Step by step guide: Formation of Quadratic Equations by Factorisations

Grade 10 Mathematics | 40-Minute Lesson

Before Class Begins

Preparation Checklist:

- Write the key formulas on the board
- Prepare root cards for each group (e.g., "Roots: x = 2 and x = 3")
- Prepare exit tickets for distribution
- Set timer for phase transitions

[WRITE on board before class]:

"FORMING QUADRATIC EQUATIONS:
If roots are x = r and x = s:
• Factors: (x − r)(x − s)
• Equation: (x − r)(x − s) = 0
• Expanded: $x^2 - (r + s)x + rs = 0$"

PHASE 1: Problem-Solving and Discovery (15 Minutes)

Opening (2 minutes)

[SAY]:

"Good morning/afternoon, class! We've learned how to SOLVE quadratic equations to find their roots. Today we're going to do the REVERSE - we'll start with roots and CREATE the quadratic equation!"

[SAY]:

"Our key question today is: How do we apply the concept of Quadratic equations? One important application is forming equations when we know the solutions."

Anchor Activity Introduction (3 minutes)

[SAY]:

"Today's challenge: Each group will receive a pair of roots. Your task is to:

1. Form a quadratic equation using those roots
2. Write your equation in factorized form
3. Swap your equation with another group
4. Solve the equation you receive and find the roots"

[DISTRIBUTE root cards to groups]:

• Group 1: Roots are $x = 2$ and $x = 3$

• Group 2: Roots are $x = -1$ and $x = 4$

• Group 3: Roots are $x = 0$ and $x = 5$

• Group 4: Roots are $x = -2$ and $x = -3$

Group Work (7 minutes)

[SAY]:

"You have 6 minutes to form your equation and swap with another group. Think about: If $x = 2$ is a root, what factor does that give you? Begin!"

[DO]: Walk around the room, observing group discussions.

[ASK probing questions as you circulate]:

- "If $x = 2$ is a root, what does $(x - 2)$ equal?"
- "How do you write the equation in factored form?"
- "What pattern do you notice between the roots and the factors?"
- "Can you verify your equation by substituting the roots?"

[TIME CHECK]: At 5 minutes, announce: "One more minute, then swap!"

Class Discussion (3 minutes)

[SAY]:

"Let's share discoveries. Group 1, what equation did you form from roots $x = 2$ and $x = 3$?"

[Expected answer]: "$(x - 2)(x - 3) = 0$ or $x^2 - 5x + 6 = 0$"

[ASK]:

"How did you know to use $(x - 2)$ and $(x - 3)$?"

[Expected answer]: "Because when x = 2, (x − 2) = 0, and when x = 3, (x − 3) = 0!"

[ASK]:

"What patterns did you notice?"

[Expected answer]: "The sum of roots equals the negative of the middle coefficient, and the product equals the constant!"

[TRANSITION]:

"Excellent! You've discovered the key relationship. Let me formalize this."

PHASE 2: Structured Instruction (10 Minutes)

Definitions (3 minutes)

[SAY]:

"A QUADRATIC EQUATION has the form $ax^2 + bx + c = 0$ where $a \neq 0$.

The FACTORISED FORM is $(x + p)(x + q) = 0$ where:
- *$p \times q = c$ (the constant term)*
- *$p + q = b$ (the coefficient of x)"*

The Zero Product Property (3 minutes)

[SAY]:

"The key principle is the ZERO PRODUCT PROPERTY:

If $A \times B = 0$, then either $A = 0$ OR $B = 0$.

So if $(x - r)(x - s) = 0$, then:
- *$x - r = 0 \rightarrow x = r$, OR*
- *$x - s = 0 \rightarrow x = s$"*

Forming Equations from Roots (3 minutes)

[WRITE on board]:

"FORMING EQUATIONS FROM ROOTS:

If the roots are $x = r$ and $x = s$:
1. Write factors: $(x - r)$ and $(x - s)$

2. Form equation: $(x - r)(x - s) = 0$
3. Expand: $x^2 - (r + s)x + rs = 0$

Note:
• Sum of roots = $r + s = -b/a$
• Product of roots = $rs = c/a$"

Misconception Alert (1 minute)

[SAY - IMPORTANT]:

"CAUTION! The equation MUST equal zero!

If you have $6x^2 + 12x$, you must first write:
$6x^2 + 12x = 0$

THEN factorise!"

[TRANSITION]:

"Now let's practice forming quadratic equations!"

PHASE 3: Practice and Application (15 Minutes)

Worked Example (5 minutes)

[SAY]:

"Let's form a quadratic equation from the expression $6x^2 + 12x$."

