

Step by step guide: Factorisation Of Quadratic Expressions

Grade 10 Mathematics | 40-Minute Lesson

Before Class Begins

Preparation Checklist:

- Write the key formula on the board: $x^2 + bx + c = (x + m)(x + n)$ where $m + n = b$ and $m \times n = c$
- Prepare factor pair reference cards for struggling learners
- Prepare exit tickets for distribution
- Set timer for phase transitions

[WRITE on board before class]:

"FACTORISING $x^2 + bx + c$:

Find m and n where:

- $m + n = b$
- $m \times n = c$

Then: $x^2 + bx + c = (x + m)(x + n)$ "

PHASE 1: Problem-Solving and Discovery (15 Minutes)

Opening (2 minutes)

[SAY]:

"Good morning/afternoon, class! Last lesson we learned how to EXPAND brackets like $(x + 3)(x + 4)$. Today we're going to learn the REVERSE - how to FACTORISE expressions like $x^2 + 7x + 12$ back into brackets!"

[SAY]:

"Our key question today is: How do we apply the concept of Quadratic equations? Factorisation is a crucial skill for solving these equations."

Anchor Activity Introduction (3 minutes)

[SAY]:

"In your groups, I want you to explore the connection between expanding and factorising."

Part A: First, EXPAND these expressions:

- $(x + 3)(x + 4)$
- $(x - 6)(x - 5)$
- $(x + 2)(x + 3)$

Part B: Then complete the table I'm giving you and look for a PATTERN."

[DISTRIBUTE table worksheets]

Group Work (7 minutes)

[SAY]:

"After completing the table, try the REVERSE challenge:

Given $x^2 + 5x + 6$, can you find two numbers that:

- ADD to give 5?
- MULTIPLY to give 6?

You have 6 minutes. Begin!"

[DO]: Walk around the room, observing group discussions.

[ASK probing questions as you circulate]:

- "What do you notice about the sum of the constants in the factors?"
- "What do you notice about the product of the constants?"
- "For $x^2 + 5x + 6$, what two numbers add to 5 and multiply to 6?"
- "How does this help you write the factorised form?"

[TIME CHECK]: At 5 minutes, announce: "One more minute!"

Class Discussion (3 minutes)

[SAY]:

"Let's share discoveries. When you expanded $(x + 3)(x + 4)$, what did you get?"

[Expected answer]: " $x^2 + 7x + 12$ "

[ASK]:

"What's $3 + 4$? And what's 3×4 ?"

[Expected answer]: "3 + 4 = 7 (the coefficient of x) and $3 \times 4 = 12$ (the constant term)!"

[ASK]:

"So for $x^2 + 5x + 6$, what two numbers add to 5 and multiply to 6?"

[Expected answer]: "2 and 3!"

[TRANSITION]:

"Excellent! You've discovered the key to factorisation. Let me formalize this."

PHASE 2: Structured Instruction (10 Minutes)

Definitions (3 minutes)

[SAY]:

"First, let's define our terms. A QUADRATIC EXPRESSION has the form $ax^2 + bx + c$ where:

- a is the coefficient of x^2
- b is the coefficient of x
- c is the constant term"

[SAY]:

"Today we focus on expressions where $a = 1$, so we have $x^2 + bx + c$."

The Factorisation Method (5 minutes)

[SAY]:

"Here's what you discovered:"

[WRITE on board]:

"To factorise $x^2 + bx + c$:

Find two numbers m and n where:

- $m + n = b$ (the coefficient of x)
- $m \times n = c$ (the constant term)

Then: $x^2 + bx + c = (x + m)(x + n)$ "

[SAY]:

"Let's verify with our examples:

$$(x + 3)(x + 4) = x^2 + 7x + 12$$

Check: $3 + 4 = 7 \checkmark$ and $3 \times 4 = 12 \checkmark$

$$(x - 6)(x - 5) = x^2 - 11x + 30$$

Check: $(-6) + (-5) = -11 \checkmark$ and $(-6) \times (-5) = 30 \checkmark$

Misconception Alert (2 minutes)

[SAY - IMPORTANT]:

"CAUTION! When both numbers in the factors are NEGATIVE:

- The middle term is NEGATIVE (negative + negative = negative)
- But the constant term is POSITIVE (negative × negative = positive)

So $x^2 - 11x + 30$ has NEGATIVE numbers in its factors: $(x - 6)(x - 5)$ "

[TRANSITION]:

"Now let's practice factorising!"

