Grade 10 Mathematics Lesson Plan

Equivalent Vectors

Strand:	Measurement and Geometry
Sub-Strand:	Vectors I
Specific Learning Outcome:	Identify equivalent vectors in different situations
Duration:	40 minutes
Key Inquiry Question:	How is Vectors I applied in day-to-day life?
Learning Resources:	CBC Grade 10 textbooks, graph paper, rulers, pencils, colored markers

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Discovering equivalent vectors
Structured Instruction	10 minutes	Formalizing equivalent vectors
Practice and Application	10 minutes	Worked examples and identification problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Discovering Equivalent Vectors

Objective: Students will plot points on graph paper, draw two vectors at different positions, and discover that vectors can be equivalent even if they start at different locations.

Materials Needed:

- Graph paper (one sheet per student)
- Rulers
- Pencils and colored markers
- Coordinate plane drawn on board or chart paper

Activity Steps (Activity 2.9.11 from textbook):

1. Step 1: Draw the x axis and y axis on the graph paper.
2. Step 2: Plot the points A(0, 4), B(3, 4), C(0, 2) and D(3, 2).
3. Step 3: Draw a line to connect point A and B, add an arrow pointing to point B.
4. Step 4: Draw a line to connect point C and D, add an arrow pointing to point D.
5. Step 5: Look at the two arrows you have drawn. Do they look like clones (exact copies) of each other?
6. Step 6: Imagine sliding the vector AB straight down without turning it. If you move point A so it sits exactly on top of point C, where does point B land? Does it land exactly on point D?
7. Step 7: Calculate the length of both vectors and compare the direction they are pointing. What two properties do vector AB and vector CD have in common?
8. Step 8: Since these vectors share the exact same properties, discuss with your group what relationship exists between them.

Expected Discovery:

Students should discover that:

- Both vectors have the same length (magnitude = 3 units)
- Both vectors point in the same direction (horizontal to the right)
- If you slide vector AB down without rotating, it perfectly overlaps vector CD
- Vectors can be equivalent even if they start at different positions
- Two conditions for equivalence: same magnitude and same direction

Guiding Questions:

9. What is the length of vector AB? What is the length of vector CD?
10. Do they point in the same direction?
11. Can you slide vector AB to overlap vector CD without rotating?
12. Do vectors need to start at the same point to be equivalent?
13. What two properties must vectors share to be equivalent?
14. If I draw another vector from (5, 0) to (8, 0), would it be equivalent to AB and CD?
15. If I draw a vector from (0, 0) to (3, 0), would it be equivalent to AB and CD?

Teacher Role During Discovery:

- Circulate among students, ensuring they plot points correctly.
- Ask probing questions: Do the vectors have the same length? Same direction?
- For struggling students: Let us measure together. Use your ruler to measure AB and CD.
- For early finishers: Draw another vector equivalent to AB. Can you draw it at a different position?

- Guide students to articulate: Equivalent vectors have the same magnitude and direction.
- Identify 2-3 students with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing Equivalent Vectors

After students have completed the anchor activity and shared their findings, the teacher formalizes the concept of equivalent vectors.

Key Takeaway 2.9.61:

Two or more vectors are said to be equivalent if they satisfy the following conditions:

1. They have same magnitude.

2. They point in the same direction.

Important Points:

- Equivalent vectors do NOT need to start at the same point
- Equivalent vectors do NOT need to end at the same point
- Equivalent vectors MUST have the same length (magnitude)
- Equivalent vectors MUST point in the same direction
- Position independence: Equivalent vectors can be located anywhere on the graph

Visual Test for Equivalence:

If you can slide one vector (without rotating) so it perfectly overlaps another, they are equivalent.

Scaffolding Strategies to Address Misconceptions:

- Misconception: Equivalent vectors must start at the same point. Clarification: No, they can start anywhere.
- Misconception: Vectors with same magnitude but opposite directions are equivalent. Clarification: No, direction must also be the same.
- Misconception: Vectors with same direction but different magnitudes are equivalent. Clarification: No, magnitude must also be the same.

