Step by step guide: Factorisation Monic Quadratics

Grade 10 Mathematics | 40-Minute Lesson

Before Class Begins

Preparation Checklist:

- Write the general form $x^2 + bx + c$ on the board
- Prepare the four anchor activity expressions
- Prepare exit tickets for distribution
- Set timer for phase transitions
- Have worked examples ready

PHASE 1: Problem-Solving and Discovery (15 Minutes)

Opening (2 minutes)

[SAY]:

"Good morning/afternoon, class! Today we're going to learn about FACTORING QUADRATIC EXPRESSIONS. This is one of the most important skills in algebra—it helps us solve equations and simplify complex expressions!"

[SAY]:

"Here's our key question: How do we apply the concept of quadratic equations? Factoring is the reverse of expanding—we break expressions into simpler parts."

Anchor Activity Introduction (3 minutes)

[SAY]: *First, we are going to revise how to expand the product of binomials—this will help us reverse the process*

[WRITE] The following expression on the board, for students to expand

1. $(x + 3)(x + 4) = ?$

2. $(x - 6)(x - 5) = ?$

3. (x + 2)(x + 3) = ?

[Expected answer]:

1. (x + 3)(x + 4) = x^2 + 7x + 12
2. (x-6)(x-5) = x^2 - 11x + 30
3. (x+2)(x+3) = x^2 + 5x + 6

Group Work (7 minutes)

[SAY]: "We need to find a pattern between the numbers on the left(e.g. 3, 4 in the first equation) and the number on the right (E.g. 7, 12 in the second equation)."

[SAY]: In groups, fill out this table:

Factored Form	Expanded Form	Sum of Constants	Product of Constants
(x + 3)(x + 4)	$x^2 + 7x + 12$	3 + 4 = ?	3 × 4 = ?
(x − 6)(x − 5)	$x^2 - 11x + 30$	(−6) + (−5) = ?	(−6) × (−5) = ?
(x + 2)(x + 3)	?	2 + 3 = ?	2 × 3 = ?

You have 3 minutes. Begin!"

[DO]: Walk around the room, observing group discussions. Encourage students to see relationships between the sum/product of constants and the coefficients (e.g. 7, 12)

[When time is up, say]: Now try to REVERSE the process. Given an expanded form, we want to find the factored form.

For $x^2 + 5x + 6$, can you find two numbers that:

• ADD to give 5 (the coefficient of x)?

• MULTIPLY to give 6 (the constant term)?

You have 3 minutes

[TIME CHECK]: At 2 minutes, announce: "One more minute!"

Class Discussion (3 minutes)

[SAY]:

"Let's share what you discovered. How did you approach factoring $x^2 + 5x + 6$?"

[Expected answer]: "We looked for two numbers that multiply to 6 and add to 5."

[ASK]:

"What about $3x^2 - 15x$? How is this different?"

[Expected answer]: "We can take out a common factor of 3x first!"

[TRANSITION]:

"Excellent! Let me formalize these factoring methods."

"As you work, discuss:
• What approaches can you use to factor these expressions?
• How does factoring help in solving quadratic equations?
• What challenges do you face when factoring?
• Can you think of real-world scenarios where factoring is useful?

PHASE 2: Structured Instruction (10 Minutes)

The ac-Method (5 minutes)

[SAY]:

"Factoring quadratic expressions means writing them as a product of two binomials:

$x^2 + bx + c = (x + m)(x + n)$"

[WRITE on board]:

"The ac-Method:

For $x^2 + bx + c$, find two numbers m and n such that:
i. $m \times n = c$ (product of first and last coefficients)
ii. $m + n = b$ (middle coefficient)

Then split the middle term and factor by grouping."

Worked Example (5 minutes)

PHASE 3: Practice and Application (15 Minutes)

Worked Example (5 minutes)

[SAY]:

"Let's factor $x^2 + 5x + 6$ step by step."

