TEWA 1: Advanced Data Analysis

Lecture 10

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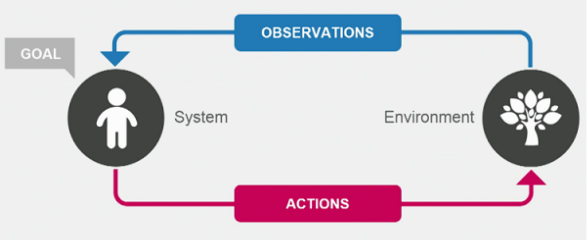
https://github.com/lei-zhang/tewa1_univie

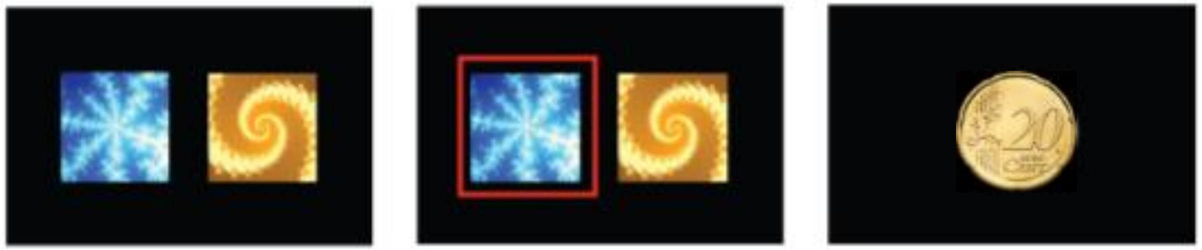
universität wien

Fakultät für Psychologie

Bayesian **warm-up**?

Rescorla-Wagner Value Update





Value update:

$$V_{t+1} = V_t + \alpha * PE_t$$

Prediction error:

$$PE_t = R_t - V_t$$

choice rule (sigmoid /softmax):

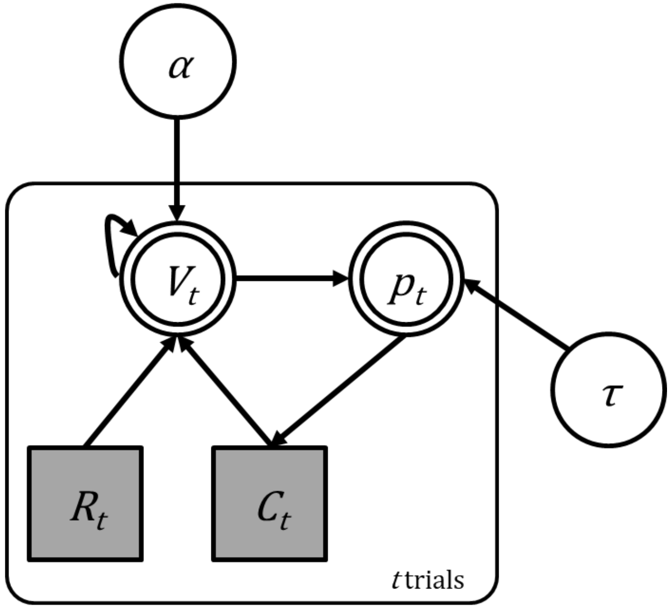
$$p(C{=}a) = \frac{1}{1+e^{\tau*(v(b)-v(a))}}$$

α - learning rate
PE - reward prediction error
V - value
R - reward
τ - softmax temperature

Rescorla & Wagner (1972)

RL – Implementation

cognitive model
statistics
computing



$$\alpha \sim Uniform(0,1)$$

$$\tau \sim Uniform(0,3)$$

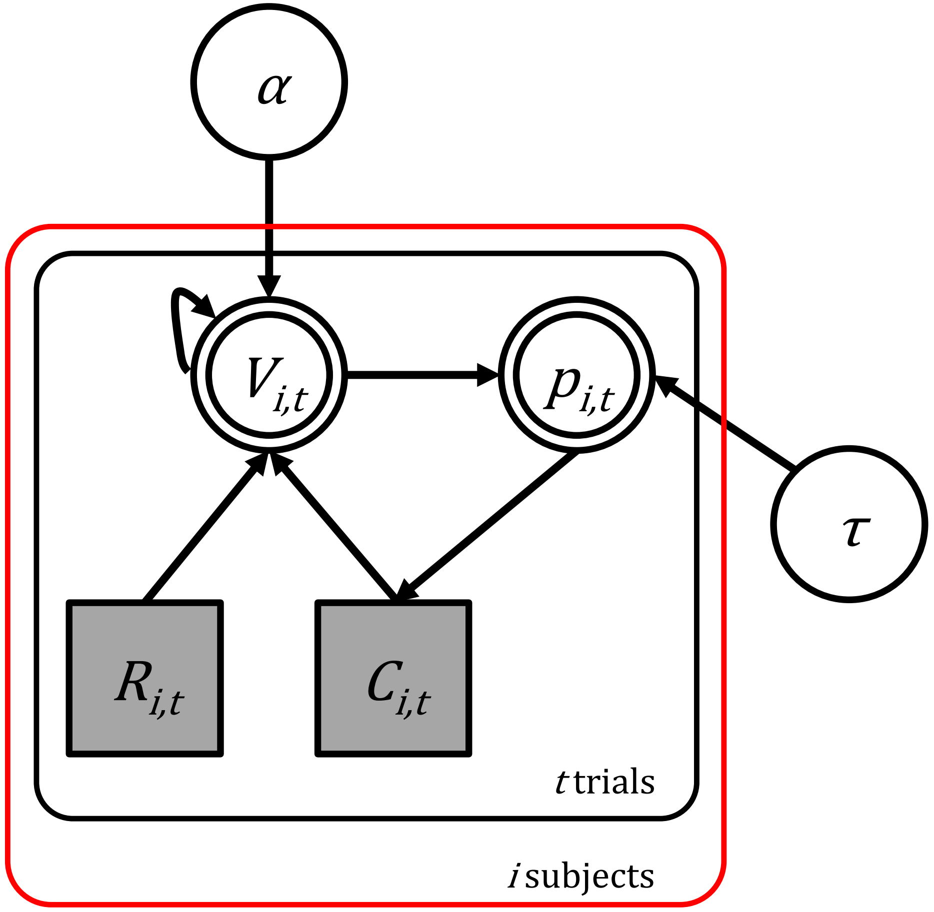
$$p_t(C=A) = \frac{1}{1+e^{\tau(V_t(B)-V_t(A))}}$$

$$V_{t+1}^{c} = V_t^C + \alpha(R_t - V_t^C)$$

```
transformed data {
  vector[2] initV;
  initV = rep_vector(0.0, 2);
}

model {
  vector[2] v[nTrials+1];
  real pe[nTrials];

  v[1] = initV;

  for (t in 1:nTrials) {
    choice[t] ~ categorical_logit( tau * v[t] );

    pe[t] = reward[t] - v[t,choice[t]];

    v[t+1] = v[t];
    v[t+1, choice[t]] = v[t, choice[t]] + lr * pe[t];
  }
}
```

Fitting Multiple Participants as ONE

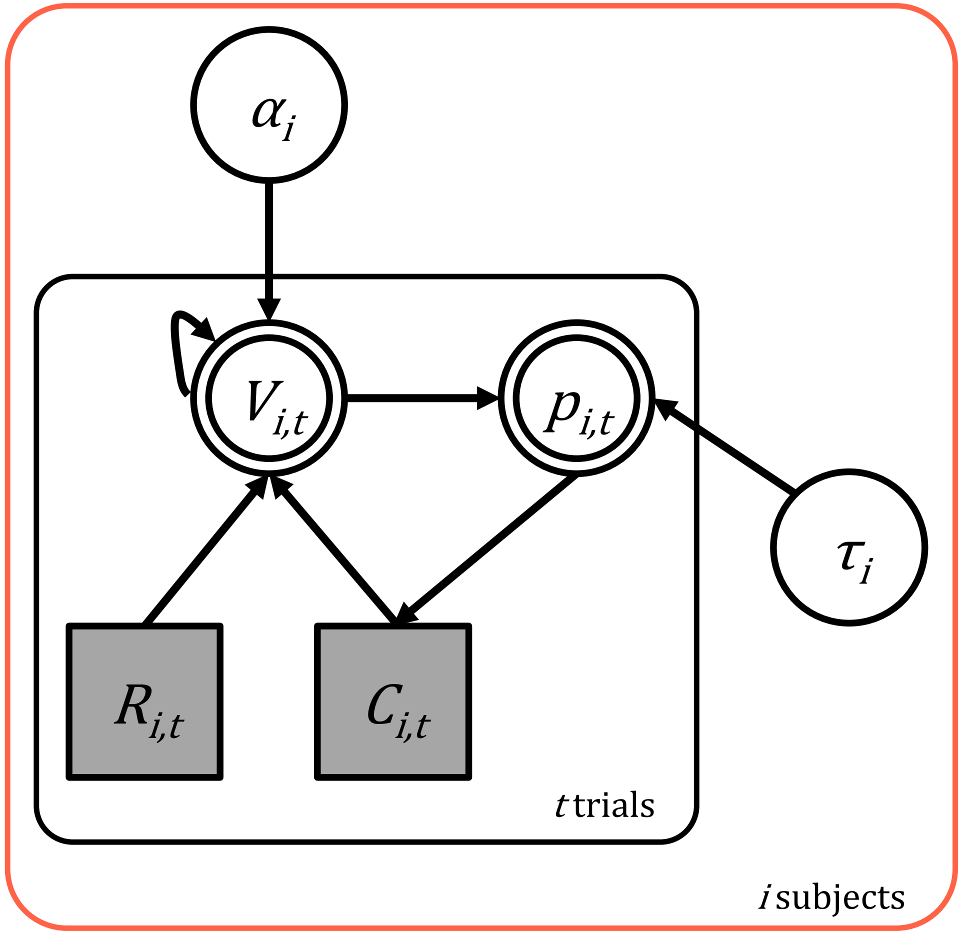
cognitive model
statistics
computing




```
model {
  for (s in 1:nSubjects) {
    vector[2] v;
    real pe;
    v = initV;

    for (t in 1:nTrials) {
      choice[s,t] ~ categorical_logit( tau * v );
      pe = reward[s,t] - v[choice[s,t]];
      v[choice[s,t]] = v[choice[s,t]] + lr * pe;
    }
  }
}
```

