TEWA 1: Advanced Data Analysis

Lecture 09

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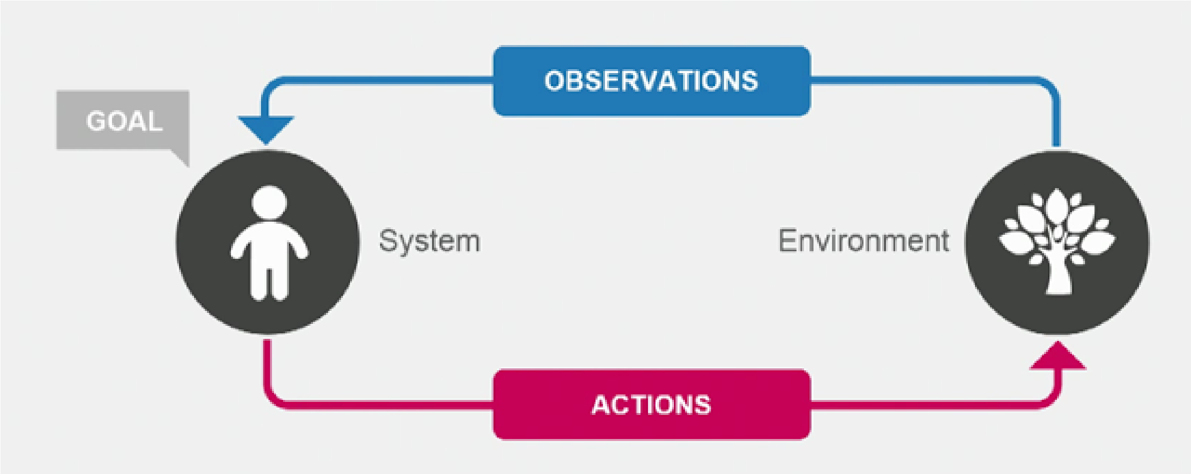
https://github.com/lei-zhang/tewa1_univie

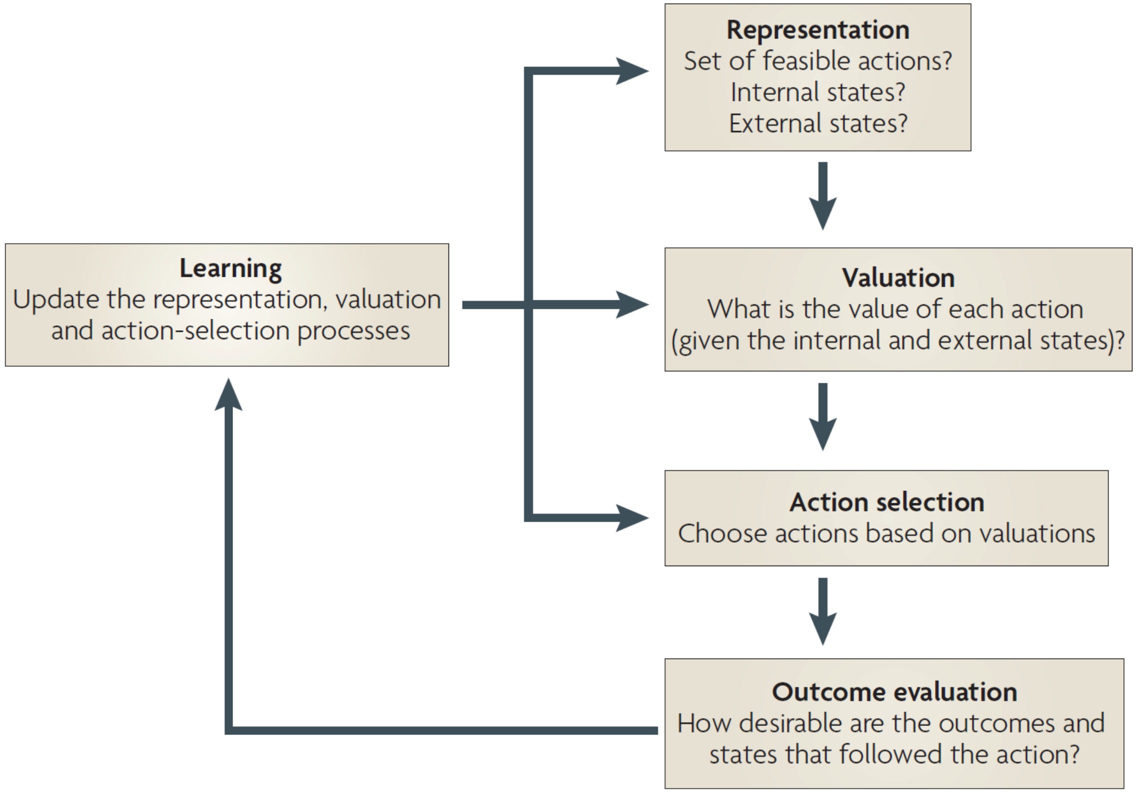
universität wien
Fakultät für Psychologie

Bayesian warm-up?

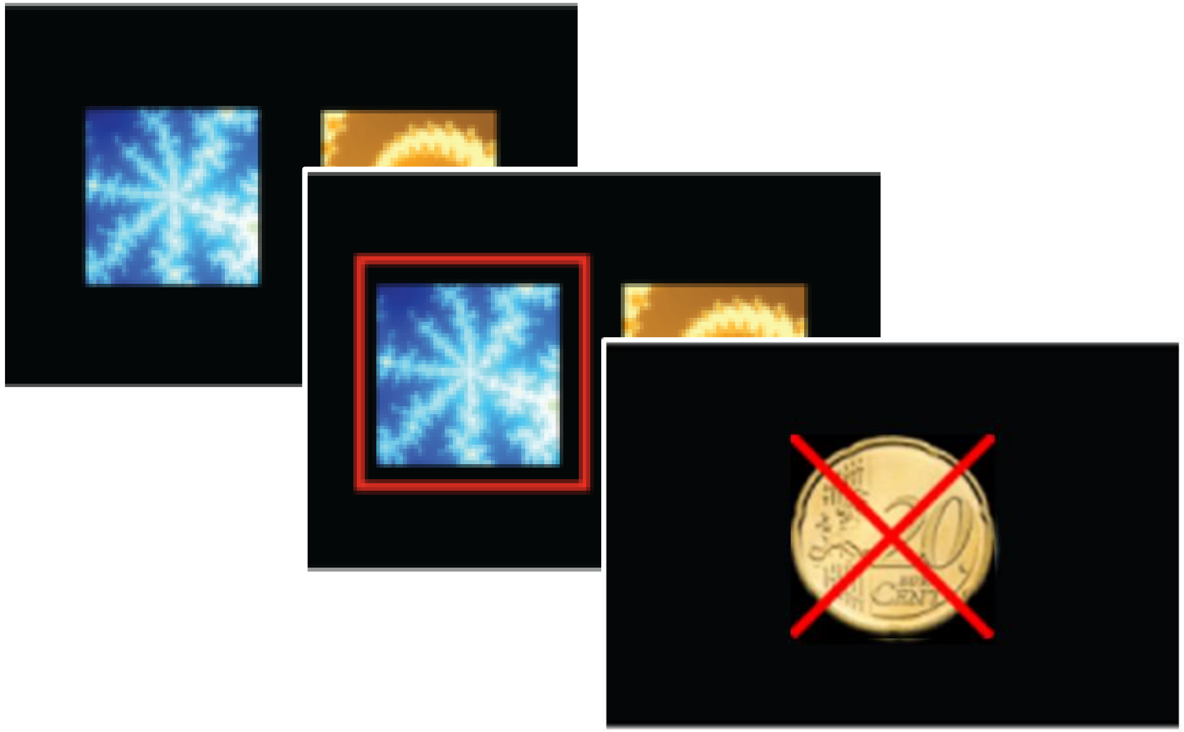
How prediction is shaped by learning?

cognitive model
statistics
computing



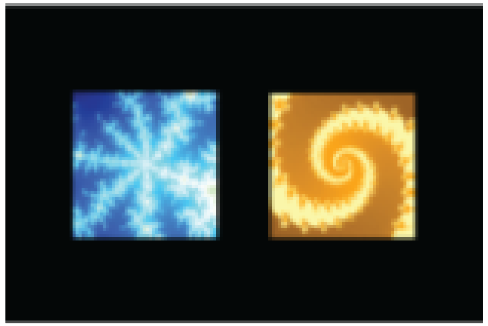


Sutton & Barto (1998); Rangel et al (2008); [Daw & Niv workshop 2015]

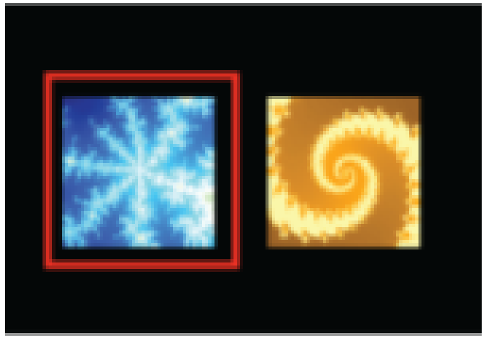




Winnings
$ 10

One simple experiment: two choice task



choice presentation



action selection

outcome

what do we know?

what can we measure?

what do we not know?

