I. Lesson Overview

Strand	**Measurement and Geometry**
Sub-Strand	Similarity and Enlargement
Specific Learning Outcome	Determine the image of an object under enlargement given the centre and the negative scale factor
Grade Level	Grade 10
Duration	40 minutes
Key Inquiry Question	How do we work out the area of polygons?
Learning Resources	CBC Grade 10 Mathematics Textbooks

II. Learning Objectives

Category	Objective
Know	Define negative scale factor enlargement as a transformation where the image is formed on the opposite side of the centre and is inverted. Distinguish between positive and negative scale factor enlargements.
Do	Construct the image of a shape under enlargement with a negative scale factor by drawing lines through the centre, measuring distances, and plotting image points on the opposite side. Calculate image coordinates using the formula (x', y') = (kx, ky) for origin-centred enlargements.
Apply	Apply negative scale factor enlargement to determine image coordinates, find unknown distances, and solve problems involving circles, triangles, and real-world contexts such as photographs and maps.

III. Materials & Resources

- CBC Grade 10 Mathematics Textbooks
- Graph paper, rulers, pencils, protractors
- Digital textbook: INNODEMS CBC Grade 10 Maths — Section 2.1.2 Enlargement

IV. Lesson Procedure

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: "What Happens When the Scale Factor Is Negative?"

Context:

In the previous lesson, students explored enlargement with positive scale factors. They learned that the image is on the same side of the centre as the object. This activity challenges them to discover what happens when the scale factor becomes negative.

Instructions (Work in pairs):

- (a) Draw and label the x-axis and y-axis on graph paper. Mark the origin O at (0, 0).
- (b) Plot the points A(2, 1), B(4, 1), and C(3, 3). Connect them to form triangle ABC.
- (c) For each vertex, draw a straight line from the point THROUGH the origin O and extend it to the OTHER SIDE of O.
- (d) On the opposite side of O, mark a point at the SAME distance from O as the original point. Label these A', B', and C'.
- (e) Connect A', B', and C' to form triangle A'B'C'.
- (f) Write down the coordinates of A', B', and C'.
- (g) Compare the coordinates of each image point to the original point. What pattern do you notice?
- (h) Compare the orientation of triangle A'B'C' to triangle ABC. Is the image the same way up or inverted?
- (i) What scale factor would produce this result? Discuss with your partner.

Teacher's Role During Discovery:

- Circulate among pairs, checking that students are extending lines THROUGH O to the opposite side.
- Ask probing questions: "Where is A' relative to O?" "Is A' on the same side as A?"
- For pairs who struggle: "If A is 2 units right and 1 unit up from O, where would a point be that is the same distance but on the opposite side?"
- For early finishers: "What if you marked points at TWICE the distance on the opposite side? What scale factor would that be?"
- Identify common errors: students extending from the point instead of through the centre.
- Select 2–3 pairs with clear findings to share with the class.

Expected Student Discoveries:

- A' = (−2, −1), B' = (−4, −1), C' = (−3, −3) — each coordinate is the negative of the original.
- The image triangle is on the OPPOSITE side of the centre from the original.
- The image triangle is INVERTED (upside down and flipped) compared to the original.
- The image is the same size as the original (because the distance is the same, just in the opposite direction).
- The scale factor that produces this result is −1 (multiply each coordinate by −1).
- The triangles are congruent (same size and shape) but the image is rotated 180° about O.

Phase 2: Structured Instruction (10 minutes)

Key Takeaways:

The Negative Scale Factor Rule:

If an enlargement has a negative scale factor, the image is formed on the OPPOSITE side of the centre and is INVERTED (upside down).

Properties of Negative Scale Factor Enlargement:

- The image appears on the opposite side of the centre of enlargement from the object.
- The image is inverted (rotated 180°) relative to the object.
- For enlargement centred at the origin with scale factor k: the image of point (x, y) is (kx, ky). When k is negative, the signs of both coordinates change.
- If k = −1: the image is the same size as the object but inverted (a half-turn about the centre).
- If |k| > 1 (e.g., k = −2): the image is larger AND inverted.
- If 0 > k > −1 (e.g., k = −½): the image is smaller AND inverted.
- Lines connecting each object point to its corresponding image point all pass through the centre of enlargement.
- The object and image remain similar — same shape, proportional sides, equal corresponding angles.

