

# **BINOMIALS ETC.**

10.10.2018

# PROBLEM SET 2

- \* Due Friday 10/12 before class!
- \* Office hours:
  - \* (me) today 1:30-3pm
  - \* (Manu) thursday 4-5:30pm

# RECAP

- \* bernoulli distribution

- \*  $X \sim \text{Bernoulli}(q)$

- \* binomial distribution

- \*  $Y \sim \text{Binomial}(n, q)$

- \* p-values

- \* what is the probability (under the null hypothesis) that you would see something *at least* as extreme as what you did see?

# BINOMIAL DISTRIBUTION

- \* If you flip a weighted coin (where  $\text{Pr}(\text{heads}) = q$ )  $n$  times, what's the probability that you get  $k$  heads?



# BINOMIAL DISTRIBUTION

- \* Simpler question: if you flip a coin twice, what's the probability of heads both times?
- \* What's the probability of one heads and one tails?
- \* What's the probability of tails both times?

# BINOMIAL DISTRIBUTION

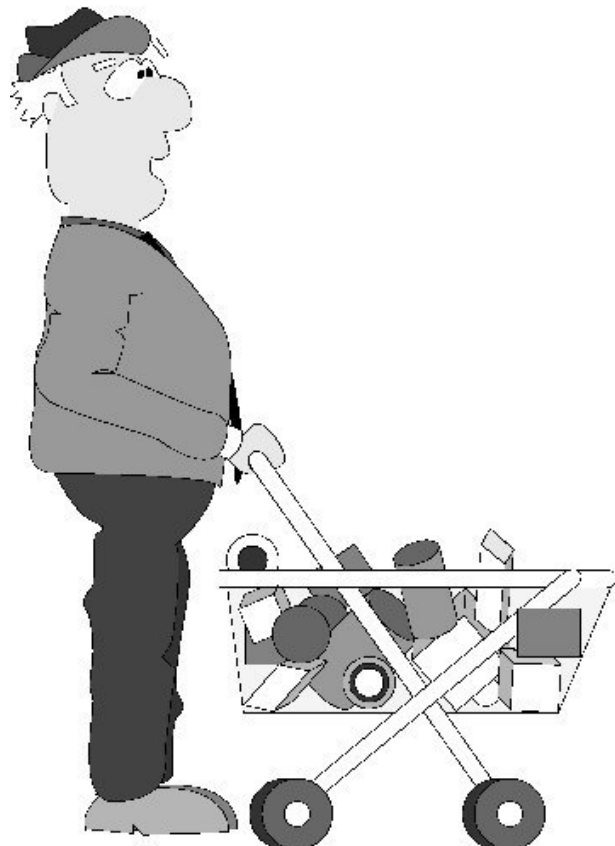
- \* The probability that two things both happen is the product of the probabilities of each thing happening
- \* (assuming that the two things are independent)
- \* This can be extended to an arbitrary number of things

# BINOMIAL DISTRIBUTION

- \* Back to the original question: if you flip a weighted coin (where  $\text{Pr}(\text{heads}) = q$ )  $n$  times, what's the probability that you get  $k$  heads?

# BINOMIAL COEFFICIENT

- \* n-choose-k (aka the binomial coefficient) is the number of ways to select k things out of n things



$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$



# BINOMIAL COEFFICIENT

- \* the n-choose-k function is available as:  
    >>> import scipy.misc  
    >>> scipy.misc.comb(n, k)

# BINOMIAL DISTRIBUTION

- \* Let  $p_1$  = the probability of flipping a coin  $k$  times and getting heads every time
- \* Let  $p_2$  = the probability of flipping a coin  $(n-k)$  times and getting tails every time
- \* Let  $c$  = the number of ways to choose  $k$  things out of  $n$  things

# BINOMIAL DISTRIBUTION

$$* \Pr(k \text{ heads in } n \text{ flips}) = p_1 * p_2 * c$$

# BINOMIAL TEST

- \* (As on monday) If we flipped a coin 100 times and got 63 heads, is it a fair coin?
- \* Formally: if the coin was fair ( $q=0.5$ ), what is the probability that we would see a result at least as extreme as 63 in 100 trials?

# BINOMIAL TEST

- \* How do we compute this probability?
- \* We can simulate, as we did on monday
- \* But, since we know the Binomial distribution we can just compute the probability for each  $k$  and sum!