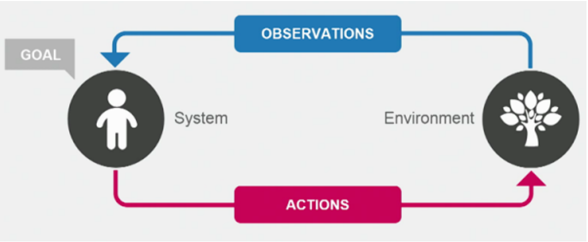
Data: choice & outcome

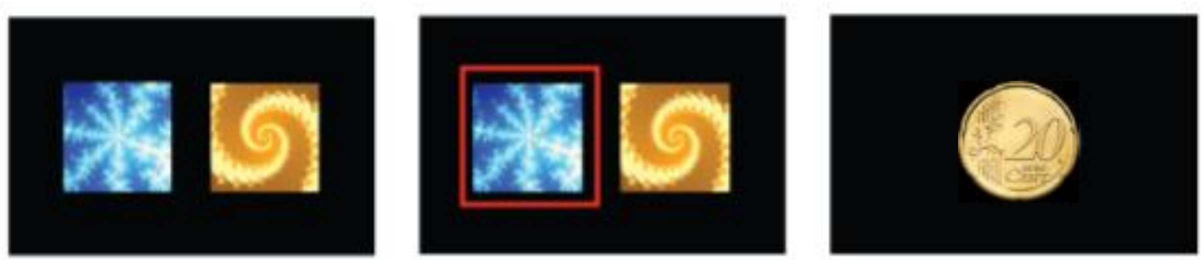
Summary stats: choice accuracy

Learning algorithm: RL update

p(choosing the better option)

Rescorla-Wagner Value Update





Cognitive Model
- cognitive process
- using internal variables and free parameters

Observation Model (Data Model)
- relate model to observed data
- has to account for noise

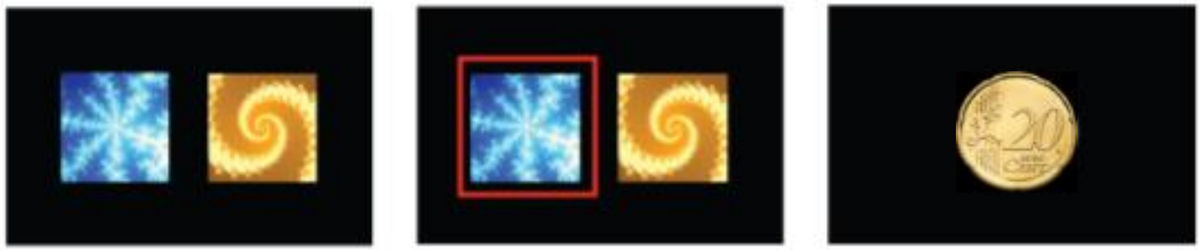
Rescorla & Wagner (1972)

Rescorla-Wagner (1972)

- The idea: error-driven learning
- Change in value is proportional to the difference between actual and predicted outcome

Robert A. Rescorla

Allan R. Wagner



Value update: $V_t = V_{t-1} + \alpha * \mathrm{PE}_{t-1}$

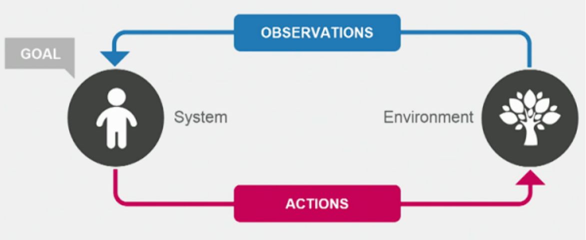
Prediction error: $\mathrm{PE}_{t-1} = R_{t-1} - V_{t-1}$

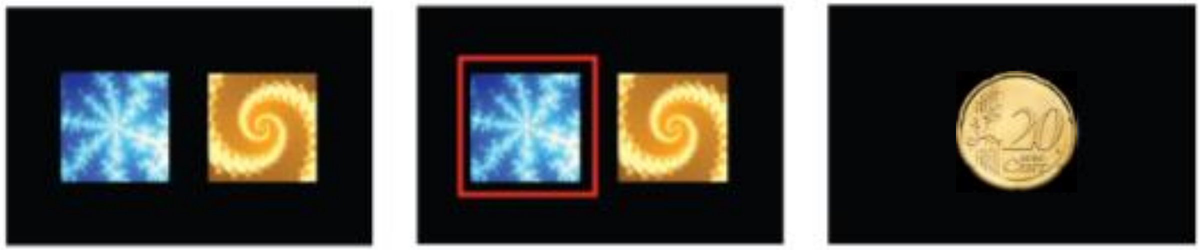
α - learning rate
PE - reward prediction error
V - value
R - reward

*Expectations on the next trial = the expectation on the current trial + learning rate * prediction error (reward – current expectation)*

Rescorla & Wagner (1972)

Rescorla-Wagner Value Update





Value update:

$$V_{t+1} = V_t + \alpha * PE_t$$

Prediction error:

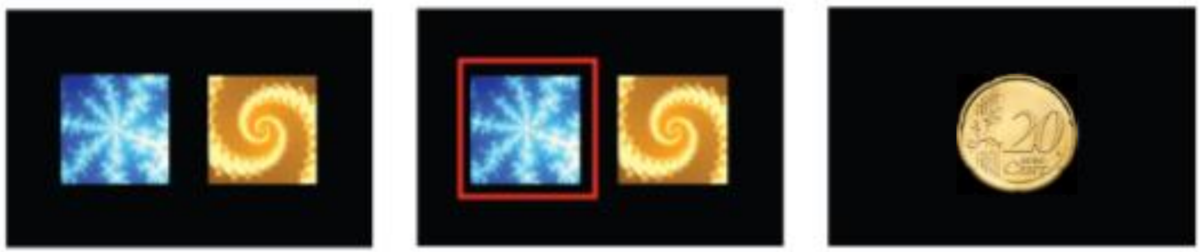
$$PE_t = R_t - V_t$$

choice rule:

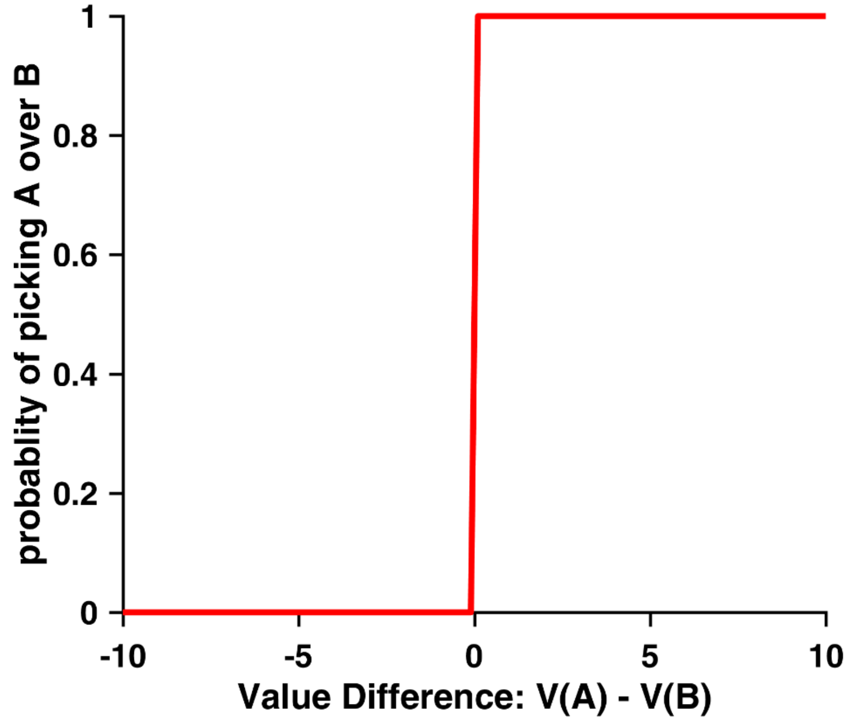
greedy / ε-greedy / softmax

Rescorla & Wagner (1972)

