Grade 10 Mathematics Lesson Plan

Surface Area of Composite Solids

Strand:	**Measurement and Geometry**
Sub-Strand:	Surface Area of Composite Solids
Specific Learning Outcome:	Calculate the surface area of composite solids.
Duration:	40 minutes
Key Inquiry Question:	How do we determine the surface area and volume of solids? Why do we determine the surface area and volume of solids?
Learning Resources:	CBC Grade 10 textbooks, nets or templates for cylinders and cones/hemispheres, scissors, glue or tape, rulers, string, calculators

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Building tree models and calculating surface area
Structured Instruction	10 minutes	Formalizing composite solids surface area and addressing misconceptions
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Tree Model Surface Area

Objective: Students will discover how to calculate the surface area of composite solids by building a tree model using a cylinder (trunk) and cone or hemisphere (tree top), and calculating the total surface area while excluding hidden surfaces.

Materials Needed:

- Nets or templates for cylinders (tree trunk)

- Nets or templates for cones or hemispheres (tree top)
- Scissors
- Glue or tape
- Rulers
- String
- Calculators
- Worksheets for recording calculations

Steps for the Activity:

1. Step 1: Create the Model. Each student or group builds a tree model using a cylinder for the trunk and either a cone (pine tree) or a hemisphere (bushy tree) for the top. For example, a pine tree would be a cone on top of a cylinder.
2. Step 2: Label Dimensions. Measure and label your dimensions (or use pre-set values). For example: Radius of trunk = 3 cm, Height of trunk = 10 cm, Radius of cone = 3 cm, Slant height of cone = 5 cm.
3. Step 3: Surface Area Calculation. Surface area of cylinder: Lateral area 2 π r h, Bottom circle π r squared. Do NOT count the top circle — it is covered by the cone. Surface area of cone: Lateral area π r l. Do NOT count the base of the cone — it is attached to the trunk. Add all visible surfaces: S.A total = Lateral area of cone + Lateral area of cylinder + Base of cylinder.
4. Step 4: Sample Calculation. With the above values: Cylinder lateral: 2 π times (3 cm) times (10 cm) = 60 cm times 3.14 = 188.40 cm squared. Cylinder base: π times (3 cm) squared = 9 cm times 3.14 = 28.26 cm squared. Cone lateral: π (3 cm) times (5 cm) = 15 cm times 3.14 = 47.10 cm squared. S.A = 188.40 cm squared + 28.26 cm squared + 47.10 cm squared = 263.76 cm squared.

Discussion Questions:

5. Why do we not count the base of the cone or the top of the cylinder?
6. What happens if the cone is bigger than the cylinder?
7. How would the surface area change if the tree had branches modeled as small cylinders?
8. What surfaces are visible in a composite solid?
9. How do we identify hidden surfaces?

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand how to build the tree model.
- Ask probing questions: What surfaces are visible? Which surfaces are hidden?

- For struggling groups: Let us start by finding the lateral area of the cylinder. Then find the base area. What about the cone?
- For early finishers: How would the surface area change if you used a hemisphere instead of a cone?
- Guide students to articulate: The surface area of a composite solid equals the sum of all visible surfaces.
- Identify 2-3 groups with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing Composite Solids Surface Area and Addressing Misconceptions

After students have completed the anchor activity and shared their findings, the teacher formalizes the concept of composite solids and how to calculate their surface area.

Key Takeaway: What is a Composite Solid?

A solid is a three-dimensional shape. Solids are objects with three dimensions: Width, Length, and Height. They have surface area and volumes.

When two or more different solids are placed together, the result is composite solids. The surface area of composite solids can be found by adding areas of the parts of the solids, excluding hidden surfaces where the solids connect.

Key Principle:

Surface Area of Composite Solid = Sum of all visible surfaces

Important: Do NOT count surfaces that are hidden or covered by other parts of the composite solid.

Example: Cone on Top of Cylinder

- Visible surfaces:
    - Lateral surface of cone
    - Lateral surface of cylinder
    - Base of cylinder

- Hidden surfaces (DO NOT count):
    - Base of cone (attached to cylinder)
    - Top of cylinder (covered by cone)

Scaffolding Strategies to Address Misconceptions:

- Misconception: I should add the surface areas of both solids completely. Clarification: No, you must exclude hidden surfaces where the solids connect.
- Misconception: All surfaces are visible. Clarification: No, some surfaces are hidden when solids are joined together.
- Misconception: I can just use one formula. Clarification: No, you must calculate the surface area of each part separately, then add them together.
- Misconception: The order of calculation does not matter. Clarification: While the order does not affect the final answer, it is best to work systematically: identify all visible surfaces, calculate each area, then sum them.

