Grade 10 Mathematics Lesson Plan

Multiplying Vectors by Scalars

Strand:	**Measurement and Geometry**
Sub-Strand:	Vectors I
Specific Learning Outcome:	Multiplying vectors by scalars
Duration:	40 minutes
Key Inquiry Question:	How is Vectors I applied in day-to-day life?
Learning Resources:	CBC Grade 10 textbooks, graph paper, rulers, pencils, colored markers

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Discovering scalar multiplication
Structured Instruction	10 minutes	Formalizing scalar multiplication rules
Practice and Application	10 minutes	Worked examples and simplifying vector expressions
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Discovering Scalar Multiplication

Objective: Students will draw vectors on graph paper and discover how multiplying a vector by a scalar affects its magnitude and direction.

Materials Needed:

- Graph paper (one sheet per student)
- Rulers
- Pencils and colored markers
- Coordinate plane drawn on board or chart paper

Activity Steps (Activity 2.9.9 from textbook):

1. Step 1: On a graph paper, draw the x axis and y axis.
2. Step 2: Draw a directed line passing through point A(0, 2) and B(2, 2).
3. Step 3: From point B(2, 2), draw a directed line to point C(4, 2).
4. Step 4: Determine the coordinate representations of vector AB and vector AC.
5. Step 5: How does vector AB relate to vector AC?
6. Step 6: Discuss and share your findings with the rest of the class.

Expected Discovery:

Students should discover that:

- Vector AB has coordinates (2, 0) - moves 2 units right
- Vector AC has coordinates (4, 0) - moves 4 units right
- AC is exactly twice as long as AB
- AC equals 2 times AB
- Both vectors point in the same direction (to the right)
- Multiplying a vector by 2 doubles its length but keeps the same direction

Guiding Questions:

7. What are the coordinates of vector AB?
8. What are the coordinates of vector AC?
9. How does the length of AC compare to AB?
10. Do the vectors point in the same direction or different directions?
11. If AB equals (2, 0), what is 2 times AB?
12. What happens to a vector when you multiply it by 2? By 3?
13. What do you think happens if you multiply by negative 2?
14. What happens if you multiply by 0?

Teacher Role During Discovery:

- Circulate among students, ensuring they draw vectors correctly with arrows.
- Ask probing questions: How does AC compare to AB? What is the relationship?
- For struggling students: Let us measure. AB is 2 units long. How long is AC? Is it twice as long?
- For early finishers: What if we multiply by 3? Draw a vector 3 times as long. What if we multiply by negative 1?
- Guide students to articulate: Multiplying a vector by a positive number makes it longer. The direction stays the same.

- Identify 2-3 students with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing Scalar Multiplication

After students have completed the anchor activity and shared their findings, the teacher formalizes the concept of multiplying vectors by scalars.

Key Takeaway 1: Positive Scalar

When we multiply a vector a by a positive scalar, say 2, the length of the vector doubles, making it 2a. The direction of the vector remains unchanged, but its magnitude increases.

Example: If vector PQ is represented as a, then PN equals a plus a equals 2a. This means PN has the same direction as PQ, but its magnitude is twice that of PQ.

Key Takeaway 2: Negative Scalar

Consider the vector AB, denoted as a. The vector points to the right and has a magnitude of a.

The vector AC is obtained by multiplying AB by negative 2, giving: AC equals negative 2 times a equals negative 2a

This means that AC has twice the magnitude of AB, but its direction is reversed.

Multiplying a vector by a negative scalar reverses its direction, making it point in the opposite direction.

Key Takeaway 3: Zero Scalar

When a vector a is multiplied by 0, its magnitude becomes 0, resulting in a zero vector.

a times 0 equals 0

Scalar Multiplication Rules:

- If k is positive: k times a has the same direction as a, but magnitude is multiplied by k
- If k is negative: k times a has the opposite direction to a, and magnitude is multiplied by absolute value of k
- If k equals 0: k times a equals zero vector
- If k equals 1: 1 times a equals a (vector stays the same)
- If k equals negative 1: negative 1 times a equals negative a (vector reverses direction)

Scaffolding Strategies to Address Misconceptions:

- Misconception: Multiplying by 2 adds 2 to each component. Clarification: No, it multiplies each component by 2.
- Misconception: Negative scalar makes the vector smaller. Clarification: No, it reverses direction and may change magnitude.
- Misconception: Multiplying by 0 keeps the vector the same. Clarification: No, it creates a zero vector with no magnitude.
- Misconception: Direction always stays the same. Clarification: No, negative scalars reverse the direction.

