

# Step by step guide: Quadratic Identities

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## Grade 10 Mathematics | 40-Minute Lesson

### Before Class Begins

#### Preparation Checklist:

- Write the three identities on the board (covered until Phase 2)
- Prepare group discussion prompts
- Prepare exit tickets for distribution
- Set timer for phase transitions
- Have worked examples ready

### PHASE 1: Problem-Solving and Discovery (15 Minutes)

#### Opening (2 minutes)

##### [SAY]:

*"Good morning/afternoon, class! Today we're going to discover some powerful shortcuts in algebra called QUADRATIC IDENTITIES. These are special formulas that make expanding and factoring much faster!"*

##### [SAY]:

*"Here's our key question: How do we apply the concept of quadratic equations? Let's explore together."*

#### Anchor Activity Introduction (3 minutes)

##### [SAY]:

*"Form groups of at least 4 people. Your task has two parts:*

*Part 1: Define and discuss these terms:*

1. Quadratic identities
2. Difference of squares
3. Perfect squares
4. Factorization of quadratic expressions

*Part 2: Copy and observe these three identities:*

- (i)  $(a + b)^2 = a^2 + 2ab + b^2$
- (ii)  $(a - b)^2 = a^2 - 2ab + b^2$
- (iii)  $(a - b)(a + b) = a^2 - b^2$ "

### **Group Work (7 minutes)**

**[SAY]:**

*"Discuss in your groups:*

- *Can you verify these identities by expanding?*
- *How do the identities help us solve expressions faster?*
- *What real-world applications might use these identities?*

*You have 6 minutes. Begin!"*

**[DO]:** Walk around the room, observing group discussions.

**[ASK probing questions as you circulate]:**

- "Can you expand  $(a + b)^2$  to verify the identity?"
- "What do you notice about the middle term?"
- "Why is  $(a - b)(a + b)$  called the difference of squares?"
- "What happens to the middle terms when you expand  $(a - b)(a + b)$ ?"
- "How is  $(a + b)^2$  different from  $(a - b)^2$ ?"

**[TIME CHECK]:** At 5 minutes, announce: "One more minute!"

### **Class Discussion (3 minutes)**

**[SAY]:**

*"Let's share what you discovered. What is an identity?"*

**[Expected answer]:** "An equation that is true for all values of the variable."

**[ASK]:**

*"When you expanded  $(a - b)(a + b)$ , what happened to the middle terms?"*

**[Expected answer]:** "They cancelled out! We got  $-ab + ab = 0$ "

**[TRANSITION]:**

*"Excellent! Let me formalize these three powerful identities."*

## PHASE 2: Structured Instruction (10 Minutes)

### The Three Quadratic Identities (6 minutes)

[REVEAL identities on board]:

[SAY]:

"Quadratic identities are special formulas that simplify our work with quadratic expressions.  
There are THREE essential identities:"

[WRITE Identity 1]:

"Identity 1: Perfect Square (Sum)

$$(a + b)^2 = a^2 + 2ab + b^2$$

The middle term is POSITIVE and equals TWICE the product of a and b."

[WRITE Identity 2]:

"Identity 2: Perfect Square (Difference)

$$(a - b)^2 = a^2 - 2ab + b^2$$

The middle term is NEGATIVE."

[WRITE Identity 3]:

"Identity 3: Difference of Squares

$$(a + b)(a - b) = a^2 - b^2$$

There is NO middle term! The middle terms cancel out."

### Factoring with Difference of Squares (2 minutes)

[SAY]:

"The difference of squares works BOTH ways:

Expanding:  $(a + b)(a - b) = a^2 - b^2$

Factoring:  $a^2 - b^2 = (a + b)(a - b)$

When you see  $a^2 - b^2$ , you can immediately factor it!"

### Addressing Misconceptions (2 minutes)

[SAY - IMPORTANT]:

"COMMON MISTAKE:  $(a + b)^2$  is NOT equal to  $a^2 + b^2$ !"

You MUST include the middle term  $2ab$ !

Let me prove it:  $(3 + 2)^2 = 5^2 = 25$

But  $3^2 + 2^2 = 9 + 4 = 13 \neq 25$

The correct answer:  $3^2 + 2(3)(2) + 2^2 = 9 + 12 + 4 = 25 \checkmark$ "

#### [TRANSITION]:

"Now let's practice using these identities!"

### PHASE 3: Practice and Application (15 Minutes)

#### Worked Example (4 minutes)

##### [SAY]:

"Let's factor  $x^2 - 16$  using the difference of squares."

