

Grade 10 Mathematics Lesson Plan

Area of Kites

Strand:	Measurement and Geometry
Sub-Strand:	Area of Polygons: Area of Quadrilaterals
Specific Learning Outcome:	Determine the area of quadrilaterals in different situations. Explore the area of polygons as used in real-life situations.
Duration:	40 minutes
Key Inquiry Question:	How do we work out the area of polygons?
Learning Resources:	CBC Grade 10 textbooks, graph paper, rulers, compasses, protractors, pencils, erasers

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Constructing a kite using geometry tools
Structured Instruction	10 minutes	Formalizing the formula: $\text{Area} = (1/2) \times d_1 \times d_2$
Practice and Application	10 minutes	Worked example using diagonals, perimeter, and trigonometry
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Constructing a Kite Using Geometry Tools

Students work in groups to construct a kite accurately using a ruler, compass, and protractor. They will discover the properties of a kite by measuring sides, angles, and diagonals.

Materials Needed:

- Graph paper or plain paper
- Ruler

- Compass
- Protractor
- Pencil and eraser

Steps for the Activity:

1. Step 1: Drawing the diagonal. Draw a vertical line of length 10 cm (this will be the longer diagonal, d_1). Label the midpoint of this line as O.
2. Step 2: Draw the Perpendicular Diagonal. At O, use a protractor to draw a perpendicular line. Mark 4 cm on each side of O (total shorter diagonal $d_2 = 8$ cm).
3. Step 3: Mark the Kite Vertices. Label the four points where the diagonals intersect as A, B, C, and D. Connect A to B, B to C, C to D and D to A.
4. Step 4: Check Properties. Measure adjacent sides to ensure two pairs are equal. Verify opposite angles (one pair should be equal). Confirm diagonals are perpendicular.
5. Step 5: Design your own kite patterns on graph paper and justify why their design would be aerodynamic and stable in the air.

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand how to use the compass and protractor correctly.
- Ask probing questions: What do you notice about the adjacent sides? Are the diagonals perpendicular?
- For struggling groups: Let us draw the first diagonal together. Make sure it is 10 cm long.
- For early finishers: Can you calculate the area of your kite using the diagonals?
- Guide students to articulate: Two pairs of adjacent sides are equal, and the diagonals are perpendicular.
- Identify 2-3 groups with accurate constructions to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Formula: $\text{Area} = \left(\frac{1}{2}\right) \times d_1 \times d_2$

After students have completed the anchor activity and shared their findings, the teacher formalizes the area formula for kites.

Key Takeaway:

A kite is a quadrilateral with two pairs of adjacent sides equal in length and one pair of opposite angles equal.

Properties of a Kite:

- Two pairs of adjacent sides are equal in length.
- One pair of opposite angles are equal.
- Diagonals are perpendicular (they intersect at 90 degrees).
- Since the diagonals are perpendicular, right-angled triangles are formed.
- We can use trigonometric ratios (sine, cosine, tangent) to find missing angles and side lengths.

Formula:

The area of a kite is given by:

$$\text{Area} = (1/2) \times d_1 \times d_2$$

where d_1 and d_2 are the lengths of the two diagonals.

Using Trigonometry with Kites:

If a kite has diagonals d_1 and d_2 , we can find interior angles using:

- $\sin \theta = \text{opposite side} / \text{hypotenuse}$
- $\cos \theta = \text{adjacent side} / \text{hypotenuse}$
- $\tan \theta = \text{opposite side} / \text{adjacent side}$

If a kite has diagonals 14 cm and 10 cm, then each right-angled triangle within the kite has:

$$\text{Base} = d_1 / 2 = 14 / 2 = 7 \text{ cm}$$

$$\text{Height} = d_2 / 2 = 10 / 2 = 5 \text{ cm}$$

The angle θ between the diagonal and a side can be found using:

$$\tan \theta = 5 / 7$$

$$\theta = \tan^{-1}(5/7) \text{ approximately } 35.5 \text{ degrees}$$

Scaffolding Strategies to Address Misconceptions:

- Misconception: All sides of a kite are equal. Clarification: No, only two pairs of adjacent sides are equal.
- Misconception: The diagonals of a kite are equal. Clarification: No, the diagonals are usually not equal, but they are perpendicular.
- Misconception: I can use the formula for a rhombus. Clarification: Yes, the formula is the same because both have perpendicular diagonals.
- Misconception: A kite is the same as a square. Clarification: No, a square has all sides equal and all angles equal, while a kite has only two pairs of adjacent sides equal.

Phase 3: Practice and Application (10 minutes)

Worked Example (Textbook Example 2.5.11):

A kite has diagonals of 16 cm and 12 cm.

- Find the area of the kite
- If one pair of adjacent sides is 10 cm, find the perimeter of the kite.
- Find the angles of the kite using trigonometry.

