Grade 10 Mathematics Lesson Plan

Magnitudes of Vectors

Strand:	Measurement and Geometry
Sub-Strand:	Vectors I
Specific Learning Outcome:	Work out magnitude of a vector in different situations
Duration:	40 minutes
Key Inquiry Question:	How is Vectors I applied in day-to-day life?
Learning Resources:	CBC Grade 10 textbooks, graph paper, rulers, pencils, colored markers, calculators

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Discovering vector magnitudes
Structured Instruction	10 minutes	Formalizing magnitude formula using Pythagorean theorem
Practice and Application	10 minutes	Worked examples and calculating magnitudes
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Discovering Vector Magnitudes

Objective: Students will plot points, draw vectors, measure lengths with a ruler, and discover the relationship between horizontal displacement, vertical displacement, and vector magnitude using the Pythagorean theorem.

Materials Needed:

- Graph paper (one sheet per student)
- Rulers with centimeter markings
- Pencils and colored markers
- Calculators
- Coordinate plane drawn on board or chart paper

Activity Steps (Activity 2.9.6 from textbook):

1. Step 1: Draw the x axis and y axis on the graph paper.
2. Step 2: Mark the coordinate (0,0) as the initial point O.
3. Step 3: From Point O, move 3 units to the right along the x axis and 4 units upward in the y axis. Mark this new position as Point A.
4. Step 4: Draw a directed line from point O to point A to represent OA.
5. Step 5: Use a ruler to measure the length of OA in centimeters.
6. Step 6: Analyze the relationship between the x displacement (3 units), y displacement (4 units), and the length of OA.
7. Step 7: Discuss and share your findings with your classmates.

Guiding Questions:

8. What is the horizontal displacement from O to A?
9. What is the vertical displacement from O to A?
10. What is the measured length of OA?
11. Do you see a pattern? Does 3^2 plus 4^2 equal the length squared?
12. What mathematical theorem does this remind you of?
13. How can we calculate the length of any vector if we know its horizontal and vertical components?

Teacher Role During Discovery:

- Circulate among students, ensuring they plot points correctly and measure accurately.
- Ask probing questions: What is the length you measured? How does it relate to 3 and 4?
- For struggling students: Let us check together. 3 squared is 9, 4 squared is 16. What is 9 plus 16?
- For early finishers: Can you try another vector, say from O to B(5,12)? What is its magnitude?
- Guide students to articulate: The length of the vector is the square root of the sum of the squares of its components.
- Identify 2-3 students with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing Vector Magnitude

After students have completed the anchor activity and shared their findings, the teacher formalizes the concept of vector magnitude.

Key Takeaway: What is Magnitude?

The magnitude of a vector AB is denoted as |AB|. The magnitude represents the distance between point A and point B, or the length of the vector.

Magnitude Formula:

If a vector has components (x, y), where x is the horizontal displacement and y is the vertical displacement, we determine the magnitude by applying the Pythagorean theorem:

|AB| equals square root of (x squared plus y squared)

Or written as: |AB| equals $\sqrt{(x^2 + y^2)}$

Important Note:

The magnitude of a vector is always positive since x and y components are squared, resulting in x^2 and y^2, both of which are non-negative. Even if x or y is negative, squaring makes it positive.

Scaffolding Strategies to Address Misconceptions:

- Misconception: Magnitude can be negative. Clarification: No, magnitude is always positive or zero because we square the components.
- Misconception: I just add x plus y to get magnitude. Clarification: No, you must square each component, add them, then take the square root.
- Misconception: The order of squaring matters. Clarification: No, x^2 plus y^2 equals y^2 plus x^2 (addition is commutative).
- Misconception: I do not need a calculator. Clarification: For most problems, you will need a calculator to find the square root.

Phase 3: Practice and Application (10 minutes)

Worked Examples from Textbook:

Example 2.9.25: Determine the magnitude of AB where the horizontal displacement is 7 and vertical displacement is 24.

Solution:

|AB| equals $\sqrt{(7)^2 + (24)^2}$

equals $\sqrt{49 + 576}$

equals $\sqrt{625}$

equals 25

Hence, the magnitude of AB is 25.

