

# Grade 10 Mathematics Lesson Plan

## Area of Sectors

<b>Strand:</b>	Measurement and Geometry
<b>Sub-Strand:</b>	Area of a Part of a Circle: Area of a Sector of a Circle
<b>Specific Learning Outcome:</b>	Work out the area of a sector of a circle in real-life situations. Apply the area of a part of a circle in real-life situations. Explore the use of the area of a part of a circle in real-life situations.
<b>Duration:</b>	40 minutes
<b>Key Inquiry Question:</b>	How do we use the concept of the area of a part of a circle in real life?
<b>Learning Resources:</b>	CBC Grade 10 textbooks, graph paper, razorblade or scissors, ruler, protractor, calculators

## Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Discovering the sector formula by cutting and measuring
Structured Instruction	10 minutes	Formalizing the sector formula and addressing misconceptions
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

### Phase 1: Problem-Solving and Discovery (15 minutes)

#### Anchor Activity: Discovering the Sector Formula

Objective: Students will discover the formula for finding the area of a sector by cutting out a sector from a circle and exploring the relationship between the angle and the area.

#### Materials Needed:

- Graph paper
- Razorblade or a pair of scissors
- Ruler
- Protractor
- Calculator

### **Steps for the Activity:**

1. Step 1: Draw a Circle. Draw a circle of radius 7 cm on a graph paper.
2. Step 2: Cut Out the Circle. Cut out the circle along its boundary.
3. Step 3: Mark the Centre. Mark the centre of the circle.
4. Step 4: Measure and Cut the Sector. Measure an angle of 30 degrees at the centre and cut out as shown.
5. Step 5: Estimate the Area. Estimate the area by counting the number of squares enclosed by the arc and the two radii of the circle.
6. Step 6: Express as a Fraction. Express the angle of the sector (30 degrees) as a fraction of the angle at the centre of the circle (360 degrees).
7. Step 7: Calculate. Multiply the fraction obtained in Step 6 by the area of the circle.
8. Step 8: Discuss and Share. Discuss and share the result with other groups.

### **Discussion Questions:**

9. What is a sector?
10. How did you calculate the area of the sector?
11. What is the relationship between the angle and the area?
12. Did different groups get the same formula?

### **Teacher Role During Discovery:**

- Circulate among groups, ensuring students understand how to measure the angle and cut the sector accurately.
- Ask probing questions: What fraction of the circle is this sector? How do we find the area?
- For struggling groups: Let us start by finding the area of the whole circle. What fraction of 360 degrees is 30 degrees? Now multiply the fraction by the area of the circle.
- For early finishers: Can you write a general formula for finding the area of any sector?
- Guide students to articulate: The area of a sector equals the fraction ( $\text{angle}/360$ ) times the area of the circle.
- Identify 2-3 groups with clear findings to share with the class.

## **Phase 2: Structured Instruction (10 minutes)**

### **Formalizing the Sector Formula and Addressing Misconceptions**

After students have completed the anchor activity and shared their findings, the teacher formalizes the formula for finding the area of a sector.

#### **Key Takeaway: What is a Sector?**

A sector is a region bounded by two radii and an arc.

Minor sector is one whose area is less than a half of the area of the circle.

Major sector is one whose area is greater than a half of the area of the circle.

#### **Formula:**

Area of a Sector =  $(\theta / 360)$  times  $\pi r^2$

Where:

- $\theta$  is in degrees
- $r$  is the radius of the circle
- $\pi$  approximately equals 3.142 or 22/7

#### **Scaffolding Strategies to Address Misconceptions:**

- Misconception: I can use the angle directly without dividing by 360. Clarification: No, you must express the angle as a fraction of 360 degrees first.
- Misconception: The formula works for any angle unit. Clarification: No, the angle must be in degrees. If given in radians, convert first or use a different formula.
- Misconception: A sector is the same as a segment. Clarification: No, a sector includes the two radii, while a segment is the region between a chord and an arc.
- Misconception: I can use diameter instead of radius. Clarification: No, the formula uses radius. If given diameter, divide by 2 first.

## **Phase 3: Practice and Application (10 minutes)**

### **Worked Example 1:**

Find the area of a sector of a circle of radius 7 cm if the angle subtended at the centre is 90 degrees.

Solution:

The values given are,  $\theta = 90$  degrees,  $r = 7$  cm

$$\text{Area} = (\theta / 360) \times \pi r^2$$

$$\text{Area} = (90 / 360) \times (22 / 7) \times (7^2)$$

$$= (1 / 4) \times (22 / 7) \times 49$$

$$= (1 / 4) \times 22 \times 7$$

$$= 38.5 \text{ cm}^2$$

### **Worked Example 2:**

Find the area of a sector of a circle where  $\theta = 45$  degrees,  $r = 10$  cm (use  $\pi = 3.142$ ).