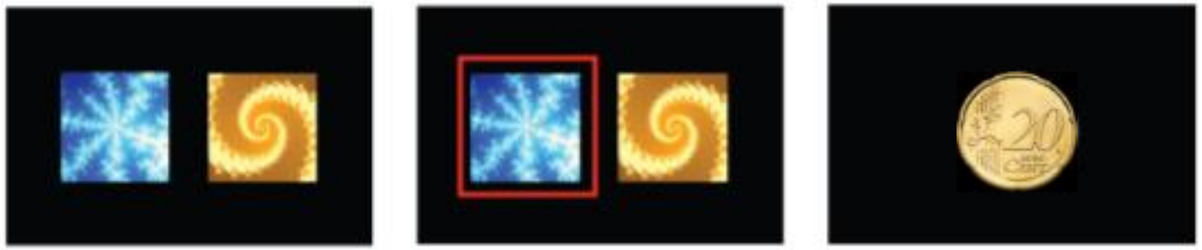
Choice rule: greedy



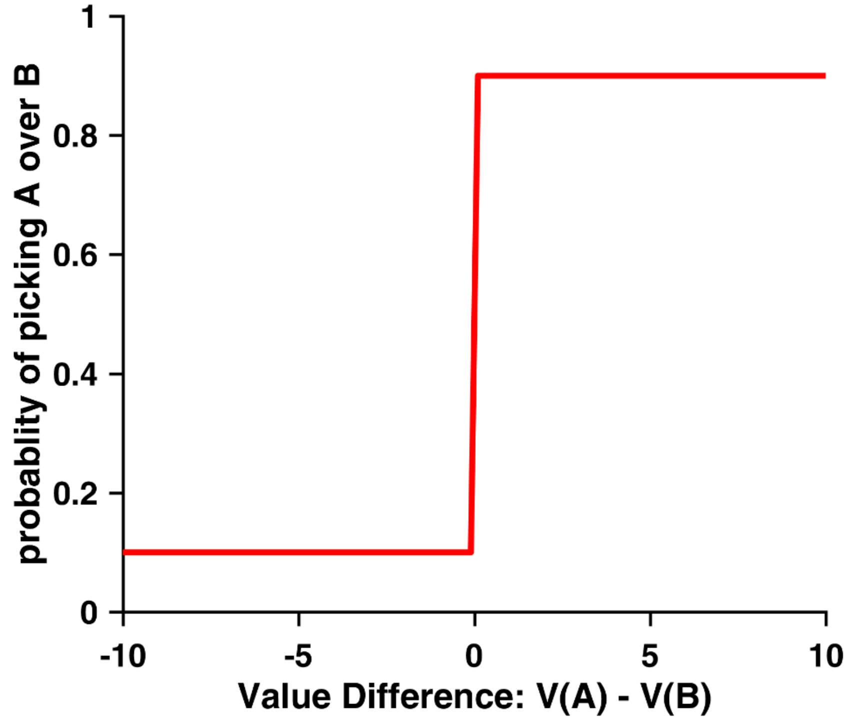
$$p(C=a)=\begin{cases}1, V(a)>V(b)\\0, V(a)<V(b)\end{cases}$$



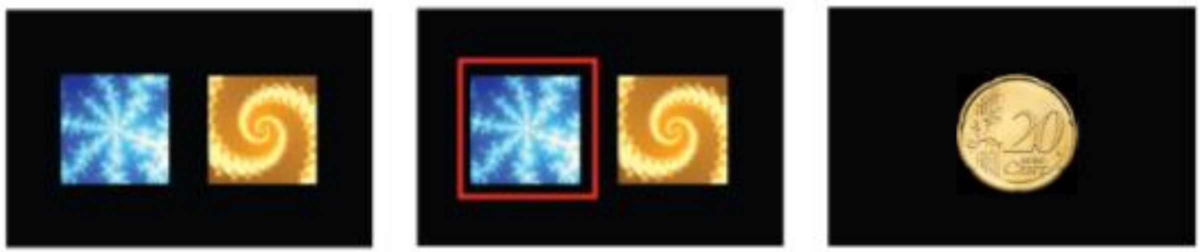
Choice rule: ε-greedy



$$p(C=a)=\begin{cases}1-\varepsilon, & V(a)>V(b)\\ \varepsilon, & V(a)<V(b)\end{cases}$$



Choice rule: softmax

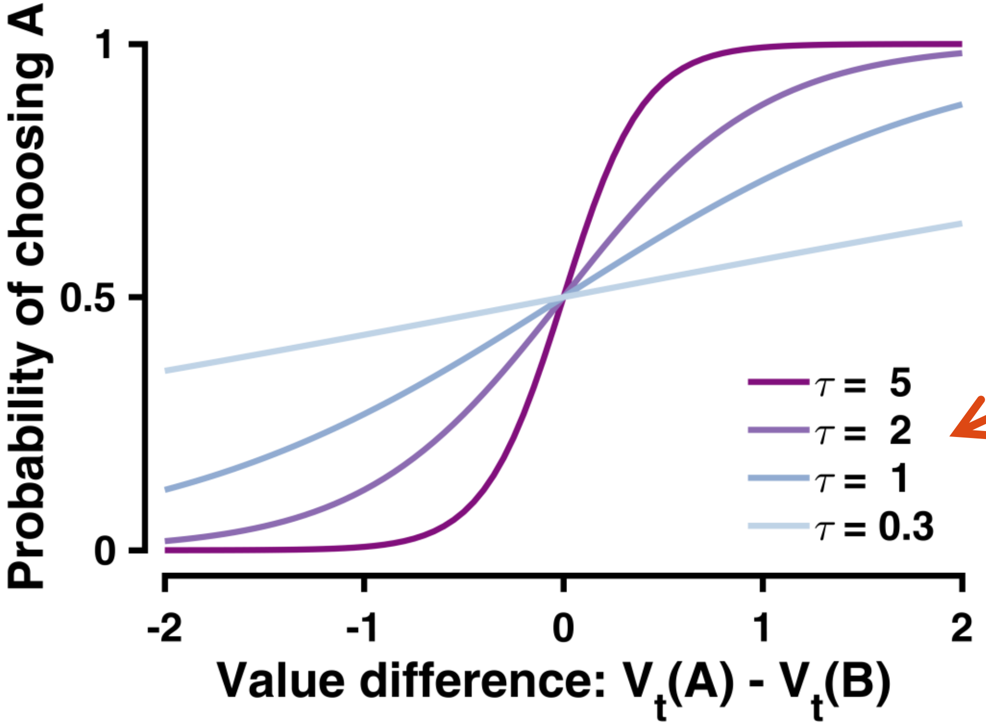


V()

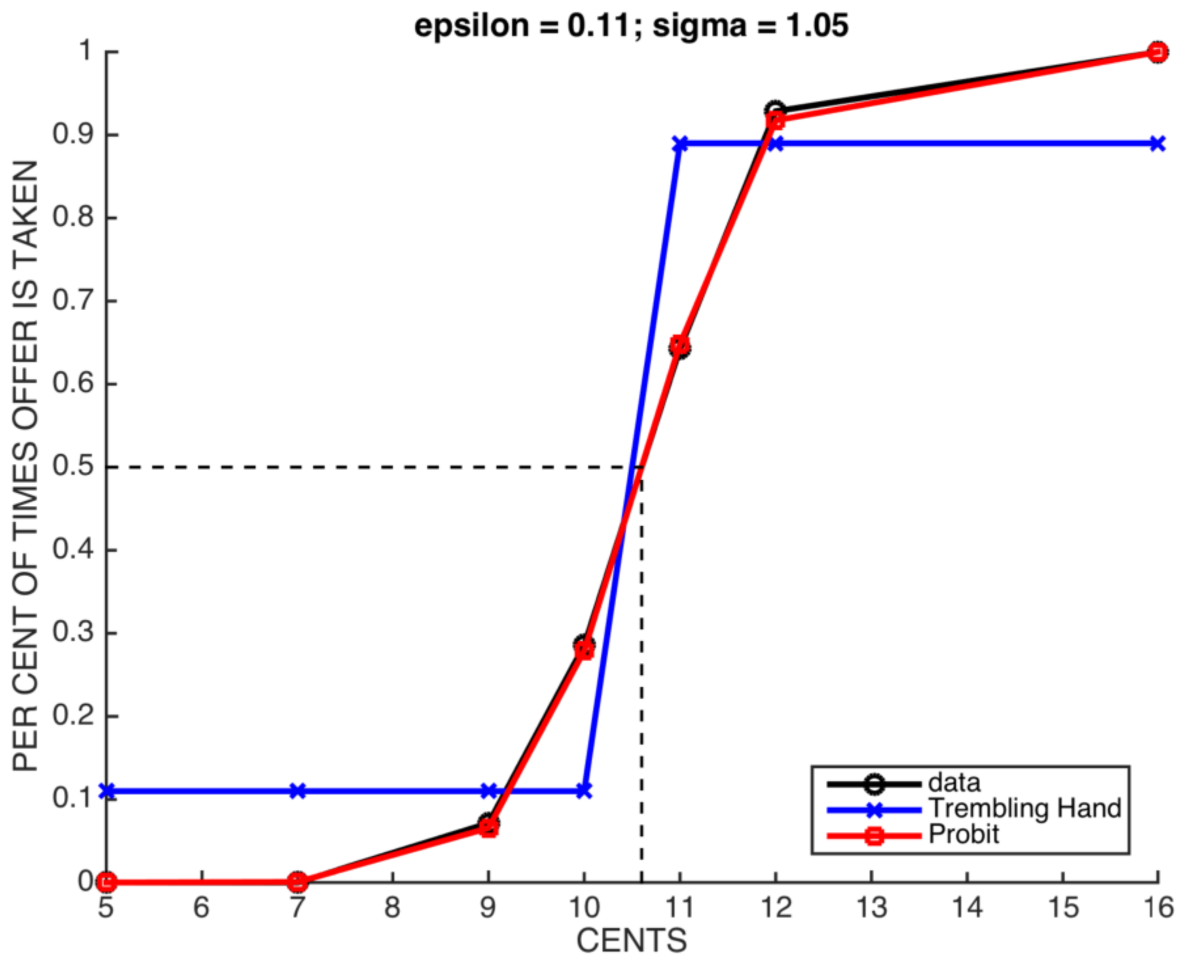
V()

$$\boxed{p_t(A) = \frac{e^{\tau * V_t(A)}}{e^{\tau * V_t(A)} + e^{\tau * V_t(B)}}}$$

