Grade 10 Mathematics Lesson Plan

Volume of Cones

Strand:	**Measurement and Geometry**
Sub-Strand:	Volume: Volume of a Cone
Specific Learning Outcome:	Calculate the volume of prisms, pyramids, cones, frustums and spheres. Explore the use of the surface area and volume of solids in real-life situations.
Duration:	40 minutes
Key Inquiry Question:	How do we determine the surface area and volume of solids? Why do we determine the surface area and volume of solids?
Learning Resources:	CBC Grade 10 textbooks, sheets of paper or cardboard, scissors, glue/tape, rulers, cylinder (cup or bottle), empty ice cream cones, water, calculators

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Constructing a cone and water pouring to discover volume formula
Structured Instruction	10 minutes	Formalizing the cone volume formula and addressing misconceptions
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Constructing a Cone

Objective: Students will discover the formula for finding the volume of a cone by constructing a cone from paper and performing a water pouring activity to discover that three cones fill a cylinder with the same base and height.

Materials Needed:

- Sheets of paper or cardboard
- Scissors
- Glue or tape
- Rulers
- Cylinder (cup or bottle) for comparison
- Empty ice cream cones
- Water
- Calculators
- Worksheets for recording observations and calculations

Method 1: Paper Cone Construction (5 minutes)

1. Step 1: Take a piece of paper and cut a circle of any radius.
2. Step 2: Cut out a sector and roll the remaining part into a cone shape.
3. Step 3: Measure the radius and height of your cone.
4. Step 4: Observe the cone structure: circular base and curved surface.

Method 2: Water Pouring Activity (8 minutes)

5. Step 1: Take an empty ice cream cone and a cylindrical cup with the same height and base as the cone.
6. Step 2: Fill the cone with water.
7. Step 3: Pour the water from the cone into the cylinder.
8. Step 4: Repeat until the cylinder is full.
9. Step 5: Count: How many cones of water fill the cylinder?
10. Step 6: Observe: It takes exactly 3 full cones to fill the cylinder!
11. Step 7: Discover: This is why the formula includes (1 / 3)!

Mathematical Insight:

V cone = (1 / 3) V cylinder = (1 / 3) π r squared h

The cone is one-third of the volume of a cylinder with the same base and height.

Discussion Questions:

12. How can we turn a flat circle into a cone?
13. How many cones of water fill the cylinder?
14. Why does the formula include (1 / 3)?
15. How is a cone related to a cylinder?
16. Can you think of real-world examples of cones?

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand cone construction.
- Ask probing questions: How many cones fill the cylinder? Why?
- For struggling groups: Let us count together. How many cones have we poured so far?
- For early finishers: How would the volume change if the radius doubled? If the height doubled?
- Guide students to articulate: The volume of a cone is one-third the volume of a cylinder with the same base and height.
- Identify 2-3 groups with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Cone Volume Formula and Addressing Misconceptions

After students have completed the anchor activity and shared their findings, the teacher formalizes the formula for finding the volume of a cone.

Key Takeaway: What is a Cone?

A cone is a three-dimensional solid with a circular base and a curved surface that tapers to a point called the apex or vertex.

Formula:

Volume of a Cone = (1 / 3) times π r squared times h

Where:

- r is the radius of the circular base
- h is the height of the cone (perpendicular distance from base to apex)
- π approximately equals 3.14 or 22 / 7

Why (1 / 3)?

Three cones with the same base and height fit exactly inside a cylinder with the same base and height. Therefore, the volume of a cone is one-third the volume of a cylinder.

Relationship to Cylinder:

- V cone = (1 / 3) V cylinder
- V cylinder = π r squared h
- Therefore, V cone = (1 / 3) π r squared h

Scaffolding Strategies to Address Misconceptions:

- Misconception: A cone is the same as a cylinder. Clarification: No, a cone has a circular base and tapers to a point, while a cylinder has two parallel circular bases.
- Misconception: The volume formula is π r squared h. Clarification: No, you must multiply by (1 / 3). The formula is (1 / 3) π r squared h.
- Misconception: The height is the slant height. Clarification: No, the height is the perpendicular distance from the base to the apex, not the slant height along the curved surface.
- Misconception: I can use the diameter instead of the radius. Clarification: No, the formula uses the radius. If you have the diameter, divide by 2 to get the radius.

Phase 3: Practice and Application (10 minutes)

Worked Example:

Find the volume of a cone with radius 14 cm and height 28 cm (correct to 1 decimal place).

Solution:

Step 1: Find the area of the base.

Area of a Circle = π r squared

= (22 / 7) times 14 cm times 14 cm

= 616 cm squared

Step 2: Calculate the volume.

V = (1 / 3) times π r squared times h

= (1 / 3) times 616 cm squared times 28 cm

= (1 / 3) times 17,248 cm cubed

= 5,749.3 cm cubed

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. An ice cream cone has a radius of 3 cm and a height of 8 cm. Estimate how much ice cream it can hold.

2. Two cones have the same height of 84 cm but different radii. The first cone has a radius of 14 cm, and the second cone has a radius of 42 cm. Calculate and compare the volumes of the two cones. Which one has a larger volume?

Answer Key:

1. V = (1 / 3) times π times 3 squared times 8 = (1 / 3) times 3.14 times 9 times 8 = 75.36 cm cubed.

2. First cone: V = (1 / 3) times π times 14 squared times 84 = 17,248 cm cubed. Second cone: V = (1 / 3) times π times 42 squared times 84 = 155,232 cm cubed. The second cone has a larger volume.

Differentiation Strategies

For Struggling Learners:

- Provide pre-made cone models with labeled dimensions.

- Use cones with simple dimensions for initial practice.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators for calculations.
- Break down the formula into steps: Find base area, multiply by height, divide by 3.

For On-Level Learners:

- Encourage students to verify their formula with the water pouring activity.
- Ask students to explain the relationship between cones and cylinders.
- Provide mixed practice with different dimensions.
- Challenge students to find the volume when only the diameter and height are given.

For Advanced Learners:

- Challenge students to derive the formula using the relationship with cylinders.
- Explore real-world applications: ice cream cones, funnels, traffic cones, conical tents.
- Investigate the relationship between radius, height, and volume.
- Apply the concept to hollow cones (outer radius minus inner radius).
- Solve optimization problems: Given a fixed volume, what dimensions minimize the surface area?

Extension Activity

Real-World Application: Cone-Shaped Funnel Investigation

Work in groups

Situation: Design a cone-shaped funnel for a specific purpose (e.g., pouring water, transferring grain, filtering liquids).

Tasks:

17. Choose a real-world purpose for your funnel.
18. Decide on the dimensions: radius and height.
19. Calculate the volume of your funnel using the cone formula.
20. If the funnel needs to hold 500 cubic cm of liquid, what dimensions would work?
21. Compare different funnel designs: tall and narrow vs. short and wide.
22. Present your findings with diagrams, measurements, and calculations.

Key Takeaway:

Students should understand how the volume of cones is used in real-world contexts such as food packaging (ice cream cones), kitchen tools (funnels), traffic safety (traffic cones), camping (conical tents), and engineering (conical structures).

Teacher Reflection Prompts

- Did students successfully construct cones and perform the water pouring activity?
- Were students able to discover that 3 cones fill a cylinder?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand the relationship between cones and cylinders?
- What adjustments would improve this lesson for future classes?