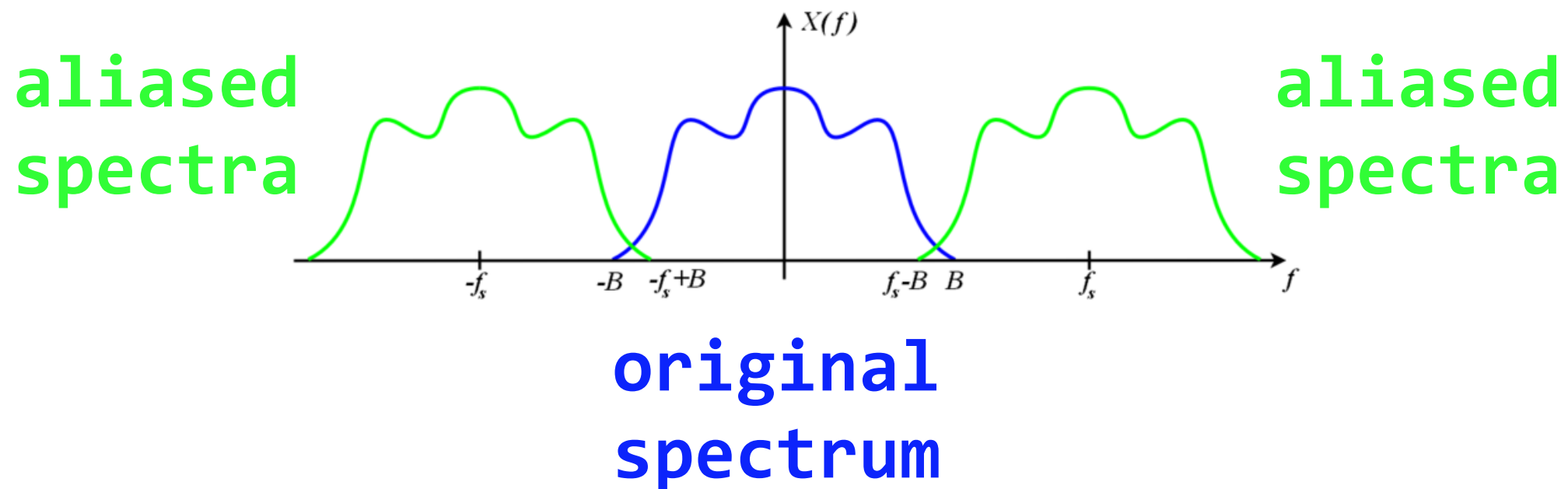
ALIASING

- * sampling is like multiplying your timeseries by a “comb” function
- * ... which is equivalent to convolving the fourier transform of your timeseries by a comb function

Time Function	Frequency Function
<p>Boxcar $G(t) = \begin{cases} 1, & t < \tau/2 \\ 0, & t > \tau/2 \end{cases}$</p> 	<p>Sinc $S(f) = \tau \operatorname{sinc}(f\tau) = (1/\pi f) \sin(\pi f \tau)$</p> 
<p>Triangle $G(t) = \begin{cases} 1- t /\tau, & t < \tau \\ 0, & t > \tau \end{cases}$</p> 	<p>Sinc² $S(f) = \tau \operatorname{sinc}^2(f\tau) = (1/\pi^2 f^2 \tau) \sin^2(\pi f \tau)$</p> 
<p>Gaussian $G(t) = e^{-1/2 t^2}$</p> 	<p>Gaussian $S(f) = \tau(2\pi)^{1/2} e^{-(\pi f \tau)^2}$</p> 
<p>Impulse $G(t) = \delta(t)$ $\begin{matrix} \infty \\ \uparrow \\ 0 \end{matrix}$ $t \neq 0$</p> 	<p>DC Shift $S(f) = 1$</p> 
<p>Sinusoid $G(t) = \cos \omega_0 t$</p> 	<p>Single Freq. $S(f) = 1/2(\delta(f+f_0) + \delta(f-f_0))$</p> 
<p>Comb. $G(t) = \operatorname{comb}(t) = \sum_{n=-\infty}^{\infty} \delta(t-n\tau)$</p> 	<p>Comb. $S(f) = \sum_{n=-\infty}^{\infty} \delta(f-n/\tau)$</p> 

ALIASING

- * which means that the fourier transform of the subsampled timeseries can have high frequencies “invading” lower frequencies



