Step by Step Guide: Area Scale Factor

Pre-Class Preparation Checklist

- Ensure each pair has graph paper or plain paper, rulers, and pencils.
- Prepare a large grid on the board or projector for demonstrating the two squares.
- Have the digital textbook section open: innodems.github.io/CBC-Grade-10-Maths/subsec-scale-factor.html
- Prepare printed handouts with the anchor activity instructions (steps a–g).
- Have pre-drawn squares on grid paper for struggling learners.
- Write on a card or slide: "Area Scale Factor = (Linear Scale Factor)^2".
- Prepare the parallelogram PQRS / P′Q′R′S′ diagram from the textbook for the instruction phase.

Phase 1: Problem-Solving and Discovery (15 minutes)

Opening and Recall (2 minutes)

[SAY] "In our previous lessons, we learned about enlargement and scale factors. We know that when we enlarge a shape, the side lengths change by the linear scale factor. But what happens to the AREA? Does it change by the same factor?"

[WAIT] Allow students to think for 10 seconds.

[SAY] "Let’s investigate! Today we will discover the relationship between the linear scale factor and how the area changes."

[ASK] "Who can remind us: what is the formula for the area of a square?"

[WAIT] Expected: "Side times side" or "side squared."

[SAY] "Perfect. We’ll use that knowledge in our investigation."

Anchor Activity Launch (3 minutes)

[SAY] "Work in pairs. Each pair needs paper and a ruler."

[DO] Distribute materials and display the instructions on the board or handout.

[SAY] "Step (a): Draw a square with side length 4 cm. Label it Square A."

[SAY] "Step (b): Draw another square with side length 8 cm. Label it Square B."

[SAY] "Step (c): Calculate the area of both squares."

[SAY] "Step (d): Divide the area of Square B by the area of Square A. What ratio do you get?"

[SAY] "Step (e): Now take one side of Square B, divide by one side of Square A, and SQUARE the result."

[SAY] "Step (f): Compare your answers from (d) and (e). What do you notice?"

[SAY] "Step (g): Discuss your findings with your partner and be ready to share."

Student Work Time (8 minutes)

[DO] Circulate among pairs. Check that students are drawing squares accurately and calculating areas correctly.

[ASK] To pairs calculating areas: "What is the area of Square A? And Square B?"

[DO] For struggling pairs: "Remember, area of a square is side times side. So 4 times 4 is...? And 8 times 8 is...?"

[ASK] To pairs who found the ratio: "What ratio did you get when you divided 64 by 16?"

[ASK] "And when you divided 8 by 4 and squared it, what did you get?"

[ASK] "Are those two answers the same? Why do you think that is?"

[ASK] To early finishers: "Try a third square with side 12 cm. Does the same pattern hold when you compare it to Square A?"

[DO] Note which pairs have clear explanations for the sharing phase.

Class Sharing (2 minutes)

[SAY] "Let’s hear from a few pairs. What did you find?"

[WAIT] Call on 2–3 pairs to share.

[SAY] "So Square A has area 16 cm^2 and Square B has area 64 cm^2. The ratio of areas is 64 divided by 16, which equals 4."

[SAY] "And the ratio of side lengths is 8 divided by 4, which equals 2. When we square that: 2 squared equals 4."

[SAY] "They're the SAME! The ratio of the areas equals the SQUARE of the ratio of the side lengths."

[ASK] "So when the side length doubles, does the area double?"

[WAIT] Expected: "No, it quadruples!"

[SAY] "Exactly! The area QUADRUPLES. This is a very important relationship that we'll now formalise."

Phase 2: Structured Instruction (10 minutes)

Formalising the Area Scale Factor (4 minutes)

[SAY] "Let's formalise what you discovered. We call this relationship the AREA SCALE FACTOR."

[WRITE] On the board: "Area Scale Factor (ASF)"

[WRITE] "Definition: Area scale factor is the ratio of the area of the image to the area of the object."

