# Step by Step Guide: Volume Scale Factor

## Pre-Class Preparation Checklist

- Ensure each pair has paper, rulers, and pencils for drawing cuboids.
- If available, prepare unit cubes (e.g., sugar cubes or linking cubes) for hands-on demonstration.
- Have the digital textbook section open: innodems.github.io/CBC-Grade-10-Maths/subsec-scale-factor.html
- Prepare printed handouts with the anchor activity instructions (steps a–i).
- Write on a card or slide: "VSF = (LSF)$^3$" and the complete scale factor summary table.
- Prepare the scale factor summary table (LSF = k, ASF = $k^2$, VSF = $k^3$) on a poster or slide.
- Have calculators available for cube root calculations.

## Phase 1: Problem-Solving and Discovery (15 minutes)

### Opening and Recall (2 minutes)

[SAY] "In our last lesson, we learned about area scale factor. Who can remind us: if the linear scale factor is k, what is the area scale factor?"

[WAIT] Expected: "k squared."

[SAY] "Excellent! The area scale factor is k squared. Now, shapes don't just have length and area — they also have VOLUME. What do you think happens to the volume when we scale a 3D object?"

[WAIT] Allow students to predict for 15 seconds.

[SAY] "Great guesses! Let's investigate with cuboids and find out."

### Anchor Activity Launch (3 minutes)

[SAY] "Work in pairs. Each pair needs paper, a ruler, and a pencil."

[DO] Distribute materials and display the instructions on the board or handout.

[SAY] "Step (a): Draw a cuboid with dimensions 8 cm by 5 cm by 4 cm. Label it Cuboid A."

[SAY] "Step (b): Draw another cuboid with dimensions 24 cm by 15 cm by 12 cm. Label it Cuboid B."

[SAY] "Step (c): Calculate the volume of each cuboid — that's length times width times height."

[SAY] "Step (d): Divide the volume of Cuboid B by the volume of Cuboid A."

[SAY] "Steps (e), (f), (g): For each dimension — length, width, and height — divide Cuboid B's measurement by Cuboid A's, then CUBE the result."

[SAY] "Step (h): Compare your answer from step (d) with steps (e), (f), and (g). What do you notice?"

[SAY] "Step (i): Discuss your findings with your partner and be ready to share."

**Student Work Time (8 minutes)**

[DO] Circulate among pairs. Check that students are calculating volumes correctly.

[ASK] To pairs calculating: "What is the volume of Cuboid A? Remember: length times width times height."

[DO] For struggling pairs: "8 times 5 is 40. Now 40 times 4 is...? Good, 160 cm cubed."

[ASK] To pairs who found the volume ratio: "What did you get when you divided 4,320 by 160?"

[ASK] "And when you took 24 divided by 8 and cubed it, what did you get?"

[ASK] "Are all four results the same? Why do you think that is?"

[ASK] To early finishers: "What is the ratio of the SURFACE AREAS of the two cuboids? How does that compare?"

[DO] Note which pairs have clear explanations for the sharing phase.

**Class Sharing (2 minutes)**

[SAY] "Let's hear from a few pairs. What did you find?"

[WAIT] Call on 2–3 pairs to share.

[SAY] "Cuboid A has volume 160 cm cubed. Cuboid B has volume 4,320 cm cubed. The volume ratio is 4,320 divided by 160, which equals 27."

[SAY] "The length ratio is 24 over 8, which is 3. Cubed: 3 cubed equals 27. Same for width and height!"

[SAY] "All four results are 27. The volume ratio equals the CUBE of the linear scale factor."

[ASK] "So when each dimension triples, does the volume triple?"

[WAIT] Expected: "No, it increases by 27 times!"

[SAY] "Exactly! The volume increases by 3 cubed, which is 27. This is the VOLUME SCALE FACTOR."

## Phase 2: Structured Instruction (10 minutes)

**Formalising the Volume Scale Factor (4 minutes)**

[SAY] "Let's formalise what you discovered. We call this the VOLUME SCALE FACTOR."

