Step by step guide: Exploring Logarithms in Real-Life Applications

Grade 10 Mathematics | 40-Minute Lesson

Before Class Begins

Preparation Checklist:

- Display charts showing pH scale, Richter scale examples
- Ensure scientific calculators and logarithm tables are available
- Prepare exit tickets for distribution
- Write key formulas on the board: $pH = -\log[H^+]$, $R = \log(I/I_0)$
- Set timer for phase transitions

PHASE 1: Problem-Solving and Discovery (15 Minutes)

Opening (2 minutes)

[SAY]:

"Good morning/afternoon, class! Today we're going to discover how logarithms are used in the REAL WORLD. You might be surprised to learn that logarithms are everywhere—from measuring earthquakes to testing the acidity of your swimming pool!"

[SAY]:

"Here's our key question: How do we use real numbers in day-to-day activities? Let's find out!"

Individual Research Task (8 minutes)

[SAY]:

"I want each of you to explore how logarithms are used in different fields. Choose ONE area that interests you:

• Engineering: Signal processing, decibels
• Finance: Compound interest, investment growth
• Science: pH scale, radioactive decay

• Geology: Richter scale for earthquakes
• Astronomy: Stellar magnitude
• Medicine: Drug concentration in the body

Find at least ONE specific example and write down the formula. You have 7 minutes. Begin!"

[DO]: Walk around the room, guiding students and asking questions.

[ASK probing questions as you circulate]:

- "Why do you think scientists use logarithms for the pH scale?"
- "What happens to the Richter scale number when earthquake intensity increases 10 times?"
- "How do logarithms help with very large numbers in astronomy?"
- "What pattern do you see in these applications?"

Group Discussion (5 minutes)

[SAY]:

"Now share your findings with your group. Discuss:
• What common patterns do you see across different applications?
• Why are logarithms useful for these applications?
• How do logarithms help us work with very large or very small numbers?

You have 4 minutes."

[TIME CHECK]: At 3 minutes, announce: "One more minute!"

[TRANSITION]:

"Excellent research! Let me now formalize the key applications you discovered."

PHASE 2: Structured Instruction (10 Minutes)

Why Logarithms? (3 minutes)

[SAY]:

"Why do scientists and engineers love logarithms? Because they help us:

1. COMPRESS large ranges of numbers into manageable scales
2. SOLVE exponential equations—find unknown exponents
3. MODEL phenomena that grow or decay exponentially
4. SIMPLIFY multiplication into addition"

Key Applications (5 minutes)

[SAY]:

"Let me show you FOUR important applications:"

[WRITE on board]:

"1. pH SCALE (Chemistry)
Formula: $pH = -\log[H^+]$
Measures: Acidity or alkalinity
pH 7 is neutral, below 7 is acidic, above 7 is alkaline"

"2. RICHTER SCALE (Geology)
Formula: $R = \log(I/I_0)$
Measures: Earthquake intensity
Each unit increase = 10× more intense!"

"3. DECIBEL SCALE (Sound)
Formula: $dB = 10 \log(I/I_0)$
Measures: Sound intensity
Normal conversation ≈ 60 dB, rock concert ≈ 120 dB"

"4. POPULATION GROWTH
Formula: $P = P_0e^{rt}$
Use logarithms to solve for time t"

Important Properties (2 minutes)

[SAY]:

"Remember these properties for solving problems:

- $\log(ab) = \log(a) + \log(b)$
- $\log(a/b) = \log(a) - \log(b)$
- $\log(a^n) = n\cdot\log(a)$
- $\log(\sqrt{a}) = \frac{1}{2}\cdot\log(a)$"

[SAY - IMPORTANT]:

"Remember: The negative sign in $pH = -\log[H^+]$ is part of the formula. And each unit on the Richter scale means 10 times more intense!"

[TRANSITION]:

"Now let's apply these formulas to real problems!"

PHASE 3: Practice and Application (15 Minutes)

Problem 1: pH Calculation (4 minutes)

[SAY]:

"Let's find the pH of a solution where $[H^+] = 3.2 \times 10^{-4}$."