[WRITE] "Key Rule: Area Scale Factor = (Linear Scale Factor)^2"

[SAY] "In your investigation, the linear scale factor was 2 — because the side went from 4 to 8. The area scale factor was 2 squared, which is 4."

[WRITE] "ASF = (LSF)^2"

[WRITE] "LSF = √(ASF)"

[SAY] "These two formulas are inverses. If you know the linear scale factor, square it to get the area scale factor. If you know the area scale factor, take the square root to get the linear scale factor."

Illustrated Example: Parallelogram (3 minutes)

[SAY] "Let me show you an example from the textbook with parallelograms."

[DO] Draw or display the parallelogram PQRS (base 5 cm, height 7 cm) and its image P'Q'R'S' (base 10 cm, height 14 cm).

[WRITE] "Area of PQRS = 5 × 7 = 35 cm^2"

[WRITE] "Area of P'Q'R'S' = 10 × 14 = 140 cm^2"

[WRITE] "Area Scale Factor = 140 ÷ 35 = 4"

[SAY] "The linear scale factor is 10 divided by 5, which is 2. And 2 squared is 4. It matches!"

[ASK] "What if the linear scale factor was 3? What would the area scale factor be?"

[WAIT] Expected: "9"

[SAY] "Yes! 3 squared is 9. The area would be 9 times larger."

Addressing Misconceptions (3 minutes)

[SAY] "Now, a very common mistake. If I tell you the linear scale factor is 5, some students say the area scale factor is also 5. Is that correct?"

[WAIT] Expected: "No!"

[SAY] "Correct! The area scale factor is 5 squared, which is 25. The area is 25 times larger, NOT 5 times larger."

[SAY] "And here's the reverse. If I tell you the area scale factor is 25, what is the linear scale factor?"

[WAIT] Expected: "5, because the square root of 25 is 5."

[SAY] "Exactly. To go from area scale factor BACK to linear scale factor, we take the square root."

[ASK] "If the area scale factor is 16, what is the linear scale factor?"

[WAIT] Expected: "4."

[SAY] "Perfect. $\sqrt{16} = 4$."

Phase 3: Practice and Application (10 minutes)

Worked Example 1: Square Enlargement (2 minutes)

[SAY] "Let's work through Problem 1 together. A square has area 28 cm² and is enlarged with linear scale factor 4."

[WRITE] "LSF = 4"

[WRITE] "ASF = $(LSF)^2 = 4^2 = 16$"

[WRITE] "Area of Image ÷ Area of Object = ASF"

[WRITE] "Area of Image ÷ 28 = 16"

[WRITE] "Area of Image = 16 × 28 = 448 cm²"

[SAY] "So the image area is 448 cm². Notice: the side lengths are 4 times larger, but the area is 16 times larger."

Worked Example 2: Circle Radii from Area Ratio (3 minutes)

[SAY] "Problem 2: The ratio of the areas of two circles is 25 to 64. Find the ratio of their radii."

[WRITE] "Area Scale Factor = 25/64"

[SAY] "To find the linear scale factor — which gives us the ratio of radii — we take the square root."

[WRITE] "LSF = √(25/64) = 5/8"

[WRITE] "Ratio of radii = 5 : 8"

[SAY] "Part (b): The smaller circle has radius 15 cm. Find the radius of the larger one."

[WRITE] "15 / radius of larger = 5/8"

[WRITE] "Radius of larger = (8 × 15) / 5 = 24 cm"

[SAY] "So the larger circle has radius 24 cm."

Student Practice: Similar Hexagons (5 minutes)

[SAY] "Now try Problem 3 on your own. Two similar hexagons have sides 9 cm and 6 cm. Hexagon A has area 450 cm². Find the area of hexagon B."

[DO] Give students 3 minutes to work. Circulate and assist.

[ASK] To students working: "What is the linear scale factor from A to B?" "What do you do with it to get the area scale factor?"

