Step by Step Guide

Apply Quadratic Identities in Numerical Cases

Pre-Class Preparation Checklist

☐ Ensure each group has CBC Grade 10 Mathematics Textbooks

☐ Prepare the garden scenario on the board or projector

☐ Write the three quadratic identities on a chart or slide (keep covered initially)

☐ Prepare exit ticket slips (3 questions each)

☐ Have reference cards ready for struggling learners

☐ Arrange desks for group work (groups of 4–5)

Phase 1: Problem-Solving and Discovery (15 minutes)

[SAY] "Good morning, class! Today we are going to become garden designers. Imagine you have been hired to design a vegetable garden for a client. The client wants a rectangular garden with an area of exactly 48 square feet, and the length must be twice the width. Your job is to figure out the exact dimensions."

[DO] Display or write the scenario on the board:

- Area = 48 square feet
- Length = 2 × Width
- Find the exact dimensions

[SAY] "Work in your groups. Let w represent the width. Write an equation for the area, then solve it. You have 8 minutes."

[DO] Start timer. Circulate among groups.

Probing Questions While Circulating:

- "How did you set up your equation?"
- "What operation did you use to isolate w^2?"
- "Can you verify your answer makes sense?"
- "What if the area changed to 72 square feet? How would your dimensions change?"

[WAIT] Allow 8 minutes for group work.

[SAY] "Let's hear from a few groups. Group 1, how did you set up your equation?"

Expected Student Responses:

- "We wrote $A = l \times w$, then substituted $l = 2w$ to get $48 = 2w \times w$"
- "We simplified to $2w^2 = 48$, then $w^2 = 24$, so $w = \sqrt{24} \approx 4.9$ feet"
- "The length is $2 \times 4.9 = 9.8$ feet"

[SAY] "Excellent work! You've just solved a quadratic equation that came from a real-world problem. Now, what if I told you there are powerful shortcuts called quadratic identities that can help us solve problems like this even faster? Let's explore them."

Phase 2: Structured Instruction (10 minutes)

[DO] Reveal the three quadratic identities chart.

[SAY] "There are three quadratic identities that are incredibly useful tools. Let me show you each one with a numerical example."

[WRITE] Identity 1: Perfect Square (Sum)

[SAY] "$(a + b)^2 = a^2 + 2ab + b^2$. Let's use this to calculate 51 squared without a calculator."

[WRITE] $51^2 = (50 + 1)^2 = 50^2 + 2(50)(1) + 1^2 = 2500 + 100 + 1 = 2{,}601$

[SAY] "See how we broke 51 into 50 + 1? That's the key — choose values of a and b that make the arithmetic easy."

[WRITE] Identity 2: Perfect Square (Difference)

[SAY] "$(a - b)^2 = a^2 - 2ab + b^2$. Let's calculate 49 squared."

[WRITE] $49^2 = (50 - 1)^2 = 50^2 - 2(50)(1) + 1^2 = 2500 - 100 + 1 = 2{,}401$

[WRITE] Identity 3: Difference of Squares

[SAY] "$(a + b)(a - b) = a^2 - b^2$. This is perfect for multiplying numbers that are equally spaced from a round number."

[WRITE] $51 \times 49 = (50 + 1)(50 - 1) = 50^2 - 1^2 = 2500 - 1 = 2{,}499$

Addressing the Common Misconception:

[SAY] "Be very careful! A common mistake is to think $(a + b)^2 = a^2 + b^2$. Let me prove this is WRONG with numbers."

[WRITE] $(3 + 2)^2 = 5^2 = 25$

[WRITE] But $3^2 + 2^2 = 9 + 4 = 13 \neq 25$

[SAY] "The missing piece is $2ab = 2(3)(2) = 12$. So 13 + 12 = 25. Never forget the middle term!"

[SAY] "Now let's connect this back to our garden. If a square garden has side $(x + 3)$, the area is $(x + 3)^2 = x^2 + 6x + 9$. When $x = 5$, that's 25 + 30 + 9 = 64 square metres. Check: $8^2 = 64$. It works!"

Phase 3: Practice and Application (15 minutes)

[SAY] "Now it's your turn to apply these identities. Work through these five problems. For each one, identify which identity to use, then solve step by step."

[DO] Display the five practice problems. Allow 10 minutes for individual/pair work.