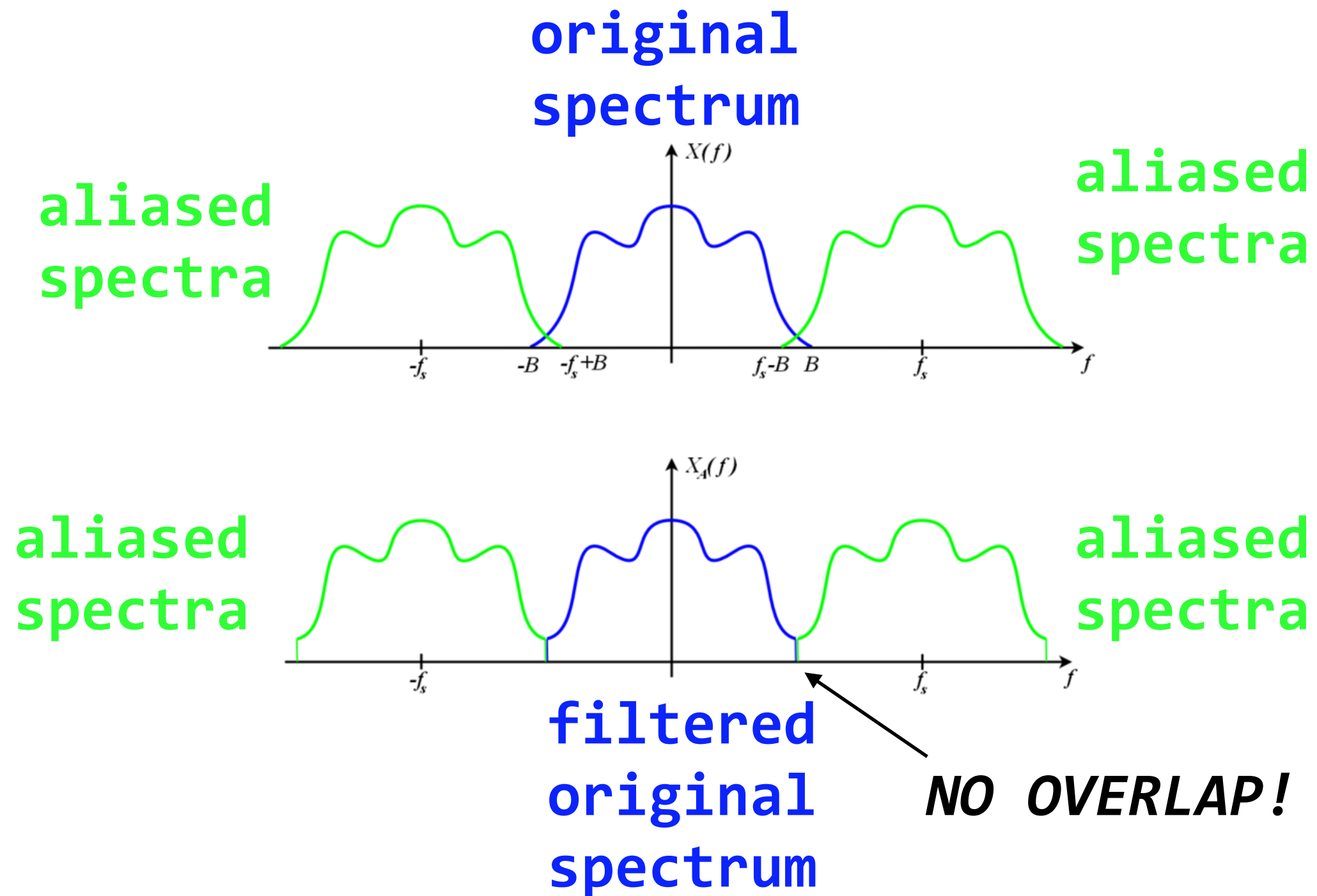
ANTI_ALIASING

- * how do we solve this?

ANTI_ALIASING

- * we can use an **antialiasing** filter
- * e.g.: the original signal is sampled at 20 kHz, we want to downsample to 2 kHz
- * the new 2 kHz shouldn't contain any frequencies above Nyquist (1 kHz)
- * so we **low-pass filter** the original signal at 1 kHz, and then subsample

ANTI_ALIASING



ANTI_ALIASING IN IMAGES

Original Image



Subsampled



Properly downsampled



ANTI_ALIASING

- * there are functions in **scipy.signal** for doing good downsampling/resampling
- * **signal.decimate** is great for downsampling
- * **signal.resample** can do downsampling or upsampling

END