Grade 10 Mathematics Lesson Plan

Surface Area of Frustums

Strand:	**Measurement and Geometry**
Sub-Strand:	Surface Area of a Frustum
Specific Learning Outcome:	Determine the surface area of prisms, pyramids, cones, frustums and spheres.
Duration:	40 minutes
Key Inquiry Question:	How do we determine the surface area and volume of solids? Why do we determine the surface area and volume of solids?
Learning Resources:	CBC Grade 10 textbooks, printable cone nets, paper cups, rulers, scissors, tape or glue, calculators

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Modeling a frustum and calculating surface area
Structured Instruction	10 minutes	Formalizing the frustum surface area formula and addressing misconceptions
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Modeling a Frustum Using Paper Cups or Cone Nets

Objective: Students will discover the formula for finding the surface area of a frustum by constructing a frustum model from paper cups or cone nets and calculating its surface area.

Materials Needed:

- Printable nets of a cone (to cut and create a frustum)
- Rulers or measuring tape
- Scissors

- Tape or glue
- Formula sheet
- Worksheets for sketching and calculations
- Paper cups or cone-shaped fruit juice glasses (optional)

Steps for the Activity:

1. Step 1: Understand What a Frustum Is. A frustum is formed when the top part of a cone is cut off parallel to the base. Surface area includes curved surface area (side) and area of both circular bases.
2. Step 2: Build the Frustum Model. Take the cone net and cut off the top part (smaller cone) parallel to the base. Assemble the remaining portion to form a frustum. Alternatively, use actual paper or plastic cups and measure directly.
3. Step 3: Label Dimensions. Radius of the larger base (R), Radius of the smaller top base (r), Slant height (l) of the frustum. If not provided, measure the height and use the Pythagorean theorem.
4. Step 4: Calculate Surface Area. Use the surface area formula: Total Surface Area = π (R + r) l + π R squared + π r squared. Where π (R + r) l is curved surface, π R squared is area of bottom base, π r squared is area of top base.
5. Step 5: Work Through Example. Given Top radius (r) = 3 cm, Bottom radius (R) = 5 cm, Slant height (l) = 6 cm. Surface Area = π (5 + 3)(6) + π (5 squared) + π (3 squared) = 3.14 times 8 times 6 + 3.14 times 25 + 3.14 times 9 = 150.72 + 78.50 + 28.26 = 257.48 cm squared.

Discussion Questions:

6. What is a frustum?
7. What happens when you cut the top part of a cone parallel to the base?
8. What surfaces make up a frustum?
9. What would happen to the surface area if the top radius increased?
10. Why is it necessary to measure the slant height, not the vertical height?
11. Can you find any real-life objects shaped like a frustum?

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand how to cut and assemble the frustum.
- Ask probing questions: What surfaces do you see? How can we find the area of each surface?

- For struggling groups: Let us start by finding the area of the bottom base. Then find the area of the top base. What about the curved surface?
- For early finishers: Given a full cone, how much surface area is lost when the top is cut off?
- Guide students to articulate: The surface area of a frustum equals the curved surface area plus the areas of both circular bases.
- Identify 2-3 groups with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Frustum Surface Area Formula and Addressing Misconceptions

After students have completed the anchor activity and shared their findings, the teacher formalizes the formula for finding the surface area of a frustum.

Key Takeaway: What is a Frustum?

A frustum is formed when a cone or pyramid is cut parallel to its base, removing the top portion. This results in a truncated shape with two parallel bases, one smaller than the other.

Real-World Examples:

- Lampshades
- Truncated cones in engineering
- Buckets
- Tunnels
- Cooling towers in power plants

Properties of a Frustum:

- Two Circular Bases: A frustum has a larger base and a smaller base (both circular)
- Slant Height l: The distance between the two bases along the side of the frustum
- Height h: The vertical distance between the two bases
- Curved Surface Area (CSA): The side surface that connects the two bases
- Total Surface Area (TSA): The sum of the CSA and the areas of the two circular bases

Formula:

Total Surface Area = π (R + r) l + π R squared + π r squared

Where:

- R is the radius of the larger base
- r is the radius of the smaller top base
- l is the slant height of the frustum

Components:

- π (R + r) l: Curved surface area
- π R squared: Area of bottom base
- π r squared: Area of top base

Scaffolding Strategies to Address Misconceptions:

- Misconception: A frustum is the same as a cone. Clarification: No, a frustum is formed when the top part of a cone is cut off parallel to the base.
- Misconception: I only need to find the curved surface area. Clarification: No, you must add the areas of both circular bases to the curved surface area.
- Misconception: The slant height and vertical height are the same. Clarification: No, the slant height is the distance along the side of the frustum, while the vertical height is the perpendicular distance between the two bases.
- Misconception: I can use the cone formula directly. Clarification: No, a frustum has two bases, so you must account for both the top and bottom circular areas.

