I. Lesson Overview

Strand	**Measurement and Geometry**
Sub-Strand	Similarity and Enlargement
Specific Learning Outcome	Determining the area scale factors of figures
Grade Level	Grade 10
Duration	40 minutes
Key Inquiry Question	How is similarity and enlargement applied in day-to-day life?
Learning Resources	CBC Grade 10 Mathematics Textbooks

II. Learning Objectives

Category	Objective
Know	Define area scale factor as the ratio of the area of the image to the area of the object. State that the area scale factor equals the square of the linear scale factor: ASF = $(LSF)^2$.
Do	Calculate the area scale factor given two similar figures. Determine the area of an image or object using the relationship between linear scale factor and area scale factor. Find the linear scale factor from a given area scale factor by taking the square root.
Apply	Apply area scale factor to solve problems involving similar hexagons, circles, cones, parallelograms, and real-world contexts such as maps, photographs, and architectural models.

III. Materials & Resources

- CBC Grade 10 Mathematics Textbooks

IV. Lesson Procedure

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: "Comparing Squares — What Happens to the Area?"

Instructions (Work in pairs):

- (a) Draw a square with a side length of 4 cm and label it as Square A.
- (b) Draw another square with a side length of 8 cm and label it as Square B.
- (c) Calculate the area of Square A and Square B.
- (d) Find the ratio of the areas by dividing the area of Square B by the area of Square A.
- (e) Take one side length of Square B, divide it by one side length of Square A, and then square the result.
- (f) Compare your answers from steps (d) and (e). What do you notice?
- (g) Discuss and share your findings with the rest of the class.

Teacher's Role During Discovery:

- Circulate among pairs, ensuring students are drawing accurate squares and calculating areas correctly.
- Ask probing questions: "What is the area of Square A?" "What is the area of Square B?" "What ratio did you get?"
- For pairs who finish early: "Try drawing a third square with side length 12 cm. Does the same pattern hold when you compare it to Square A?"
- For struggling pairs: "Remember, the area of a square is side × side. What is 4 × 4? What is 8 × 8?"
- Identify 2–3 pairs with clear findings to share with the class.
- Guide students toward noticing that the ratio of areas equals the square of the ratio of side lengths.

Expected Student Discoveries:

- Area of Square A = 4 × 4 = 16 cm².
- Area of Square B = 8 × 8 = 64 cm².
- Ratio of areas: 64 ÷ 16 = 4.
- Ratio of side lengths: 8 ÷ 4 = 2. Squaring: $2^2 = 4$.
- The ratio of the areas (4) equals the square of the ratio of the side lengths ($2^2 = 4$).
- When side lengths double, the area quadruples — the area does NOT simply double.

Phase 2: Structured Instruction (10 minutes)

Key Takeaways:

Definition:

Area scale factor is the ratio of the area of the image to the area of the object. Area scale factor is the square of the linear scale factor.

Key Formulas:

Formula	Description
Area Scale Factor (ASF) = (Linear Scale Factor)²	The area scale factor is the square of the linear scale factor.
ASF = Area of Image ÷ Area of Object	The area scale factor can be found by dividing the area of the image by the area of the object.
Linear Scale Factor = √(Area Scale Factor)	To find the linear scale factor from the area scale factor, take the square root.
Area of Image = ASF × Area of Object	To find the area of the image, multiply the area of the object by the area scale factor.

Illustrated Example (from Digital Textbook):

Parallelogram PQRS has base 5 cm and height 7 cm. Its enlarged image P′Q′R′S′ has base 10 cm and height 14 cm (linear scale factor = 2).

