Step by step guide: Using Reciprocals of Real Numbers in Mathematical Computations

Lesson Information

Topic	Using Reciprocals of Real Numbers in Mathematical Computations
Grade Level	10
Duration	40 minutes (15 + 10 + 15 + Exit Ticket)
Materials	CBC Grade 10 Mathematics Textbooks, exercise books, pens

Pre-Class Preparation Checklist

- Ensure each group has exercise books and pens
- Write the three key applications on the board (covered until Phase 2)
- Prepare worked examples for display
- Have extension activity problems ready for advanced learners
- Prepare exit ticket questions on the board or printed handouts

Phase 1: Problem-Solving and Discovery (15 minutes)

Opening Hook (2 minutes)

[SAY] "Good morning, class! Let me ask you a quick question. If I have 4 cups of sugar and a recipe needs 2/3 of a cup per serving, how many servings can I make? Don't calculate yet—just think about what operation you would use."

[WAIT] 10 seconds for students to think

[ASK] "Who thinks they know? What operation would you use?"

[SAY] "Most of you said division—and you're right! But today we're going to learn a powerful trick: instead of dividing, we can MULTIPLY by something called a reciprocal. This makes many calculations much easier."

Anchor Activity (10 minutes)

[SAY] "Now, get into your groups. Each group needs an exercise book and a pen. You're going to explore reciprocals through a hands-on activity."

[DO] Display or read the activity instructions:

1. Write different numbers—positive, negative, decimals, and fractions—in your exercise book. For example: 2, −4, 0.25, 5/8

2. Find the reciprocal of each number. Remember: the reciprocal of 5/8 is found by 1 ÷ 5/8 = 1 × 8/5 = 8/5

3. Multiply each original number by its reciprocal and observe the result

4. Discuss: Are there cases where reciprocals do NOT exist?

5. Identify a real-world problem where reciprocals are useful

6. Prepare to share your findings with the class

[DO] Circulate among groups. Use these probing questions:

- "What happens every time you multiply a number by its reciprocal?"
- "Can you find the reciprocal of zero? Try it—what happens?"
- "How did you handle the decimal 0.25? Did you convert it first?"
- "What is the reciprocal of a negative number? Is it positive or negative?"

Group Sharing (3 minutes)

[SAY] "Time's up! Let's hear from each group. Group 1, what did you discover when you multiplied a number by its reciprocal?"

[WAIT] for response (Expected: "The answer is always 1")

[ASK] "Did any group find a number that does NOT have a reciprocal?"

[WAIT] for response (Expected: "Zero")

[SAY] "Excellent discoveries! You've found two critical properties: a number times its reciprocal always equals 1, and zero has no reciprocal because we cannot divide by zero. Now let's formalize what you've discovered."

Phase 2: Structured Instruction (10 minutes)

Key Application 1: Converting Division into Multiplication (3 minutes)

[SAY] "The first and most important use of reciprocals is this: instead of dividing by a number, we can MULTIPLY by its reciprocal."

[WRITE] On the board: a ÷ b = a × 1/b

[SAY] "Let's see this with whole numbers first. 8 ÷ 2 = 8 × 1/2 = 4. Simple, right?"

[WRITE] On the board: $8 \div 2 = 8 \times 1/2 = 4$

[SAY] "Now with fractions—this is where it becomes really powerful. To divide 5/6 by 2/3, we flip the second fraction and multiply."

[WRITE] On the board: $5/6 \div 2/3 = 5/6 \times 3/2 = 15/12 = 5/4$

Key Application 2: Solving Equations (3 minutes)

[SAY] "The second application: when a variable is multiplied by a number, we use the reciprocal to isolate it."

[WRITE] On the board: $3x = 12$

[SAY] "To get x alone, I multiply both sides by the reciprocal of 3, which is 1/3."

[WRITE] $x = 12 \times 1/3 = 4$

[ASK] "Can someone tell me what the reciprocal of 3/5 would be? And how would you solve $3/5 \times x = 9$?"

[WAIT] for response (Expected: "Reciprocal is 5/3, so $x = 9 \times 5/3 = 15$")

Key Application 3: Proportions and Ratios (2 minutes)

[SAY] "The third application connects to real life. Remember our opening question about the recipe? If a recipe uses 2/3 of a cup of sugar per serving, how many servings from 4 cups?"

[WRITE] $4 \div 2/3 = 4 \times 3/2 = 12/2 = 6$ servings

[SAY] "By using the reciprocal, we turned a tricky division into a simple multiplication. This works for cooking, construction, farming—any situation where you need to divide by a fraction."

Addressing Misconceptions (2 minutes)

[SAY] "Before we practice, let me address three common mistakes:"

[WRITE] On the board:

- Zero has NO reciprocal (1/0 is undefined)
- The reciprocal of a negative number is negative ($-4 \rightarrow -1/4$)
- For decimals, convert to a fraction first ($0.25 = 1/4 \rightarrow$ reciprocal = 4)

Phase 3: Practice and Application (15 minutes)

Guided Practice (5 minutes)

[SAY] "Let's work through some real-world problems together. Follow along in your notebooks."

[SAY] "Problem 1: A printing machine can print 5/6 of a page per second. How long to print 20 pages?"

[ASK] "What operation do we need? And what's the reciprocal of 5/6?"