Comparison: Positive vs Negative Scale Factors

Feature	Positive Scale Factor	Negative Scale Factor
Position of image	Same side as object	Opposite side of centre

Orientation	Same as object (upright)	Inverted (upside down)
$\|k\| > 1$	Image is larger	Image is larger AND inverted
$\|k\| < 1$	Image is smaller	Image is smaller AND inverted

Connecting to Student Discoveries:

- Reference the coordinates students found: A(2,1) → A'(−2,−1). Each coordinate was multiplied by −1.
- Highlight that the image appeared on the opposite side of O — this is the defining feature of a negative scale factor.
- Show that the image was inverted — if the original triangle pointed up, the image points down.
- Address misconception: "negative" does not mean the shape gets smaller. The negative sign controls DIRECTION (opposite side), while the absolute value controls SIZE.

Phase 3: Practice and Application (10 minutes)

Problem 1: Enlargement with Scale Factor −1

Enlarge triangle ABC by scale factor of −1 about the point O (the origin).

Solution:

- The centre of enlargement is O, the origin.
- Draw a line from point A through O and extend the line to the opposite side of O.
- Measure the distance from O to A. Since the scale factor is −1, and OA = 5 units, then OA' = |−1| × 5 = 5 units on the opposite side.
- Similarly, draw lines from B through O and C through O, extending to the opposite side.
- Measure OB and OC, then mark B' and C' at the same distances on the opposite side of O.
- Join A', B', and C' to form the image triangle A'B'C'.
- The image is the same size as the original but inverted and on the opposite side of O.

Problem 2: Circle Enlargement

The radius of a circle is 22.4 cm. The circle is enlarged by a scale factor of 0.25. Find the circumference of the image circle.

Solution:

- Original radius: r = 22.4 cm.
- New radius: r' = 0.25 × 22.4 = 5.6 cm.
- Circumference = 2πr = 2 × (22/7) × 5.6 = 35.2 cm.
- Alternatively: circumference scales linearly, so C' = 0.25 × (2 × 22/7 × 22.4) = 35.2 cm.

Problem 3: Coordinate Enlargement with Scale Factor ½

The vertices of triangle △ABC are A(−2, 9), B(8, 7), and C(5, 4). The triangle is enlarged with centre (0, 0) and scale factor ½. Find the coordinates of the image.

Solution:

Using (x', y') = (kx, ky) with k = ½:

Original Point	Calculation	Image Point
A(−2, 9)	(½ × −2, ½ × 9)	A'(−1, 9/2)
B(8, 7)	(½ × 8, ½ × 7)	B'(4, 7/2)
C(5, 4)	(½ × 5, ½ × 4)	C'(5/2, 2)

Phase 4: Assessment — Exit Ticket (5 minutes)

Assessment Questions:

1. A triangle with vertices X(4, 0), Y(6, 3), and Z(5, 4) is enlarged. If the centre of enlargement is (1, 1), find the coordinates of the image when the scale factor is: (a) −2 (b) ½

2. Points A(2, 6), B(4, 6), and C(4, 2) are the vertices of a triangle. Taking point (0, 2) as the centre of enlargement, find the coordinates of its image when the scale factor is −1.

3. Points P(1, 4), Q(3, 4), and R(3, 1) are vertices of a triangle. Taking the origin as the centre of enlargement, find the image when the scale factor is: (a) −¼ (b) −3 (c) 2

4. A rectangle measures 5 cm by 9 cm. Find the corresponding measurements of the image after an enlargement with scale factor −2.

5. A photograph is enlarged so that its width increases from 10 cm to 25 cm. If the original height is 15 cm, find the new height.

6. A map has a scale of 1 : 50,000. If the distance between two cities on the map is 8 cm, find the actual distance.

Answer Key:

- 1(a) Using Image = Centre + k(Point − Centre) with centre (1,1) and k = −2:
  X' = (1 + (−2)(4−1), 1 + (−2)(0−1)) = (1−6, 1+2) = (−5, 3)
  Y' = (1 + (−2)(6−1), 1 + (−2)(3−1)) = (1−10, 1−4) = (−9, −3)
  Z' = (1 + (−2)(5−1), 1 + (−2)(4−1)) = (1−8, 1−6) = (−7, −5)
- 1(b) With k = ½:
  X' = (1 + ½(4−1), 1 + ½(0−1)) = (1+1.5, 1−0.5) = (2.5, 0.5)
  Y' = (1 + ½(6−1), 1 + ½(3−1)) = (1+2.5, 1+1) = (3.5, 2)
  Z' = (1 + ½(5−1), 1 + ½(4−1)) = (1+2, 1+1.5) = (3, 2.5)
- 2. Centre (0, 2), k = −1:
  A' = (0 + (−1)(2−0), 2 + (−1)(6−2)) = (−2, −2)
  B' = (0 + (−1)(4−0), 2 + (−1)(6−2)) = (−4, −2)
  C' = (0 + (−1)(4−0), 2 + (−1)(2−2)) = (−4, 2)
- 3(a) Origin centre, k = −¼:
  P' = (−¼, −1), Q' = (−¾, −1), R' = (−¾, −¼)
- 3(b) k = −3:
  P' = (−3, −12), Q' = (−9, −12), R' = (−9, −3)
- 3(c) k = 2:
  P' = (2, 8), Q' = (6, 8), R' = (6, 2)
- 4. Scale factor −2: dimensions = |−2| × 5 = 10 cm by |−2| × 9 = 18 cm. The image is inverted.
- 5. Scale factor = 25/10 = 2.5. New height = 2.5 × 15 = 37.5 cm.
- 6. Scale 1:50,000. Actual distance = 8 × 50,000 = 400,000 cm = 4 km.

V. Differentiation Strategies

Learner Level	Strategy
Struggling Learners	Provide pre-drawn coordinate grids with the original triangle already plotted. Use colour coding: draw lines from object to centre in blue, and from centre to image in red, to emphasise the "opposite side" concept. Start with scale factor −1 only (same size, just inverted). Provide a step-by-step checklist: (1) Draw line from point through centre, (2) Measure distance, (3) Mark same distance on opposite side, (4) Repeat for all points, (5) Connect image points.
On-Level Learners	Complete all anchor activity steps and practice problems independently. Work through both construction method and coordinate method. Use the digital textbook interactive checkpoints for additional practice. Encourage peer explanation of why the image is inverted.
Advanced Learners	Extension Activity: Investigate the combined effect of two successive enlargements. If triangle ABC is enlarged by scale factor −2 about the origin, then

	the image is enlarged by scale factor −½ about the origin, what is the final image? Prove that two successive enlargements with scale factors k_1 and k_2 about the same centre produce the same result as a single enlargement with scale factor $k_1 \times k_2$. Explore: What happens to the area when the scale factor is −3? (Area scales by $k^2 = 9$.)

VI. Extension Activity

Successive Enlargements and Area Investigation:

1. Triangle PQR has vertices P(1, 1), Q(3, 1), R(2, 3). Perform two successive enlargements about the origin:

- (a) First enlargement: scale factor −2. Find P', Q', R'.
- (b) Second enlargement of the image: scale factor −½. Find P'', Q'', R''.
- (c) Compare P'', Q'', R'' with the original P, Q, R. What single scale factor would produce the same result?

Solution:

- (a) k = −2: P'(−2, −2), Q'(−6, −2), R'(−4, −6).
- (b) k = −½ applied to image: P''(1, 1), Q''(3, 1), R''(2, 3).
- (c) The final image equals the original! The combined scale factor is (−2) × (−½) = 1.

2. Area Investigation: If a triangle has area 6 cm² and is enlarged by scale factor −3, what is the area of the image?

- Area scales by $k^2 = (-3)^2 = 9$. New area = 9 × 6 = 54 cm².
- Note: The area is always positive regardless of the sign of the scale factor.

3. A building casts a shadow. If the building is 12 m tall and the shadow tip is 8 m from the base, a model of the building uses scale factor −1/100. What are the model dimensions and where does the model shadow fall?

- Model height = 12/100 = 0.12 m = 12 cm. Model shadow = 8/100 = 0.08 m = 8 cm. The negative sign means the model is inverted (upside down).

VII. Assessment Methods

Type	Method
Formative	Observation during pair work: Are students extending lines through the centre to the opposite side? Questioning: "Which side of the centre is the image on?" "Why is the image inverted?" Monitoring coordinate calculations during practice.
Summative	Exit ticket with 6 questions covering: negative scale factor construction, coordinate enlargement with non-origin centre, circle enlargement, mixed positive/negative scale factors, and real-world applications (photographs, maps).

VIII. Teacher Reflection

1. Did students understand the key difference between positive and negative scale factors (same side vs opposite side)?

2. Were students able to construct the image on the opposite side of the centre accurately?

3. How effectively did the anchor activity build on prior knowledge of positive scale factor enlargement?

4. Did students grasp that "negative" refers to direction (inversion), not size reduction?

5. Were students able to apply the coordinate formula (kx, ky) with negative values of k?

6. How well did the comparison table (positive vs negative) clarify the concepts?

7. What adjustments would improve the lesson for future delivery?