Step by Step Guide: Congruence In Triangles

Pre-Class Preparation Checklist

- Prepare handouts with a set of 8–10 triangles of various sizes and shapes (include pairs that are congruent by SSS, SAS, ASA, and RHS).
- Ensure construction paper, pencils, rulers, protractors, and scissors are available for each group.
- Prepare a second handout with the worked example: triangles ABC and PQR with AB = PR = 3 cm, BC = PQ = 8 cm, $\angle B = \angle P = 60°$.
- Write the five congruence criteria on a chart or prepare to build the table on the board.
- Prepare graph paper for the coordinate geometry assessment question.
- Have compasses available for the equilateral triangle construction.
- Have the digital textbook open: Section 2.2.4 Congruence in Triangles.
- Prepare exit ticket handouts with 2 questions.

Phase 1: Problem-Solving and Discovery (15 minutes)

Opening and Connection to Prior Learning (3 minutes)

[SAY] "In our previous lessons, we studied reflection and mirror lines. Today, we're going to explore an important concept that is closely connected to reflection — congruence."

[ASK] "What do you think congruent means? Have you heard this word before?"

[WAIT] Expected: "Same shape and size." "Identical." "Equal."

[SAY] "Good! Two figures are congruent if they have exactly the same shape and the same size. Today, we'll focus specifically on congruence in TRIANGLES."

[ASK] "If I wanted to prove that two triangles are identical, would I need to check ALL six measurements — three sides and three angles?"

[WAIT] Expected: "Maybe not all of them?" "I think some are enough."

[SAY] "That's exactly what we're going to discover. Let's find out the MINIMUM conditions needed to prove two triangles are congruent."

Anchor Activity Launch (2 minutes)

[DO] Distribute the handout with triangles and the construction materials.

[SAY] "On your handout, you have several triangles. Your task is to:"

[SAY] "First, trace these triangles onto construction paper and cut them out."

[SAY] "Second, find pairs of congruent triangles by placing one on top of the other."

[SAY] "Third, for each congruent pair, determine which of these four criteria applies."

[WRITE] On the board:
"Criterion 1: Three sides equal.
Criterion 2: Two sides and the included angle equal.
Criterion 3: One side and two included angles equal.
Criterion 4: Hypotenuse and one side of a right-angled triangle equal."

[SAY] "Work in your groups. You have 8 minutes."

Student Work Time (8 minutes)

[DO] Circulate among groups.

[ASK] To a group comparing triangles: "How did you decide those two are congruent?"

[WAIT] Expected: "We put one on top of the other and they matched perfectly."

[ASK] "Which measurements match? Did you check the sides? The angles?"

[SAY] "Good. Now, which criterion does this pair fit? Look at the four criteria on the board."

[DO] For struggling groups: "Start with the easiest criterion. Measure all three sides of each triangle. Can you find two triangles where all three sides are the same length?"

[ASK] To a group that found a SAS pair: "You said two sides and an angle match. WHERE is the angle? Is it BETWEEN the two sides?"

[WAIT] Expected: "Yes, the angle is between the two equal sides."

[SAY] "That’s important! The angle must be the INCLUDED angle — the one between the two sides."

[DO] For early finishers: "Can you find a pair where two angles and one side match? Which criterion is that?"

Class Sharing (2 minutes)

[SAY] "Let’s share what you found. Which pairs did you identify as congruent?"

[DO] Invite 2–3 groups to present their findings.

[ASK] "For each pair, which criterion did you use? How do you know?"

[SAY] "Excellent work! You've discovered the conditions for congruence through hands-on investigation. Now let's formalise what you found."

Phase 2: Structured Instruction (10 minutes)

Formalising the Congruence Criteria (5 minutes)

[SAY] "Congruence in triangles depends on the measure of the sides and angles. Two triangles are said to be congruent if all pairs of corresponding sides and corresponding angles are equal."

[SAY] "But as you discovered, you don't need to check ALL six measurements. Here are the five criteria."

[WRITE] Build the table on the board:

[WRITE] "SSS (Side-Side-Side): All three sides of one triangle equal the three corresponding sides of the other."

[SAY] "This is the most straightforward. If all sides match, the triangles must be identical."

[WRITE] "SAS (Side-Angle-Side): Two sides and the INCLUDED angle are equal."

[SAY] "The word INCLUDED is critical. The angle must be BETWEEN the two sides. If the angle is not between the two sides, it's NOT SAS."

[DO] Draw two triangles on the board showing the included angle clearly.

[WRITE] "ASA (Angle-Side-Angle): Two angles and the INCLUDED side are equal."

[SAY] "Here, the side must be BETWEEN the two angles."

[WRITE] "RHS (Right angle-Hypotenuse-Side): Both triangles have a right angle, and the hypotenuse and one other side are equal."

[SAY] "This only works for right-angled triangles. You need the right angle, the hypotenuse, and one other side."

[WRITE] "AAS (Angle-Angle-Side): Two angles and a NON-included side are equal."

[SAY] "This is similar to ASA, but the side is NOT between the two angles."

Important Warnings (2 minutes)

[SAY] "Two important warnings. First: AAA — Angle-Angle-Angle — is NOT a congruence criterion."

[ASK] "Why not? Can two triangles have the same three angles but be different sizes?"

[WAIT] Expected: "Yes, they would be similar but not congruent."

[SAY] "Exactly! AAA proves similarity, not congruence."

[SAY] "Second warning: SSA — Side-Side-Angle where the angle is NOT included — is NOT a valid criterion. This is called the ambiguous case because you can sometimes construct two different triangles with the same SSA."

[SAY] "Remember: the congruence symbol is ≅. And the ORDER of vertices matters! △ABC ≅ △PQR means A corresponds to P, B to Q, and C to R."

