Grade 10 Mathematics Lesson Plan

Area of Annular Sectors

Strand:	**Measurement and Geometry**
Sub-Strand:	Area of a Part of a Circle: Area of an Annular Sector
Specific Learning Outcome:	Determine the area of an annular sector in different situations. Apply the area of a part of a circle in real-life situations. Explore the use of the area of a part of a circle in real-life situations.
Duration:	40 minutes
Key Inquiry Question:	How do we use the concept of the area of a part of a circle in real life?
Learning Resources:	CBC Grade 10 textbooks, two circular paper cutouts (one smaller, one larger), scissors, protractor, ruler, colored markers, calculators

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Discovering annular sectors by cutting concentric sectors
Structured Instruction	10 minutes	Formalizing the annular sector formula and addressing misconceptions
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Discovering Annular Sectors

Objective: Students will discover the formula for finding the area of an annular sector by cutting out sectors from two concentric circles and exploring the relationship between the areas.

Materials Needed:

- Two circular paper cutouts (one smaller, one larger)
- Scissors
- Protractor
- Ruler
- Colored markers
- Calculator

Steps for the Activity:

1. Step 1: Take Two Circular Cutouts. Take two circular cutouts of different sizes but with the same center.
2. Step 2: Mark the Central Angle. Use a protractor to mark the same central angle θ on both circles.
3. Step 3: Cut Out the Sectors. Cut out the corresponding sectors from both circles.
4. Step 4: Observe the Shape. Place the smaller sector on the larger one and observe the remaining shape.
5. Step 5: Measure and Calculate. Measure and calculate the area of each sector using the formula and compare with your actual cutout.
6. Step 6: Discuss. Discuss with other groups how to get the area of the figure you have formed.

Discussion Questions:

7. What shape did you create when you placed the smaller sector on the larger one?
8. How did you calculate the area of this shape?
9. What is the relationship between the two sectors?
10. Can you write a formula for finding the area of an annular sector?

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand how to mark the same angle on both circles.
- Ask probing questions: What is the difference between the two sectors? How do we find the area of the remaining shape?
- For struggling groups: Let us find the area of the larger sector first. Then find the area of the smaller sector. What do we do next?
- For early finishers: Can you write a general formula for finding the area of any annular sector?

- Guide students to articulate: The area of an annular sector equals the area of the larger sector minus the area of the smaller sector.
- Identify 2-3 groups with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Annular Sector Formula and Addressing Misconceptions

After students have completed the anchor activity and shared their findings, the teacher formalizes the formula for finding the area of an annular sector.

Key Takeaway: What is an Annular Sector?

An annular sector is the region enclosed between two concentric sectors of a circle with different radii but the same central angle. It is similar to a sector but with a smaller sector removed from a larger one.

Having the knowledge of the area of a sector and the area of an annulus it is very easy to identify the area of an annular sector.

Formula:

Area of an annular sector is:

A = (θ / 360) times π (R squared minus r squared)

Where:

- θ is the central angle in degrees
- R is the outer radius
- r is the inner radius
- π approximately equals 3.142 or 22/7

Scaffolding Strategies to Address Misconceptions:

- Misconception: I can use different angles for the two sectors. Clarification: No, both sectors must have the same central angle to form an annular sector.
- Misconception: The two circles can have different centers. Clarification: No, the circles must be concentric (share the same center).

- Misconception: I subtract the radii first, then square. Clarification: No, you must square each radius first, then subtract.
- Misconception: An annular sector is the same as an annulus. Clarification: No, an annular sector is part of an annulus, defined by a central angle.

Phase 3: Practice and Application (10 minutes)

Worked Example 1: Wind Turbine Blade

A wind turbine blade sweeps through a central angle of 140 degrees. The length of the blade is 50 m, and the inner radius (distance from the pivot to the base of the blade) is 10 m. Find the swept area.

