I. Lesson Overview

Strand	Measurement and Geometry
Sub-Strand	Similarity and Enlargement
Specific Learning Outcome	Apply similarity and enlargement to real-life situations
Grade Level	Grade 10
Duration	40 minutes
Key Inquiry Question	In what ways could the area and volume scale factors impact the design and functionality of a park, and how would you ensure that the park meets the needs of the community while being sustainably designed?
Learning Resources	CBC Grade 10 Mathematics Textbooks, rulers, calculators, graph paper, coloured pencils

II. Learning Objectives

Category	Objective
Know	Define linear scale factor (LSF) as the ratio of corresponding lengths in similar figures. State the relationships: Area Scale Factor (ASF) = $(LSF)^2$ and Volume Scale Factor (VSF) = $(LSF)^3$. Explain how architects and designers use scale factors to enlarge blueprints while maintaining proportions.
Do	Calculate the linear scale factor between a blueprint and its actual-size enlargement. Determine the area scale factor and use it to find the actual area of a park from its blueprint area. Calculate the volume scale factor and use it to find the actual volume of structures or water features from blueprint dimensions. Convert between linear, area, and volume scale factors.
Apply	Solve a real-world park design problem involving enlargement of a blueprint to actual size. Analyse how area and volume scale factors impact the design, cost, and functionality of community spaces. Evaluate sustainability considerations when scaling up park designs.

III. Materials & Resources

- CBC Grade 10 Mathematics Textbooks
- Rulers and measuring tapes
- Calculators
- Graph paper and coloured pencils (for blueprint sketching)
- Printed handout with the park design scenario

IV. Lesson Procedure

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: "Designing a New Park"

Scenario:

A local community is planning to design a new park. The architects have created a small blueprint of the park that measures 10 feet by 15 feet. They need to enlarge the blueprint to an actual size that fits a vacant lot that is 200 feet by 300 feet. The architects will use a scale factor to determine how much they need to enlarge their blueprint to fit the available space while maintaining the same proportions and layout.

Instructions (Work in pairs):

- (a) Write down the dimensions of the blueprint and the dimensions of the actual park.
- (b) Calculate the linear scale factor (LSF) by dividing the actual length by the blueprint length. Verify by dividing the actual width by the blueprint width. Are they the same?
- (c) Calculate the area of the blueprint and the area of the actual park.
- (d) Find the area scale factor (ASF) by dividing the area of the actual park by the area of the blueprint.
- (e) Compare the ASF with the square of the LSF. What do you notice?
- (f) The blueprint shows a rectangular swimming pool measuring 1 foot by 2 feet by 0.5 feet deep. Using the LSF, calculate the actual dimensions of the swimming pool.
- (g) Calculate the volume of the blueprint pool and the volume of the actual pool.
- (h) Find the volume scale factor (VSF) by dividing the actual pool volume by the blueprint pool volume.
- (i) Compare the VSF with the cube of the LSF. What do you notice?
- (j) Discuss: How do these scale factors affect the amount of materials, water, and cost needed for the real park? Share your findings with the class.

Teacher's Role During Discovery:

- Circulate among pairs, ensuring students correctly identify the linear scale factor from both length and width ratios.
- Ask probing questions: "What is 200 divided by 10?" "Is the width ratio the same?" "Why must they be equal for similarity?"
- For struggling pairs: "Start with the lengths. The blueprint is 10 feet and the actual park is 200 feet. How many times bigger is 200 than 10?"
- For early finishers: "If a circular fountain on the blueprint has a diameter of 0.5 feet, what would its actual diameter be? What about its actual area?"
- Guide students toward discovering that ASF = $(LSF)^2$ and VSF = $(LSF)^3$.
- Identify 2–3 pairs with clear findings to share with the class.