[WRITE] On the board: "Volume Scale Factor (VSF)"

[WRITE] "Definition: A volume scale factor is the cube of the linear scale factor, representing the ratio by which the volume of a scaled object changes compared to the original object."

[WRITE] "Key Rule: Volume Scale Factor = (Linear Scale Factor)$^3$"

[WRITE] "VSF = (LSF)$^3$"

[WRITE] "LSF = $\sqrt[3]{(VSF)}$"

[SAY] "In your investigation, the linear scale factor was 3. The volume scale factor was 3 cubed, which is 27."

[SAY] "To go from VSF back to LSF, we take the CUBE root — not the square root like we did for area."

**Complete Scale Factor Summary (3 minutes)**

[SAY] "Let me put all three scale factors together so you can see the complete picture."

[WRITE] Draw the summary table on the board:

[WRITE] "Length (1D): LSF = k"

[WRITE] "Area (2D): ASF = k$^2$"

[WRITE] "Volume (3D): VSF = k$^3$"

[SAY] "If k = 2: lengths double, areas multiply by 4, and volumes multiply by 8."

[SAY] "If k = 3: lengths triple, areas multiply by 9, and volumes multiply by 27."

[ASK] "If k = 10, what is the volume scale factor?"

[WAIT] Expected: "1,000."

[SAY] "Yes! 10 cubed is 1,000. The volume is 1,000 times larger."

**Addressing Misconceptions (3 minutes)**

[SAY] "Common mistake number one: confusing volume scale factor with linear scale factor. If LSF = 4, the VSF is NOT 4. It is 4 cubed, which is 64."

[SAY] "Common mistake number two: confusing volume scale factor with area scale factor. If LSF = 4, the ASF is 16 but the VSF is 64. They are different!"

[SAY] "Now the reverse. If the volume scale factor is 125, what is the linear scale factor?"

[WAIT] Expected: "5, because the cube root of 125 is 5."

[SAY] "Correct! And what would the area scale factor be?"

[WAIT] Expected: "25, because 5 squared is 25."

[SAY] "Perfect. You can always move between the three: take the cube root of VSF to get LSF, then square LSF to get ASF."

## Phase 3: Practice and Application (10 minutes)

**Worked Example 1: Similar Cylinders (3 minutes)**

[SAY] "Let's work through Problem 1 together. Two similar cylinders have heights 4 m and 5 m."

[SAY] "Part (a): Find the ratio of their volumes."

[WRITE] "LSF = 4/5"

[WRITE] "VSF = $(4/5)^3$ = 64/125"

[WRITE] "Ratio of volumes = 64 : 125"

[SAY] "Part (b): The smaller cylinder has volume 1,536 m cubed. Find the volume of the larger."

[WRITE] "Volume of smaller / Volume of larger = 64/125"

[WRITE] "1536 / Volume of larger = 64/125"

[WRITE] "Volume of larger = (125 × 1536) / 64 = 3,000 $m^3$"

[SAY] "The larger cylinder has volume 3,000 m cubed."

**Worked Example 2: Containers — Finding Heights and Areas (3 minutes)**

[SAY] "Problem 2 is more challenging. Two similar containers have volumes 288 cm cubed and 4,500 cm cubed."

[SAY] "Part (a): Find the ratio of their heights. We need to go from VOLUME back to LENGTH."

[WRITE] "VSF = 288/4500 = 8/125"

[WRITE] "LSF = $\sqrt[3]{8/125}$ = 2/5"

[WRITE] "Ratio of heights = 2 : 5"

[SAY] "Part (b): The smaller container has area 140 cm squared. Find the area of the larger."

[SAY] "Now we go from LSF to ASF."

[WRITE] "ASF = $(2/5)^2$ = 4/25"

[WRITE] "4/25 = 140 / Area of larger"

[WRITE] "Area of larger = (25 × 140) / 4 = 875 cm²"

[SAY] "Notice how we moved from volume to length to area — all using the same linear scale factor."