Problems:

1. Use a quadratic identity to calculate 103^2 without a calculator.

2. Use a quadratic identity to calculate 97^2 without a calculator.

3. Use the difference of squares identity to calculate 63×57.

4. A rectangular field has dimensions $(x + 5)$ metres by $(x - 5)$ metres. Express the area using a quadratic identity. If $x = 12$, find the area.

5. A farmer wants to build a square pen with side length $(2n + 3)$ metres. Find the area as a quadratic expression. If $n = 4$, what is the area?

Probing Questions While Circulating:

• "Which identity did you choose for this problem? Why?"

• "How did you decide what values to use for a and b?"

• "Can you verify your answer using regular multiplication?"

• "For the field problem, which identity pattern do you recognise?"

[DO] After 10 minutes, review answers as a class.

[SAY] "Let's go through the answers together."

Answer Key:

[WRITE] 1. $103^2 = (100 + 3)^2 = 10000 + 600 + 9 = 10{,}609$

[WRITE] 2. $97^2 = (100 - 3)^2 = 10000 - 600 + 9 = 9{,}409$

[WRITE] 3. $63 \times 57 = (60 + 3)(60 - 3) = 3600 - 9 = 3{,}591$

[WRITE] 4. Area = $(x + 5)(x - 5) = x^2 - 25$. When $x = 12$: $144 - 25 = 119$ m²

[WRITE] 5. Area = $(2n + 3)^2 = 4n^2 + 12n + 9$. When $n = 4$: $64 + 48 + 9 = 121$ m²

PHASE 4: Assessment / Checkpoint (8 Minutes)

Checkpoint exploration (5 minutes)

[DO] Project the digital textbook on the screen. Navigate to the "Checkpoint" section.

[SAY] "This is our digital mathematics textbook. It has something special called checkpoints. Watch what happens when I click this button..."

[DO] Click "Show new example question" on Checkpoint

[SAY] "See? A new number appeared! And if I click again..."

[DO] Click the button again to show randomization

[SAY] "A different number! This means you can practice with hundreds of different examples. The computer never runs out of problems to give you."

[SAY] "Now it's your turn. With your partner, open the digital textbook and find the checkpoint.

[SAY] Click “Show new example question” to load the problem

[SAY] Solve the displayed question

[SAY] Click “submit” to check your answer

[SAY] If incorrect, carefully read the feedback and analyse the error before trying a new question. The immediate feedback from checkpoint submissions allows students to identify and correct errors in real-time.

[SAY] Complete at least 5 questions

[DO] Circulate among pairs. Ask probing questions, for example, what patterns do you notice?

Independent Work (Exit ticket) (5 minutes)

[SAY] "For our final activity, I’d like you to complete this exit ticket independently. This helps me understand what you’ve learned today."

[DO] Distribute exit ticket slips.

Exit Ticket Questions:

1. Use a quadratic identity to evaluate 202^2.

2. Calculate 48×52 using the difference of squares identity.

3. A square playground has side length $(y + 7)$ metres. Write the area as a quadratic expression and find the area when $y = 3$.

[WAIT] Allow 5 minutes for completion. Collect slips.

Exit Ticket Answer Key:

1. $202^2 = (200 + 2)^2 = 40000 + 800 + 4 = 40{,}804$

2. $48 \times 52 = (50 - 2)(50 + 2) = 2500 - 4 = 2{,}496$

3. Area $= (y + 7)^2 = y^2 + 14y + 49$. When $y = 3$: $9 + 42 + 49 = 100$ m^2

Differentiation Notes

Struggling Learners:

• Provide a reference card with the three identities and worked numerical examples

• Start with simpler numbers: $11^2 = (10+1)^2$, $19^2 = (20-1)^2$, 21×19

• Use colour-coded steps to show how a and b map into each identity

• Pair with a peer mentor during practice

Advanced Learners:

• Simplify $(2x + 3y)^2 - (2x - 3y)^2$ using identities

• Challenge: A farmer has 100m of fencing. Using quadratic identities, find dimensions that maximise the enclosed area

• Prove that the product of two consecutive odd numbers is always one less than a perfect square

Post-Lesson Reflection Prompts

1. Did students successfully connect the garden scenario to quadratic identities?

2. Which identity did students find most challenging to apply numerically?

3. Were students able to distinguish between $(a + b)^2$ and $(a + b)(a - b)$?

4. How effectively did the real-world context motivate student engagement?

5. Did the differentiation strategies adequately support all learner levels?

6. What adjustments would improve the transition from discovery to formal instruction?

7. How can I better scaffold the connection between algebraic identities and numerical computation?