Step by Step Guide: Rotation and Congruence

Pre-Class Preparation Checklist

- Prepare graph paper (one sheet per student).
- Ensure rulers and protractors are available for each student.
- Prepare handouts with pre-drawn triangle ABC and point D.
- Optional: Prepare tracing paper for verification.
- Bring real-life objects or pictures: clock, wheel, rotating door, gears.
- Write the key definitions on chart paper: congruence, rotation, direct congruence.
- Prepare the properties table on chart paper or be ready to build it on the board.
- Have the digital textbook open: Section 2.3.4 Rotation and Congruence.
- Prepare exit ticket handouts with 3 questions.

Phase 1: Problem-Solving and Discovery (15 minutes)

Opening Hook (2 minutes)

[DO] Hold up a clock or show a picture of a clock.

[SAY] "When the clock hand moves from 12 to 3, it rotates 90 degrees. Does the hand change its length or shape?"

[WAIT] Expected: "No!" "It stays the same."

[SAY] "Exactly! The hand rotates, but it stays the same length and shape. Today, we are going to discover that rotation produces CONGRUENT figures — figures that are identical in size and shape."

[ASK] "Can you think of other objects that rotate but stay the same shape? Maybe a wheel? A door?"

[WAIT] Expected: "Wheel." "Rotating door." "Fan blades." "Gears."

[SAY] "Great examples! Let us investigate how rotation preserves shape and size."

Anchor Activity Launch (3 minutes)

[DO] Distribute graph paper, rulers, and protractors to each student.

[SAY] "Here is your challenge: You have triangle ABC and point D. Your job is to rotate the triangle −90° (90° clockwise) about point D."

[SAY] "Here is what you will do:"

[SAY] "Step 1: Copy triangle ABC and point D on your graph paper."

[SAY] "Step 2: Draw a dotted line from vertex A to point D."

[SAY] "Step 3: Place your protractor at line AD with the centre at D."

[SAY] "Step 4: Measure 90° clockwise from AD. (Negative angle means clockwise.)"

[SAY] "Step 5: Use your ruler to draw DA-prime such that AD = DA-prime. Mark point A-prime."

[SAY] "Step 6: Repeat for vertices B and C to find B-prime and C-prime."

[SAY] "Step 7: Connect A-prime, B-prime, and C-prime to form triangle A-primeB-primeC-prime."

[SAY] "Step 8: Measure the side lengths and angles of both triangles and record your findings."

[SAY] "Work in your groups. You have 8 minutes."

Student Work Time (8 minutes)

[DO] Circulate among groups.

[ASK] To a group drawing dotted lines: "Are you connecting each vertex to point D?"

[WAIT] Expected: "Yes!" or "We are not sure which point is D."

[SAY] "Point D is your centre of rotation. Draw a line from A to D, then from B to D, then from C to D."

[ASK] To another group using the protractor: "Are you measuring 90° clockwise from line AD?"

[WAIT] Expected: "Yes!" "We measured 90°."

[SAY] "Good! Now mark point A-prime at the same distance from D as A, but 90° clockwise."

[ASK] "Are the side lengths of the two triangles the same?"

[WAIT] Expected: "Yes!" "They are equal."

[SAY] "Excellent! What about the angles?"

[WAIT] Expected: "The angles are the same too."

[SAY] "Perfect! This tells us something important about rotation."

[DO] For struggling groups: "Let us focus on vertex A. Draw a line from A to D. Now place your protractor at D with the baseline on AD. Measure 90° clockwise. Mark A-prime at the same distance from D."

[DO] For early finishers: "What if you rotated 180° instead? Would the triangles still be congruent?"

Class Sharing (2 minutes)

[SAY] "Let's share your results. Are the side lengths of the two triangles the same?"

[WAIT] Expected: "Yes!" "AB = A-primeB-prime, BC = B-primeC-prime, AC = A-primeC-prime."

[SAY] "And what about the angles?"

[WAIT] Expected: "The angles are the same too."

[SAY] "Perfect! You've discovered that rotation preserves side lengths and angles. Let's formalise this concept."

Phase 2: Structured Instruction (10 minutes)

Formalising the Concept (4 minutes)

[SAY] "You discovered that the two triangles have the same side lengths and angles. Let's define these terms formally."

[WRITE] "CONGRUENCE refers to a relationship between two figures or objects whereby they are identical in size and shape."

[WRITE] "ROTATION is a type of transformation that repositions an object but preserves the shape and size of the object."

[SAY] "Thus, rotation produces congruent figures."

[WRITE] "ΔABC and ΔA-primeB-primeC-prime are similar in size and shape. Therefore, they are said to be DIRECTLY CONGRUENT."

Building the Properties Table (4 minutes)

[SAY] "Let's build a table of properties preserved under rotation."

[WRITE] Build the table on the board:

[WRITE] "Side Lengths — All corresponding sides have equal lengths: AB = A-primeB-prime, BC = B-primeC-prime, AC = A-primeC-prime."