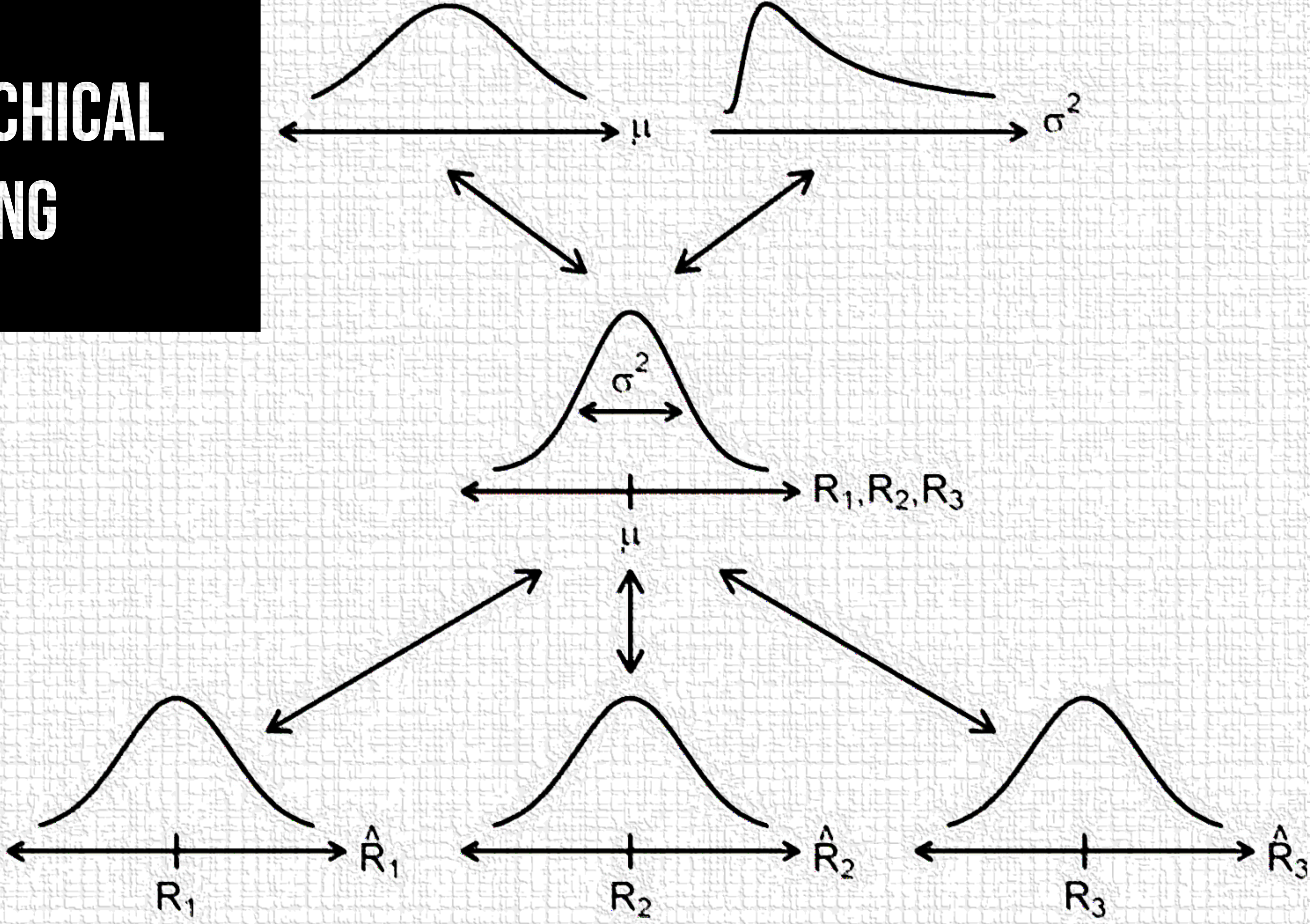
Fitting Multiple Participants Independently

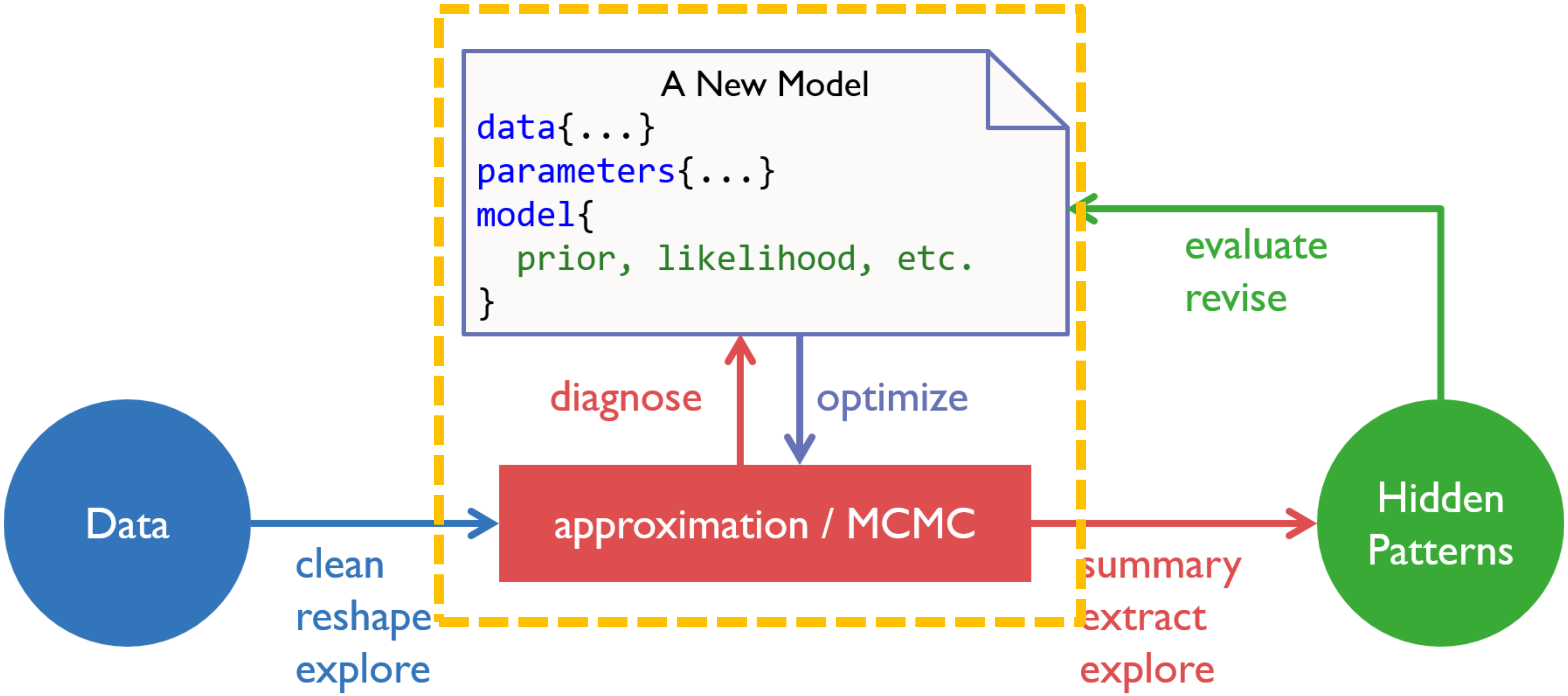
cognitive model
statistics
computing




```
model {
  for (s in 1:nSubjects) {
    vector[2] v;
    real pe;
    v = initV;

    for (t in 1:nTrials) {
      choice[s,t] ~ categorical_logit( tau[s] * v );
      pe = reward[s,t] - v[choice[s,t]];
      v[choice[s,t]] = v[choice[s,t]] + lr[s] * pe;
    }
  }
}
```