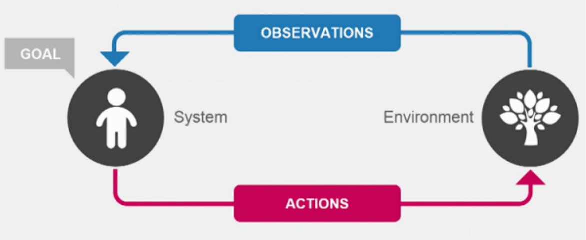
$$= \frac{1}{1 + e^{-\tau * (V_t(A) - V_t(B))}}$$

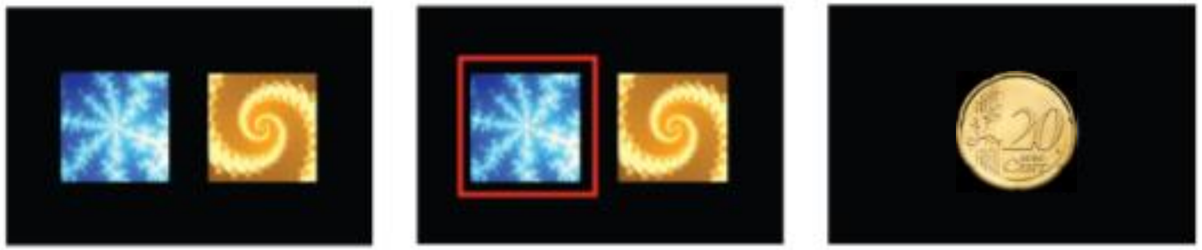


Choice rule: direct comparison



Rescorla-Wagner Value Update





Value update:

$$V_{t+1} = V_t + \alpha * PE_t$$

Prediction error:

$$PE_t = R_t - V_t$$

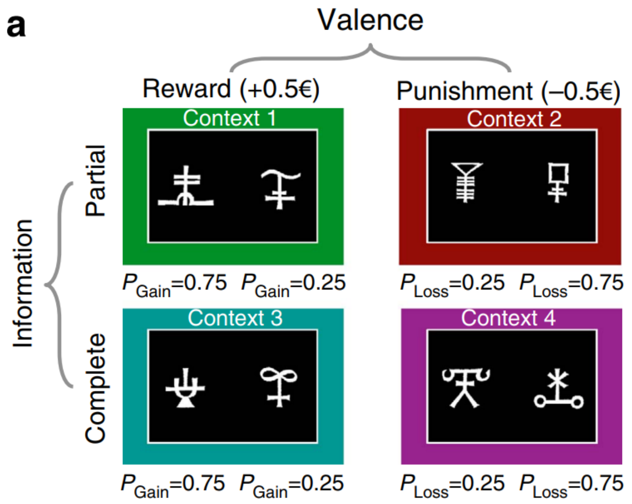
choice rule (sigmoid /softmax):

$$p(C=a) = \frac{1}{1+e^{\tau*(v(b)-v(a))}}$$

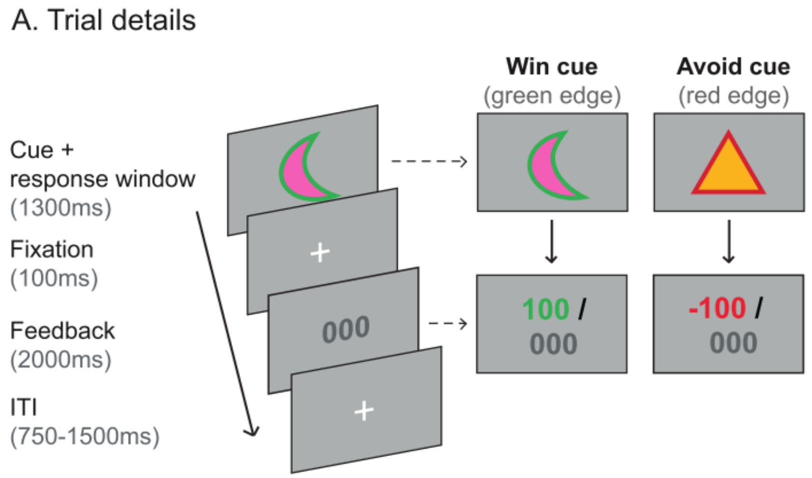
α - learning rate
PE - reward prediction error
V - value
R - reward
τ - softmax temperature

Rescorla & Wagner (1972)

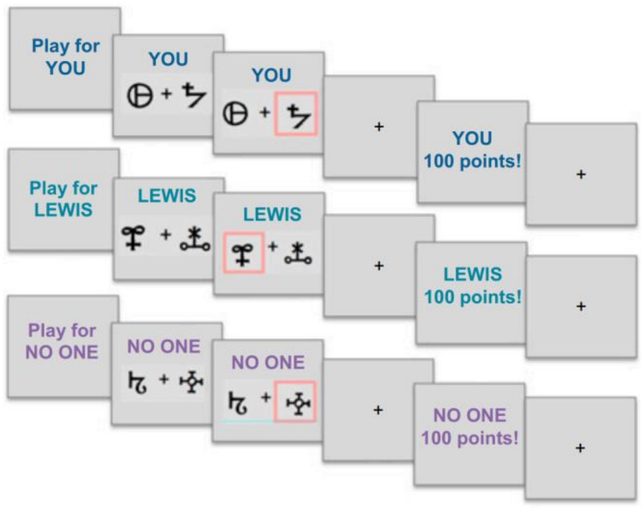
Generalizing RL framework



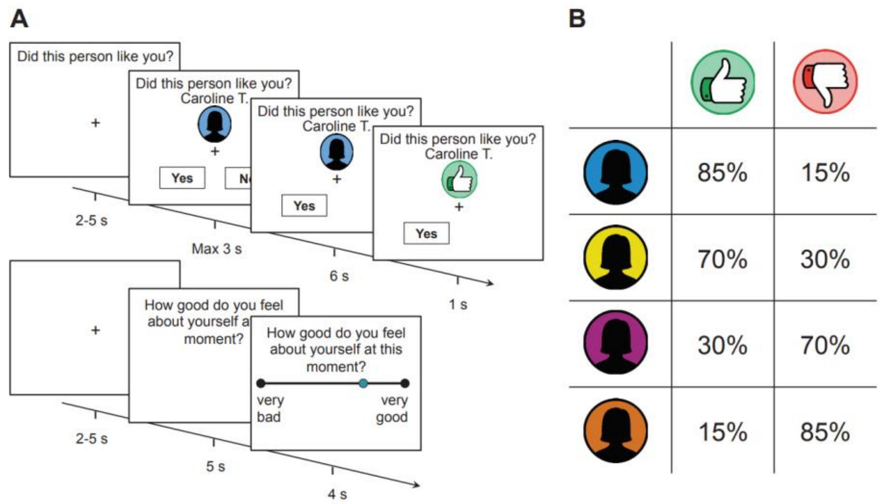
[Palminteri et al. (2015)]



[Swart et al. (2017)]