[DO] After 3 minutes, work through the solution on the board:

[WRITE] "LSF (A to B) = 6/9 = 2/3"

[WRITE] "ASF = (2/3)² = 4/9"

[WRITE] "Area of B / Area of A = 4/9"

[WRITE] "Area of B / 450 = 4/9"

[WRITE] "Area of B = (4 × 450) / 9 = 200 cm²"

[SAY] "Hexagon B is smaller, so its area should be less than 450. And 200 is indeed less. This makes sense because the linear scale factor from A to B is less than 1."

Phase 4: Assessment — Exit Ticket (5 minutes)

[SAY] "For our exit ticket, answer these four questions on a separate piece of paper. You have 5 minutes."

[SAY] "Question 1: Two similar regular hexagons have corresponding sides of 4 cm and 9 cm. Part (a): Find the ratio of their areas. Part (b): Find the area of the larger hexagon if the smaller has area 64 cm²."

[SAY] "Question 2: The ratio of the areas of two similar cones is 9 to 36. Part (a): Find the area of the smaller cone if the bigger cone has area 320 m². Part (b): Find the ratio of their base radii. Part (c): If the slanting height of the smaller cone is 7 m, find the slanting height of the larger cone."

[SAY] "Question 3: A parallelogram has length 15 cm and area 240 cm². Calculate the length of a similar parallelogram whose area is 375 cm²."

[SAY] "Question 4: A circle has area 49 m². A second circle has a radius that is 4 times the radius of the first. Find the area of the second circle."

[DO] Collect exit tickets as students finish.

Answer Key:

- 1(a) LSF = 4/9. ASF = $(4/9)^2$ = 16/81. Ratio of areas = 16 : 81.
- 1(b) 64 / Area of larger = 16/81. Area of larger = (64 × 81) / 16 = 324 cm².
- 2(a) Area ratio = 9:36 = 1:4. Area of smaller = 320 / 4 = 80 m².
- 2(b) ASF = 1/4. LSF = √(1/4) = 1/2. Ratio of base radii = 1 : 2.
- 2(c) LSF = 1/2. Slanting height of larger = 7 × 2 = 14 m.
- 3. ASF = 375/240 = 25/16. LSF = √(25/16) = 5/4. Length = (5/4) × 15 = 18.75 cm.
- 4. LSF = 4. ASF = 4^2 = 16. Area = 16 × 49 = 784 m².

Differentiation Notes

Struggling Learners:

Provide pre-drawn squares on grid paper so students can count unit squares. Use colour coding for the two squares. Start with simple integer scale factors (2, 3) before fractions. Provide a formula card: "$ASF = (LSF)^2$" and "$LSF = \sqrt{ASF}$". Walk through Problem 1 step-by-step before independent work.

On-Level Learners:

Complete all problems independently. Work through both forward (LSF → ASF → Area) and reverse (Area ratio → ASF → LSF) problems. Use digital textbook checkpoints 2.1.27, 2.1.28, and 2.1.29 for additional practice.

Advanced Learners:

Investigate the relationship between LSF, ASF, and Volume Scale Factor ($VSF = (LSF)^3$). Challenge problem: A map has scale 1:25,000. A lake on the map has area 8 cm^2. Find the actual area of the lake in m^2. Explore: If the area of a shape triples, by what factor does each side length increase? ($\sqrt{3} \approx 1.73$)

Post-Lesson Reflection

1. Did students discover through the anchor activity that the area ratio equals the square of the side length ratio?

2. Were students able to articulate the difference between linear scale factor and area scale factor?

3. How effectively did the parallelogram example from the digital textbook reinforce the concept?

4. Did students grasp the reverse process: finding LSF from ASF by taking the square root?

5. Were students able to apply the concept to varied shapes (hexagons, circles, cones, parallelograms)?

6. What common misconceptions arose (e.g., thinking ASF = LSF instead of $ASF = LSF^2$)?

7. What adjustments would improve the lesson for future delivery?