

# STATISTICS WITH GAUSSIANS

10.22.2018

*Department of Neuroscience PIZZA WITH PROFESSORS presents:*

# ***Student Organizations...***

## ***How can I (you) get involved?***

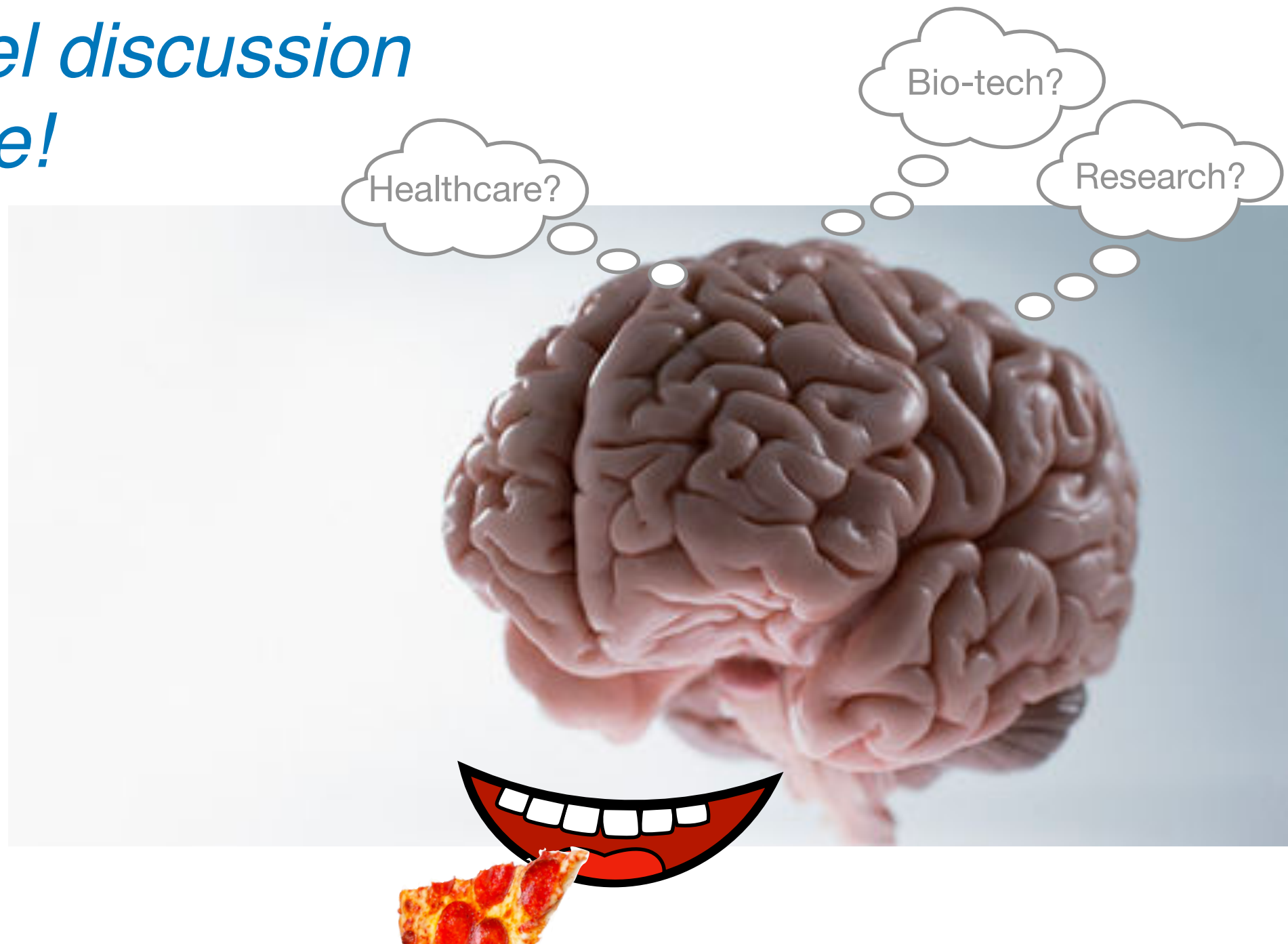
*Pizza and panel discussion*

*All are welcome!*

*Meet friends!*

*Get involved!*

**Monday**  
**October 22**  
**5:00 p.m.**  
**NHB 1.720**



# PROBLEM SET 3

- \* is posted on the github
- \* is due 11/2



# RECAP

- \* central limit theorem!
- \* gaussian distribution
- \* gaussian transformations
- \* estimating mean & variance from sample

# SCIPY.STATS.NORM

- \* `scipy.stats` has many useful functions
- \* `from scipy.stats import norm`
  - \* `norm.pdf` - the probability density function
  - \* `norm.cdf` - the cumulative density function
  - \* `norm.sf` - “survival function” (1 - cdf)
  - \* `norm.isf` - inverse survival function
- \* *all functions assume a standard normal unless you give them extra arguments!*

# GAUSSIAN STANDARD ERROR

- \* if we have samples from a Gaussian we can always use bootstrapping to find the standard error
- \* but, like the binomial distribution, there is also an analytic solution for the standard error:

$$SE = \frac{s}{\sqrt{n}}$$

# GAUSSIAN CONFIDENCE INTERVAL

- \* suppose we have  $X \sim N(0,1)$ , a standard normal RV
- \* what's its 95% confidence interval? (i.e. what is the interval  $[a,b]$  where 95% of the samples from  $X$  will fall between  $a$  and  $b$ ?)

# GAUSSIAN CONFIDENCE INTERVAL

- \* for  $[a,b]$  we want  $a$  to be the 2.5th %-ile  
and  $b$  to be the 97.5th %-ile
- \* so:



# GAUSSIAN CONFIDENCE INTERVAL

- \* for  $[a,b]$  we want  $a$  to be the 2.5th %-ile  
and  $b$  to be the 97.5th %-ile
- \* so:  $a=\text{norm.isf}(0.975)$ ,  $b=\text{norm.isf}(0.025)$

# GAUSSIAN CONFIDENCE INTERVAL

- \* for a standard normal, the 95% confidence interval is almost exactly  $[-1.96, 1.96]$

# GAUSSIAN CONFIDENCE INTERVAL

\* suppose we have a sample from a gaussian RV with sample mean  $\mu$  and sample variance  $s^2$ , what's the 95% confidence interval for the true mean?

\*

# GAUSSIAN CONFIDENCE INTERVAL

- \* suppose we have a sample from a gaussian RV with sample mean  $\mu$  and sample variance  $s^2$ , what's the 95% confidence interval for the true mean?
- \*  $[\mu - 1.96*SE, \mu + 1.96*SE]$

# GAUSSIAN Z-TEST

- \* recall the definition of the p-value: the probability of observing a result *at least as extreme* as your actual result
- \* to test whether a particular value is unlikely under a gaussian distribution, we can evaluate the CDF of that distribution!

# GAUSSIAN Z-TEST

- \* e.g. we know that a specific voxel recorded with fMRI has a background response of  $\mu=0.5$  and  $\sigma^2=1.7$
- \* we record a response of  $x=2.4$ . is this significantly higher than the background response?

# GAUSSIAN Z-TEST

- \* first we compute a “z-score” for  $x$ :
- \*  $z(x) = (x - \mu) / \sigma$
- \* then we can compute  $\Pr[X \geq x]$  using **norm.sf**

**END**