Grade 10 Mathematics Lesson Plan

Area of Common Regions Between Circles

Strand:	**Measurement and Geometry**
Sub-Strand:	Area of a Part of a Circle: Area of Common Regions Between Circles
Specific Learning Outcome:	Determine the area of common region between two intersecting circles. Apply the area of a part of a circle in real-life situations. Explore the use of the area of a part of a circle in real-life situations.
Duration:	40 minutes
Key Inquiry Question:	How do we use the concept of the area of a part of a circle in real life?
Learning Resources:	CBC Grade 10 textbooks, compass, ruler, pencil, graph paper, calculator

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Discovering common regions by drawing overlapping circles
Structured Instruction	10 minutes	Formalizing the common region formula and addressing misconceptions
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Discovering Common Regions Between Circles

Objective: Students will discover the formula for finding the area of the common region between two intersecting circles by drawing overlapping circles and exploring the relationship between the segments.

Materials Needed:

- Compass
- Ruler
- Pencil
- Graph paper
- Calculator

Steps for the Activity:

1. Step 1: Draw First Circle. Use a compass to draw a circle with a radius of 5 cm centered at point O.
2. Step 2: Draw Second Circle. From a point 2 cm to the right of O, draw another circle with the same radius (5 cm). This should create an overlapping region.
3. Step 3: Name Intersection Points. Name the intersection points of the two circles as P and Q.
4. Step 4: Shade the Region. Lightly shade the overlapping region between the two circles.
5. Step 5: Find the Area. Find the area of the common region. How does the distance between centers affect the common area?
6. Step 6: Discuss. Discuss your result with other learners in class.

Discussion Questions:

7. What is the common region between two circles?
8. What shapes do you see in the common region?
9. How did you calculate the area of the common region?
10. How does the distance between centers affect the size of the common region?

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand how to draw overlapping circles accurately.
- Ask probing questions: What shapes make up the common region? How can we find the area of each shape?
- For struggling groups: Let us identify the two segments. How do we find the area of each segment? What do we do next?
- For early finishers: Can you write a general formula for finding the area of the common region?
- Guide students to articulate: The area of the common region equals the sum of the areas of the two segments.
- Identify 2-3 groups with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Common Region Formula and Addressing Misconceptions

After students have completed the anchor activity and shared their findings, the teacher formalizes the formula for finding the area of the common region between two intersecting circles.

Key Takeaway: What is a Common Region?

The common region between two intersecting circles refers to the overlapping area shared by both circles.

It is formed when two circles of different or equal radii intersect at two distinct points.

Formula:

The area of the common region can be found by:

11. 1. Calculating the area of the two circular segments formed by the chord joining the intersection points
12. 2. Sum the areas of the two segments

A = A segment 1 + A segment 2

Where each segment area = Area of sector minus Area of triangle

Scaffolding Strategies to Address Misconceptions:

- Misconception: The common region is a single segment. Clarification: No, the common region is made up of two segments, one from each circle.
- Misconception: I multiply the areas of the segments. Clarification: No, you add the areas of the two segments.
- Misconception: The two segments are always equal. Clarification: No, the segments are only equal if the circles have the same radius.
- Misconception: I can find the area without using trigonometry. Clarification: For most problems, you need trigonometry to find the angles and segment areas.

Phase 3: Practice and Application (10 minutes)

Worked Example:

Two circles of radii 8 cm and 6 cm with centres O1 and O2 respectively. The circles intersect at points A and B. The lines O1O2 and AB are perpendicular to each other. If the common chord AB is 9 cm, calculate the area of the shaded region.

Solution:

Step 1: Find the distances from centers to the chord.

From triangle AO1M:

O1M = square root of (8 squared minus 4.5 squared) = square root of 43.75 = 6.14 cm

From triangle AO2M:

O2M = square root of (6 squared minus 4.5 squared) = square root of 15.75 = 3.969 cm

Step 2: Find the area of segment AP1B.

sin angle AO2M = 4.5 / 6 = 0.75

angle AO2M = sin inverse of 0.75 = 48.59 degrees

angle AO2B = 2 times 48.59 = 97.18 degrees

Area of sector = (97.18 / 360) times 3.142 times 6 squared = 30.53 cm squared

Area of triangle = (1 / 2) times 9 times 3.969 = 17.86 cm squared

Area of segment AP1B = 30.53 minus 17.86 = 12.67 cm squared

Step 3: Find the area of segment AP2B.

sin angle AO1M = 4.5 / 8 = 0.5625

angle AO1M = sin inverse of 0.5625 = 34.23 degrees

angle AO1B = 2 times 34.23 = 68.46 degrees

Area of sector = (68.46 / 360) times 3.142 times 8 squared = 38.24 cm squared

Area of triangle = (1 / 2) times 9 times 6.614 = 29.76 cm squared

Area of segment AP2B = 38.24 minus 29.76 = 8.48 cm squared

Step 4: Find the total area.

Area of common region = 12.67 + 8.48 = 21.15 cm squared

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. Two circular traffic islands of radius 10 meters overlap so that the centers are 12 meters apart. The angle subtended at the center of each circle by the chord of intersection is 120 degrees. Find the area of the overlapping region.

Answer Key:

1. For each circle: Area of sector = (120 / 360) times π times 10 squared = 104.72 m squared. Using trigonometry to find the triangle area: Area of triangle = (1 / 2) times 10 times 10 times sin 120 degrees = 43.30 m squared. Area of segment = 104.72 minus 43.30 = 61.42 m squared. Total area of common region = 2 times 61.42 = 122.84 m squared.

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn overlapping circles with intersection points marked.
- Use circles with equal radii for initial practice.

- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators for trigonometric functions.
- Break down the formula into steps: Find segment 1 area, find segment 2 area, add.

For On-Level Learners:

- Encourage students to verify their formula with different circle sizes.
- Ask students to explain how the distance between centers affects the common area.
- Provide mixed practice with different types of overlapping circle problems.
- Challenge students to solve problems with circles of different radii.

For Advanced Learners:

- Challenge students to derive the formula algebraically.
- Explore real-world applications: Olympic symbol, traffic islands, Venn diagrams, planetary alignments.
- Investigate problems where the chord length is given instead of the angles.
- Apply the concept to three or more overlapping circles.
- Solve optimization problems: Given fixed radii, what distance between centers maximizes the common area?

Extension Activity

Real-World Application: Olympic Symbol Investigation

Work in groups

Situation: In the Olympic symbol, circles of radius 5 cm overlap, forming intersections. If the central angle corresponding to the common region is 90 degrees, find the area of the intersection.

Tasks:

13. Draw a diagram of two overlapping circles representing part of the Olympic symbol.
14. Calculate the area of the common region between two circles.
15. If the Olympic symbol has 5 circles with multiple intersections, estimate the total area of all intersections.

16. Research the actual dimensions of the Olympic symbol and compare with your calculations.
17. Present your findings with diagrams and calculations.

Key Takeaway:

Students should understand how the area of common regions between circles is used in real-world contexts such as the Olympic symbol, traffic planning (circular traffic islands), astronomy (planetary alignments), and data visualization (Venn diagrams).

Teacher Reflection Prompts

- Did students successfully discover the common region formula through the anchor activity?
- Were students able to draw overlapping circles and identify intersection points accurately?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand that the common region is made up of two segments?
- What adjustments would improve this lesson for future classes?