Phase 3: Practice and Application (10 minutes)

Worked Example:

Find the surface area of a galvanized iron bucket.

Solution:

Complete the cone from which the bucket is made by adding a smaller cone of height x cm.

From the concept of similarity and enlargement:

R / r = H / h and (H - h) / (R - r) = h / r

x / 15 = (x + 20) / 20

20x = 15x + 300

300 = 5x

x = 60 cm

Surface area of frustum = Area of curved surface of bigger cone - Area of curved surface of smaller cone

= π R L - π r l

Surface area (Large) = (22 / 7) times 20 times square root of (80 squared + 20 squared)

= 5183.33 cm squared

Surface area (Small) = (22 / 7) times 15 times square root of (60 squared + 15 squared)

= 2915.62 cm squared

Difference in Surface Areas = 5183.33 - 2915.62

= 2267.71 cm squared

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. A frustum of a square pyramid has: Top square side length 4 m, Bottom square side length 6 m, Slant height (along one face) 5 m. Calculate the total surface area of the frustum.

2. A conical frustum has a bottom radius of 6 cm, no top (the top is flat), and a slant height of 10 cm. i. Explain the difference between the curved surface area and the total surface area of a frustum. ii. Find only the curved surface area of the frustum.

3. A frustum is formed by cutting a cone with a height of 24 cm into two parts. The smaller cone has a height of 9 cm. If the base radius of the original cone is 16 cm, calculate the total surface area of the frustum.

4. If the curved surface area of a frustum is 330 cm squared, the top radius is 5 cm, and the bottom radius is 10 cm, find the slant height of the frustum.

Answer Key:

1. For a square pyramid frustum: Lateral surface area = (1 / 2) times (perimeter of top + perimeter of bottom) times slant height = (1 / 2) times (16 + 24) times 5 = 100 m squared. Base areas = 4 squared + 6 squared = 16 + 36 = 52 m squared. Total = 152 m squared.

2. i. Curved surface area is only the side surface. Total surface area includes the curved surface plus both circular bases. ii. Curved surface area = π times 6 times 10 = 188.50 cm squared.

3. Using similarity: r / 16 = 9 / 24, so r = 6 cm. Find slant heights using Pythagorean theorem. Calculate using formula: Total Surface Area = π (R + r) l + π R squared + π r squared.

4. Curved surface area = π (R + r) l. 330 = 3.14 times (10 + 5) times l. 330 = 47.1 times l. l = 7 cm.

Differentiation Strategies

For Struggling Learners:

- Provide pre-cut frustum models for students to measure.
- Use frustums with simple dimensions for initial practice.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators for calculations.
- Break down the formula into steps: Find bottom base area, find top base area, find curved surface area, add.

For On-Level Learners:

- Encourage students to verify their formula with different frustum dimensions.
- Ask students to explain the difference between a frustum and a cone.
- Provide mixed practice with frustums with and without top bases.
- Challenge students to find the surface area when only height and radii are given.

For Advanced Learners:

- Challenge students to derive the formula using the concept of similarity and enlargement.
- Explore real-world applications: lampshades, buckets, cooling towers, tunnels.
- Investigate problems where the slant height must be calculated using the Pythagorean theorem.
- Apply the concept to composite solids involving frustums.
- Solve optimization problems: Given a fixed surface area, what dimensions maximize the volume?

Extension Activity

Real-World Application: Designing Frustum Cups

Work in groups

Situation: Design your own frustum cups with chosen dimensions for a school event.

Tasks:

12. Choose dimensions for your frustum cup (top radius, bottom radius, slant height).
13. Calculate the surface area of your cup using the frustum formula.
14. Determine how much material (paper or plastic) is needed to make 100 cups.
15. If material costs Ksh 2 per square cm, calculate the total cost.
16. Compare your design with classmates and discuss which design is most cost-effective.
17. Present your findings with diagrams, measurements, and calculations.

Key Takeaway:

Students should understand how the surface area of frustums is used in real-world contexts such as manufacturing (buckets, cups, lampshades), engineering (cooling towers, tunnels), and design (architectural elements).

Teacher Reflection Prompts

- Did students successfully construct frustum models and calculate surface area?
- Were students able to identify the surfaces of a frustum?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand the difference between a frustum and a cone?
- What adjustments would improve this lesson for future classes?