- Area of PQRS = 5 × 7 = 35 cm²
- Area of P′Q′R′S′ = 10 × 14 = 140 cm²
- Area Scale Factor = 140 ÷ 35 = 4
- Check: (Linear Scale Factor)² = 2² = 4 ✔

Connecting to Student Discoveries:

- Reference the squares: "You found that when the side doubled (factor of 2), the area quadrupled (factor of 4). That's because 2² = 4."
- Emphasise: "The area does NOT scale by the same factor as the length. It scales by the SQUARE of the linear factor."
- Address common misconception: "If the linear scale factor is 3, the area scale factor is NOT 3 — it is 3² = 9."
- Show the reverse: "If the area scale factor is 25, the linear scale factor is √25 = 5."

Phase 3: Practice and Application (10 minutes)

Problem 1: Square Enlargement

A square whose area is 28 cm² is given an enlargement with a linear scale factor of 4. Find the area of the image.

Solution:

Step	Working
Linear Scale Factor (LSF)	4
Area Scale Factor (ASF)	$(LSF)^2 = 4^2 = 16$
Set up equation	Area of Image ÷ 28 = 16
Solve	Area of Image = 16 × 28 = 448 cm²

Problem 2: Circle Radii from Area Ratio

Given that the ratio of the area of two circles is 25/64:

- (a) Find the ratio of their radii.
- (b) If the smaller one has a radius of 15 cm, find the radius of the larger one.

Solution:

(a) Area scale factor = 25/64. Linear scale factor = $\sqrt{(25/64)} = 5/8$. Therefore the ratio of the two radii = 5 : 8.

(b) If the radius of the smaller circle is 15 cm:

Radius of smaller ÷ Radius of larger = 5/8

15 ÷ Radius of larger = 5/8

Radius of larger = (8 × 15) ÷ 5 = 24 cm

Problem 3: Similar Hexagons

Two similar hexagons have corresponding sides of 9 cm and 6 cm. The area of hexagon A is 450 cm². Calculate the area of hexagon B.

Solution:

Step	Working
Linear Scale Factor	9/6 = 3/2 (from A to B, but B is smaller, so B to A ratio is 6/9 = 2/3)
Area Scale Factor (A to B)	$(6/9)^2 = 36/81$
Set up equation	Area of B ÷ Area of A = 36/81, so Area of B ÷ 450 = 36/81
Solve	Area of B = (36 × 450) ÷ 81 = 200 cm^2

Phase 4: Assessment — Exit Ticket (5 minutes)

Assessment Questions:

1. The corresponding sides of two similar regular hexagons are 4 cm and 9 cm respectively.

- (a) Find the ratio of their areas.
- (b) Calculate the area of the larger hexagon if the area of the smaller hexagon is 64 cm^2.

2. The ratio of the area of two similar cones is 9 : 36.

- (a) Find the area of the smaller cone if the area of the bigger cone is 320 m^2.
- (b) Find the ratio of their base radii.
- (c) If the slanting height of the smaller cone is 7 m, find the slanting height of the larger cone.

3. The length of a parallelogram is 15 cm and its area is 240 cm^2. Calculate the length of a similar parallelogram whose area is 375 cm^2.

4. The area of a circle is 49 m^2. A second circle has a radius that is 4 times the radius of the first circle. What is the area of the second circle?

Answer Key:

- 1(a) Linear scale factor = 4/9 (smaller to larger). Area scale factor = $(4/9)^2 = 16/81$. Ratio of areas = 16 : 81.
- 1(b) Area of smaller / Area of larger = 16/81. So 64 / Area of larger = 16/81. Area of larger = (64 × 81) / 16 = 324 cm^2.
- 2(a) Area ratio = 9 : 36 = 1 : 4. Area of smaller / 320 = 1/4. Area of smaller = 320 / 4 = 80 m^2.

- 2(b) Area scale factor = 9/36 = 1/4. Linear scale factor = $\sqrt{(1/4)} = 1/2$. Ratio of base radii = 1 : 2.
- 2(c) Linear scale factor (smaller to larger) = 1/2. So 7 / slanting height of larger = 1/2. Slanting height of larger = 7 × 2 = 14 m.
- 3. Area scale factor = 375/240 = 25/16. Linear scale factor = $\sqrt{(25/16)} = 5/4$. Length of similar parallelogram = (5/4) × 15 = 18.75 cm.
- 4. Linear scale factor = 4. Area scale factor = $4^2 = 16$. Area of second circle = 16 × 49 = 784 m^2.