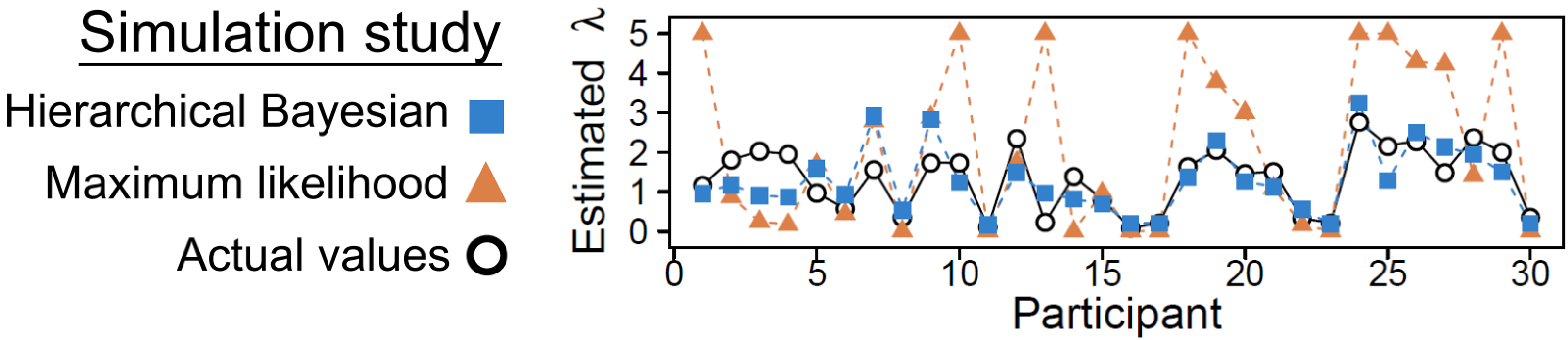
HIERARCHICAL MODELING




A New Model
data{...}
parameters{...}
model{
prior, likelihood, etc.
}
diagnose
optimize
evaluate
revise
Data
clean
reshape
explore
approximation / MCMC
summary
extract
explore
Hidden Patterns

Why Hierarchical Bayesian Cognitive Modeling?

cognitive model
statistics
computing



Ahn et al. (2011)

Why Hierarchical Bayesian Cognitive Modeling?

Fixed effects

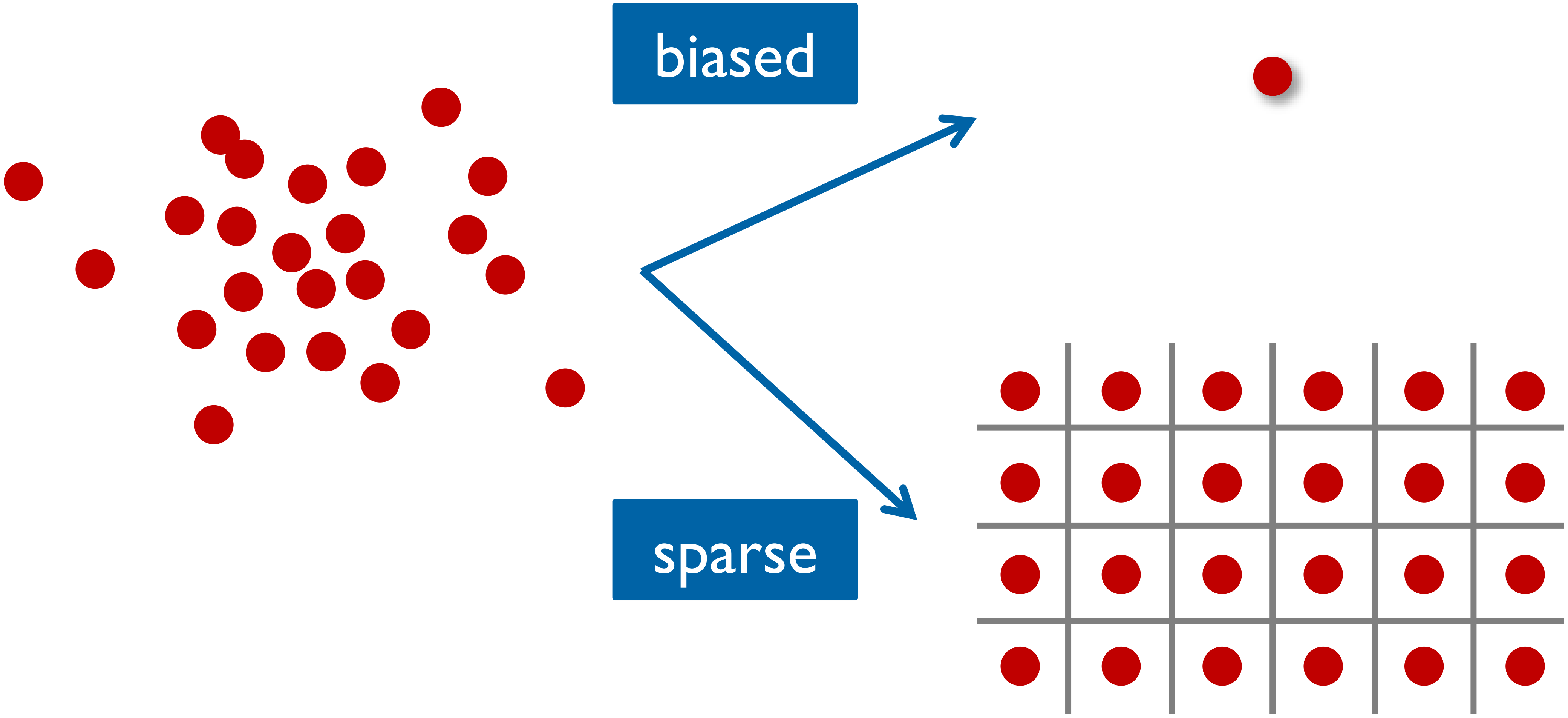
- all subjects are fitted with the same set of parameters
- worse model fit than "random effects"

Random effects

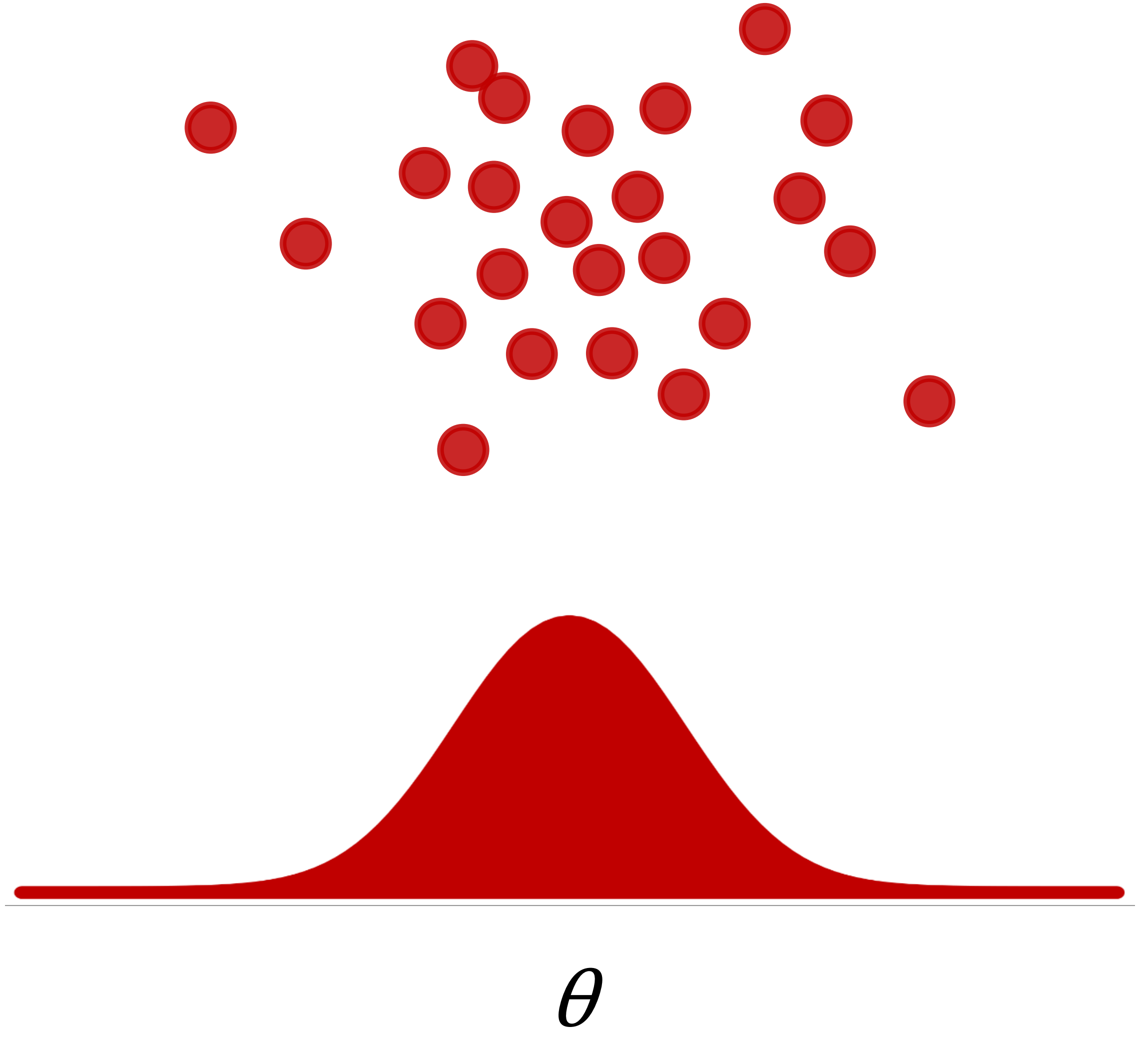
- each subject is fitted independently of the others
- best model fit for each subject
- parameter estimates can be noisy