**Quick Check (4 minutes)**

[SAY] "Quick check before the exit ticket. If two similar cones have a linear scale factor of 1/3, and cone A has volume 150 cm cubed, what is the volume of cone B?"

[DO] Give students 1 minute to calculate.

[WRITE] "VSF = $(1/3)^3$ = 1/27"

[WRITE] "150 / Volume of B = 1/27"

[WRITE] "Volume of B = 27 × 150 = 4,050 cm³"

[SAY] "This is Example 2.1.32 from the digital textbook. You can practise more with the checkpoints online."

## Phase 4: Assessment — Exit Ticket (5 minutes)

[SAY] "For our exit ticket, answer these four questions on a separate piece of paper. You have 5 minutes."

[SAY] "Question 1: Two similar containers have heights of 6 cm and 9 cm. If the smaller holds 400 ml, what is the capacity of the larger container?"

[SAY] "Question 2: Two similar cans have volumes of 192 cm cubed and 648 cm cubed. If the smaller can has height 14 cm, find the height of the larger can."

[SAY] "Question 3: The ratio of corresponding sides of two similar rectangular tanks is 3 to 5. The smaller tank has volume 8 cm cubed. Calculate the volume of the larger tank."

[SAY] "Question 4: A small cube has side length 3 cm. A larger cube has sides 6 times longer. Part (a): Volume of the small cube? Part (b): Volume of the larger cube? Part (c): By what factor has the volume increased?"

[DO] Collect exit tickets as students finish.

### Answer Key:

- 1. LSF = 6/9 = 2/3. VSF = $(2/3)^3$ = 8/27. 400/Larger = 8/27. Larger = (27 × 400)/8 = 1,350 ml.
- 2. VSF = 192/648 = 8/27. LSF = $\sqrt[3]{(8/27)}$ = 2/3. 14/Larger height = 2/3. Larger height = 21 cm.
- 3. LSF = 3/5. VSF = $(3/5)^3$ = 27/125. 8/Larger = 27/125. Larger = (125 × 8)/27 ≈ 37.04 cm³.
- 4(a) Volume of small cube = $3^3$ = 27 cm³.
- 4(b) Larger side = 18 cm. Volume = $18^3$ = 5,832 cm³.
- 4(c) Factor = 5,832/27 = 216 = $6^3$.

## Differentiation Notes

**Struggling Learners:**

Provide a formula card with all three scale factors: LSF = k, ASF = $k^2$, VSF = $k^3$. Use physical unit cubes to build small and large cuboids so students can count and compare volumes. Start with integer scale factors (2, 3) before fractions. Walk through Problem 1 step-by-step before independent work.

**On-Level Learners:**

Complete all problems independently. Work through both forward (LSF → VSF → Volume) and reverse (Volume ratio → VSF → LSF) problems. Use digital textbook checkpoints 2.1.33–2.1.37 for additional practice. Convert between VSF, ASF, and LSF within problems.

**Advanced Learners:**

Complete the Extension Activity involving architectural models, map scales, photograph enlargements, and similar triangle shadow problems. Challenge: A swimming pool model has volume 0.5 litres and LSF = 50. Find the actual pool volume. (VSF = $50^3$ = 125,000. Actual = 125,000 × 0.5 = 62,500 litres.)

## Post-Lesson Reflection

1. Did students discover through the anchor activity that the volume ratio equals the cube of the linear scale factor?

2. Were students able to articulate the difference between linear, area, and volume scale factors?

3. How effectively did the cuboid comparison build on prior knowledge of area scale factor?

4. Did students grasp the reverse process: finding LSF from VSF by taking the cube root?

5. Were students able to convert between VSF, ASF, and LSF within a single problem?

6. What common misconceptions arose (e.g., confusing VSF with ASF)?

7. What adjustments would improve the lesson for future delivery?