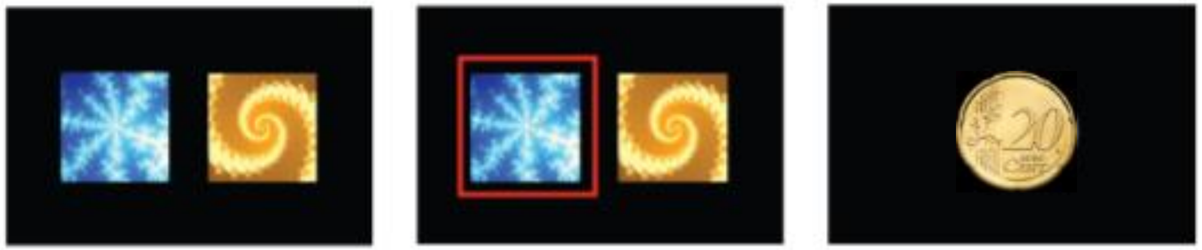


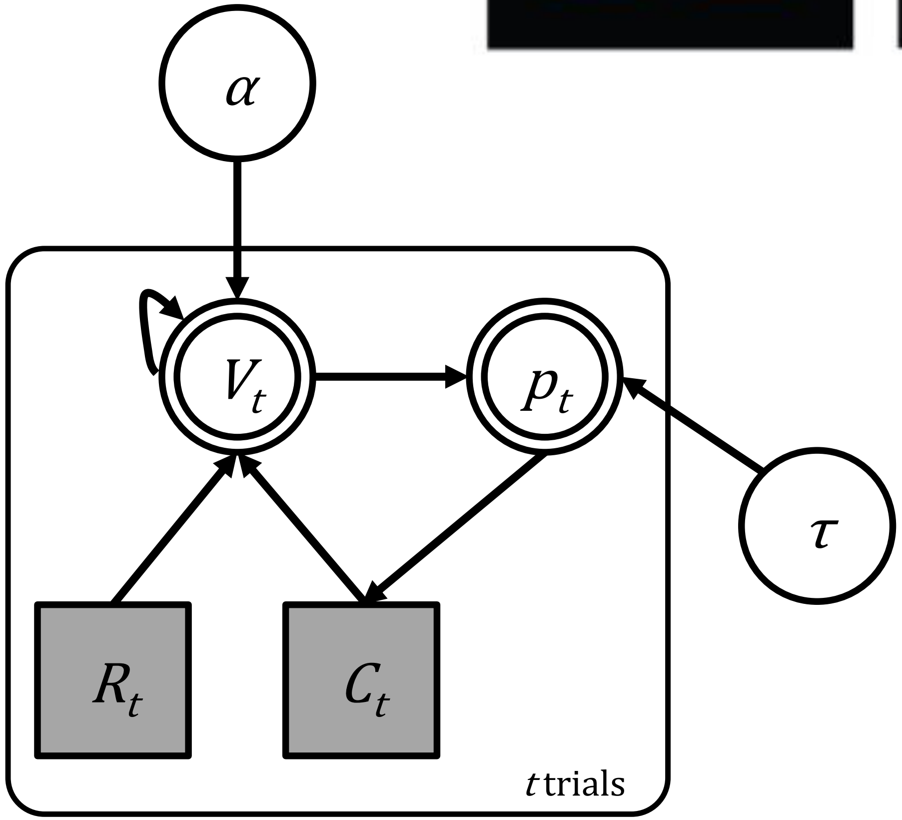
[Lockwood et al. (2016)]



[Will et al. (2017)]

RL – Implementation





$$\alpha \sim Uniform(0,1)$$

$$\tau \sim Uniform(0,3)$$

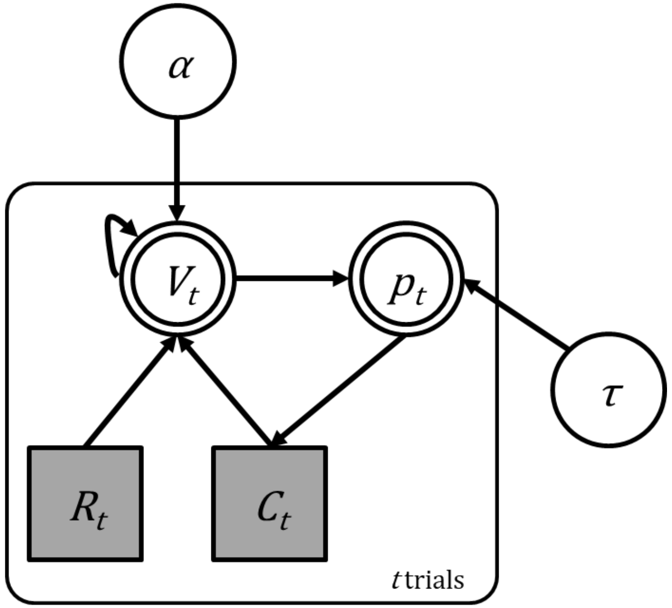
$$p_t(C=A) = \frac{1}{1+e^{\tau(V_t(B)-V_t(A))}}$$

$$V_{t+1}^{c} = V_t^C + \alpha\left(R_t - V_t^C\right)$$

deterministic variable

RL – Implementation

cognitive model
statistics
computing



$$\alpha \sim Uniform(0,1)$$

$$\tau \sim Uniform(0,3)$$

$$p_t(C=A) = \frac{1}{1+e^{\tau(V_t(B)-V_t(A))}}$$

$$V_{t+1}^{c} = V_t^C + \alpha(R_t - V_t^C)$$

```
transformed data {
  vector[2] initV;
  initV = rep_vector(0.0, 2);
}

model {
  vector[2] v[nTrials+1];
  real pe[nTrials];

  v[1] = initV;

  for (t in 1:nTrials) {
    choice[t] ~ categorical_logit( tau * v[t] );

    pe[t] = reward[t] - v[t,choice[t]];

    v[t+1] = v[t];
    v[t+1, choice[t]] = v[t, choice[t]] + lr * pe[t];
  }
}
```

RL – Implementation

cognitive model
statistics
computing

```
model {
  vector[2] v[nTrials+1];
  real pe[nTrials];

  v[1] = initV;

  for (t in 1:nTrials) {
    choice[t] ~ categorical_logit( tau * v[t] );
    pe[t] = reward[t] - v[t,choice[t]];

    v[t+1] = v[t];
    v[t+1, choice[t]] = v[t, choice[t]] + lr * pe[t];
  }
}
```

```
model {
  vector[2] v;
  real pe;

  v = initV;

  for (t in 1:nTrials) {
    choice[t] ~ categorical_logit( tau * v );
    pe = reward[t] - v[choice[t]];

    v[choice[t]] = v[choice[t]] + lr * pe;
  }
}
```

RL – Fitting with Stan

cognitive model
statistics
computing

```
…/06.reinforcement_learning/_scripts/reinforcement_learning_single_parm_main.R
```

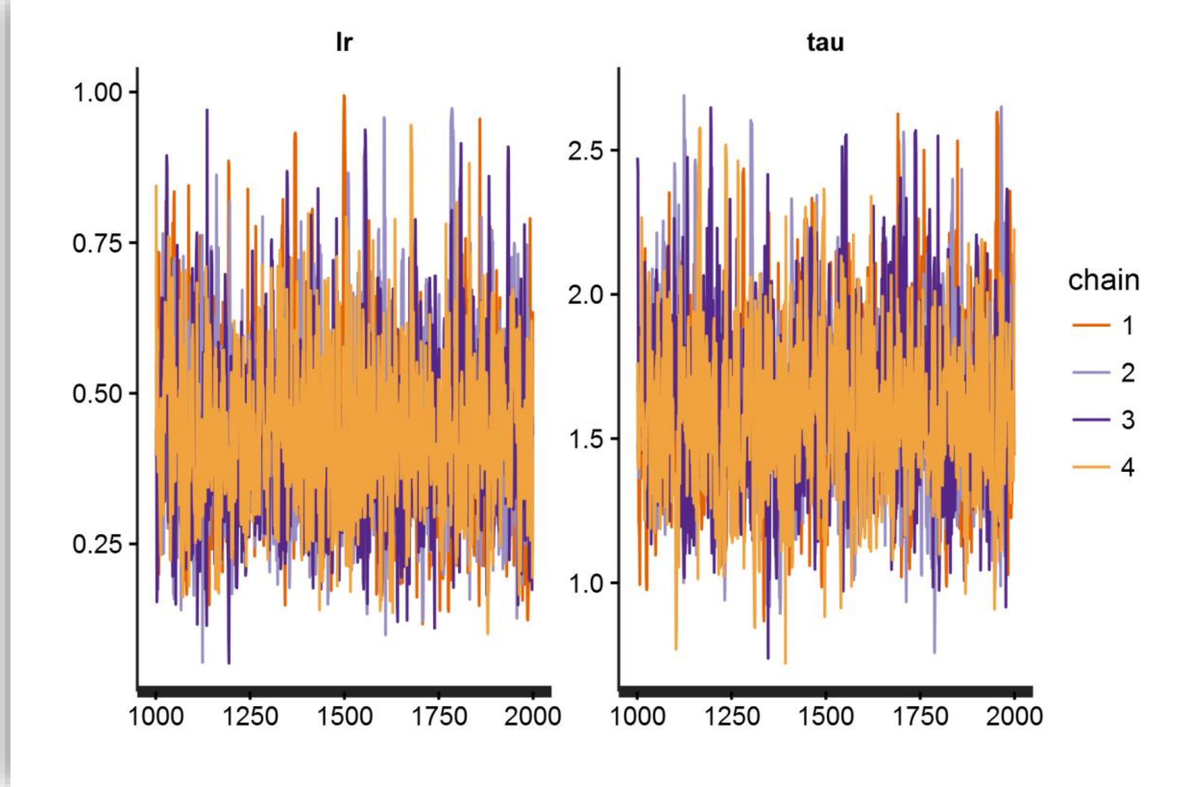
TASK: fit the model for single participants