[WRITE step by step]:

"$pH = -\log[H^+]$
$pH = -\log(3.2 \times 10^{-4})$
$pH = -[\log(3.2) + \log(10^{-4})]$
$pH = -[0.5051 + (-4)]$
$pH = -[0.5051 - 4]$
$pH = -(-3.4949)$
$pH \approx 3.49$"

[ASK]:

"Is this solution acidic or alkaline?"

[Expected answer]: "Acidic, because $pH < 7$!"

Problem 2: Richter Scale (3 minutes)

[SAY]:

"An earthquake is 1000 times more intense than the reference. What's its magnitude?"

[WRITE]:

"$R = \log(I/I_0)$
$R = \log(1000)$
$R = \log(10^3)$
$R = 3$"

[SAY]:

"The magnitude is 3.0 on the Richter scale. Notice: $1000 = 10^3$, so the answer is simply 3!"

Problem 3: Compound Calculation (4 minutes)

[SAY]:

"Let's use logarithm tables to evaluate $\sqrt{[(6.28 \times 42.5) / 9.81]}$"

[WRITE step by step]:

"Let $x = \sqrt{[(6.28 \times 42.5) / 9.81]}$

$\log(x) = ½ \times \log[(6.28 \times 42.5) / 9.81]$
$\log(x) = ½ \times [\log(6.28) + \log(42.5) - \log(9.81)]$
$\log(x) = ½ \times [0.7980 + 1.6284 - 0.9917]$
$\log(x) = ½ \times 1.4347$
$\log(x) = 0.7174$
$x = \text{antilog}(0.7174) \approx 5.22$"

Guided Practice (4 minutes)

[SAY]:

"Try this with your partner: If a population doubles at a growth rate of 5% per year, how long does it take? Use $P = P_0e^{rt}$."

[GIVE 3 minutes, then review]:

"For doubling: $P = 2P_0$
$2P_0 = P_0e^{rt}$
$2 = e^{rt}$
$\ln(2) = rt$
$t = \ln(2)/r = 0.693/0.05 \approx 13.86$ *years"*

[TRANSITION]:

"Now I want to see what each of you has learned."

PHASE 4: Assessment / Checkpoint (8 Minutes)

Independent Work (5 minutes)

[DISPLAY questions]:

"1. Name two real-life applications of logarithms.
2. Find the pH if $[H^+] = 5 \times 10^{-6}$.
3. What is the Richter magnitude if $I = 10{,}000 \times I_0$?"

[SAY]:

"You have 5 minutes. Begin."

Collection and Closure (2 minutes)

[SAY]:

"Time's up. Please pass your exit tickets forward."

[COLLECT all tickets]

[SAY]:

"Today you discovered that logarithms are everywhere in the real world:
• pH scale measures acidity
• Richter scale measures earthquakes
• Decibel scale measures sound
• Population models predict growth

Logarithms help us work with numbers that span huge ranges—from tiny hydrogen ion concentrations to massive earthquake energies!"

[SAY]:

"Great work today! Keep looking for logarithms in the world around you."

Differentiation Notes

For Struggling Learners:

- Provide formula cards with examples
- Focus on one application (pH or Richter)
- Use calculators for verification
- Pair with stronger students during research

For Advanced Learners:

[GIVE these extensions]:

- Decibel problem: If conversation is 60 dB and a concert is 120 dB, how many times more intense is the concert? (Answer: 10^6 = 1,000,000 times)
- Half-life problem: If $t_{1/2}$ = 5 years, find decay constant λ. (Answer: λ = 0.1386/year)
- Create your own real-world logarithm problem

Answer Key

Exit Ticket Answers:

1. Two applications: pH scale, Richter scale, decibel scale, compound interest, radioactive decay, stellar magnitude (any two)

2. pH = $-\log(3.2 \times 10^{-4})$: pH = $-[0.5051 - 4] = 3.49$

3. R = log(1000): R = 3.0

4. $\sqrt{[(6.28 \times 42.5)/9.81]}$: ≈ 5.22

5. Doubling time at 5%: $t = \ln(2)/0.05 \approx 13.86$ years

Post-Lesson Reflection Prompts

1. What went well? Did students engage with the research activity?

2. What would I change? Were the applications relatable to students?

3. Student Understanding: Could students apply the formulas correctly?

4. Next Steps: Which applications need more practice?