Expected Student Discoveries:

Calculation	Result
LSF (length)	200 ÷ 10 = 20
LSF (width)	300 ÷ 15 = 20
Both ratios equal?	Yes — confirms similarity. LSF = 20.
Area of blueprint	10 × 15 = 150 sq ft
Area of actual park	200 × 300 = 60,000 sq ft
ASF	60,000 ÷ 150 = 400
$(LSF)^2$	20^2 = 400. ASF = $(LSF)^2$ ✔
Blueprint pool volume	1 × 2 × 0.5 = 1 cu ft
Actual pool dimensions	20 × 40 × 10 feet
Actual pool volume	20 × 40 × 10 = 8,000 cu ft

- VSF = 8,000 ÷ 1 = 8,000.
- $(LSF)^3 = 20^3$ = 8,000. VSF = $(LSF)^3$ ✔
- The area of the park is 400 times the area of the blueprint, but the volume of structures is 8,000 times the blueprint volume.
- This means materials, water, and costs scale dramatically — volume increases much faster than area.

Phase 2: Structured Instruction (10 minutes)

Key Takeaways:

Definition:

When architects enlarge a blueprint to create the actual structure, every dimension is multiplied by the same linear scale factor. This ensures that the enlarged design is SIMILAR to the original blueprint — all proportions and the layout are preserved. The area and volume, however, do not scale by the same factor as the length.

Scale Factor Relationships:

Dimension	Scale Factor	Park Design Example (LSF = 20)
Length (1D)	Linear Scale Factor (LSF) = k k = Actual length ÷ Blueprint length	k = 200 ÷ 10 = 20 A 1 ft path on the blueprint becomes 20 ft.
Area (2D)	Area Scale Factor (ASF) = k^2 ASF = Actual area ÷ Blueprint area	ASF = 20^2 = 400 Blueprint area 150 sq ft ⟶ Actual area 60,000 sq ft. Affects: land coverage, paving, grass, garden beds.
Volume (3D)	Volume Scale Factor (VSF) = k^3 VSF = Actual volume ÷ Blueprint volume	VSF = 20^3 = 8,000 Blueprint pool 1 cu ft ⟶ Actual pool 8,000 cu ft. Affects: water features, soil, concrete, excavation.

Connecting to Student Discoveries:

- Reference the park scenario: "You found that the linear scale factor is 20. Every length on the blueprint is multiplied by 20 to get the actual length."
- Emphasise the area impact: "The area scale factor is 400. This means the actual park covers 400 times more ground than the blueprint. If grass costs Ksh 50 per square foot, the blueprint area would cost Ksh 7,500 but the actual park would cost Ksh 3,000,000!"
- Emphasise the volume impact: "The volume scale factor is 8,000. The swimming pool on the blueprint holds 1 cubic foot of water, but the actual pool holds 8,000 cubic feet. That's a massive increase in water, concrete, and cost."
- Address misconception: "If the linear scale factor is 20, the area does NOT increase by 20 times — it increases by 400 times. And the volume increases by 8,000 times. This is why large-scale projects cost so much more than small models suggest."
- Connect to sustainability: "When we scale up a park design, we must consider: How much water will the pool need? How much soil for the gardens? How much concrete for the paths? These are all VOLUME questions, and they scale by k cubed."

Phase 3: Practice and Application (10 minutes)

Problem 1: Park Pathways (Area Scale Factor)

On the blueprint, a winding pathway covers an area of 8 square feet. Using the area scale factor, calculate the actual area of the pathway in the real park.

Solution:

$ASF = (LSF)^2 = 20^2 = 400$.

Actual pathway area = Blueprint area × ASF = 8 × 400 = 3,200 square feet.

The actual pathway will cover 3,200 square feet.

Problem 2: Fountain Volume (Volume Scale Factor)

The blueprint shows a circular fountain basin with a volume of 0.25 cubic feet. Calculate the actual volume of the fountain basin in the real park.

Solution:

$VSF = (LSF)^3 = 20^3 = 8{,}000$.

Actual fountain volume = Blueprint volume × VSF = 0.25 × 8,000 = 2,000 cubic feet.

The actual fountain basin will hold 2,000 cubic feet of water.