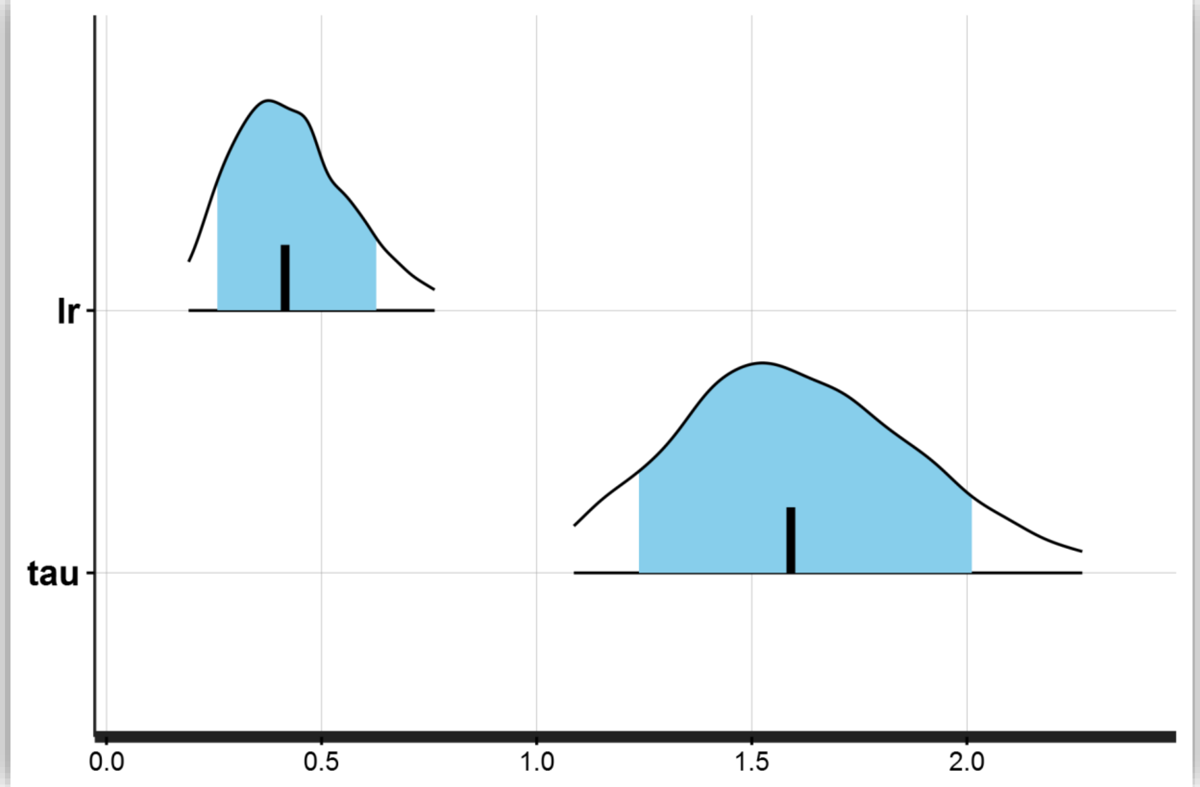
```
> source('_scripts/reinforcement_learning_single_parm_main.R’) # a function

> fit_rl1 <- run_rl_sp(multiSubj = FALSE)
```

```
> load('_data/rl_sp_ss.RData')
> head(rl_ss)
     [,1] [,2]
[1,]    2   -1
[2,]    1    1
[3,]    1    1
[4,]    1    1
[5,]    2   -1
[6,]    1    1
```

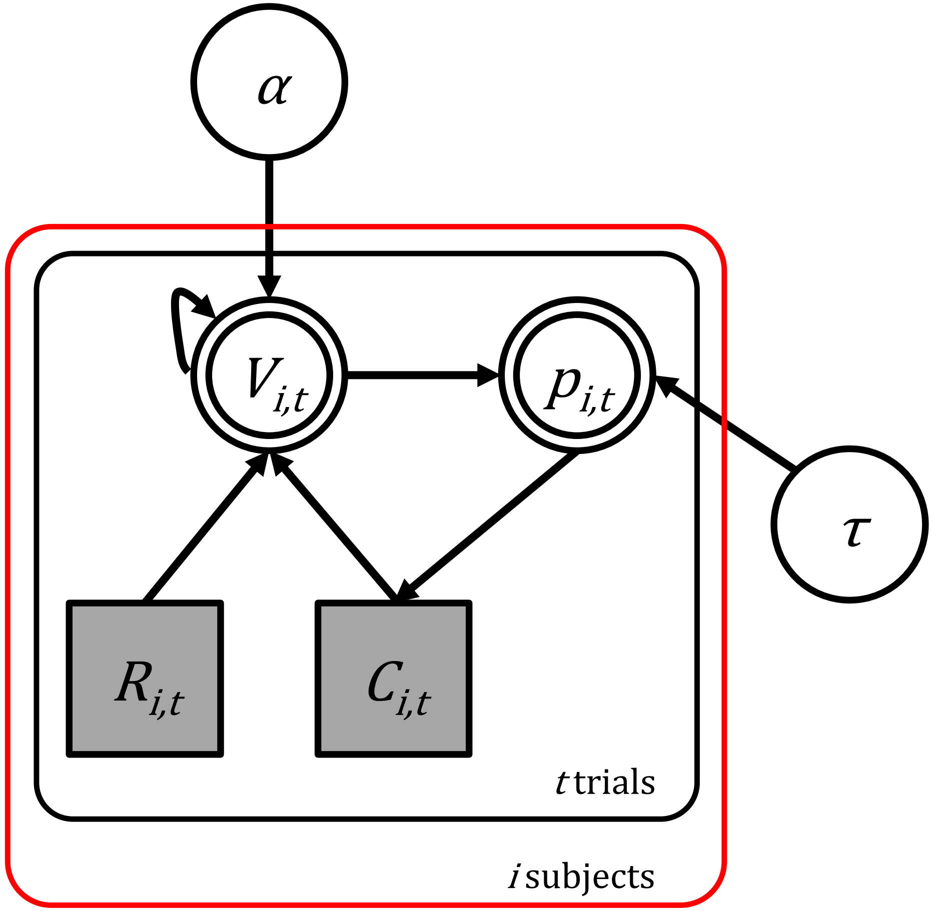
RL – MCMC Output

cognitive model
statistics
computing





Fitting Multiple Participants as ONE




```
model {
  for (s in 1:nSubjects) {
    vector[2] v;
    real pe;
    v = initV;

    for (t in 1:nTrials) {
      choice[s,t] ~ categorical_logit( tau * v );
      pe = reward[s,t] - v[choice[s,t]];
      v[choice[s,t]] = v[choice[s,t]] + lr * pe;
    }
  }
}
```

Exercise X

cognitive model
statistics
computing

```
…/06.reinforcement_learning/_scripts/reinforcement_learning_single_parm_main.R
```

TASK:

(1) complete the model (Tip: the for-loop)

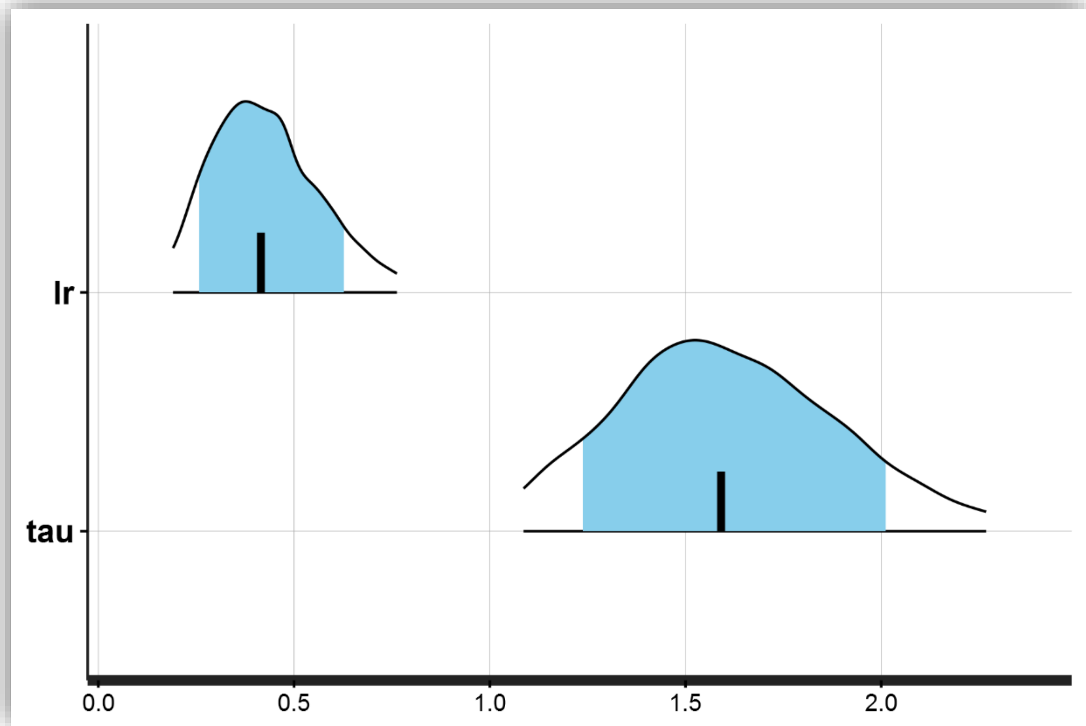
(2) fit the model for multiple participants (assuming same parameters)

```
> source('_scripts/reinforcement_learning_single_parm_main.R')

> fit_rl2 <- run_rl_sp(multiSubj = TRUE)
```