Adapted from Jan Gläscher's workshop

Fitting Multiple Participants

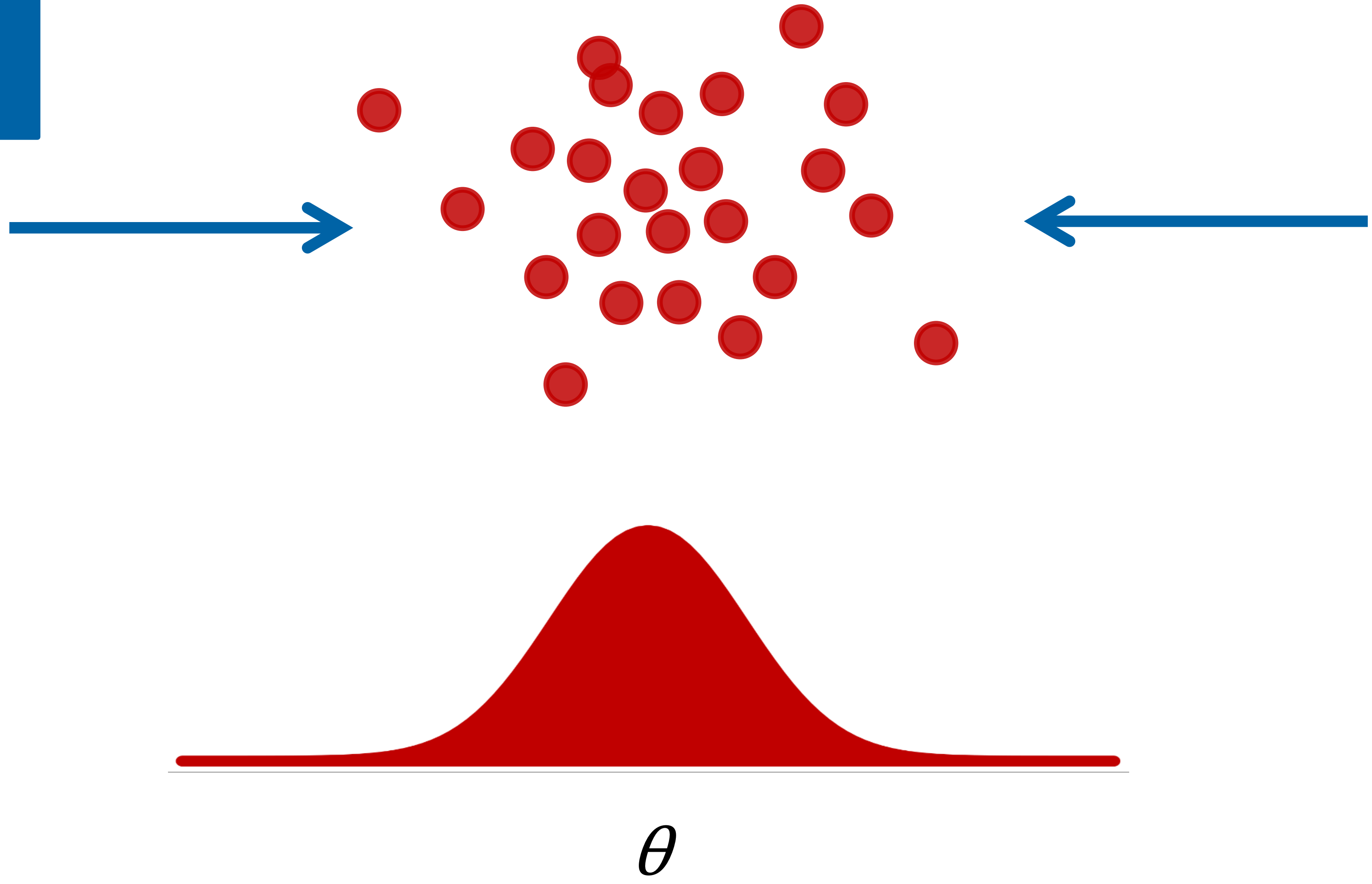


Fitting Multiple Participants



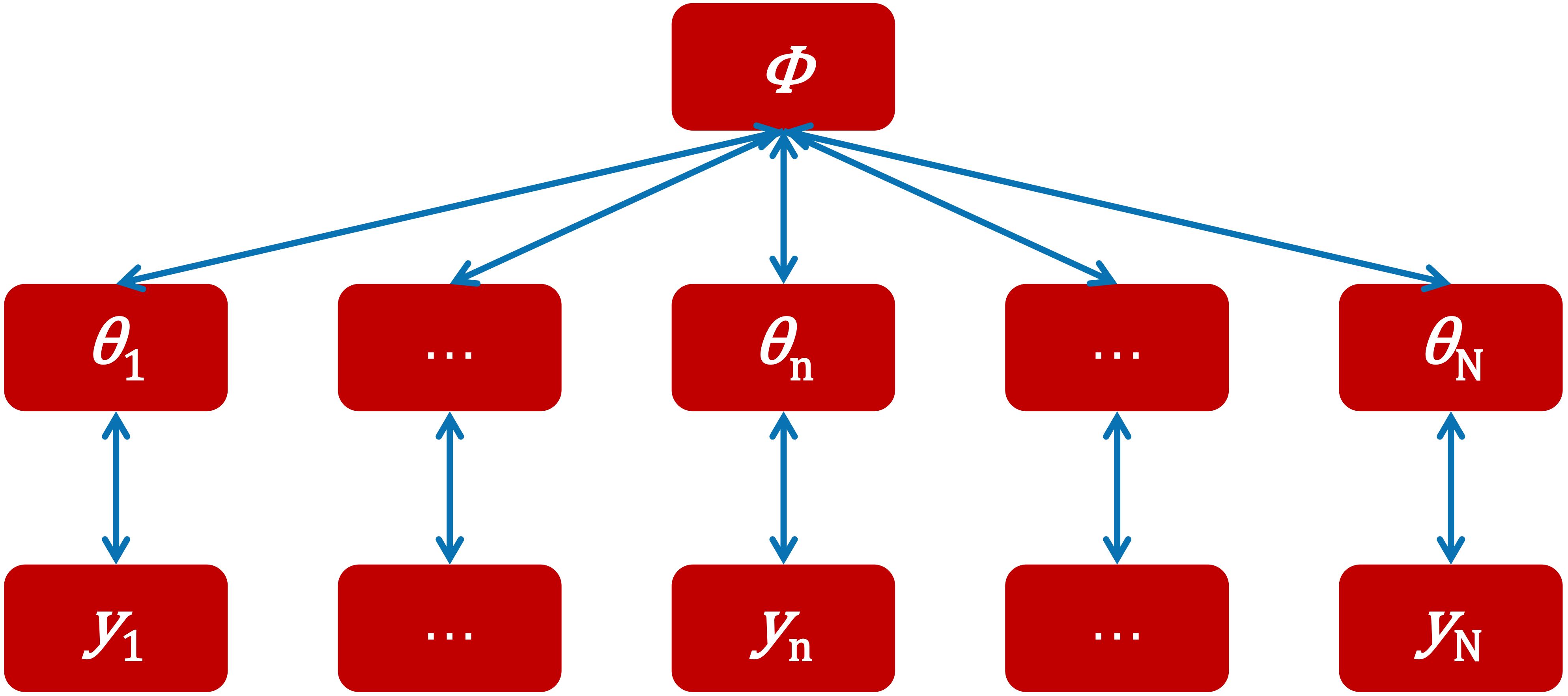
Fitting Multiple Participants

shrinkage effect



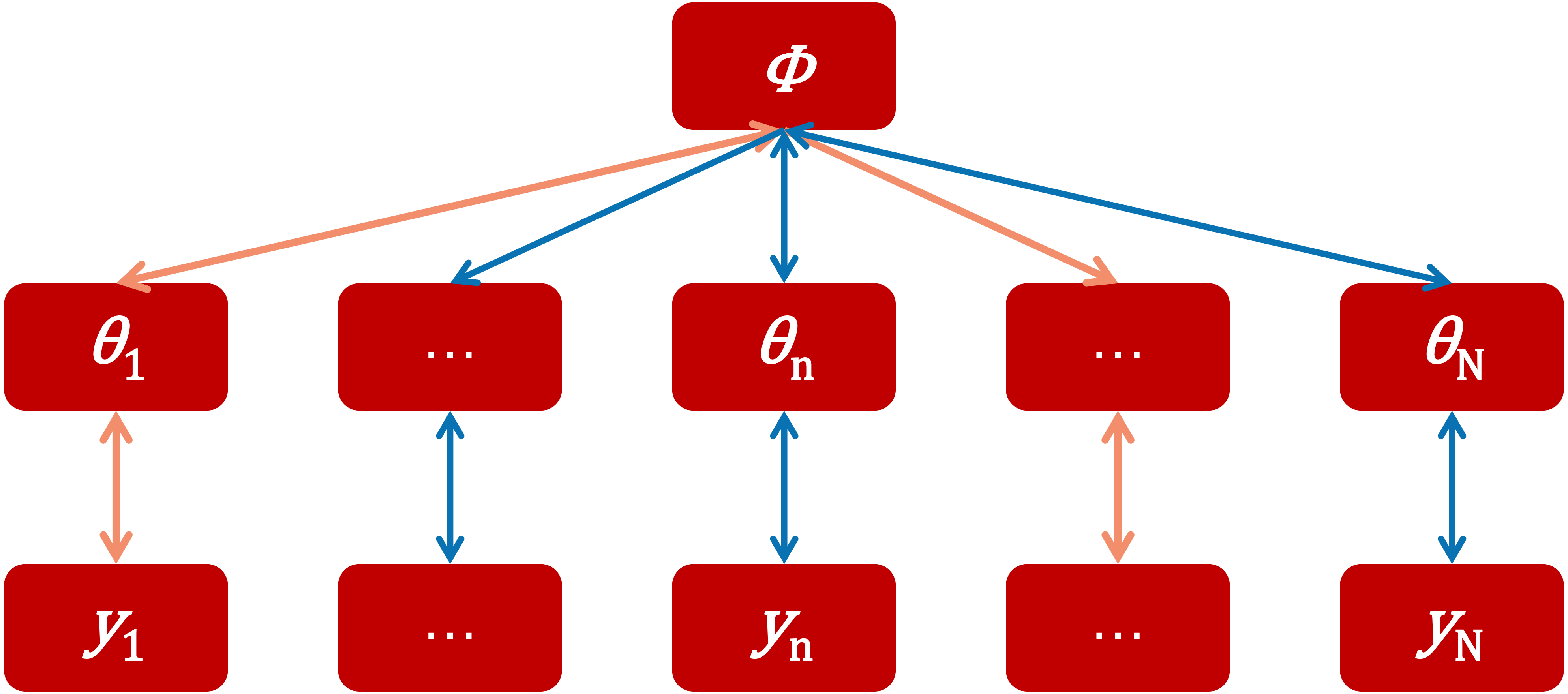
Hierarchical Structure

cognitive model
statistics
computing



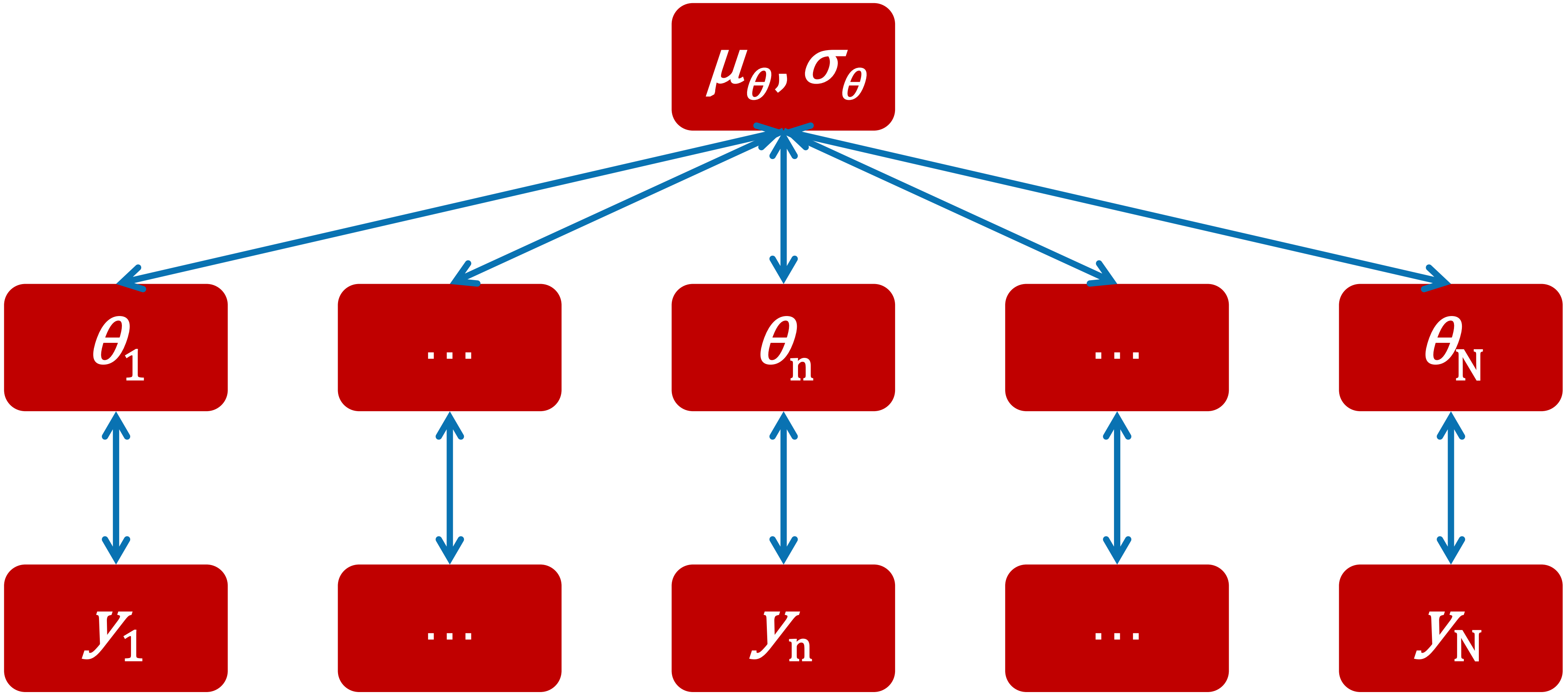