- Misconception: Equivalent vectors are the same as equal vectors. Clarification: Yes, equivalent and equal mean the same thing for vectors.

Phase 3: Practice and Application (10 minutes)

Worked Example from Textbook:

Example 2.9.63: Using Figure 2.9.64, determine whether vector AB and DC are equivalent.

Solution:

Step 1: Calculate the magnitude of both vectors.

Step 2: Check if they point in the same direction.

Step 3: If both conditions are met, they are equivalent.

Additional Practice Problems:

Problem 1: Is it possible for two vectors to have the same direction but not to be equivalent? Explain your answer.

Solution: Yes, if they have different magnitudes. For example, vectors (2, 0) and (4, 0) point in the same direction but have different lengths.

Problem 2: Draw two vectors that have the same magnitude and direction but start at different points.

Solution: Draw vector from (0, 0) to (3, 2) and another from (5, 5) to (8, 7). Both have magnitude sqrt(13) and point in the same direction.

Problem 3: Are vectors (3, 4) and (3, 4) equivalent?

Solution: Yes, they have the same magnitude (5) and same direction.

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. Define equivalent vectors in your own words.

2. Draw two equivalent vectors on a coordinate plane that start at different points.

3. Are vectors from A(1, 2) to B(4, 6) and from C(0, 0) to D(3, 4) equivalent? Explain.

4. Can two vectors have the same magnitude but not be equivalent? Give an example.

Answer Key:

1. Equivalent vectors have the same magnitude and direction, but can start at different positions.

2. Example: Vector from (0, 0) to (2, 3) and vector from (5, 5) to (7, 8).

3. Yes, both have magnitude 5 and point in the same direction (3 right, 4 up).

4. Yes, vectors (2, 0) and (0, 2) have the same magnitude (2) but point in different directions.

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn vectors with labeled magnitudes and directions.
- Use color coding: same color for equivalent vectors.
- Provide tracing paper to physically slide vectors and check overlap.
- Start with horizontal and vertical vectors only.
- Use physical arrows (cut from paper) to demonstrate sliding.
- Pair struggling students with confident problem solvers.

For On-Level Learners:

- Encourage students to create their own equivalent vector problems.
- Ask students to explain why position does not matter for equivalence.
- Provide mixed practice with diagonal vectors.

- Challenge students to find all equivalent vectors in a complex diagram.

For Advanced Learners:

- Explore equivalent vectors in three dimensions.
- Investigate how equivalent vectors relate to parallel lines.
- Apply equivalent vectors to physics problems (forces at different locations).
- Challenge: Prove that if two vectors are equivalent, their components are equal.
- Explore applications in computer graphics (identical transformations at different locations).

Extension Activity

Real-World Application: Forces in Physics

Work in pairs or small groups

Situation: Two people push a car with the same force in the same direction, but at different locations on the car.

Tasks:

16. Draw a simple diagram of a car (rectangle).
17. Person A pushes at the front with a force of 100 N to the right. Draw this force vector.
18. Person B pushes at the back with a force of 100 N to the right. Draw this force vector.
19. Are these two force vectors equivalent? Explain. (Answer: Yes, same magnitude and direction)
20. What is the total force on the car? (Answer: 200 N to the right)
21. If Person B pushes with 100 N to the left instead, are the vectors still equivalent? (Answer: No, opposite directions)
22. Draw another scenario: Two people pulling a rope in the same direction. Are their force vectors equivalent?
23. Present your findings to the class.

Real-World Applications of Equivalent Vectors:

- Physics: Forces applied at different locations on an object.
- Engineering: Parallel forces in structural analysis.
- Computer Graphics: Identical transformations at different screen positions.

- Navigation: Multiple ships traveling in the same direction at the same speed.
- Wind Patterns: Wind vectors at different locations with same speed and direction.

Teacher Reflection Prompts

- Did students successfully discover the conditions for equivalent vectors?
- Were students able to identify equivalent vectors correctly?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand that position does not matter for equivalence?
- What adjustments would improve this lesson for future classes?