Solution:

$$\text{Area} = (\theta / 360) \times \pi r^2$$

$$\text{Area} = (45 / 360) \times 3.142 \times (10^2)$$

$$= (1 / 8) \times 3.142 \times 100$$

$$= 39.275 \text{ cm}^2$$

### **Worked Example 3: Windscreen Wiper Problem**

The shaded region shows the area swept out on a flat windscreens by a wiper. The larger sector has radius 20 cm and angle 120 degrees. The smaller sector has radius 16 cm and angle 120 degrees. Calculate the area of this region.

Solution:

The area of the region is gotten by subtracting the Area of the smaller sector from Area of the larger sector.

Area of the larger sector:

$$R = 20 \text{ cm}, \theta = 120 \text{ degrees}$$

$$\begin{aligned} A &= (120 / 360) \times (22 / 7) \times 20^2 \\ &= (1 / 3) \times (22 / 7) \times 400 \\ &= 419.05 \text{ cm squared} \end{aligned}$$

Area of the smaller sector:

$$r = 16 \text{ cm}, \theta = 120 \text{ degrees}$$

$$\begin{aligned} A &= (120 / 360) \times (22 / 7) \times 16^2 \\ &= (1 / 3) \times (22 / 7) \times 256 \\ &= 268.19 \text{ cm squared} \end{aligned}$$

Therefore:

$$\text{Area of the region} = 419.05 - 268.19 = 150.86 \text{ cm squared}$$

#### Phase 4: Assessment (5 minutes)

**Exit Ticket:**

Students complete the following questions individually.

1. A sector of a circle of radius  $r$  is subtended at the centre by an angle of  $\theta$ . Calculate the area of the sector if: (a)  $r = 10$  m,  $\theta = 264$  degrees (b)  $r = 8.4$  cm,  $\theta = 40$  degrees (c)  $r = 1.4$  cm,  $\theta = 80$  degrees

2. A sector has an angle of  $\pi/3$  radians and a radius of 8 cm. Find its area.

3. A goat is tethered at the corner of a fenced rectangular grazing field. If the length of the rope is 21 m, what is its grazing area?

#### **Answer Key:**

1. (a)  $A = (264 / 360)$  times  $\pi$  times 10 squared =  $(264 / 360)$  times  $(22 / 7)$  times 100 = 733.33 m squared. (b)  $A = (40 / 360)$  times  $\pi$  times 8.4 squared =  $(40 / 360)$  times  $(22 / 7)$  times 70.56 = 24.64 cm squared. (c)  $A = (80 / 360)$  times  $\pi$  times 1.4 squared =  $(80 / 360)$  times  $(22 / 7)$  times 1.96 = 1.37 cm squared.

2. For radians, use  $A = (1 / 2)$  times  $r$  squared times  $\theta$ .  $A = (1 / 2)$  times 8 squared times  $(\pi / 3)$  =  $(1 / 2)$  times 64 times  $(\pi / 3)$  = 33.51 cm squared.

3. The goat can graze in a quarter circle (90 degrees) with radius 21 m.  $A = (90 / 360)$  times  $\pi$  times 21 squared =  $(1 / 4)$  times  $(22 / 7)$  times 441 = 346.5 m squared.

#### **Differentiation Strategies**

##### **For Struggling Learners:**

- Provide pre-drawn circles with sectors already outlined.
- Use simpler angles (e.g., 90 degrees, 180 degrees) for initial practice.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators.
- Break down the formula into steps: Find the fraction, find the area of the circle, multiply.

##### **For On-Level Learners:**

- Encourage students to verify their formula with different angles.
- Ask students to explain the relationship between the angle and the area.
- Provide mixed practice with different types of sector problems.
- Challenge students to solve problems involving major sectors.

### **For Advanced Learners:**

- Challenge students to derive the formula for sectors in radians.
- Explore real-world applications: pizza slices, pie charts, windscreen wipers, grazing areas.
- Investigate the relationship between sector area and arc length.
- Solve optimization problems: Given a fixed radius, what angle maximizes or minimizes the sector area?
- Apply the concept to composite shapes involving multiple sectors.

### **Extension Activity**

#### **Real-World Application: Pizza Slice Design Project**

Individual or Group Work

Situation: A pizza restaurant wants to design different pizza slice sizes for their customers.

Task:

13. Design a circular pizza with radius 30 cm.
14. Calculate the area of one slice if the pizza is cut into 8 equal slices.
15. Calculate the area of one slice if the pizza is cut into 6 equal slices.
16. If a customer wants a slice that is 50 cm squared, what angle should the slice have?
17. Present your findings with diagrams and calculations.

### **Key Takeaway:**

Students should understand how the area of a sector is used in real-world contexts such as food service, agriculture (grazing areas), engineering (windscreen wipers), and data visualization (pie charts).

### **Teacher Reflection Prompts**

- Did students successfully discover the sector formula through the anchor activity?
- Were students able to cut and measure the sector accurately?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand the difference between a sector and a segment?
- What adjustments would improve this lesson for future classes?