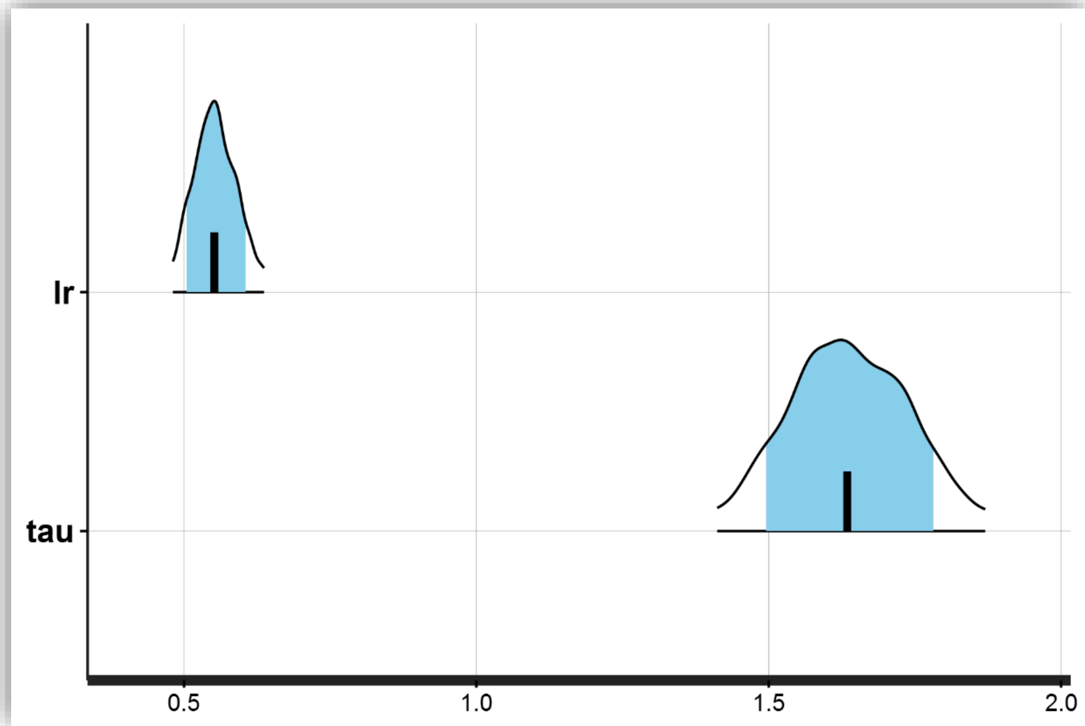
Exercise X

cognitive model
statistics
computing

N = 1

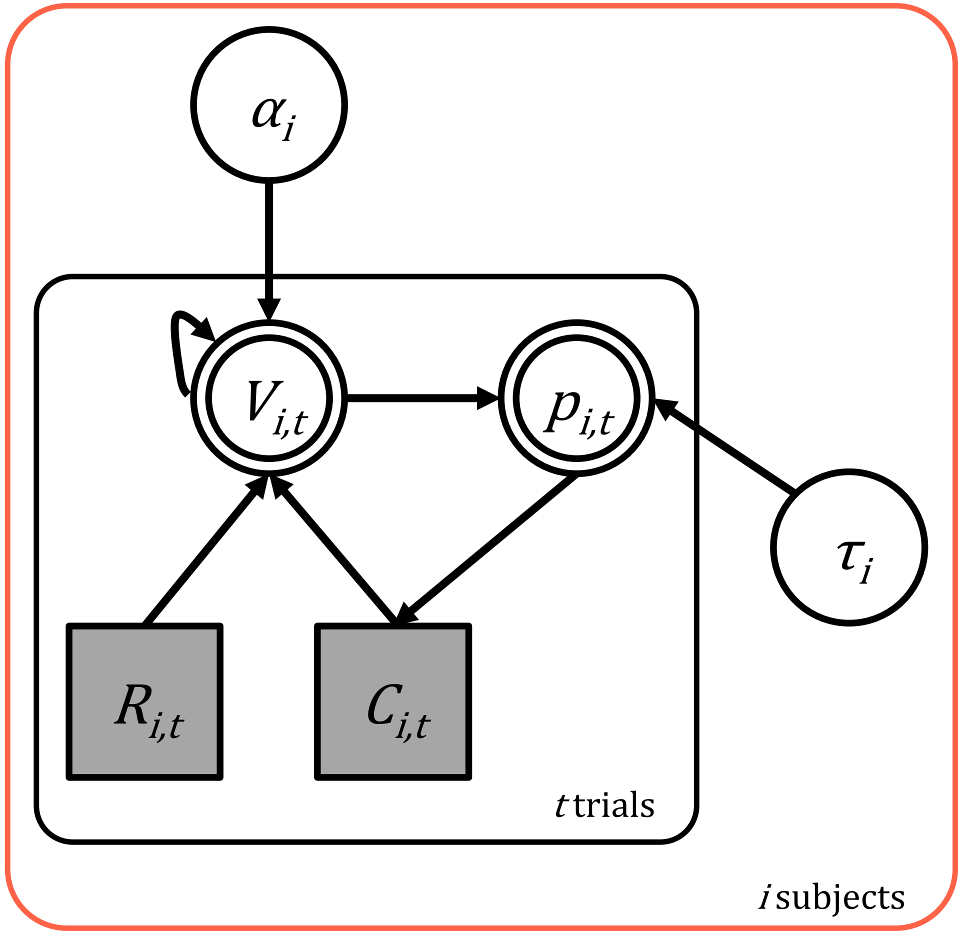


N = 10



Fitting Multiple Participants Independently

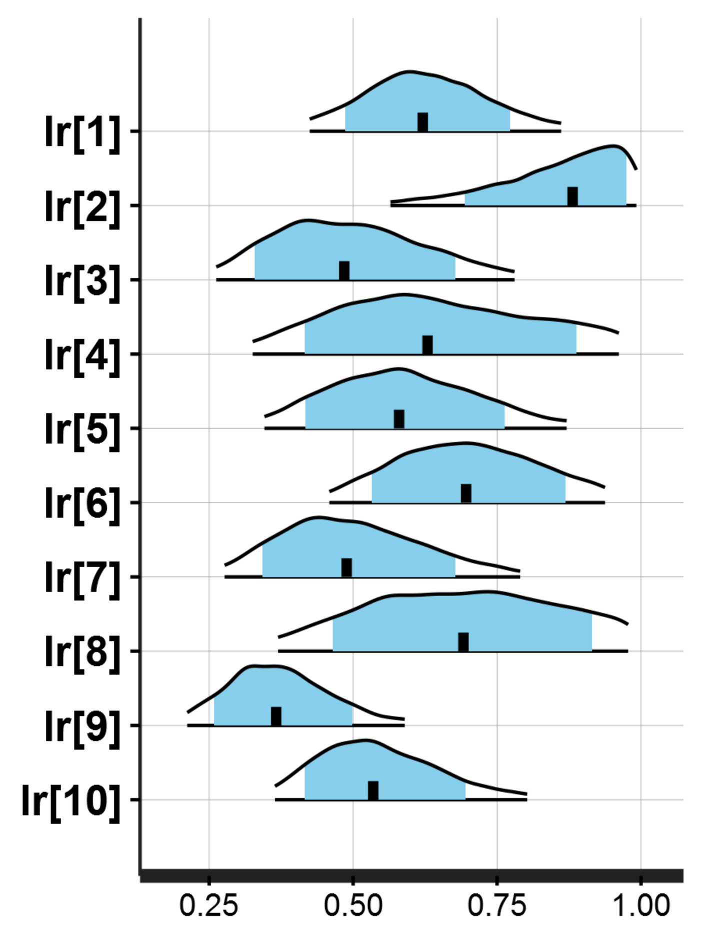
cognitive model
statistics
computing

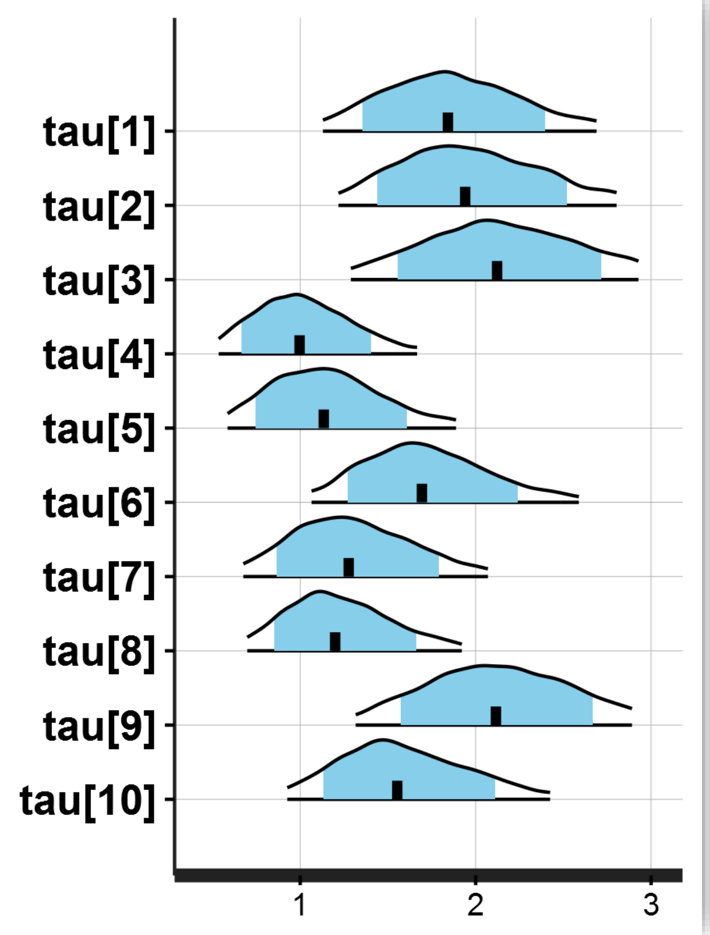


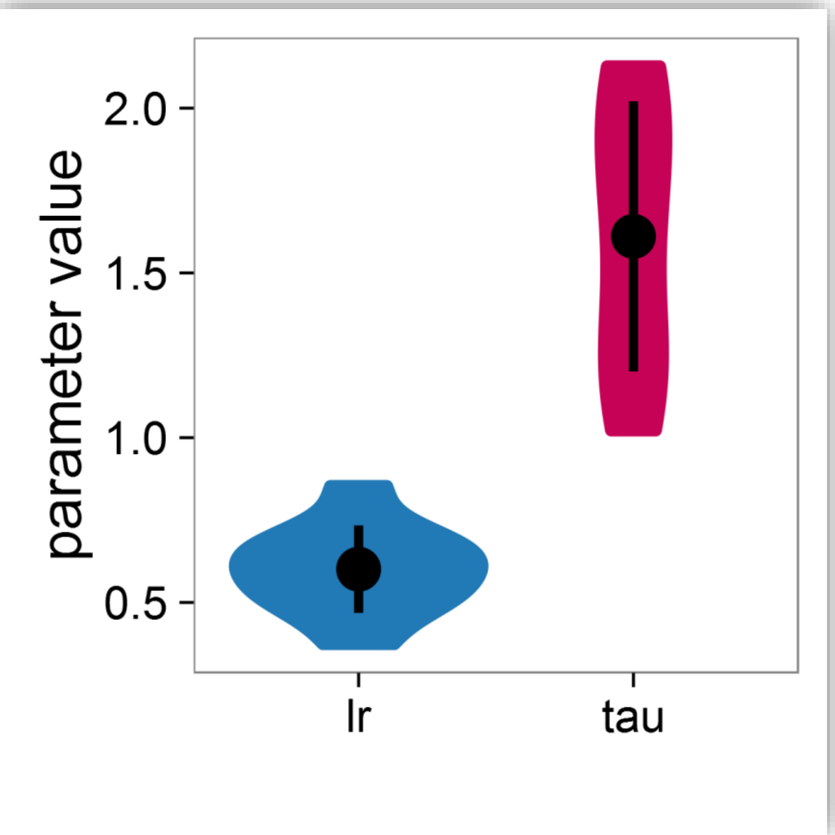

```
model {
  for (s in 1:nSubjects) {
    vector[2] v;
    real pe;
    v = initV;

