

I. Lesson Overview

Lesson Title:	Zero and Negative Indices
Strand:	Numbers and Algebra
Sub-Strand:	Indices and Logarithms
Grade Level:	10
Estimated Duration:	40 minutes

Key Inquiry Question

How do we use real numbers in day-to-day activities?

II. Learning Objectives & Standards

Learning Objectives

Upon completion of this lesson, students will be able to:

1. **Know (Conceptual Understanding):** Understand that any non-zero base raised to the power of zero equals one, and that a negative index represents a reciprocal.
2. **Do (Procedural Skill):** Simplify expressions involving zero and negative indices using the rules $a^0 = 1$ and $a^{-n} = 1/a^n$.
3. **Apply (Application/Problem-Solving):** Apply zero and negative indices to solve real-world problems involving depreciation, decay, and halving patterns.

Curriculum Alignment

Strand:	Indices and Logarithms
Sub-Strand:	Indices
Specific Learning Outcome:	Understand and apply the rules that any non-zero base raised to the power of zero equals one and that a negative index represents a reciprocal.

III. Materials & Resources

Textbooks:	CBC Grade 10 Mathematics Learner's Book CBC Grade 10 Mathematics Teacher's Book
Materials:	Pen, paper, calculator

IV. Lesson Procedure

Phase 1: Problem-Solving and Discovery / Engage & Explore (15 minutes)

Objective: To discover the pattern that leads to zero and negative indices through exploration.

Materials Required: Pen, paper, and calculator for each student.

Anchor Activity: Pattern Discovery

Instructions: Look at the pattern below and complete the missing values:

2^4	16
2^3	8
2^2	4
2^1	2
2^0	?
2^{-1}	?

Discussion Questions:

- What pattern do you notice as the exponent decreases by 1?
- What happens when the exponent reaches zero?
- Can you try the same pattern with 3^n ?
- What do you predict 2^{-1} and 2^{-2} would equal?

Expected Discovery:

Students should notice that each time the exponent decreases by 1, the value is divided by 2. Following this pattern:

- $2^0 = 1$ (dividing 2 by 2)
- $2^{-1} = 1/2$ (dividing 1 by 2)
- $2^{-2} = 1/4$ (dividing $1/2$ by 2)

Teacher's Role: The teacher circulates among students, asking probing questions (e.g., "What operation connects each row?", "Why does $2^0 = 1$ make sense?"). The teacher uses student discoveries to bridge to formal instruction.

Phase 2: Structured Instruction / Explain (10 minutes)

Objective: To formalize the rules for zero and negative indices.

Key Takeaways:

Rule 1: Zero Index

Any non-zero number raised to the power zero is always 1.

Formula: $a^0 = 1$ (where $a \neq 0$)

Examples:

- $5^0 = 1$
- $100^0 = 1$
- $(-3)^0 = 1$

Rule 2: Negative Index

A negative exponent means taking the reciprocal of the base.

Formula: $a^{-n} = 1/a^n$

Examples:

- $2^{-1} = 1/2^1 = 1/2$
- $3^{-2} = 1/3^2 = 1/9$
- $5^{-3} = 1/5^3 = 1/125$

Why These Rules Work:

Using the quotient law: $a^n \div a^n = a^{n-n} = a^0$

But $a^n \div a^n = 1$, so $a^0 = 1$

Similarly: $a^0 \div a^n = a^{0-n} = a^{-n} = 1/a^n$

Addressing Misconceptions: "Remember: 0^0 is undefined. The zero index rule only applies when the base is NOT zero. Also, a negative index does NOT make the answer negative—it creates a fraction!"

Phase 3: Practice and Application / Elaborate (15 minutes)

Objective: To apply zero and negative indices to simplify expressions and solve real-world problems.

Varied Problems:

1. Carpenter's Plank Problem:

A carpenter is cutting wooden planks for desks. The length of each plank decreases by half with each cut.