Connecting to Reflection (3 minutes)

[SAY] "Remember our work on reflection? When you reflect a triangle, the image is congruent to the original. But there's a difference."

[SAY] "If the image fits directly on the original WITHOUT flipping, we call it direct congruence."

[SAY] "If you need to FLIP the image to fit it on the original, we call it opposite congruence. Reflected triangles have opposite congruence."

[ASK] "In your anchor activity, did any pairs need flipping to match?"

[WAIT] Expected: "Yes, some pairs needed to be turned over."

[SAY] "Those pairs have opposite congruence — just like a reflection."

Phase 3: Practice and Application (10 minutes)

Worked Example: SAS Criterion (3 minutes)

[SAY] "Let's work through the textbook example together."

[SAY] "We have triangle ABC and triangle PQR. From the figure: AB = PR = 3 cm, BC = PQ = 8 cm, and angle B = angle P = 60 degrees."

[WRITE] "AB = PR = 3 cm ✔"

[WRITE] "BC = PQ = 8 cm ✔"

[WRITE] "∠B = ∠P = 60° ✔"

[ASK] "Is angle B the INCLUDED angle? Is it between sides AB and BC?"

[WAIT] Expected: "Yes."

[SAY] "Yes! Angle B is between AB and BC. So we have two sides and the included angle."

[WRITE] "Therefore, △ABC ≅ △PQR by SAS criterion."

[SAY] "Notice the vertex order: A corresponds to P, B corresponds to Q, C corresponds to R."

Student Practice: Identifying Criteria (4 minutes)

[SAY] "Now try these three problems. For each, determine if the triangles are congruent and state the criterion."

[WRITE] "Problem 1: △DEF: DE = 5, EF = 7, DF = 9. △GHI: GH = 5, HI = 7, GI = 9."

[WRITE] "Problem 2: △LMN and △XYZ: ∠L = ∠X = 50°, LM = XY = 6 cm, ∠M = ∠Y = 70°."

[WRITE] "Problem 3: △PQR right-angled at Q, PR = 10, QR = 6. △STU right-angled at T, SU = 10, TU = 6."

[SAY] "Work individually first, then check with your partner. 3 minutes."

[WAIT] 3 minutes.

[SAY] "Let’s check. Problem 1?"

[WAIT] Expected: "SSS — all three sides are equal."

[SAY] "Correct! Problem 2?"

[WAIT] Expected: "ASA — two angles and the included side."

[SAY] "Right! The side LM is between angles L and M. Problem 3?"

[WAIT] Expected: "RHS — right angle, hypotenuse, and one side."

[SAY] "Excellent! All correct."

Quick Misconception Check (3 minutes)

[ASK] "True or false: If two triangles have all three angles equal, they must be congruent."

[WAIT] Expected: "False — that's AAA, which only proves similarity."

[SAY] "Correct! AAA gives similarity, not congruence."

[ASK] "True or false: For SAS, the angle can be ANY angle in the triangle."

[WAIT] Expected: "False — it must be the included angle."

[SAY] "Right! The angle must be BETWEEN the two equal sides."

[ASK] "Can RHS be used for any triangle?"

[WAIT] Expected: "No, only for right-angled triangles."

[SAY] "Exactly. RHS requires a right angle."

Phase 4: Assessment — Exit Ticket (5 minutes)

[SAY] "For our exit ticket, answer these two questions on a separate piece of paper. You have 5 minutes."

[SAY] "Question 1: A(0, 4), B(−3, 0) and C(0, 2) are the coordinates of triangle ABC. Reflect the triangle over the mirror line x = 0. Prove that the triangle and its image are congruent and state the congruence criterion."

[SAY] "Hint: Use the distance formula to calculate the lengths of all sides of both triangles."

[SAY] "Question 2: Construct an equilateral triangle UVW with sides 6 cm. X is the midpoint of UW and VX is perpendicular to UW. Show that triangle UVX is congruent to triangle VWX. State the criterion."

[DO] Collect exit tickets as students finish.

Answer Key:

Question 1: Reflect over x = 0 (y-axis): A′(0,4), B′(3,0), C′(0,2). Calculate: AB = A′B′ = 5, BC = B′C′ = $\sqrt{13}$, AC = A′C′ = 2. All three sides equal ⟶ SSS criterion.

Question 2: UX = XW = 3 cm (midpoint), $\angle VXU = \angle VXW = 90°$, UV = VW = 6 cm (equilateral). Right angle + hypotenuse + side $\rightarrow$ RHS criterion.

Differentiation Notes

Struggling Learners:

Provide pre-cut triangles. Use colour-coded sides and angles. Start with SSS only. Provide a reference card with all five criteria. Allow physical overlaying. Pair with stronger students.

On-Level Learners:

Complete all practice problems. Write formal congruence statements with correct vertex order. Attempt coordinate geometry proof. Verify criterion choice by checking all conditions.

Advanced Learners:

Investigate why AAA fails. Explore the ambiguous SSA case. Prove altitude in isosceles triangle creates congruent triangles. Create own congruence proof with quadrilateral diagonals.

Post-Lesson Reflection

1. Did the hands-on tracing and cutting activity help students discover the criteria?

2. Were students able to distinguish between SSS, SAS, ASA, RHS, and AAS?

3. Did students understand the importance of "included" in SAS and ASA?

4. Were students able to write congruence statements with correct vertex correspondence?

5. Did students grasp direct vs. opposite congruence?

6. Were students able to apply criteria to the coordinate geometry problem?

7. What common errors arose (e.g., confusing SAS with SSA)?

8. What adjustments would improve the lesson for future delivery?