Grade 10 Mathematics Lesson Plan

Independent Events

Strand:	Statistics and Probability
Sub-Strand:	Probability 1: Independent Events
Specific Learning Outcome:	Determine the probability of mutually exclusive and independent events, Determine the probability of independent events using tree diagrams
Duration:	40 minutes
Key Inquiry Questions:	How is probability applied in real life situations?
Learning Resources:	CBC Grade 10 textbooks, coins, dice, tree diagram templates, chart paper

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Club Membership Analysis

Objective: Students work in groups to explore the concept of independent events, distinguish them from mutually exclusive events, and calculate probabilities.

Work in groups to complete the following tasks:

Task 1: Define Independent Events

Based on your prior knowledge or intuition, write a definition of "independent events" in your own words.

Task 2: State One Example

Give one example of two events that are independent. Explain why one event does not affect the other.

Task 3: Club Membership Problem

A student can choose to join either the Science Club or the Drama Club, but not both.

a) If the probability of joining Science Club is 40% and Drama Club is 30%, what is the probability that a student joins either club?

b) Are these events mutually exclusive or independent? Explain.

Task 4: Compare and Discuss

Compare and discuss your answers with other groups.

Discussion prompts for teachers:

- What definition did your group write?
- What examples did you think of? Why are they independent?
- How did you calculate the probability in Task 3?
- Are the club events mutually exclusive, independent, or both?
- What's the difference between mutually exclusive and independent?
- Can events be both mutually exclusive AND independent?

Phase 2: Structured Instruction (10 minutes)

Key Takeaways

1. Definition of Independent Events

Two events are independent if the occurrence of one does not affect the probability of the other occurring.

2. Multiplication Rule for Independent Events

If A and B are independent events, then:

$P(A \text{ and } B) = P(A) \times P(B)$

$P(A \cap B) = P(A) \times P(B)$

Explanation: The probability of both events occurring together is the product of their individual probabilities.

3. Key Difference: Independent vs Mutually Exclusive

- Mutually Exclusive: Events cannot happen at the same time. $P(A \text{ and } B) = 0$
- Independent: One event does not affect the other. $P(A \text{ and } B) = P(A) \times P(B)$
- Note: Events can be mutually exclusive OR independent, but usually not both!

4. Real-Life Examples of Independent Events

- A student bringing lunch from home and another student buying lunch from cafeteria
- A student answering a question correctly in English class and another student dropping their pencil in science class
- Flipping a coin and rolling a die
- Rain falling and a person being late to work

Phase 3: Practice and Application (15 minutes)

Worked Example 3.2.15 (Coin and Die)

Problem: A coin is flipped, and a six-sided die is rolled. What is the probability of getting heads and rolling a 6?

Solution:

Step 1: Identify the sample spaces

Coin outcomes: {H, T}

Die outcomes: {1, 2, 3, 4, 5, 6}

Step 2: Calculate individual probabilities

$P(H) = 1/2$

$P(6) = 1/6$

Step 3: Check if events are independent

Flipping the coin does not affect the die roll, and vice versa. They are independent events.

Step 4: Apply the multiplication rule

$P(H \text{ and } 6) = P(H) \times P(6)$

$= 1/2 \times 1/6$

$= 1/12$

Answer: The probability of getting heads and a 6 is 1/12 or 8.33%

Worked Example 3.2.16 (Rain and Lateness)

Problem: The probability that it rains on a given day is 40%, and the probability that a person is late to work is 20%.

Let $P(R)$ = probability of rain, $P(L)$ = probability of being late

Let $P(R^c)$ = probability of no rain, $P(L^c)$ = probability of not being late

Tasks:

a) Find $P(R)$

b) Find $P(L)$

c) Find the probability that it rains and the person is late

d) Find the probability that it does not rain but the person is late

e) Find the probability that it rains or the person is late

Solution:

a) $P(R) = 0.4$ or 40%

b) $P(L) = 0.2$ or 20%

c) Since rain and being late are independent:

$P(R \cap L) = P(R) \times P(L)$

$= 0.4 \times 0.2 = 0.08$

Answer: 0.08 or 8%

d) First find $P(R^c) = 1 - P(R) = 1 - 0.4 = 0.6$

Since events are independent:

$P(R^c \cap L) = P(R^c) \times P(L)$

$= 0.6 \times 0.2 = 0.12$

Answer: 0.12 or 12%

e) Using the general addition rule:

$P(R \cup L) = P(R) + P(L) - P(R \cap L)$

$= 0.4 + 0.2 - 0.08 = 0.52$

Answer: 0.52 or 52%

Phase 4: Assessment (5 minutes)

Exit Ticket

1. A coin is tossed twice. What is the probability of getting heads on both tosses?

2. A die is rolled, and a coin is tossed. What is the probability of rolling a 6 on the die and getting tails on the coin?

3. A bakery in Makongeni produces cakes. The probability that a cake is decorated with chocolate icing is 0.7. If two cakes are made independently, what is the probability that both cakes are decorated with chocolate icing?

4. A seed has a 60% chance of germinating. If two seeds are planted independently, what is the probability that both seeds germinate?

Differentiation Strategies

For Struggling Learners:

- Use tree diagrams to visualize all possible outcomes and identify independent events.
- Start with simple two-event problems before moving to complex scenarios.
- Provide multiplication formula cards: P(A and B) = P(A) × P(B).
- Use concrete materials: actual coins and dice for hands-on experiments.
- Create comparison charts: Mutually Exclusive vs Independent.
- Work in pairs with peer support.
- Provide step-by-step templates for calculations.

For Advanced Students:

- Explore three or more independent events.
- Investigate conditional probability and how it relates to independence.
- Calculate probabilities involving complements of events.
- Combine independent events with mutually exclusive events in complex problems.
- Research real-world applications in genetics, quality control, or weather forecasting.
- Create their own word problems involving independent events.

Extension Activity: Tree Diagrams for Independent Events

Scenario: Use tree diagrams to visualize and calculate probabilities of independent events.

Tasks:

1. Experiment: Toss a coin twice. Draw a tree diagram showing all possible outcomes.
2. First branch: H or T (first toss). Second branches: H or T (second toss) from each first branch.
3. Label each branch with its probability (1/2 for each).
4. Calculate probability of each final outcome by multiplying along the branches.
5. Verify: P(HH) = 1/2 × 1/2 = 1/4. Do the same for HT, TH, TT.
6. Question: What is P(getting at least one head)? Use the tree diagram to find all favorable outcomes.
7. Extended challenge: Draw a tree diagram for rolling a die and tossing a coin. Calculate P(even number and heads).
8. Present findings: How do tree diagrams help visualize independent events?