Problem 3: Garden Bed (Combined Calculation)

On the blueprint, a rectangular garden bed measures 0.75 feet long, 0.5 feet wide, and 0.15 feet deep (filled with soil).

- (a) Calculate the actual dimensions of the garden bed.
- (b) Calculate the actual area of the garden bed (for planting).
- (c) Calculate the actual volume of soil needed to fill the garden bed.

Solution:

Step	Working
(a) Actual dimensions	Length = 0.75 × 20 = 15 feet Width = 0.5 × 20 = 10 feet Depth = 0.15 × 20 = 3 feet

(b) Actual area	Method 1: Blueprint area × ASF = (0.75 × 0.5) × 400 = 0.375 × 400 = 150 sq ft Method 2: Actual L × Actual W = 15 × 10 = 150 sq ft ✔
(c) Actual volume of soil	Method 1: Blueprint volume × VSF = (0.75 × 0.5 × 0.15) × 8,000 = 0.05625 × 8,000 = 450 cu ft Method 2: 15 × 10 × 3 = 450 cu ft ✔
Verification	ASF = 150 / 0.375 = 400 = 20^2 ✔ VSF = 450 / 0.05625 = 8,000 = 20^3 ✔

Problem 4: Sustainability Analysis

The blueprint shows a children's sandpit with a volume of 0.4 cubic feet. Sand costs Ksh 200 per cubic foot.

- (a) Calculate the actual volume of sand needed.
- (b) Calculate the total cost of sand for the actual sandpit.
- (c) If the community budget for the sandpit is Ksh 500,000, is this sufficient?

Solution:

(a) Actual volume = 0.4 × 8,000 = 3,200 cubic feet.

(b) Total cost = 3,200 × Ksh 200 = Ksh 640,000.

(c) No. The budget of Ksh 500,000 is NOT sufficient. The community needs an additional Ksh 140,000, or they must reduce the sandpit size.

Phase 4: Assessment — Exit Ticket (5 minutes)

Assessment Questions:

1. A blueprint of a community garden measures 5 feet by 8 feet. The actual garden will be 100 feet by 160 feet.

- (a) What is the linear scale factor?
- (b) What is the area scale factor?
- (c) If a flower bed on the blueprint has an area of 2 square feet, what is the actual area of the flower bed?

2. On the park blueprint (LSF = 20), a storage shed is shown as a box measuring 0.3 feet long, 0.2 feet wide, and 0.15 feet tall. What is the actual volume of the storage shed?

3. A landscape architect creates a model of a park at a scale of 1 : 50. A pond in the model has a volume of 0.064 cubic metres. What is the actual volume of the pond?

4. Two similar park designs have an area scale factor of 25. If the smaller park covers 1,200 square feet, what is the area of the larger park? What is the linear scale factor between them?

Answer Key:

- 1(a) LSF = 100 ÷ 5 = 20 (or 160 ÷ 8 = 20). LSF = 20.
- 1(b) ASF = 20^2 = 400.
- 1(c) Actual flower bed area = 2 × 400 = 800 square feet.
- 2. Actual dimensions: 0.3 × 20 = 6 ft, 0.2 × 20 = 4 ft, 0.15 × 20 = 3 ft. Volume = 6 × 4 × 3 = 72 cubic feet. (Or: Blueprint volume = 0.3 × 0.2 × 0.15 = 0.009 cu ft. Actual = 0.009 × 8,000 = 72 cu ft.)
- 3. LSF = 50. VSF = 50^3 = 125,000. Actual volume = 0.064 × 125,000 = 8,000 cubic metres.
- 4. Area of larger park = 1,200 × 25 = 30,000 square feet. LSF = $\sqrt{25}$ = 5.