##### [WRITE step by step]:

"Step 1: Recognize this is a difference of squares

$x^2 - 16$  is in the form  $a^2 - b^2$

Step 2: Identify a and b

$a^2 = x^2$ , so  $a = x$

$b^2 = 16$ , so  $b = 4$

Step 3: Apply the identity

$x^2 - 16 = x^2 - 4^2 = (x - 4)(x + 4)"$

#### Guided Practice (5 minutes)

##### [SAY]:

"Try these with your partner:

a) Factor:  $9y^2 - 25$

b) Expand:  $(2x + 3)^2"$

##### [GIVE 4 minutes, then review]:

$$\begin{aligned}a) \quad & 9y^2 - 25 \\&= (3y)^2 - 5^2 \\&= (3y - 5)(3y + 5)\end{aligned}$$

$$\begin{aligned}b) \quad & (2x + 3)^2 \\&= (2x)^2 + 2(2x)(3) + 3^2 \\&= 4x^2 + 12x + 9\end{aligned}$$

### Independent Practice (6 minutes)

[SAY]:

"Now try these on your own:

- a) Factor:  $x^2 - 49$
- b) Factor:  $x^2 + 6x + 9$
- c) Expand:  $(3y - 2)^2$

[GIVE 5 minutes, then quickly check]:

- a)  $(x - 7)(x + 7)$
- b)  $(x + 3)^2$  [This is a perfect square trinomial!]
- c)  $9y^2 - 12y + 4$

[TRANSITION]:

"Now I want to see what each of you has learned."

### PHASE 4: Assessment / Checkpoint (8 Minutes)

#### Checkpoint exploration (5 minutes)

[DO] Project the digital textbook on the screen. Navigate to the "Checkpoint" section.

[SAY] "This is our digital mathematics textbook. It has something special called checkpoints. Watch what happens when I click this button..."

[DO] Click "Show new example question" on Checkpoint

[SAY] "See? A new number appeared! And if I click again..."

[DO] Click the button again to show randomization

[SAY] "A different number! This means you can practice with hundreds of different examples. The computer never runs out of problems to give you."

**[SAY]** "Now it's your turn. With your partner, open the digital textbook and find the checkpoint.

**[SAY]** Click "Show new example question" to load the problem

**[SAY]** Solve the displayed question

**[SAY]** Click "submit" to check your answer

**[SAY]** If incorrect, carefully read the feedback and analyse the error before trying a new question. The immediate feedback from checkpoint submissions allows students to identify and correct errors in real-time.

**[SAY]** Complete at least 5 questions

**[DO]** Circulate among pairs. Ask probing questions, for example, what patterns do you notice?

### Independent Work (5 minutes)

**[DISPLAY questions]:**

"1. Factor:  $4a^2 - 9$

2. Expand:  $(x + 7)^2$

3. A square garden has sides of length  $(x + 5)$  meters. Write an expression for the area."

**[SAY]:**

"You have 5 minutes. Begin."

### Collection and Closure (2 minutes)

**[SAY]:**

"Time's up. Please pass your exit tickets forward."

**[COLLECT all tickets]**

**[SAY]:**

"Today you learned the THREE quadratic identities:

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$

*Remember: Always include the middle term in perfect squares!"*

**[SAY]:**

*"Great work today! For homework, practice more factoring problems."*

## Differentiation Notes

### For Struggling Learners:

- Provide identity reference cards
- Use color coding to highlight a, b, and the middle term
- Start with numerical examples (e.g.,  $5^2 - 3^2 = (5-3)(5+3)$ )
- Allow peer support during practice

### For Advanced Learners:

#### [GIVE these extensions]:

- Factor  $x^4 - 16$  completely
- Prove that  $(a+b)^2 - (a-b)^2 = 4ab$
- Use difference of squares to calculate  $51 \times 49$  mentally

## Answer Key

### Exit Ticket Answers:

1.  $4a^2 - 9$ :  $(2a - 3)(2a + 3)$
2.  $(x + 7)^2$ :  $x^2 + 14x + 49$
3. Area of garden:  $(x + 5)^2 = x^2 + 10x + 25$  square meters

### Extension Answers:

1.  $x^4 - 16 = (x^2 - 4)(x^2 + 4) = (x - 2)(x + 2)(x^2 + 4)$
2.  $(a+b)^2 - (a-b)^2 = a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) = 4ab$
3.  $51 \times 49 = (50+1)(50-1) = 50^2 - 1 = 2500 - 1 = 2499$

## Post-Lesson Reflection Prompts

1. **What went well?** Did students recognize the patterns in the identities?
2. **What would I change?** Was the misconception about  $(a+b)^2$  addressed effectively?

**3. Student Understanding:** Could students distinguish between the three identities?

**4. Next Steps:** Which students need more practice with factoring?