

GAUSSIANS

10.19.2018

Department of Neuroscience PIZZA WITH PROFESSORS presents:

Student Organizations...

How can I (you) get involved?

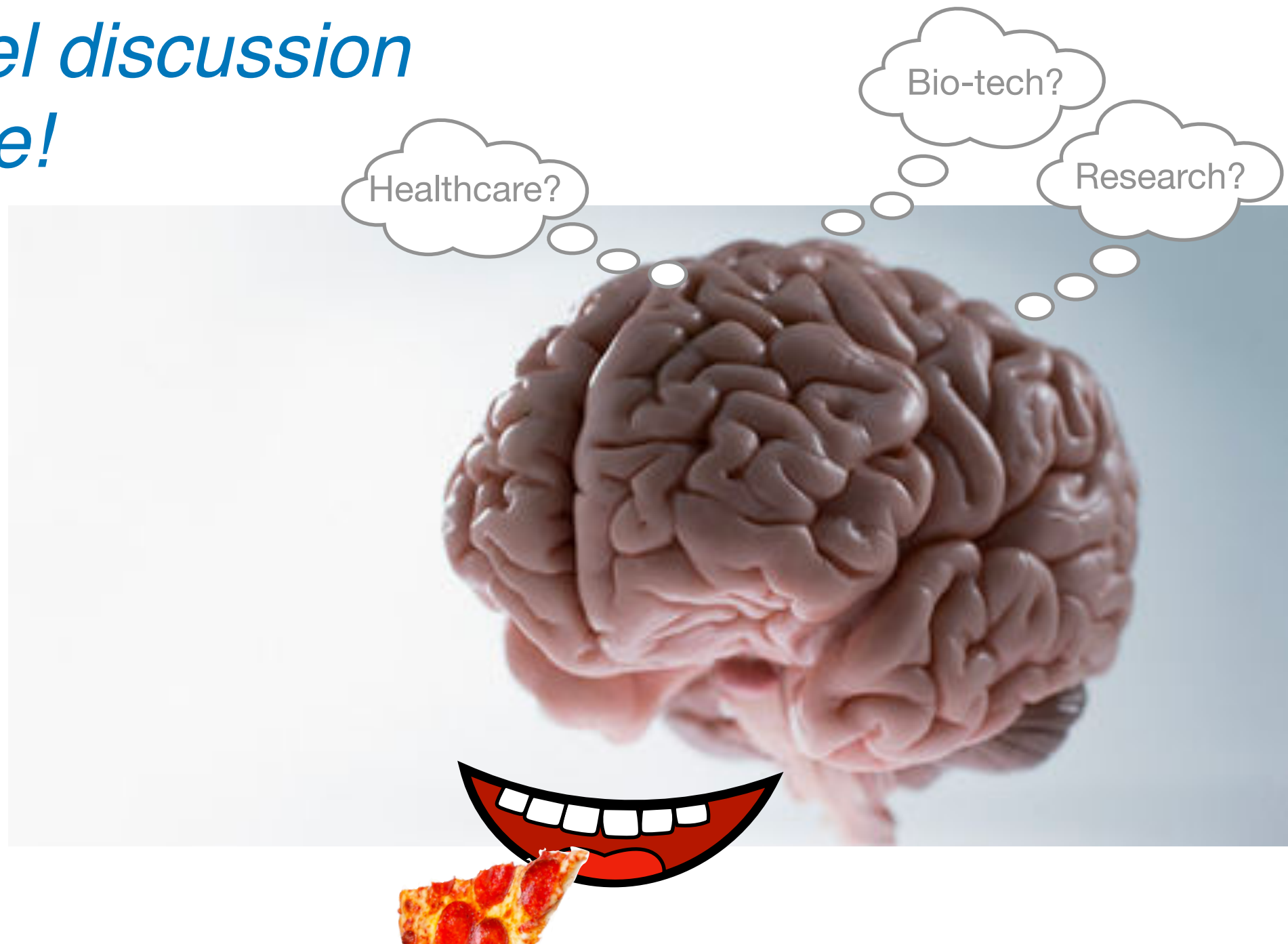
Pizza and panel discussion

All are welcome!

Meet friends!

Get involved!

Monday
October 22
5:00 p.m.
NHB 1.720



RECAP

- * bootstrap
- * confidence interval
- * standard error (standard deviation of bootstrapped statistic)
- * analytic standard error for binomial distribution
- * central limit theorem

CENTRAL LIMIT THEOREM

- * DEMO: let's take a bunch of random values
(from the same distribution)
- * and take their average
- * let's do this many times

CENTRAL LIMIT THEOREM

- * as the number of samples we take increases, the distribution of their averages converges to..
- * a Gaussian distribution!

CENTRAL LIMIT THEOREM

- * *and it doesn't matter what distribution you start with*
- * *THE DISTRIBUTION YOU START WITH CAN BE TOTALLY BIZARRE AND NOTHING LIKE A GAUSSIAN DISTRIBUTION*
- * **AND IT STILL WORKS!!!!!!**



CENTRAL LIMIT THEOREM

- * this is why gaussian distributions are everywhere (fMRI data, behavioral data, calcium imaging data, etc. etc. etc.), and we will learn about them

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Carl Gauss

A handwritten signature of Carl Gauss in cursive script.

* “Gaussian distribution”
aka
“Normal distribution”

* $X \sim N(\mu, \sigma^2)$: X is a normal RV with mean μ
and variance σ^2

$$PDF(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

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- * “Standard normal” distribution has mean $\mu=0$, and variance $\sigma^2=1$

$$PDF \propto e^{-x^2}$$

- * `np.random.randn` generates random variables from standard normal

GAUSSIAN TRANSFORMATIONS

- * suppose you have samples from $X \sim N(0,1)$
but you want samples from $Y \sim N(5,1)$
- * i.e. you want samples from a distribution
with $\mu=5$
- *

GAUSSIAN TRANSFORMATIONS

- * suppose you have samples from $X \sim N(0,1)$
but you want samples from $Y \sim N(5,1)$
- * i.e. you want samples from a distribution
with $\mu=5$
- * add the mean! $(X + b) \sim N(b,1)$

GAUSSIAN TRANSFORMATIONS

- * suppose you have samples from $X \sim N(0,1)$
but you want samples from $Y \sim N(0,5)$
 - * i.e. you want samples with variance 5
 - * multiply by $\sqrt{5}$!
- $$(a * X) \sim N(0, \sqrt{a})$$

ESTIMATING MEAN & VARIANCE

- * “sample mean” is just the mean of your sample
- * but “sample variance” (s^2) is *not* just the variance of your sample! (omg)
- * i.e. $s^2 \neq \sigma^2$

ESTIMATING MEAN & VARIANCE

* sample variance (s^2) is defined as:

$$s^2 = \frac{1}{n-1} \sum_i (x_i - \bar{x})^2$$

*

ESTIMATING MEAN & VARIANCE

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* which is normalized by $n-1$ instead of n

GAUSSIAN STANDARD ERROR

- * if we have samples from a Gaussian we can always use bootstrapping to find the standard error
- * but, like the binomial distribution, there is also an analytic solution for the standard error:

$$SE = \frac{s}{\sqrt{n}}$$

NEXT TIME

- * t-tests!
- * confidence intervals!
- * correlation!

END