Solution:

- Find the Area of the Kite

Let $d_1 = 16$ cm and $d_2 = 12$ cm

$$\text{Area} = \left(\frac{1}{2}\right) \times d_1 \times d_2$$

$$\text{Area} = \left(\frac{1}{2}\right) \times 16 \text{ cm} \times 12 \text{ cm}$$

$$\text{Area} = 192 / 2 \text{ cm squared}$$

$$\text{Area} = 96 \text{ cm squared}$$

- Find the Perimeter of the Kite

The diagonals bisect each other at right angles, so each half-diagonal forms a right-angled triangle.

$$d_1 / 2 = 16 / 2 = 8 \text{ cm}$$

$$d_2 / 2 = 12 / 2 = 6 \text{ cm}$$

Using the Pythagoras Theorem:

$$a^2 + b^2 = c^2$$

$$8^2 + 6^2 = c^2$$

$$64 + 36 = c^2$$

$$c^2 = 100$$

$$c = \sqrt{100}$$

$$c = 10 \text{ cm}$$

Since a kite has two pairs of equal sides, the perimeter is:

$$P = 2(a + b) = 2(10 + 10) \text{ cm} = 40 \text{ cm}$$

c) Finding the Angles Using Trigonometry

Using trigonometry in the right-angled triangle:

$$\tan \theta = \text{Opposite} / \text{Adjacent} = 6 / 8 = 0.75$$

$$\theta = \tan^{-1}(0.75)$$

$$\theta = 36.87^\circ$$

A kite is a quadrilateral, meaning it has four angles.

The sum of the interior angles of any quadrilateral is given by the formula:

$$\text{Sum of Interior Angles} = (n - 2) \times 180^\circ \text{ where } n = 4 \text{ (since a kite has 4 sides)}$$

$$(4 - 2) \times 180^\circ = 2 \times 180^\circ = 360^\circ$$

Since the kite is symmetric, the larger angle (alpha) is:

$$2\theta + 2\alpha = 360 \text{ degrees}$$

$$2(36.87) \text{ degrees} + 2\alpha = 360 \text{ degrees}$$

$$73.74 \text{ degrees} + 2\alpha = 360 \text{ degrees}$$

$$2\alpha = 286.26 \text{ degrees}$$

$$\alpha = 143.13 \text{ degrees}$$

The angles of the kite are: 143.13 degrees, 143.13 degrees, 36.87 degrees, and 36.87 degrees

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. Find the area of a kite with diagonals 10 cm and 8 cm.
2. A playground has a kite-shaped design with diagonals measuring 15 meters and 12 meters. Find its area.
3. A rescue helicopter designates an emergency landing zone in the shape of a kite. The diagonals measure 60 m and 45 m. Calculate the available landing space.
4. Engineers are designing a kite-shaped solar panel with diagonals of 30 m and 18 m. What is the total solar-collecting area?
5. A relief team sets up a kite-shaped safe zone for disaster survivors. The diagonals measure 50 m and 40 m. What is the area available for the survivors?

Answer Key:

1. Area = $(1/2) \times 10 \times 8 = 40$ cm squared.
2. Area = $(1/2) \times 15 \times 12 = 90$ m squared.
3. Area = $(1/2) \times 60 \times 45 = 1,350$ m squared.

4. $\text{Area} = (1/2) \times 30 \times 18 = 270 \text{ m squared.}$

5. $\text{Area} = (1/2) \times 50 \times 40 = 1,000 \text{ m squared.}$

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn kites with diagonals already labeled.
- Use simpler numbers for diagonal lengths.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators.

For On-Level Learners:

- Encourage students to draw their own kites from word problems.
- Ask students to explain which formula they chose and why.
- Provide mixed practice with both finding area and finding missing dimensions.

For Advanced Learners:

- Challenge students to derive the formula themselves using the concept of perpendicular diagonals.
- Explore real-world applications: kite design, architecture, solar panel design.
- Investigate the relationship between the area of a kite and other quadrilaterals.
- Apply trigonometry to find all angles when given only the diagonals.

Extension Activity

Real-World Application: Designing Kite-Shaped Structures

Students work in groups to design a kite-shaped structure (solar panel, landing zone, playground, safe zone) and calculate its area.

Materials: Graph paper, rulers, protractors, calculators, colored pencils

Tasks:

6. Choose a real-world application that uses kite shapes (solar panel, landing zone, playground, safe zone, etc.).
7. Draw the kite on graph paper with appropriate dimensions.

8. Measure or specify the lengths of both diagonals.
9. Calculate the area using the formula $\text{Area} = (1/2) \times d_1 \times d_2$.
10. If applicable, use trigonometry to find the angles of the kite.
11. Present your findings to the class, explaining your design choices and calculations.

Key Takeaway:

Students should understand how the area formula for kites is used in real-world professions such as engineering, architecture, and disaster management to calculate areas of structures, landing zones, and safe zones.

Teacher Reflection Prompts

- Did students successfully construct the kite and identify the perpendicular diagonals in the anchor activity?
- Were students able to discover the properties of a kite by measuring sides and angles?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand when to use the kite formula versus other quadrilateral formulas?
- What adjustments would improve this lesson for future classes?