Example 2.9.27: Given that a equals (2, 4), b equals (negative 2, 2.5), c equals (6, negative 4) and r equals a plus 2b minus c. Find |r|

Solution:

r equals a plus 2b minus c

r equals (2, 4) plus 2(negative 2, 2.5) minus (6, negative 4)

r equals (2, 4) plus (negative 4, 5) minus (6, negative 4)

r equals (2 plus negative 4 minus 6, 4 plus 5 minus negative 4)

r equals (negative 8, 13)

|r| equals $\sqrt{(\text{negative } 8)^2 + 13^2}$

equals $\sqrt{64 + 169}$

equals $\sqrt{233}$

equals 15.26

Additional Practice Problems:

Problem 1: Find the magnitude of vector (negative 6, 8).

Solution: $\sqrt{((\text{negative } 6)^2 + 8^2)}$ equals $\sqrt{(36 + 64)}$ equals $\sqrt{100}$ equals 10

Problem 2: Find the magnitude of vector (8, 15).

Solution: $\sqrt{(8^2 + 15^2)}$ equals $\sqrt{(64 + 225)}$ equals $\sqrt{289}$ equals 17

Problem 3: Find the magnitude of vector (3, 7).

Solution: $\sqrt{(3^2 + 7^2)}$ equals $\sqrt{(9 + 49)}$ equals $\sqrt{58}$ approximately equals 7.62

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. Find the magnitude of vector (negative 5, 12).

2. A vector has horizontal displacement 9 and vertical displacement 12. What is its magnitude?

3. If vector p equals (4, negative 3) and vector q equals (1, 2), find the magnitude of p plus q.

Answer Key:

1. $\sqrt{((\text{negative } 5)^2 + 12^2)}$ equals $\sqrt{(25 + 144)}$ equals $\sqrt{169}$ equals 13

2. $\sqrt{(9^2 + 12^2)}$ equals $\sqrt{(81 + 144)}$ equals $\sqrt{225}$ equals 15

3. p plus q equals (5, negative 1), so |p plus q| equals $\sqrt{(5^2 + (\text{negative } 1)^2)}$ equals $\sqrt{26}$ approximately equals 5.10

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn coordinate planes with vectors already drawn.
- Use color coding: one color for horizontal, another for vertical, third for the vector.
- Provide step-by-step calculation templates with spaces to fill in.
- Allow use of calculators for all calculations.
- Start with vectors that have perfect square magnitudes (3-4-5, 5-12-13, etc.).
- Pair struggling students with confident problem solvers.

For On-Level Learners:

- Encourage students to create their own magnitude problems.
- Ask students to explain why magnitude is always positive.
- Provide mixed practice with different vector representations.
- Challenge students to find patterns in Pythagorean triples.

For Advanced Learners:

- Explore three-dimensional vector magnitudes: $|v|$ equals $\sqrt{(x^2 + y^2 + z^2)}$.
- Investigate unit vectors (vectors with magnitude 1).
- Apply magnitudes to real-world distance and speed problems.
- Challenge: If $|a|$ equals 5 and $|b|$ equals 3, what are the possible values of $|a$ plus $b|$?
- Explore the relationship between magnitude and direction.

Extension Activity

Real-World Application: Navigation and Distance

Work in pairs or small groups

Situation: You are a pilot flying an aircraft. Your flight path can be represented as vectors.

Tasks:

14. You fly vector a equals (120, 90) representing 120 km East and 90 km North. Calculate the straight-line distance you traveled.
15. Then you fly vector b equals (negative 50, 60) representing 50 km West and 60 km North. Calculate the distance of this leg.

16. What is your total displacement vector from the starting point? (Add a plus b)
17. What is the straight-line distance from your starting point to your final position?
18. If your average speed is 200 km per hour, how long did the entire journey take?
19. Create your own flight path scenario with at least three vectors.
20. Present your scenario and calculations to the class.

Real-World Applications of Vector Magnitudes:

- Navigation: Calculating straight-line distances between locations.
- Physics: Determining the speed of an object (magnitude of velocity vector).
- Engineering: Calculating the strength of forces acting on structures.
- Computer Graphics: Determining distances between objects in games and animations.
- Sports: Analyzing the speed and distance of a ball or athlete.

Teacher Reflection Prompts

- Did students successfully measure vector lengths and discover the Pythagorean relationship?
- Were students able to apply the magnitude formula correctly?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand why magnitude is always positive?
- What adjustments would improve this lesson for future classes?