    for (t in 1:nTrials) {
      choice[s,t] ~ categorical_logit( tau[s] * v );
      pe = reward[s,t] - v[choice[s,t]];
      v[choice[s,t]] = v[choice[s,t]] + lr[s] * pe;
    }
  }
}
```

Individual Fitting

cognitive model
statistics
computing

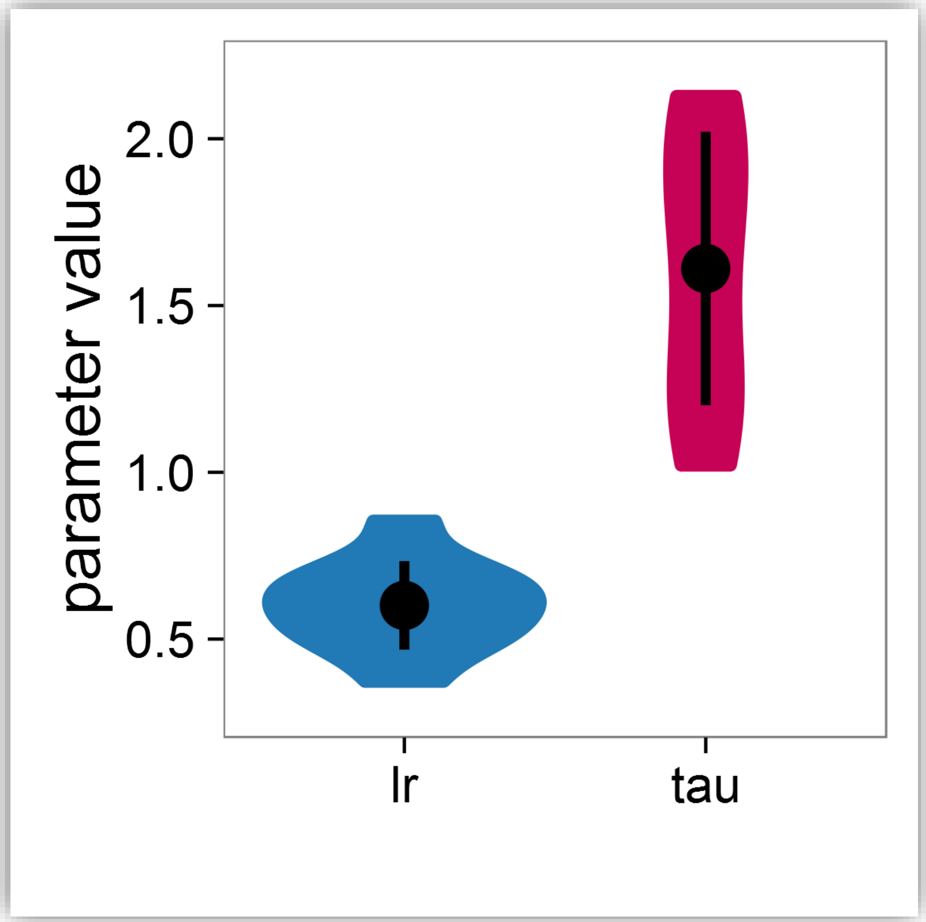




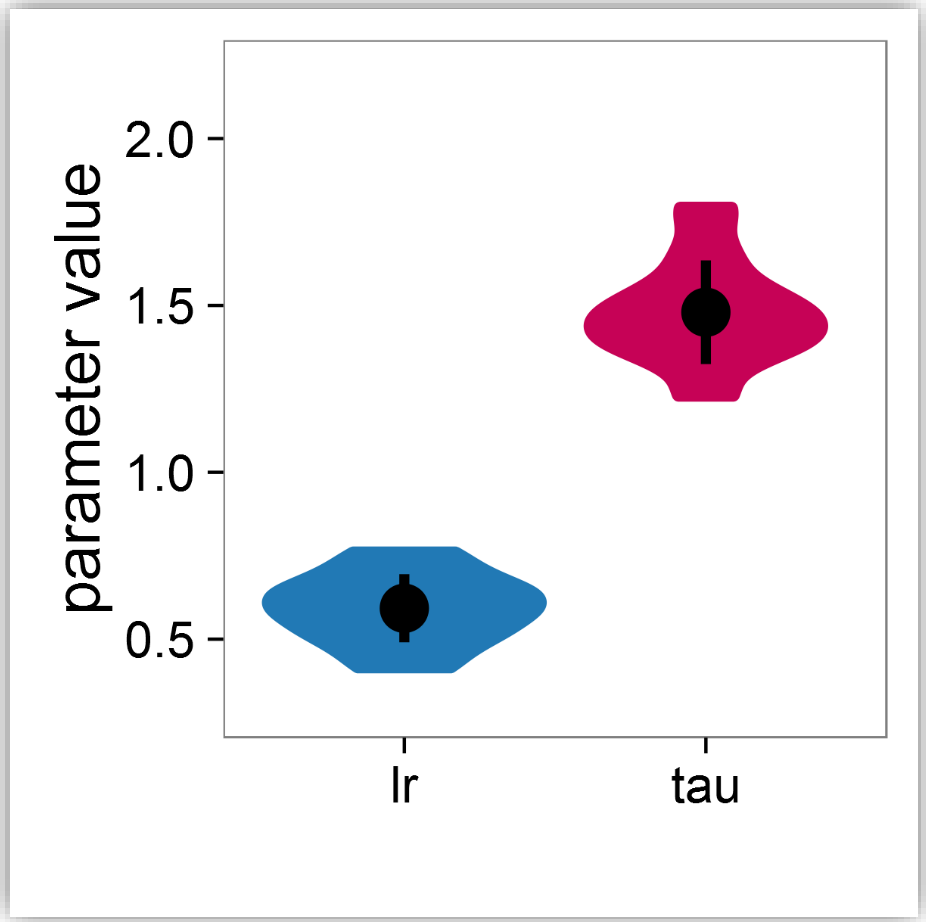


Comparing with True Parameters

Posterior Means



True Parameters




ANY
QUESTIONS
?

Happy Computing!