[WRITE] "Angles — All corresponding angles remain equal: ∠A = ∠A-prime, ∠B = ∠B-prime, ∠C = ∠C-prime."

[WRITE] "Shape — The shape of the figure is preserved (triangle remains a triangle)."

[WRITE] "Size (Area) — The area of the figure remains the same."

[SAY] "Notice: Rotation is a RIGID TRANSFORMATION. It preserves distances and angles."

[ASK] "What changes when we rotate a figure?"

[WAIT] Expected: "The position." "The orientation."

[SAY] "Exactly! Only the position (orientation) changes. Everything else stays the same."

Important Notes (2 minutes)

[SAY] "Three important points:"

[WRITE] "1. Rotation is a rigid transformation (isometry) — it preserves distances and angles."

[WRITE] "2. Direct congruence means the figure and its image have the same orientation."

[WRITE] "3. Negative angles = clockwise; positive angles = anticlockwise."

[SAY] "Remember: Rotation preserves EVERYTHING except position."

Phase 3: Practice and Application (10 minutes)

Worked Example 1: Triangle Rotation −45° (3 minutes)

[SAY] "Let's work through the textbook example. Triangle ABC is mapped onto A-primeB-primeC-prime after a rotation of −45° and centre D."

[SAY] "What can we observe?"

[WAIT] Expected: "The triangles have the same shape and size."

[SAY] "Correct! Let's list the observations:"

[WRITE] "• ΔABC and ΔA-primeB-primeC-prime have the same shape and size."

[WRITE] "• The length of the corresponding sides are the same."

[WRITE] "• Every corresponding internal angle remains the same."

[SAY] "Therefore, ΔABC and ΔA-primeB-primeC-prime are said to be directly congruent."

Worked Example 2: Square Rotation (3 minutes)

[SAY] "A square ABCD is rotated 90° anticlockwise about its centre O to form A-primeB-primeC-primeD-prime. Are they congruent?"

[SAY] "Work with your partner. 2 minutes."

[WAIT] 2 minutes.

[SAY] "Let's check. Are they congruent?"

[WAIT] Expected: "Yes!" "All sides are equal." "All angles are 90°."

[SAY] "Excellent! Since rotation preserves distances and angles, all corresponding sides and angles are equal. Therefore, the two squares are congruent."

Real-Life Connection (2 minutes)

[DO] Show real-life objects: clock, wheel, rotating door.

[ASK] "Is the clock hand congruent at different positions?"

[WAIT] Expected: "Yes!" "It's the same length."

[SAY] "Exactly! Rotation preserves shape and size. The hand is congruent at all positions."

[ASK] "What about a wheel? Is it congruent at all rotations?"

[WAIT] Expected: "Yes!"

[SAY] "Perfect! Rotation and congruence are everywhere in real life."

Quick Check (2 minutes)

[ASK] "True or False: Rotation changes the size of a figure."

[WAIT] Expected: "False!"

[SAY] "Correct! Rotation does NOT change the size. It preserves everything except position."

Phase 4: Assessment — Exit Ticket (5 minutes)

[SAY] "For our exit ticket, answer these three questions. You have 5 minutes."

[SAY] "Question 1: Identify the axes of rotational symmetry and their order for: cylinder, rectangular pyramid, sphere, cube."

[SAY] "Question 2: True or False: Rotation changes the size of a figure. Explain."

[SAY] "Question 3: Triangle PQR is rotated 180° about point M to form P-primeQ-primeR-prime. Are they congruent? Why?"

[DO] Collect exit tickets as students finish.

Answer Key:

- 1. Cylinder: 1 main axis (infinite order) + infinite other axes (order 2). Rectangular pyramid: 0 axes (unless square base). Sphere: Infinite axes (infinite order). Cube: 9 axes (3 main order 4, 4 diagonal order 3, 6 edge order 2).
- 2. False. Rotation does NOT change size. It preserves distances, angles, shape, and size. Only position changes.
- 3. Yes, they are congruent. Rotation preserves side lengths and angles, so all corresponding parts are equal.

Differentiation Notes

Struggling Learners:

Provide pre-drawn triangles and points. Use tracing paper to physically rotate. Demonstrate first vertex together. Provide reference card with steps. Allow protractor templates. Pair with stronger students.

On-Level Learners:

Complete all practice problems. Rotate using ruler and protractor. Verify congruence by measuring. Identify axes and order for 3D shapes. Recognize rotation in real life.

Advanced Learners:

Investigate 360° rotation. Explore composition of rotations. Prove algebraically using coordinates. Design logo with order 6. Investigate rotation matrices.

Post-Lesson Reflection

1. Did the rotation activity help students discover congruence?

2. Were students able to use ruler and protractor accurately?

3. Did students understand that rotation preserves side lengths and angles?

4. Were students able to deduce that rotated figures are congruent?

5. Did students recognize rotation and congruence in real life?

6. What common errors arose (e.g., incorrect angle measurement)?

7. What adjustments would improve the lesson for future delivery?