[WRITE step by step]:

"Step 1: Set the expression equal to 0
$6x^2 + 12x = 0$

Step 2: Identify the common factor
Both terms have 6x as a common factor

Step 3: Factor out the common factor
$6x^2 + 12x = 6x(x + 2)$

Step 4: Write the factored equation
$6x(x + 2) = 0$

The roots are:
$6x = 0 \rightarrow x = 0$
$x + 2 = 0 \rightarrow x = -2$"

[SAY]:

"Verify: $6(0)^2 + 12(0) = 0$ ✓
$6(-2)^2 + 12(-2) = 24 - 24 = 0$ ✓ "

Guided Practice (8 minutes)

[SAY]:

"Try these with your partner:
1. Expand $(x + 4)(x + 5)$ and write the equation
2. Factor $4x^2 + 8x$ and find the roots"

[GIVE 5 minutes, then review]:

"1. $(x + 4)(x + 5)$:
$= x^2 + 5x + 4x + 20$
$= x^2 + 9x + 20$
Equation: $x^2 + 9x + 20 = 0$
Roots: $x = -4$ or $x = -5$

2. $4x^2 + 8x = 0$:
$4x(x + 2) = 0$
Roots: $x = 0$ or $x = -2$"

[TRANSITION]:

"Now I want to see what each of you has learned."

PHASE 4: Assessment / Checkpoint (8 Minutes)

Checkpoint exploration (5 minutes)

[DO] Project the digital textbook on the screen. Navigate to the "Checkpoint" section.

[SAY] "This is our digital mathematics textbook. It has something special called checkpoints. Watch what happens when I click this button..."

[DO] Click "Show new example question" on Checkpoint

[SAY] "See? A new number appeared! And if I click again..."

[DO] Click the button again to show randomization

[SAY] "A different number! This means you can practice with hundreds of different examples. The computer never runs out of problems to give you."

[SAY] "Now it's your turn. With your partner, open the digital textbook and find the checkpoint.

[SAY] Click “Show new example question” to load the problem

[SAY] Solve the displayed question

[SAY] Click “submit” to check your answer

[SAY] If incorrect, carefully read the feedback and analyse the error before trying a new question. The immediate feedback from checkpoint submissions allows students to identify and correct errors in real-time.

[SAY] Complete at least 5 questions

[DO] Circulate among pairs. Ask probing questions, for example, what patterns do you notice?

Independent Work (5 minutes)

[DISPLAY questions]:

"1. Expand $(x + 3)^2$ and write the equation
2. Factor $3x^2 + 6x$ and find the roots"

[SAY]:

"You have 5 minutes. Show your working. Begin."

Collection and Closure (2 minutes)

[SAY]:

"Time's up. Please pass your exit tickets forward."

[COLLECT all tickets]

[SAY]:

"Today you learned how to FORM quadratic equations:

- *From roots: If $x = r$ and $x = s$, then $(x - r)(x - s) = 0$*
- *By factoring: Extract common factors, then set equal to zero*

• *Key relationship: Sum of roots = −b/a, Product = c/a*

Remember: The equation must always equal zero!

Great work today!"

Differentiation Notes

For Struggling Learners:

- Provide a step-by-step template
- Start with simple integer roots
- Use substitution to verify answers
- Allow visual diagrams

For Advanced Learners:

[GIVE these extensions]:

- Form equation with roots $x = ½$ and $x = −3 \longrightarrow 2x^2 + 5x − 3 = 0$
- Rectangle problem: $(x + 5)(x + 2) = 40 \longrightarrow x = 3$
- Sum = 7, Product = 12 $\longrightarrow x^2 − 7x + 12 = 0$

Answer Key

Exit Ticket Answers:

1. (x + 4)(x + 5):

Expanded: $x^2 + 9x + 20$

Equation: $x^2 + 9x + 20 = 0$

Roots: $x = −4$ or $x = −5$

2. $(x + 3)^2$:

Expanded: $x^2 + 6x + 9$

Equation: $x^2 + 6x + 9 = 0$

Root: $x = −3$ (repeated)

3. $4x^2 + 8x$:

Factored: $4x(x + 2) = 0$

Roots: $x = 0$ or $x = -2$

4. $3x^2 + 6x$:

Factored: $3x(x + 2) = 0$

Roots: $x = 0$ or $x = -2$

5. $(p - q)(p - q)$:

Expanded: $p^2 - 2pq + q^2$

Equation: $p^2 - 2pq + q^2 = 0$

Root: $p = q$ (repeated)

Post-Lesson Reflection Prompts

1. What went well? Did students discover the root-factor relationship?

2. What would I change? Was the group swap activity effective?

3. Student Understanding: Could students form equations from roots?

4. Next Steps: Which students need more practice?