Step by step guide: Quadratic Identities

Grade 10 Mathematics | 40-Minute Lesson

Before Class Begins

Preparation Checklist:

- Write the three identities on the board (covered until Phase 2)
- Prepare group discussion prompts
- Prepare exit tickets for distribution
- Set timer for phase transitions
- Have worked examples ready

PHASE 1: Problem-Solving and Discovery (15 Minutes)

Opening (2 minutes)

[SAY]:

"Good morning/afternoon, class! Today we're going to discover some powerful shortcuts in algebra called QUADRATIC IDENTITIES. These are special formulas that make expanding and factoring much faster!"

[SAY]:

"Here's our key question: How do we apply the concept of quadratic equations? Let's explore together."

Anchor Activity Introduction (3 minutes)

[SAY]:

"Form groups of at least 4 people. Your task has two parts:

Part 1: Define and discuss these terms:
1. Quadratic identities
2. Difference of squares
3. Perfect squares
4. Factorization of quadratic expressions

Part 2: Copy and observe these three identities:
(i) $(a + b)^2 = a^2 + 2ab + b^2$
(ii) $(a - b)^2 = a^2 - 2ab + b^2$
(iii) $(a - b)(a + b) = a^2 - b^2$*"*

Group Work (7 minutes)

[SAY]:

"Discuss in your groups:
• Can you verify these identities by expanding?
• How do the identities help us solve expressions faster?
• What real-world applications might use these identities?

You have 6 minutes. Begin!"

[DO]: Walk around the room, observing group discussions.

[ASK probing questions as you circulate]:

- "Can you expand $(a + b)^2$ to verify the identity?"
- "What do you notice about the middle term?"
- "Why is $(a - b)(a + b)$ called the difference of squares?"
- "What happens to the middle terms when you expand $(a - b)(a + b)$?"
- "How is $(a + b)^2$ different from $(a - b)^2$?"

[TIME CHECK]: At 5 minutes, announce: "One more minute!"

Class Discussion (3 minutes)

[SAY]:

"Let's share what you discovered. What is an identity?"

[Expected answer]: "An equation that is true for all values of the variable."

[ASK]:

"When you expanded $(a - b)(a + b)$*, what happened to the middle terms?"*

[Expected answer]: "They cancelled out! We got $-ab + ab = 0$"

[TRANSITION]:

"Excellent! Let me formalize these three powerful identities."

PHASE 2: Structured Instruction (10 Minutes)

The Three Quadratic Identities (6 minutes)

[REVEAL identities on board]:

[SAY]:

"Quadratic identities are special formulas that simplify our work with quadratic expressions. There are THREE essential identities:"

[WRITE Identity 1]:

"Identity 1: Perfect Square (Sum)
$(a + b)^2 = a^2 + 2ab + b^2$

The middle term is POSITIVE and equals TWICE the product of a and b."

[WRITE Identity 2]:

"Identity 2: Perfect Square (Difference)
$(a - b)^2 = a^2 - 2ab + b^2$

The middle term is NEGATIVE."

[WRITE Identity 3]:

"Identity 3: Difference of Squares
$(a + b)(a - b) = a^2 - b^2$

There is NO middle term! The middle terms cancel out."

Factoring with Difference of Squares (2 minutes)

[SAY]:

"The difference of squares works BOTH ways:

Expanding: $(a + b)(a - b) = a^2 - b^2$
Factoring: $a^2 - b^2 = (a + b)(a - b)$

When you see $a^2 - b^2$*, you can immediately factor it!"*

Addressing Misconceptions (2 minutes)

[SAY - IMPORTANT]:

"COMMON MISTAKE: $(a + b)^2$ is NOT equal to $a^2 + b^2$!

You MUST include the middle term 2ab!

Let me prove it: $(3 + 2)^2 = 5^2 = 25$
But $3^2 + 2^2 = 9 + 4 = 13 \neq 25$

The correct answer: $3^2 + 2(3)(2) + 2^2 = 9 + 12 + 4 = 25$ ✓ "

[TRANSITION]:

"Now let's practice using these identities!"