Hierarchical Structure

cognitive model
statistics
computing

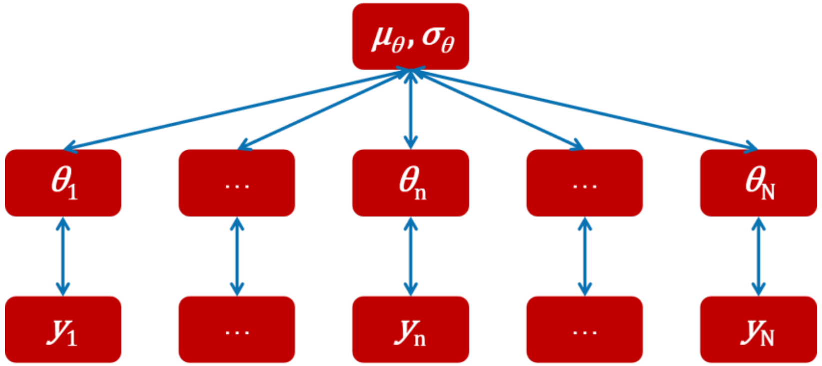


Hierarchical Structure

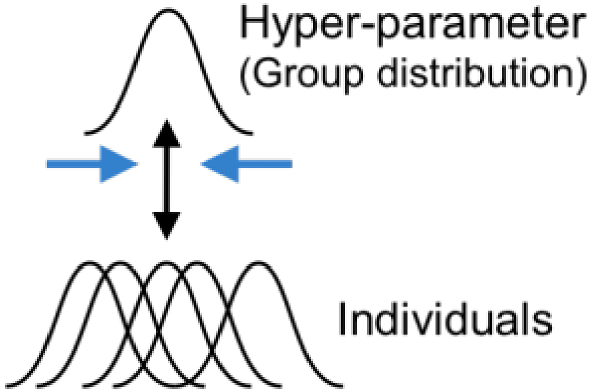
cognitive model
statistics
computing

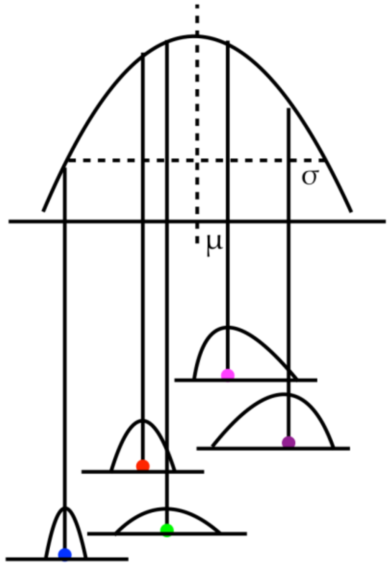


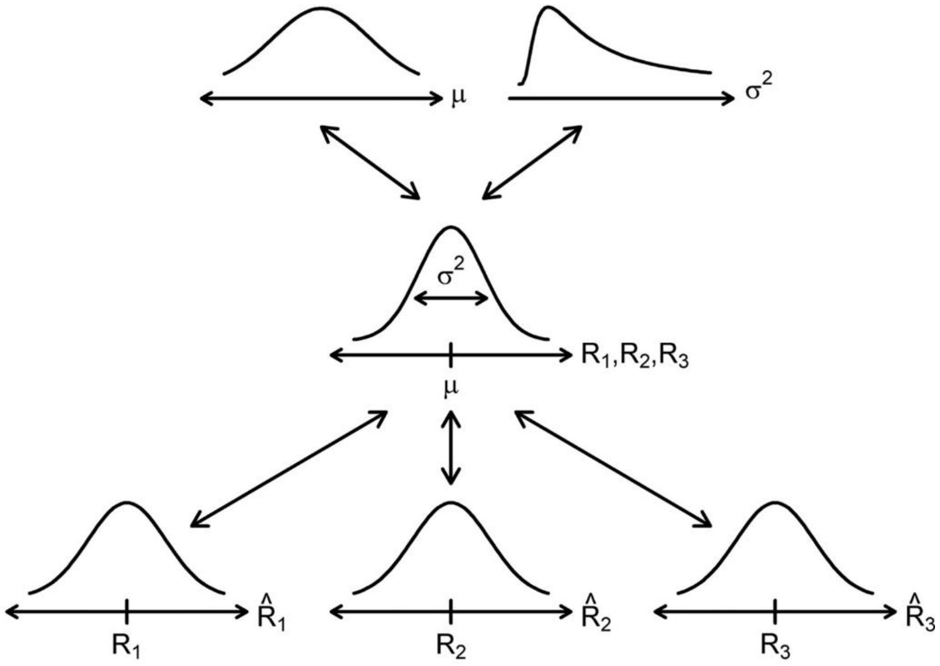
Hierarchical Structure



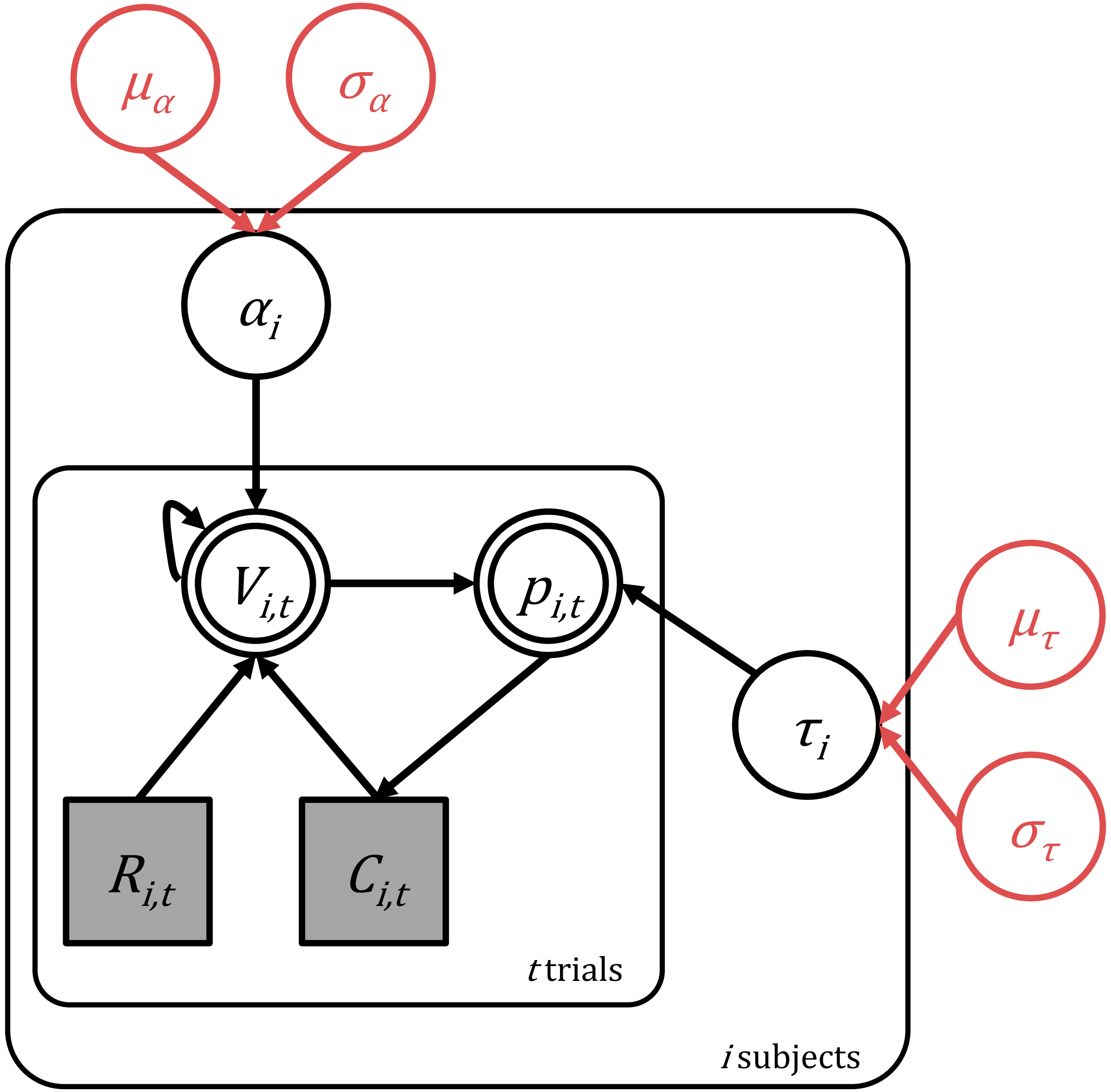