# BINOMIAL TEST

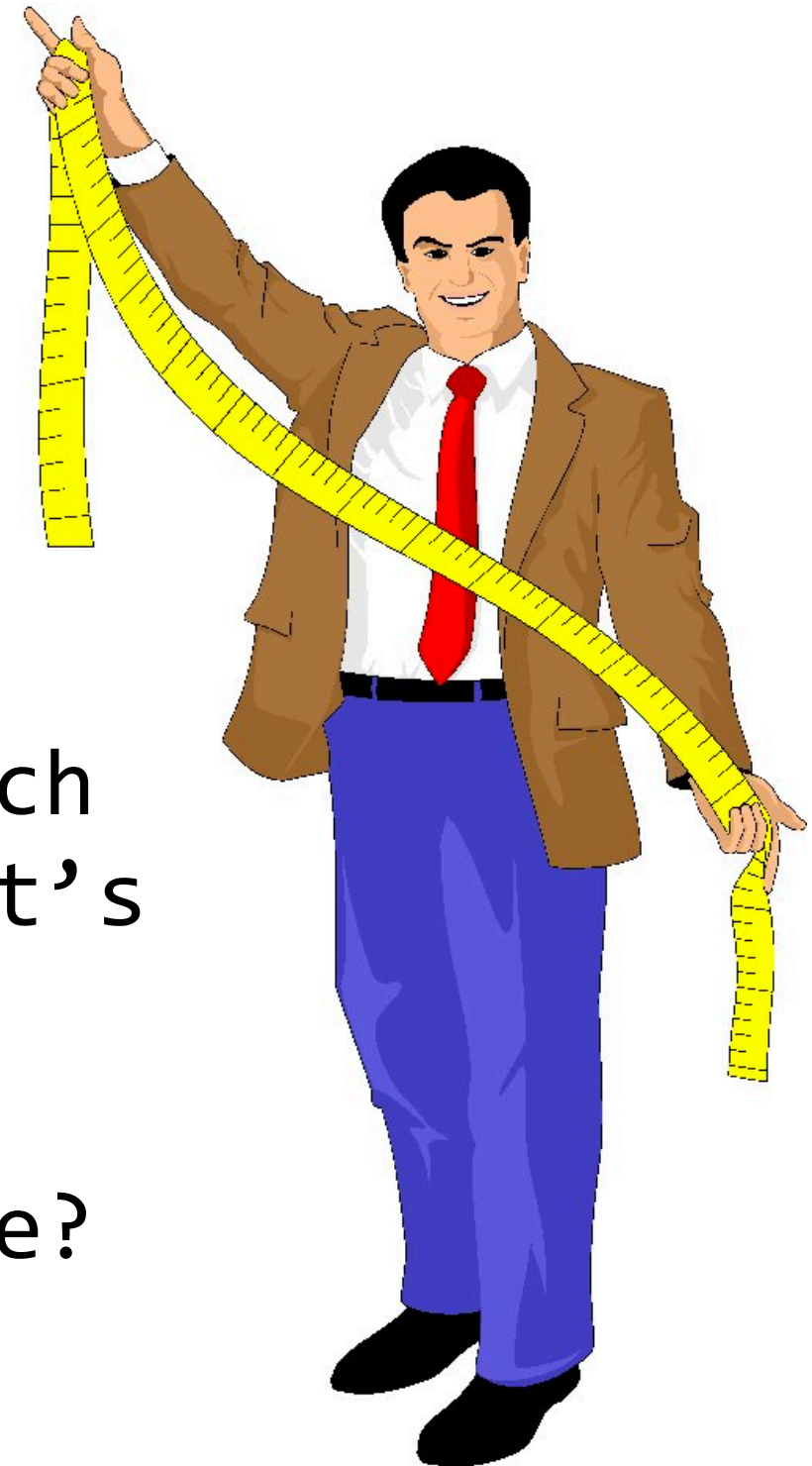
- \* (but in actuality we would always use `scipy.stats.binom_test`)

# MEAN

- \* as we've already seen when talking about numpy, the **mean** of a collection of numbers is the same as the average
- \* i.e.  $\text{mean}(\text{arr}) = \text{sum}(\text{arr}) / \text{len}(\text{arr})$

# VARIABILITY

- \* What if we want to measure how variable the data is around the mean?
- \* We could do compute how far each data point is from the mean—let's call this the deviation
- \* What will the mean deviation be?





# VARIABILITY

- \* The mean deviation is always zero!
- \* So obviously we can't just average deviations to get a sense of how variable the data is
- \* One thing we could do is take the mean *squared* deviation
- \* This is the *variance* of the data

# VARIABILITY

- \* Variance can also be obtained using `arr.var()` in numpy

# VARIABILITY

- \* Another useful number is the mean absolute deviation (or square root of the mean squared deviation)
- \* This is the *standard deviation*
- \* Standard deviation can be obtained using `arr.std()` in numpy

**END**