Phase 3: Practice and Application (10 minutes)

Worked Example:

Find the surface area of a composite solid made of a cone on top of a cylinder. Leave your answer in m squared.

Given: Cone radius = 40 cm, Cone height = 30 cm, Cylinder radius = 40 cm, Cylinder height = 50 cm

Solution:

Surface Area of Cone:

CSA cone = π r times (l = square root of (r squared + h squared))

= 3.14 times (40 cm) times square root of ((40) squared cm + (30) squared cm)

= 3.14 times 40 cm times 50 cm

= 125.60 times 50

= 6,280 cm squared

Curved Surface Area of Cylinder:

CSA cylinder = 2 π r times h

= 2 times 3.14 times (40 cm) times 50 cm

= 2 times 3.14 times 40 cm times 50 cm

= 251.20 cm squared times 50 cm

= 12,560 cm squared

Base Area of Cylinder (since only the bottom is exposed):

Base Area = π r squared

= π times (40) squared cm

= 5024 cm squared

Total Surface Area = CSA cylinder + CSA cone + Base Area

= 12,560 cm squared + 6,280 cm squared + 5024 cm squared

= 23,864 cm squared

Converting to m squared:

1 m = 100 cm, so 1 m squared = 10,000 cm squared

= 23,864 cm squared / 10,000 cm squared

= 2.3864 m squared

= 2.39 m squared (to two d.p)

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. A right circular ice cream cone with a radius of 3 cm and a height of 12 cm holds a half scoop of ice cream in the shape of a hemisphere on top. If the ice cream melts completely, will it fit inside the cone? Show all calculations to justify your answer.

2. A lampshade is in the shape of a frustum of a cone. The top and bottom circular openings have diameters of 12 cm and 20 cm, respectively. If the slant height is 15 cm, find the lateral surface area of the lampshade.

3. Mogaka, a grade 10 student, was trying to sketch an image of an ice cream cone container with ice cream. Find the surface area of the sketched image.

4. A birthday cake has a cylindrical base of radius 10 cm and height 15 cm. The top is shaped like a hemisphere with the same radius. Find the total volume of the cake.

Answer Key:

1. Volume of cone = (1 / 3) π r squared h = (1 / 3) times 3.14 times 3 squared times 12 = 113.04 cm cubed. Volume of hemisphere = (2 / 3) π r cubed = (2 / 3) times 3.14 times 3 cubed = 56.52 cm cubed. Since 56.52 < 113.04, yes, the ice cream will fit inside the cone.

2. Lateral surface area of frustum = π (R + r) l = 3.14 times (10 + 6) times 15 = 753.6 cm squared.

3. Surface area = Lateral area of cone + Curved surface area of hemisphere. Calculate using given dimensions.

4. Volume = Volume of cylinder + Volume of hemisphere = π r squared h + (2 / 3) π r cubed = 3.14 times 10 squared times 15 + (2 / 3) times 3.14 times 10 cubed = 4710 + 2093.33 = 6803.33 cm cubed.

Differentiation Strategies

For Struggling Learners:

- Provide pre-made composite solid models.
- Use simple dimensions for initial practice.

- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators for calculations.
- Break down the process: Identify visible surfaces, calculate each area, add them together.

For On-Level Learners:

- Encourage students to verify their calculations with different composite solids.
- Ask students to explain which surfaces are hidden and why.
- Provide mixed practice with different combinations of solids.
- Challenge students to design their own composite solids.

For Advanced Learners:

- Challenge students to work with three or more solids combined.
- Explore real-world applications: buildings, monuments, containers, toys.
- Investigate optimization problems: Given a fixed surface area, what dimensions maximize the volume?
- Apply the concept to irregular composite solids.
- Design and build their own composite solid models with chosen dimensions.

Extension Activity

Real-World Application: Designing Composite Solid Objects

Work in groups

Situation: Design a composite solid object for a real-world purpose (e.g., water tank, monument, toy, container, building).

Tasks:

10. Choose a real-world object that can be modeled as a composite solid.
11. Identify the basic solids that make up your object (e.g., cylinder + cone, cube + pyramid, hemisphere + cylinder).
12. Choose dimensions for your composite solid.
13. Calculate the surface area of your composite solid, excluding hidden surfaces.

14. If the object needs to be painted, calculate the total area to be painted and the cost (assume Ksh 10 per square cm).
15. Present your findings with diagrams, measurements, and calculations.

Key Takeaway:

Students should understand how the surface area of composite solids is used in real-world contexts such as architecture (buildings, monuments), manufacturing (containers, toys), engineering (water tanks, structures), and design (cakes, decorations).

Teacher Reflection Prompts

- Did students successfully build tree models and calculate surface area?
- Were students able to identify hidden surfaces?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand how to calculate surface area of composite solids?
- What adjustments would improve this lesson for future classes?