V. Differentiation Strategies

Learner Level	Strategy
Struggling Learners	Provide pre-drawn squares on grid paper so students can count unit squares to find areas. Use colour coding: shade Square A in blue and Square B in red. Start with simple integer scale factors (2, 3) before fractions. Provide a formula card: "ASF = $(LSF)^2$" and "LSF = $\sqrt{ASF}$" for reference. Walk through Problem 1 step-by-step with the pair before independent work.
On-Level Learners	Complete all anchor activity steps and practice problems independently. Work through both forward problems (LSF → ASF → Area) and reverse problems (Area ratio → ASF → LSF). Use the digital textbook interactive checkpoints (2.1.27, 2.1.28, 2.1.29) for additional practice.
Advanced Learners	Extension Activity: Investigate the relationship between linear scale factor, area scale factor, and volume scale factor. If a cube has side length 3 cm and is enlarged by LSF = 2, find the volume scale factor. Prove that Volume Scale Factor = $(LSF)^3$. Challenge: A map has scale 1:25,000. A lake on the map has area 8 cm^2. Find the actual area of the lake in m^2.

VI. Extension Activity

From Area Scale Factor to Volume Scale Factor:

1. A cube has side length 3 cm.

- (a) Calculate its volume.
- (b) The cube is enlarged by a linear scale factor of 2. Calculate the new side length and volume.

- (c) Find the ratio of the volumes. What is the volume scale factor?
- (d) Compare the volume scale factor to $(LSF)^3$. What do you notice?

Solution:

- (a) Volume = $3^3 = 27$ cm³.
- (b) New side = $2 \times 3 = 6$ cm. New volume = $6^3 = 216$ cm³.
- (c) Volume scale factor = $216 / 27 = 8$.
- (d) $(LSF)^3 = 2^3 = 8$. The volume scale factor equals the cube of the linear scale factor.

2. Real-World Application: A map has a scale of 1 : 25,000. A lake on the map has an area of 8 cm². Find the actual area of the lake.

- LSF = 25,000. ASF = $(25{,}000)^2 = 625{,}000{,}000$.
- Actual area = $625{,}000{,}000 \times 8$ cm² = 5,000,000,000 cm² = 500,000 m² = 50 hectares.

3. Summary of Scale Factor Relationships:

Dimension	Scale Factor	Relationship
Length (1D)	Linear Scale Factor (LSF)	k
Area (2D)	Area Scale Factor (ASF)	k^2
Volume (3D)	Volume Scale Factor (VSF)	k^3

VII. Assessment Methods

Type	Method
Formative	Observation during pair work: Are students calculating areas correctly? Do they see the relationship between side ratio and area ratio? Questioning: "If the side length triples, what happens to the area?" "Is the area scale factor the same as the linear scale factor?" Monitoring calculations during practice problems.
Summative	Exit ticket with 4 questions covering: area ratio of similar hexagons, area of similar cones with ratio given, finding length from area ratio of parallelograms, and area of enlarged circles. Complete answer key provided for marking.

VIII. Teacher Reflection

1. Did students discover the relationship between the ratio of side lengths and the ratio of areas through the anchor activity?

2. Were students able to articulate that the area scale factor is the SQUARE of the linear scale factor?

3. How effectively did the parallelogram example from the digital textbook reinforce the concept?

4. Did students grasp the reverse process: finding the linear scale factor from the area scale factor by taking the square root?

5. Were students able to apply the concept to varied shapes (hexagons, circles, cones, parallelograms)?

6. What common misconceptions arose (e.g., confusing ASF with LSF)?

7. What adjustments would improve the lesson for future delivery?