Phase 3: Practice and Application (10 minutes)

Worked Example from Textbook:

Example 2.9.51: Given the vectors u equals 2p plus 5q and v equals p minus 3q, express 3u plus 2v in terms of p and q.

Solution:

Substitute the given expressions for u and v.

3u plus 2v equals 3(2p plus 5q) plus 2(p minus 3q)

equals 6p plus 15q plus 2p minus 6q

Combine like terms (terms with p and terms with q).

3u plus 2v equals (6p plus 2p) plus (15q minus 6q)

equals 8p plus 9q

Additional Practice Problems:

Problem 1: If a equals (3, 4), find 2a.

Solution: 2a equals 2(3, 4) equals (6, 8)

Problem 2: If b equals (5, negative 2), find negative 3b.

Solution: negative 3b equals negative 3(5, negative 2) equals (negative 15, 6)

Problem 3: Simplify 5x plus 3y minus z plus 2(3x minus z) plus (8x minus 6y).

Solution: equals 5x plus 3y minus z plus 6x minus 2z plus 8x minus 6y

equals (5x plus 6x plus 8x) plus (3y minus 6y) plus (negative z minus 2z)

equals 19x minus 3y minus 3z

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. Simplify: 5x plus 3y minus z plus 2(3x minus z) plus (8x minus 6y)
2. Simplify: (a minus b) plus (c minus a) plus (b minus c)
3. Simplify: 4m minus 2n plus 5(k minus m) plus 2(m plus n)
4. Given x equals 3m minus n and y equals n plus 4m, express 3x in terms of m and n.
5. Given x equals 3m minus n and y equals n plus 4m, express 6x minus 9y in terms of m and n.

Answer Key:

1. 19x minus 3y minus 3z

2. 0 (zero vector)

3. m plus 5k

4. 3x equals 3(3m minus n) equals 9m minus 3n

5. 6x minus 9y equals 6(3m minus n) minus 9(n plus 4m) equals 18m minus 6n minus 9n minus 36m equals negative 18m minus 15n

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn vectors with labels.
- Use color coding: one color for original vector, another for scaled vector.
- Provide step-by-step calculation templates.
- Start with simple scalars (2, 3) before introducing negative scalars.
- Use physical manipulatives (arrows on sticks) to show scaling.
- Pair struggling students with confident problem solvers.

For On-Level Learners:

- Encourage students to create their own scalar multiplication problems.
- Ask students to explain why negative scalars reverse direction.
- Provide mixed practice with positive, negative, and zero scalars.
- Challenge students to simplify complex vector expressions.

For Advanced Learners:

- Explore scalar multiplication in three dimensions.
- Investigate unit vectors (vectors with magnitude 1).
- Apply scalar multiplication to physics problems (force, velocity).
- Challenge: Prove that scalar multiplication is distributive: k(a plus b) equals ka plus kb.
- Explore applications in computer graphics (scaling transformations).

Extension Activity

Real-World Application: Force Vectors in Physics

Work in pairs or small groups

Situation: You are pushing a box with a force vector F equals (10, 5) Newtons.

Tasks:

15. If you push twice as hard, what is the new force vector? (Answer: 2F equals (20, 10))
16. If your friend pushes in the opposite direction with the same force, what is their force vector? (Answer: negative F equals (negative 10, negative 5))
17. If you both push together (you with F and your friend with 1.5F), what is the total force? (Answer: F plus 1.5F equals 2.5F equals (25, 12.5))
18. If you stop pushing (multiply by 0), what is the force? (Answer: 0F equals (0, 0))
19. Plot all force vectors on graph paper.
20. Calculate the magnitude of each force vector using the formula: magnitude equals square root of (x squared plus y squared).
21. Present your findings to the class.

Real-World Applications of Scalar Multiplication:

- Physics: Scaling forces, velocities, and accelerations.
- Engineering: Adjusting magnitudes of structural forces.
- Computer Graphics: Scaling objects in animations and video games.
- Economics: Scaling production or consumption vectors.
- Navigation: Adjusting speed while maintaining direction.

Teacher Reflection Prompts

- Did students successfully discover how scalar multiplication affects vectors?
- Were students able to apply scalar multiplication correctly?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand the difference between positive and negative scalars?
- What adjustments would improve this lesson for future classes?