$$P(\Theta,\Phi \mid D) = \frac{P(D \mid \Theta,\Phi)P(\Theta,\Phi)}{P(D)} \propto P(D \mid \Theta)P(\Theta \mid \Phi)P(\Phi)$$





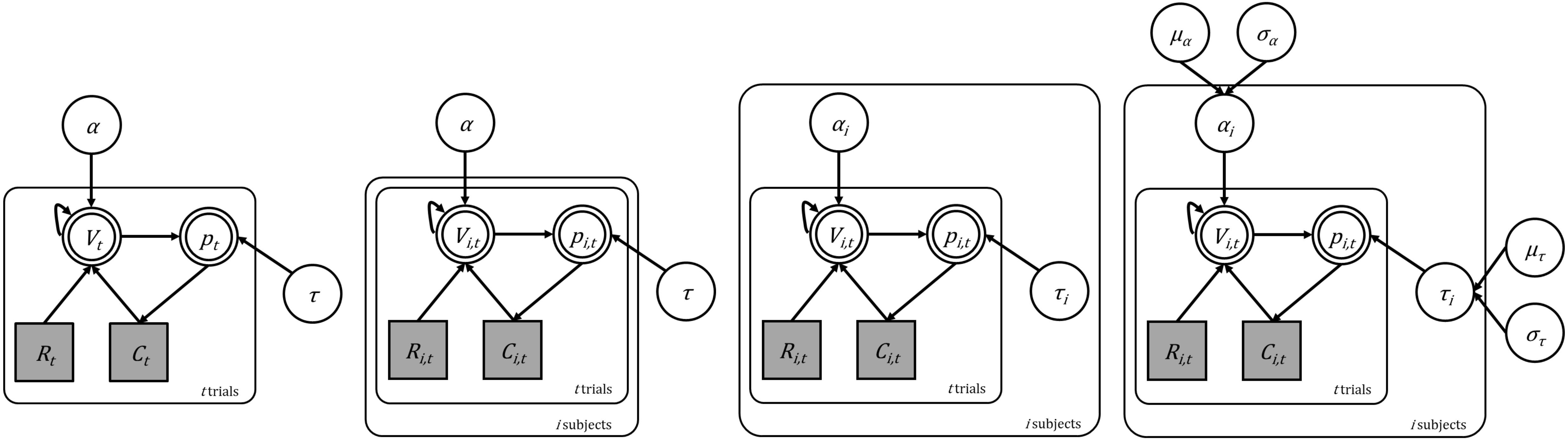


Hierarchical RL Model



HOW DID WE GET HERE?

cognitive model
statistics
computing



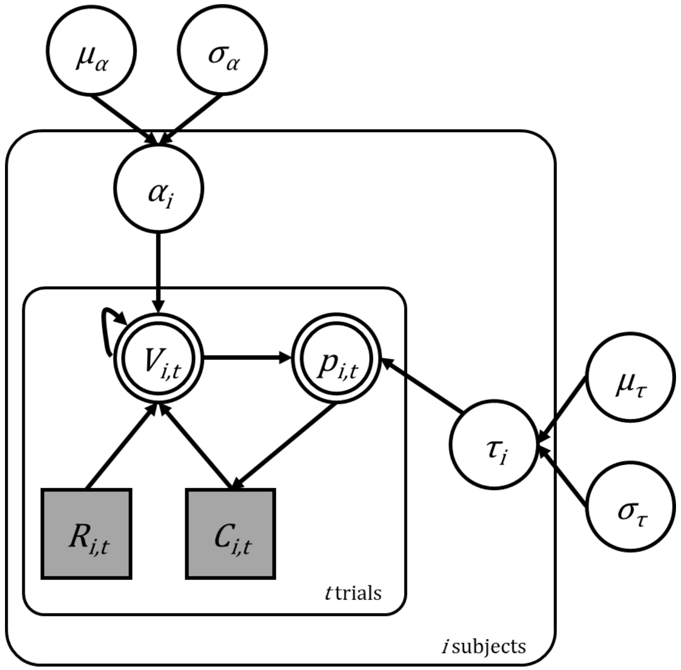
The cognitive model *per se* is the same!