Solution:

- Original plank: $2^0 = 1$ metre
- After 1st cut: $2^{-1} = 1/2$ metre
- After 2nd cut: $2^{-2} = 1/4$ metre
- After 3rd cut: $2^{-3} = 1/8$ metre
- After 4th cut: $2^{-4} = 1/16$ metre

General formula: $L = 2^{-n}$ where n = number of cuts

2. Simplify: $8b^0$

Solution: $8b^0 = 8 \times 1 = 8$

(Since $b^0 = 1$ for any non-zero b)

3. Simplify without negative indices: $6b^{-1}$

Solution: $6b^{-1} = 6 \times (1/b) = 6/b$

Teacher's Role: The teacher monitors students, helping them recognize when to apply the zero index rule versus the negative index rule.

Phase 4: Assessment / Evaluate (Exit Ticket)

Objective: To formatively assess individual student understanding.

Exit Ticket Questions:

1. In the Msomii Community Library, the maths-book shelf has 64 books. The librarian removes half of them every day.
 - a) Express the number of books left after 3 days using indices.
 - b) How many books remain after 5 days?
2. A school van costs Ksh 1,000,000. Its value decreases by half every 3 years. Express the van's value after 9 years in index form and find how much it will be worth after 15 years.
3. A patient takes 400 mg of medicine. Every 4 hours, the amount reduces to $1/2$ of what was left.
 - a) Write an index expression for the medicine remaining after 12 hours.
 - b) How much medicine remains after 20 hours?

Answer Key:

1a) Books after 3 days: $64 \times 2^{-3} = 64 \times (1/8) = 8$ books

OR: $64 = 2^6$, so $2^6 \times 2^{-3} = 2^3 = 8$ books

1b) Books after 5 days: $64 \times 2^{-5} = 64 \times (1/32) = 2$ books

2. After 9 years (3 halvings): $1,000,000 \times 2^{-3} = 1,000,000 \times (1/8) = \text{Ksh } 125,000$

After 15 years (5 halvings): $1,000,000 \times 2^{-5} = 1,000,000 \times (1/32) = \text{Ksh } 31,250$

3a) After 12 hours (3 intervals): $400 \times 2^{-3} = 400 \times (1/8) = 50$ mg

3b) After 20 hours (5 intervals): $400 \times 2^{-5} = 400 \times (1/32) = 12.5$ mg

V. Differentiation

Student Group	Strategy & Activity
Struggling Learners (Support)	Scaffolding: Provide the pattern table pre-filled up to 2^1 . Use visual aids showing halving (folding paper). Start with base 2 before other bases. Allow calculator use for verification.
On-Level Learners (Core)	The core lesson activities as described above.
Advanced Learners (Challenge)	Extension Activity: A school installs 100 energy-saving bulbs. Every year, a quarter stop working. a) Write an index expression for working bulbs after 4 years. b) How many bulbs are functional after 6 years? (Hint: Use base $3/4$ or explore $(1/4)^n$ patterns)

Extension Activity Solutions:

Energy-Saving Bulbs Problem:

- Initial bulbs: 100
- After each year, $3/4$ remain working
- After 4 years: $100 \times (3/4)^4 = 100 \times (81/256) \approx 31.6 \approx 32$ bulbs
- After 6 years: $100 \times (3/4)^6 = 100 \times (729/4096) \approx 17.8 \approx 18$ bulbs

VI. Assessment

Type	Method	Purpose
Formative (During Lesson)	- Observation during pattern discovery - Questioning during exploration - Exit Ticket	To monitor progress and adjust instruction.
Summative (After Lesson)	- Homework assignment - Future quiz/test questions	To evaluate mastery of learning objectives.

Teacher's Role: Collect and review the exit tickets to gauge student understanding and identify any common misconceptions that need to be addressed in the next lesson.

VII. Teacher Reflection

To be completed after the lesson.

1. What went well?
2. What would I change?
3. Student Understanding: What did the exit tickets reveal?
4. Next Steps: Based on assessment data, what is the plan for the next lesson?