V. Differentiation Strategies

Learner Level	Strategy
Struggling Learners	Provide a formula card: LSF = k, ASF = k^2, VSF = k^3. Use grid paper to draw the blueprint (10×15 squares) and count squares to find area. Start with the linear scale factor only before introducing area and volume. Walk through Problem 1 step-by-step with the pair. Use concrete language: "If each side is 20 times bigger, the area is 20 times 20 = 400 times bigger."
On-Level Learners	Complete all anchor activity steps and practice problems independently. Verify answers using both methods (scale factor method and direct calculation). Work through the sustainability analysis problem (Problem 4) to connect mathematics to real-world budgeting.
Advanced Learners	Challenge: "The community wants to add a hemispherical dome (half-sphere) over the fountain. On the blueprint, the dome has radius 0.25 feet. Calculate the actual surface area and volume of the dome." Investigate: "If the park budget is Ksh 10,000,000, and costs scale with

	area (for flat features) and volume (for 3D features), estimate the total cost breakdown." Design their own park blueprint with at least 5 features, calculate all actual dimensions, areas, and volumes.

VI. Extension Activity

Sustainability and Community Design Challenge:

Using the park blueprint (10 ft by 15 ft, LSF = 20), answer the following:

1. The blueprint shows a jogging track that is 0.1 feet wide and runs along the entire perimeter of the park. Calculate:

- (a) The actual width of the jogging track.
- (b) The actual perimeter of the park.
- (c) The actual area of the jogging track (approximate it as perimeter × width).

2. A rainwater harvesting tank on the blueprint has dimensions 0.5 ft × 0.4 ft × 0.3 ft. Calculate:

- (a) The actual dimensions of the tank.
- (b) The actual volume of the tank in cubic feet.
- (c) Convert the volume to litres (1 cubic foot ≈ 28.3 litres). Is this sufficient for irrigating the park gardens?

3. The community wants the park to include solar panels on the roof of a pavilion. On the blueprint, the pavilion roof measures 1.5 ft by 1 ft.

- (a) Calculate the actual roof area.
- (b) If each solar panel covers 20 square feet, how many panels can fit on the roof?

Extension Answer Key:

- 1(a) Actual width = 0.1 × 20 = 2 feet.
- 1(b) Blueprint perimeter = 2(10 + 15) = 50 ft. Actual perimeter = 50 × 20 = 1,000 feet.
- 1(c) Actual track area ≈ 1,000 × 2 = 2,000 square feet.
- 2(a) Actual dimensions: 0.5 × 20 = 10 ft, 0.4 × 20 = 8 ft, 0.3 × 20 = 6 ft.

- 2(b) Actual volume = 10 × 8 × 6 = 480 cubic feet.
- 2(c) Volume in litres = 480 × 28.3 ≈ 13,584 litres. This is a substantial amount for garden irrigation.
- 3(a) Actual roof area = (1.5 × 20) × (1 × 20) = 30 × 20 = 600 square feet.
- 3(b) Number of panels = 600 ÷ 20 = 30 solar panels.

VII. Assessment Methods

Type	Method
Formative	Observation during pair work: Can students find the LSF from both length and width? Do they discover that ASF = $(LSF)^2$ and VSF = $(LSF)^3$? Questioning: "If the linear scale factor is 20, why is the area 400 times bigger and not 20 times bigger?" "What happens to the volume when you double all dimensions?" Monitoring calculations during practice problems, especially unit awareness.
Summative	Exit ticket with 4 questions covering: LSF and ASF calculation from blueprint dimensions, volume of an actual structure from blueprint dimensions, VSF application with a different scale, and reverse calculation (ASF to LSF). Complete answer key provided for marking.

VIII. Teacher Reflection

1. Did students discover that the linear scale factor is the same for both length and width, confirming similarity?

2. Were students able to connect ASF = $(LSF)^2$ and VSF = $(LSF)^3$ through their own calculations?

3. How effectively did the park design scenario engage students in seeing mathematics as relevant to real-world design?

4. Did students grasp the practical implications — that volume (materials, water, cost) scales much faster than area?

5. Were students able to use both methods (scale factor method and direct calculation) and verify their answers?

6. Did the sustainability discussion (budgeting, water, materials) deepen understanding of why scale factors matter?

7. What common misconceptions arose (e.g., thinking area scales by k instead of k^2)?

8. What adjustments would improve the lesson for future delivery?