PHASE 3: Practice and Application (15 Minutes)

Worked Example (4 minutes)

[SAY]:

"Let's factor $x^2 - 16$ using the difference of squares."

[WRITE step by step]:

"Step 1: Recognize this is a difference of squares
$x^2 - 16$ is in the form $a^2 - b^2$

Step 2: Identify a and b
$a^2 = x^2$, so $a = x$
$b^2 = 16$, so $b = 4$

Step 3: Apply the identity
$x^2 - 16 = x^2 - 4^2 = (x - 4)(x + 4)$"

Guided Practice (5 minutes)

[SAY]:

"Try these with your partner:
a) Factor: $9y^2 - 25$
b) Expand: $(2x + 3)^2$"

[GIVE 4 minutes, then review]:

"a) $9y^2 - 25$
$= (3y)^2 - 5^2$
$= (3y - 5)(3y + 5)$

b) $(2x + 3)^2$
$= (2x)^2 + 2(2x)(3) + 3^2$
$= 4x^2 + 12x + 9$"

Independent Practice (6 minutes)

[SAY]:

"Now try these on your own:
a) Factor: $x^2 - 49$
b) Factor: $x^2 + 6x + 9$
c) Expand: $(3y - 2)^2$"

[GIVE 5 minutes, then quickly check]:

"a) $(x - 7)(x + 7)$
b) $(x + 3)^2$ [This is a perfect square trinomial!]
c) $9y^2 - 12y + 4$"

[TRANSITION]:

"Now I want to see what each of you has learned."

PHASE 4: Assessment / Checkpoint (8 Minutes)

Independent Work (5 minutes)

[DISPLAY questions]:

"1. Factor: $4a^2 - 9$

2. Expand: $(x + 7)^2$

3. A square garden has sides of length $(x + 5)$ meters. Write an expression for the area."

[SAY]:

"You have 5 minutes. Begin."

Collection and Closure (2 minutes)

[SAY]:

"Time's up. Please pass your exit tickets forward."

[COLLECT all tickets]

[SAY]:

"Today you learned the THREE quadratic identities:
• $(a + b)^2 = a^2 + 2ab + b^2$
• $(a - b)^2 = a^2 - 2ab + b^2$
• $(a + b)(a - b) = a^2 - b^2$

Remember: Always include the middle term in perfect squares!"

[SAY]:

"Great work today! For homework, practice more factoring problems."

Differentiation Notes

For Struggling Learners:

- Provide identity reference cards
- Use color coding to highlight a, b, and the middle term
- Start with numerical examples (e.g., $5^2 - 3^2 = (5-3)(5+3)$)
- Allow peer support during practice

For Advanced Learners:

[GIVE these extensions]:

- Factor $x^4 - 16$ completely
- Prove that $(a+b)^2 - (a-b)^2 = 4ab$
- Use difference of squares to calculate 51×49 mentally

Answer Key

Exit Ticket Answers:

1. $4a^2 - 9$: $(2a - 3)(2a + 3)$

2. $(x + 7)^2$: $x^2 + 14x + 49$

3. Area of garden: $(x + 5)^2 = x^2 + 10x + 25$ square meters

Extension Answers:

1. $x^4 - 16 = (x^2 - 4)(x^2 + 4) = (x - 2)(x + 2)(x^2 + 4)$

2. $(a+b)^2 - (a-b)^2 = a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) = 4ab$

3. $51 \times 49 = (50+1)(50-1) = 50^2 - 1 = 2500 - 1 = 2499$

Post-Lesson Reflection Prompts

1. What went well? Did students recognize the patterns in the identities?

2. What would I change? Was the misconception about $(a+b)^2$ addressed effectively?

3. Student Understanding: Could students distinguish between the three identities?

4. Next Steps: Which students need more practice with factoring?