Implementing Hierarchical RL Model

cognitive model

statistics

computing



$$\mu_\alpha \sim Uniform(0,1)$$
$$\sigma_\alpha \sim halfCauchy(0,1)$$
$$\mu_\tau \sim Uniform(0,3)$$
$$\sigma_\tau \sim halfCauchy(0,3)$$
$$\alpha_i \sim Normal(\mu_\alpha, \sigma_\alpha)_{\mathcal{T}(0,1)}$$
$$\tau_i \sim Normal(\mu_\tau, \sigma_\tau)_{\mathcal{T}(0,3)}$$

$$p_{i,t}(C = A) = \frac{1}{1 + e^{\tau_i(V_{i,t}(B) - V_{i,t}(A))}}$$

$$V_{i,t+1}^{c} = V_{i,t}^{C} + \alpha_i(R_{i,t} - V_{i,t}^{C})$$

```
parameters {
  real<lower=0,upper=1> lr_mu;
  real<lower=0,upper=3> tau_mu;

  real<lower=0> lr_sd;
  real<lower=0> tau_sd;

  real<lower=0,upper=1> lr[nSubjects];
  real<lower=0,upper=3> tau[nSubjects];
}

model {
  lr_sd  ~ cauchy(0,1);
  tau_sd ~ cauchy(0,3);
  lr     ~ normal(lr_mu, lr_sd) ;
  tau    ~ normal(tau_mu, tau_sd) ;

  for (s in 1:nSubjects) {
    vector[2] v;
    real pe;
    v = initV;

    for (t in 1:nTrials) {
      choice[s,t] ~ categorical_logit( tau[s] * v );
      pe = reward[s,t] - v[choice[s,t]];
      v[choice[s,t]] = v[choice[s,t]] + lr[s] * pe;
    }
  }
}
```

Exercise XI

cognitive model
statistics
computing

```
…/06.reinforcement_learning/_scripts/reinforcement_learning_multi_parm_main.R
```

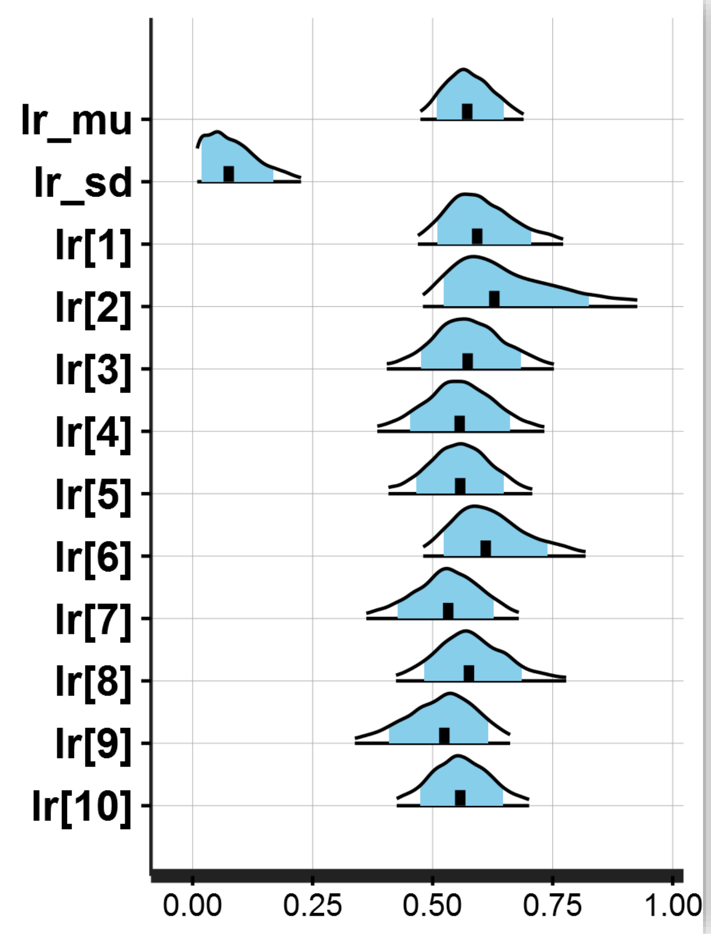
TASK: (1) complete the model (TIP: individual ~ group)

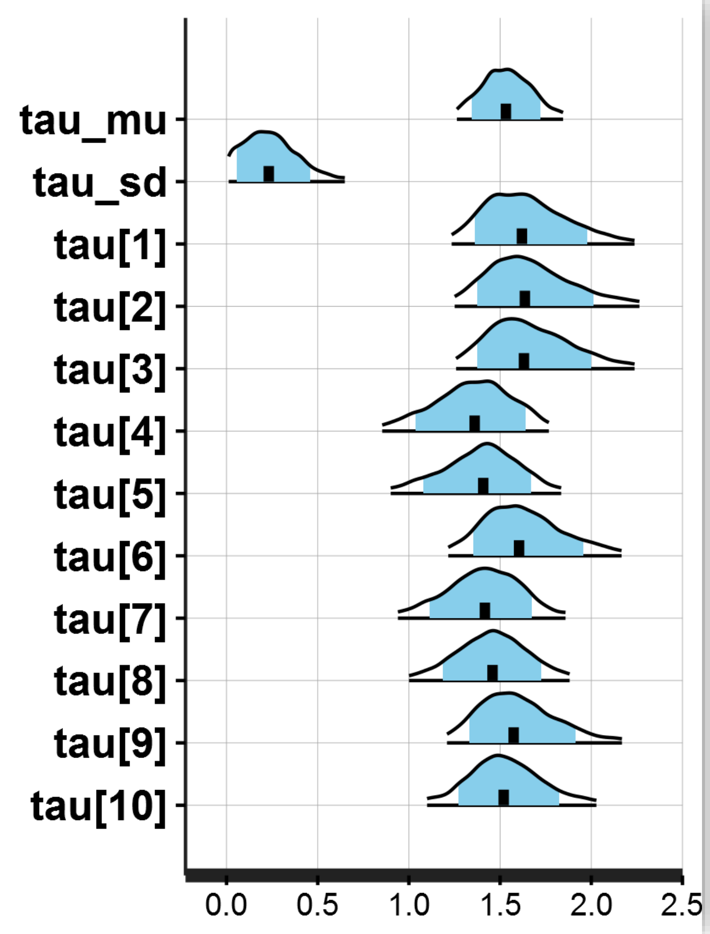
(2) fit the hierarchical RL model

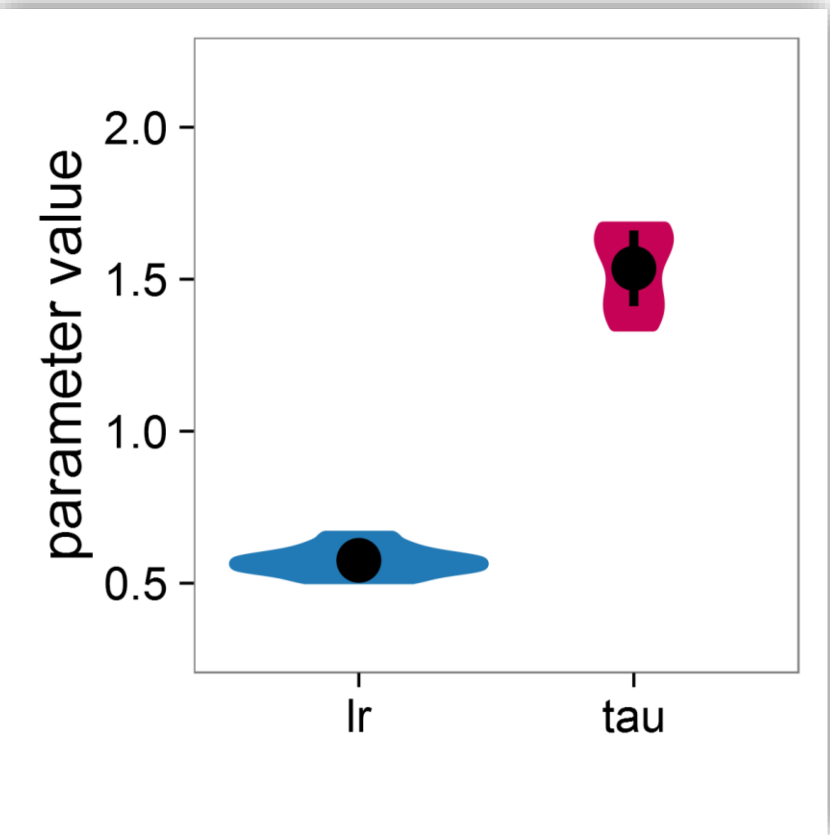
```
> source('_scripts/reinforcement_learning_multi_parm_main.R')

> fit_rl3 <- run_rl_mp( modelType ='hrch' )
```

```
In addition: Warning messages:
1: There were 97 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help. See
http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup
2: Examine the pairs() plot to diagnose sampling problems
```