[WRITE step by step]:

"Here $a = 1$, $b = 5$, $c = 6$

Step 1: Find m and n where $m \times n = ac$ and $m + n = b$
- *$ac = 1 \times 6 = 6$*
- *Need: $m \times n = 6$ AND $m + n = 5$*
- *Numbers: 2 and 3 (since $2 \times 3 = 6$ and $2 + 3 = 5$)*

Step 2: Rewrite the middle term
$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$

Step 3: Group the terms
$= (x^2 + 2x) + (3x + 6)$

Step 4: Factor each group
$= x(x + 2) + 3(x + 2)$

Step 5: Factor out the common binomial
$= (x + 2)(x + 3)$"

[SAY]:

"Let's verify: $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$ ✓ "

Worked Example 2 (5 minutes)

[**SAY]:** When there is no constant coefficient, we can factorise immediately by taking out an x, as well as any common factors

[ASK]: What is the common factor in 4x^2 + 6x?

[Expected answer]: 2x (correct if students just say x)

[SAY] Thus, we can factorise by taking out the 2x term:

[WRITE] 4x^2+6x = 2x(x+3)

Important Reminders (2 minutes)

[SAY - IMPORTANT]:

"ALWAYS:
1. Check for common factors FIRST
2. Verify your answer by expanding
3. Make sure you have the correct signs"

[TRANSITION]:

"Now let's practice factoring step by step!"

Guided Practice (5 minutes)

[SAY]:

"Try these with your partner:
a) Factor: $x^2 - 7x + 12$
b) Factor: $3x^2 - 15x$"

[GIVE 4 minutes, then review]:

"a) $x^2 - 7x + 12$
• Need: $m \times n = 12$ and $m + n = -7$
• Numbers: -3 and -4
• Result: $(x - 3)(x - 4)$

b) $3x^2 - 15x$
• Common factor: $3x$
• Result: $3x(x - 5)$"

Independent Practice (5 minutes)

[SAY]:

"Now try these on your own:
a) Factor: $x^2 + 4x + 3x + 12$
b) Factor: $x^2 + 9x + 20$"

[GIVE 4 minutes, then quickly check]:

"a) $(x + 4)(x + 3)$
b) $(x + 4)(x + 5)$"

[TRANSITION]:

"Now I want to see what each of you has learned."

[SAY]:

"Today you learned two factoring methods:
• Common Factor: Look for shared factors first
• Simple Factoring: When the leading coefficient is 1, Find m and n where $m \times n = c$ and $m + n = b$

Remember: ALWAYS verify by expanding!"

[SAY]:

"Great work today! Practice makes perfect with factoring."

PHASE 4: Assessment / Checkpoint (8 Minutes)

Checkpoint exploration (5 minutes)

[DO] Project the digital textbook on the screen. Navigate to the "Checkpoint" section.

[SAY] "This is our digital mathematics textbook. It has something special called checkpoints. Watch what happens when I click this button..."

[DO] Click "Show new example question" on Checkpoint

[SAY] "See? A new number appeared! And if I click again..."

[DO] Click the button again to show randomization

[SAY] "A different number! This means you can practice with hundreds of different examples. The computer never runs out of problems to give you."

[SAY] "Now it's your turn. With your partner, open the digital textbook and find the checkpoint.

[SAY] Click “Show new example question” to load the problem

[SAY] Solve the displayed question

[SAY] Click “submit” to check your answer

[SAY] If incorrect, carefully read the feedback and analyse the error before trying a new question. The immediate feedback from checkpoint submissions allows students to identify and correct errors in real-time.

[SAY] Complete at least 5 questions

[DO] Circulate among pairs. Ask probing questions, for example, what patterns do you notice?

Differentiation Notes

For Struggling Learners:

- Provide factor pair charts
- Use color coding to highlight common factors
- Start with expressions where a = 1
- Allow calculators for checking arithmetic

For Advanced Learners:

[GIVE these extensions]:

- Factor: $2x^2 + 7x + 3 \rightarrow (2x + 1)(x + 3)$
- Factor: $6x^2 - 11x - 10 \rightarrow (3x + 2)(2x - 5)$
- Solve by factoring: $x^2 + 5x + 6 = 0 \rightarrow x = -2$ or $x = -3$

Post-Lesson Reflection Prompts

1. What went well? Did students understand the ac-method?

2. What would I change? Was the grouping technique clear?

3. Student Understanding: Could students find the correct factor pairs?

4. Next Steps: Which students need more practice with negative coefficients?