Solution:

The larger radius (R) = 50 m

The inner radius (r) = 10 m

Angle subtended = 140 degrees

A = (140 / 360) times π (50 squared minus 10 squared)

= (7 / 18) times (22 / 7) (2500 minus 100)

= (7 / 18) times (22 / 7) times 2400

= 2933.333333

approximately equals 2933.3 m squared

The turbine sweeps an area of approximately 2933.3 m squared.

Worked Example 2:

Find the area of an annular sector where $\theta = 60$ degrees, R = 12 cm, r = 8 cm (Use $\pi = 3.142$).

Solution:

A = (θ / 360) times π (R squared minus r squared)

A = (60 / 360) times π (12 squared minus 8 squared)

= (60 / 360) times 3.142 (144 minus 64)

= (1 / 6) times 3.142 times 80

= (1 / 3) times 3.142 times 40

= 41.89333333

approximately equals 41.89 cm squared

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. A clock minute hand moves 150 degrees in 25 minutes. The minute hand is 15 cm long, and the inner radius is 5 cm. Calculate the cleaned area.

2. A windshield wiper moves through 110 degrees. The blade is 45 cm long, and the pivot distance is 15 cm. Calculate the cleaned area.

3. A car wiper has: Outer radius = 40 cm, Inner radius r = 10 cm, Central angle = 120 degrees. Find the area cleaned by the wiper.

4. A mechanical arm sweeps through 180 degrees. The outer radius is 8 m, and the inner radius is 2 m. Determine the area covered.

Answer Key:

1. A = (150 / 360) times π (15 squared minus 5 squared) = (150 / 360) times (22 / 7) (225 minus 25) = (5 / 12) times (22 / 7) times 200 = 261.90 cm squared

2. A = (110 / 360) times π (45 squared minus 15 squared) = (110 / 360) times (22 / 7) (2025 minus 225) = (11 / 36) times (22 / 7) times 1800 = 1728.57 cm squared

3. A = (120 / 360) times π (40 squared minus 10 squared) = (120 / 360) times (22 / 7) (1600 minus 100) = (1 / 3) times (22 / 7) times 1500 = 1571.43 cm squared

4. A = (180 / 360) times π (8 squared minus 2 squared) = (180 / 360) times (22 / 7) (64 minus 4) = (1 / 2) times (22 / 7) times 60 = 94.29 m squared

Differentiation Strategies

For Struggling Learners:

- Provide pre-cut circular cutouts with sectors already outlined.
- Use simpler angles and radii for initial practice.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators.
- Break down the formula into steps: Find area of larger sector, find area of smaller sector, subtract.

For On-Level Learners:

- Encourage students to verify their formula with different angles and radii.
- Ask students to explain the relationship between annular sectors, sectors, and annuli.
- Provide mixed practice with different types of annular sector problems.
- Challenge students to solve problems where they need to find missing dimensions.

For Advanced Learners:

- Challenge students to derive the formula algebraically.
- Explore real-world applications: wind turbines, windshield wipers, clock hands, mechanical arms.
- Investigate the relationship between the angle and the area swept.
- Apply the concept to optimization problems: Given fixed radii, what angle maximizes the area?
- Solve problems involving multiple annular sectors or composite shapes.

Extension Activity

Real-World Application: Car Wiper Blade Investigation

Work in groups

Materials Needed:

- A car wiper blade (or a picture of one)
- A protractor
- A ruler
- A notebook and calculator

Tasks:

11. Find the dimensions of the following: The length of the wiper blade. The pivot point to the base of the wiper blade. The angle θ is the angle through which the wiper moves.
12. Use the formula: A = (θ / 360) times π (R squared minus r squared) to calculate the area cleaned by the wiper.
13. Ask students in your group to observe whether the wiper covers all parts of the windshield equally.

Key Takeaway:

Students should understand how the area of an annular sector is used in real-world contexts such as wind turbines, windshield wipers, clock mechanisms, and mechanical arms to calculate swept areas and optimize designs.

Teacher Reflection Prompts

- Did students successfully discover the annular sector formula through the anchor activity?
- Were students able to cut and place the sectors accurately?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand the relationship between annular sectors, sectors, and annuli?
- What adjustments would improve this lesson for future classes?