Step by step guide: Factoring Quadratic

Grade 10 Mathematics | 40-Minute Lesson

Before Class Begins

Preparation Checklist:

- Write the general form $ax^2 + bx + c$ on the board
- Prepare the four anchor activity expressions
- Prepare exit tickets for distribution
- Set timer for phase transitions
- Have worked examples ready

PHASE 1: Problem-Solving and Discovery (15 Minutes)

Opening (2 minutes)

[SAY]:

"Good morning/afternoon, class! Today we're going to learn about FACTORING QUADRATIC EXPRESSIONS. This is one of the most important skills in algebra—it helps us solve equations and simplify complex expressions!"

[SAY]:

"Here's our key question: How do we apply the concept of quadratic equations? Factoring is the reverse of expanding—we break expressions into simpler parts."

Anchor Activity Introduction (3 minutes)

[SAY]:

"In your groups, I want you to define, discuss, and work on:

1. Factorization of quadratic expressions
2. Identifying common factors in expressions
3. Factorizing using the method of splitting the middle term
4. Recognizing the difference between factoring by grouping and simple factoring

Copy and try to factorize these expressions:
(i) x² + 5x + 6
(ii) x² − 7x + 12
(iii) 3x² − 15x
(iv) x² + 4x + 3x + 12"

Group Work (7 minutes)
[SAY]:

"As you work, discuss:
• What approaches can you use to factor these expressions?
• How does factoring help in solving quadratic equations?
• What challenges do you face when factoring?
• Can you think of real-world scenarios where factoring is useful?

You have 6 minutes. Begin!"

[DO]: Walk around the room, observing group discussions.

[ASK probing questions as you circulate]:

- "What two numbers multiply to give the constant and add to give the coefficient of x?"
- "Can you identify a common factor first?"
- "How do you know when to use grouping?"
- "How can you verify your answer?"
- "What pattern do you see in expression (iii)?"

[TIME CHECK]: At 5 minutes, announce: "One more minute!"

Class Discussion (3 minutes)
[SAY]:

"Let's share what you discovered. How did you approach factoring x² + 5x + 6?"

[Expected answer]: "We looked for two numbers that multiply to 6 and add to 5."

[ASK]:

"What about 3x² − 15x? How is this different?"

[Expected answer]: "We can take out a common factor of 3x first!"

[TRANSITION]:

"Excellent! Let me formalize these factoring methods."

PHASE 2: Structured Instruction (10 Minutes)

The ac-Method (5 minutes)
[SAY]:

"Factoring quadratic expressions means writing them as a product of two binomials:

ax² + bx + c = (px + m)(qx + n)"

[WRITE on board]:

"The ac-Method:

For ax² + bx + c, find two numbers m and n such that:
i. m × n = ac (product of first and last coefficients)
ii. m + n = b (middle coefficient)

Then split the middle term and factor by grouping."

Three Factoring Methods (3 minutes)
[SAY]:

"There are THREE main factoring methods:

1. COMMON FACTOR: When all terms share a factor
  Example: 3x² − 15x = 3x(x − 5)

2. SIMPLE FACTORING: When a = 1
  Example: x² + 5x + 6 = (x + 2)(x + 3)

3. GROUPING: When a ≠ 1 or you have 4 terms
  Example: x² + 4x + 3x + 12 = (x + 4)(x + 3)"

Important Reminders (2 minutes)
[SAY - IMPORTANT]:

"ALWAYS:
1. Check for common factors FIRST
2. Verify your answer by expanding
3. Make sure you have the correct signs"

[TRANSITION]:

"Now let's practice factoring step by step!"

PHASE 3: Practice and Application (15 Minutes)

Worked Example (5 minutes)
[SAY]:

"Let's factor $x^2 + 5x + 6$ step by step."

[WRITE step by step]:

"Here $a = 1$, $b = 5$, $c = 6$

Step 1: Find m and n where $m \times n = ac$ and $m + n = b$
• $ac = 1 \times 6 = 6$
• Need: $m \times n = 6$ AND $m + n = 5$
• Numbers: 2 and 3 (since $2 \times 3 = 6$ and $2 + 3 = 5$)

Step 2: Rewrite the middle term
$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$

Step 3: Group the terms
$= (x^2 + 2x) + (3x + 6)$

Step 4: Factor each group
$= x(x + 2) + 3(x + 2)$

Step 5: Factor out the common binomial
$= (x + 2)(x + 3)$"

[SAY]:

"Let's verify: $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$ ✓ "

Guided Practice (5 minutes)
[SAY]:

"Try these with your partner:
a) Factor: $x^2 - 7x + 12$
b) Factor: $3x^2 - 15x$"

[GIVE 4 minutes, then review]:

"a) x² − 7x + 12

• Need: m × n = 12 and m + n = −7

• Numbers: −3 and −4

• Result: (x − 3)(x − 4)

b) 3x² − 15x

• Common factor: 3x

• Result: 3x(x − 5)"

Independent Practice (5 minutes)

[SAY]:

"Now try these on your own:
a) Factor: x² + 4x + 3x + 12
b) Factor: x² + 9x + 20"

[GIVE 4 minutes, then quickly check]:

"a) (x + 4)(x + 3)
b) (x + 4)(x + 5)"

[TRANSITION]:

"Now I want to see what each of you has learned."

PHASE 4: Assessment / Checkpoint (8 Minutes)

Independent Work (5 minutes)

[DISPLAY questions]:

"Expand the following:
(a) (4x + 5)²
(b) (x − 7)²
(c) (x + 2)(x + 3)"

[SAY]:

"You have 5 minutes. Begin."

Collection and Closure (2 minutes)

[SAY]:

"Time's up. Please pass your exit tickets forward."

[COLLECT all tickets]

[SAY]:

"Today you learned THREE factoring methods:
• Common Factor: Look for shared factors first
• Simple Factoring: Find m and n where m × n = c and m + n = b
• Grouping: Split the middle term and group

Remember: ALWAYS verify by expanding!"

[SAY]:

"Great work today! Practice makes perfect with factoring."

Differentiation Notes

For Struggling Learners:

- Provide factor pair charts
- Use color coding to highlight common factors
- Start with expressions where a = 1
- Allow calculators for checking arithmetic

For Advanced Learners:

[GIVE these extensions]:

- Factor: $2x^2 + 7x + 3 \rightarrow (2x + 1)(x + 3)$
- Factor: $6x^2 - 11x - 10 \rightarrow (3x + 2)(2x - 5)$
- Solve by factoring: $x^2 + 5x + 6 = 0 \rightarrow x = -2$ or $x = -3$

Answer Key

Exit Ticket Answers:
(a) $(4x + 5)^2$: $16x^2 + 40x + 25$

(b) $(x - 7)^2$: $x^2 - 14x + 49$

(c) $(x + 2)(x + 3)$: $x^2 + 5x + 6$

Post-Lesson Reflection Prompts

1. What went well? Did students understand the ac-method?

2. What would I change? Was the grouping technique clear?

3. Student Understanding: Could students find the correct factor pairs?

4. Next Steps: Which students need more practice with negative coefficients?