PHASE 3: Practice and Application (15 Minutes)

Worked Example (5 minutes)

[SAY]:

"Let's factorise $x^2 + 5x + 6$ step by step."

[WRITE step by step]:

"Step 1: Identify the coefficients

Coefficient of $x^2 = 1 \checkmark$

Coefficient of x (b) = 5

Constant term (c) = 6

Step 2: Find m and n where $m + n = 5$ and $m \times n = 6$

List factor pairs of 6:

$1 \times 6 = 6$, but $1 + 6 = 7 \times$

$2 \times 3 = 6$, and $2 + 3 = 5 \checkmark$

Step 3: Write the factorised form

$$x^2 + 5x + 6 = (x + 2)(x + 3)"$$

[SAY]:

"Always VERIFY by expanding:

$$(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6 \checkmark "$$

Guided Practice (8 minutes)

[SAY]:

"Try these with your partner:

1. Factorise $x^2 + 8x + 15$

2. Factorise $x^2 - 7x + 12$ "

[GIVE 5 minutes, then review]:

"1. $x^2 + 8x + 15$:

Need: $m + n = 8$, $m \times n = 15$

Numbers: 3 and 5 ($3 + 5 = 8$, $3 \times 5 = 15$)

Answer: $(x + 3)(x + 5)$

2. $x^2 - 7x + 12$:

Need: $m + n = -7$, $m \times n = 12$

Numbers: -3 and -4 ($-3 + (-4) = -7$, $(-3)(-4) = 12$)

Answer: $(x - 3)(x - 4)"$

[TRANSITION]:

"Now I want to see what each of you has learned."

PHASE 4: Assessment / Checkpoint (8 Minutes)

Independent Work (5 minutes)

[DISPLAY questions]:

"Factorise:

1. $x^2 + 4x + 4$

2. $x^2 + 3x + 2$ "

[SAY]:

"You have 5 minutes. Show your working. Begin."

Collection and Closure (2 minutes)

[SAY]:

"Time's up. Please pass your exit tickets forward."

[COLLECT all tickets]

[SAY]:

"Today you learned how to FACTORISE quadratic expressions:

- For $x^2 + bx + c$, find m and n where $m + n = b$ and $m \times n = c$
- Then write $(x + m)(x + n)$
- Always verify by expanding!

Remember: The SUM gives the middle coefficient, the PRODUCT gives the constant.

Great work today!"

Differentiation Notes

For Struggling Learners:

- Provide factor pair tables for common numbers (6, 12, 15, etc.)
- Use visual diagrams showing the connection
- Start with expressions where both numbers are positive
- Allow verification by expansion

For Advanced Learners:

[GIVE these extensions]:

- Factorise $x^2 - x - 12 \rightarrow (x + 3)(x - 4)$
- Factorise $x^2 + 2x - 15 \rightarrow (x + 5)(x - 3)$
- Solve $x^2 + 5x + 6 = 0 \rightarrow x = -2$ or $x = -3$

Answer Key

Exit Ticket Answers:

(a) $x^2 + 4x + 4$:

$$m + n = 4, m \times n = 4$$

Numbers: 2 and 2

Answer: $(x + 2)^2$

(b) $x^2 + 8x + 15$:

$$m + n = 8, m \times n = 15$$

Numbers: 3 and 5

$$\text{Answer: } (x + 3)(x + 5)$$

(c) $x^2 - 7x + 12$:

$$m + n = -7, m \times n = 12$$

Numbers: -3 and -4

$$\text{Answer: } (x - 3)(x - 4)$$

(d) $x^2 - 6x + 9$:

$$m + n = -6, m \times n = 9$$

Numbers: -3 and -3

$$\text{Answer: } (x - 3)^2$$

(e) $x^2 + 3x + 2$:

$$m + n = 3, m \times n = 2$$

Numbers: 1 and 2

$$\text{Answer: } (x + 1)(x + 2)$$

Post-Lesson Reflection Prompts

- 1. What went well?** Did students discover the sum/product relationship?
- 2. What would I change?** Was the anchor activity effective?
- 3. Student Understanding:** Could students handle negative numbers in factors?
- 4. Next Steps:** Which students need more practice?