Hierarchical Fitting*





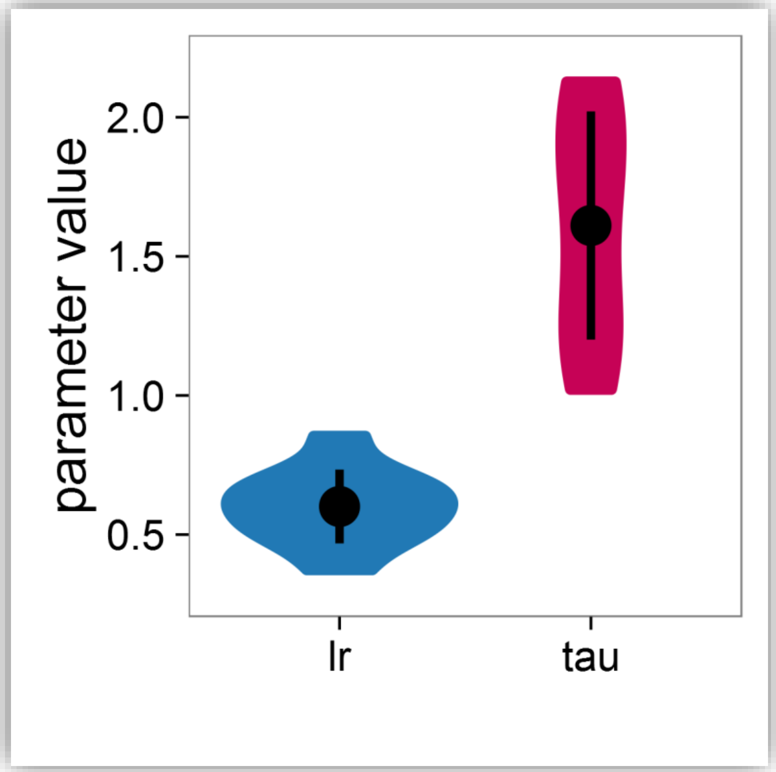


*: adapt_delta=0.999, max_treedepth=100

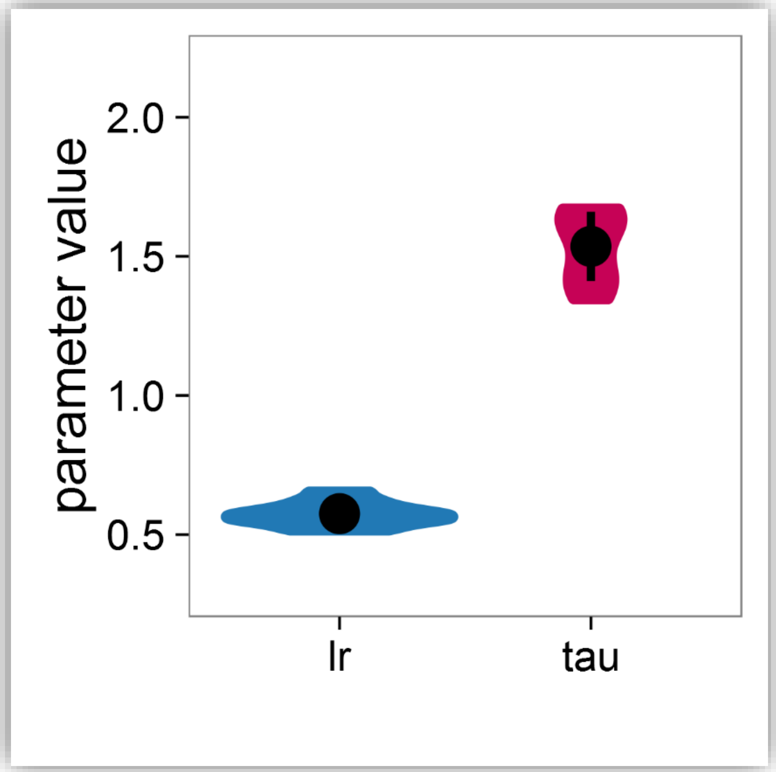
Comparing with True Parameters

cognitive model
statistics
computing

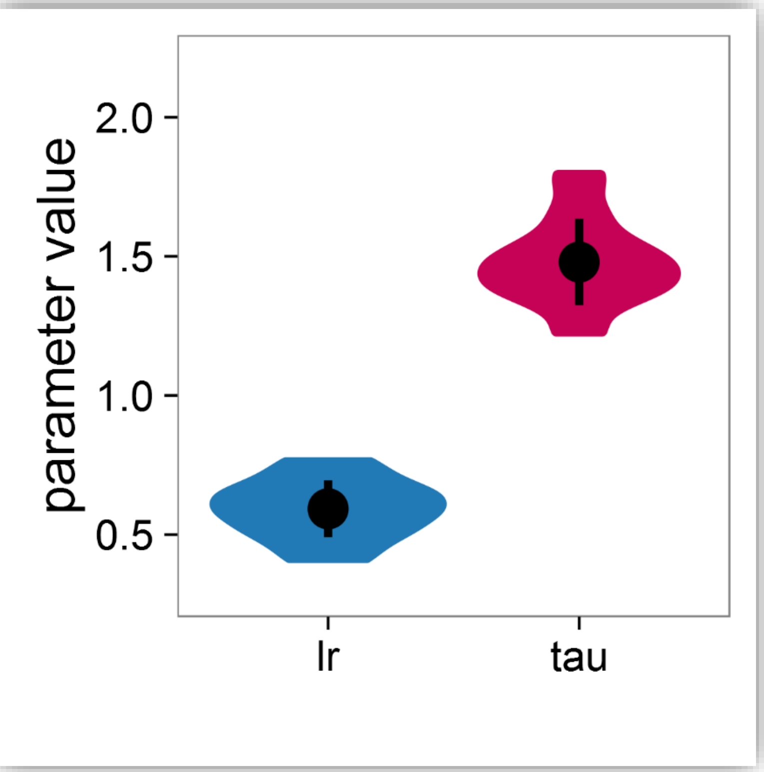
Posterior Means (indv)



Posterior Means (hrch)*



True Parameters



*: adapt_delta=0.999, max_treedepth=100

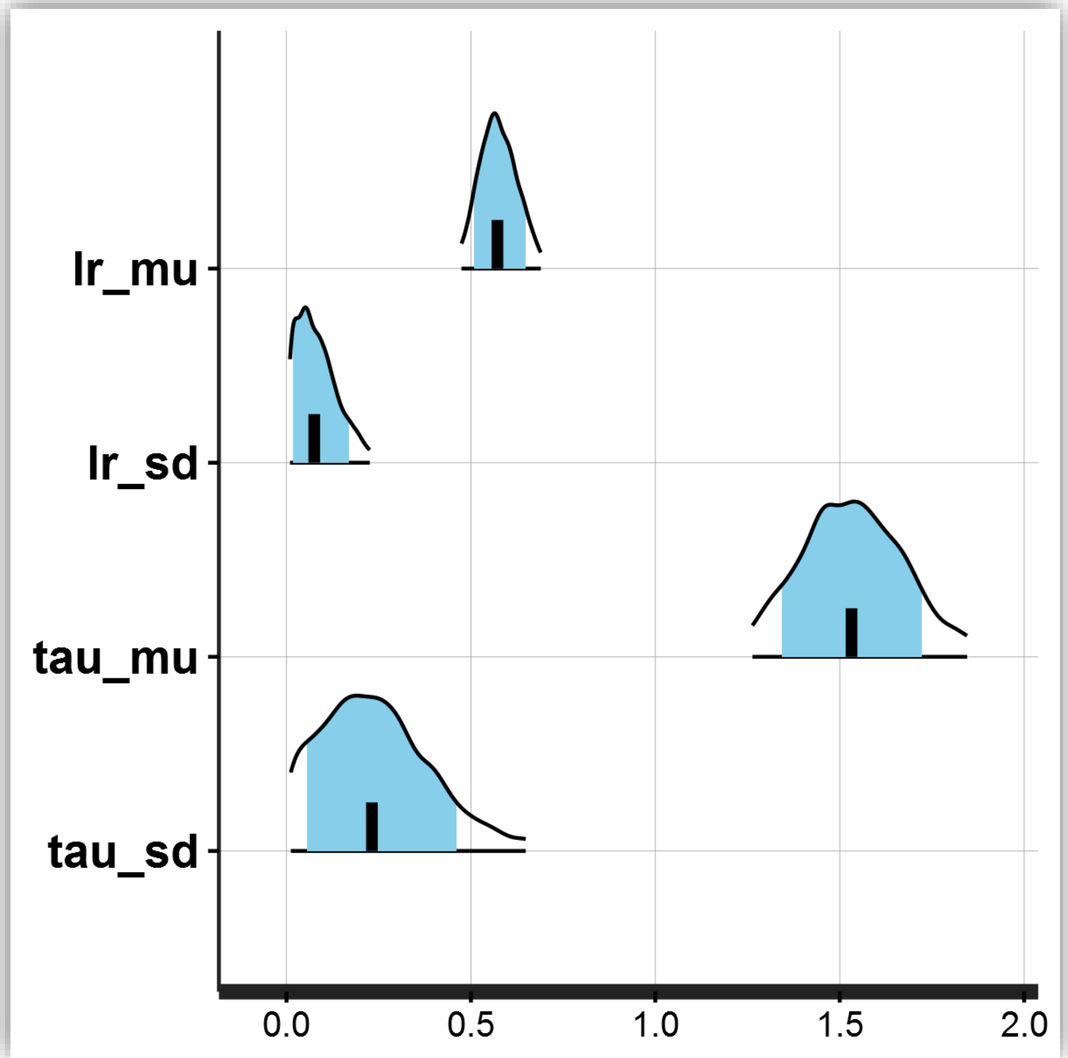
Group-level Parameters

True group parameters

```
lr  = rnorm(10, mean=0.6, sd=0.12)
tau = rnorm(10, mean=1.5, sd=0.2)
```

Estimated group parameters

	mean	2.5%	25%	50%	75%	97.5%
lr_mu	0.58	0.47	0.54	0.57	0.61	0.69
lr_sd	0.09	0.01	0.04	0.08	0.12	0.23
tau_mu	1.54	1.26	1.43	1.53	1.63	1.85
tau_sd	0.25	0.01	0.13	0.23	0.34	0.65




ANY
QUESTIONS
?

Happy Computing!