[WAIT] for response

[WRITE] Time = $20 \div 5/6 = 20 \times 6/5 = 120/5 = 24$ seconds

[SAY] "Problem 2: Kerich has 2/3 of an acre and wants plots of 1/6 acre each. How many plots?"

[ASK] "Who can set this up using reciprocals?"

[WAIT] for volunteer

[WRITE] $2/3 \div 1/6 = 2/3 \times 6/1 = 12/3 = 4$ plots

Independent Practice (7 minutes)

[SAY] "Now try these on your own. Work with your partner if you get stuck."

[DO] Display remaining problems:

- If a car travels at 80 km/h, find the time taken per km using reciprocals.
- A factory produces 300 items in 5 hours. How much time per item?
- Njoki invests Ksh 5,000 in a business that doubles every 1.5 years. What will she have after 4.5 years?

[DO] Circulate and support. For struggling students:

- "What are you dividing by? Can you write it as a fraction?"
- "Now flip that fraction—that's the reciprocal. Multiply instead."
- "Check your answer: does it make sense in the real-world context?"

Review Solutions (3 minutes)

[SAY] "Let's check our answers."

[WRITE] Solutions on the board:

Car: Time per km = 1/80 hours = 0.75 minutes per km

Factory: Rate = 60 items/hour, Time per item = 1/60 hours = 1 minute

Njoki: $4.5 \div 1.5 = 3$ doublings $\rightarrow 5{,}000 \times 2^3 = 5{,}000 \times 8 =$ Ksh 40,000

PHASE 4: Assessment / Checkpoint (8 Minutes)

Checkpoint exploration (5 minutes)

[DO] Project the digital textbook on the screen. Navigate to the "Checkpoint" section.

[SAY] "This is our digital mathematics textbook. It has something special called checkpoints. Watch what happens when I click this button..."

[DO] Click "Show new example question" on Checkpoint

[SAY] "See? A new number appeared! And if I click again..."

[DO] Click the button again to show randomization

[SAY] "A different number! This means you can practice with hundreds of different examples. The computer never runs out of problems to give you."

[SAY] "Now it's your turn. With your partner, open the digital textbook and find the checkpoint.

[SAY] Click “Show new example question” to load the problem

[SAY] Solve the displayed question

[SAY] Click “submit” to check your answer

[SAY] If incorrect, carefully read the feedback and analyse the error before trying a new question. The immediate feedback from checkpoint submissions allows Learners to identify and correct errors in real-time.

[SAY] Complete at least 5 questions

[DO] Circulate among pairs. Ask probing questions, for example, what patterns do you notice?

Independent Work (5 minutes)

[SAY] "For our final activity, complete these questions independently in your notebooks. Show all your working using reciprocals."

[WRITE] Display exit ticket questions:

1. Solve using reciprocals: a) $6 \div 2/3$ b) $4x = 12$ c) $3/5 \times x = 9$ d) $0.2 \div 4.5$

2. Solve: $(4/7 \div 2/5) \times (3/8 \div 9/16)$

3. If $x \times 3/8 = 5/12$, find x using reciprocals.

4. A rope of 18 m is cut into pieces of 3/4 m each. How many pieces?

5. A hiker walks 24 km, taking breaks every 2/3 km. How many breaks? If the interval changes to 1 1/3 km, how does this change?

6. A road is 12.6 km long, divided into segments of 0.35 km. How many segments?

[DO] Allow students to work. Collect notebooks or have students self-check.

Closure (1 minute)

[SAY] "Today we learned three powerful ways to use reciprocals: converting division into multiplication, solving equations, and working with proportions. The key rule to remember is: to divide by a fraction, flip it and multiply. For homework, practice with the textbook exercises and look for real-life situations where you use reciprocals—you'll be surprised how often they appear!"

Complete Answer Key

1a) $6 \div 2/3 = 6 \times 3/2 = 9$

1b) $4x = 12 \longrightarrow x = 12 \times 1/4 = 3$

1c) $3/5 \times x = 9 \longrightarrow x = 9 \times 5/3 = 15$

1d) $0.2 \div 4.5 = 1/5 \times 2/9 = 2/45$

2. $(4/7 \times 5/2) \times (3/8 \times 16/9) = (10/7) \times (2/3) = 20/21$

3. $x = 5/12 \times 8/3 = 40/36 = 10/9$

4. $18 \div 3/4 = 18 \times 4/3 = 24$ pieces

5. At 2/3 km: $24 \times 3/2 = 36$ breaks; At 4/3 km: $24 \times 3/4 = 18$ breaks (halved)

6. $12.6 \div 0.35 = 36$ segments

Differentiation Notes

Struggling Learners: Provide a step-by-step reference card. Begin with simple whole number divisions. Use visual fraction models. Focus on questions 1a, 1b, and 4 from the exit ticket.

On-Level Learners: Complete all practice problems and exit ticket questions. Verify answers by multiplying results back.

Advanced Learners: Extend to challenge problems: water tank rates, tailor fabric problem, and combined work rates. Create their own real-world reciprocal problems.

Post-Lesson Reflection Prompts

1. Did students successfully connect the anchor activity to the formal rules for using reciprocals?
2. Which application (division, equations, proportions) was most challenging?
3. Were the real-world problems effective in demonstrating practical value?
4. How well did differentiation strategies support diverse learners?
5. Was the 40-minute timeframe sufficient?
6. Which students need additional support in the next lesson?
7. What modifications would improve this lesson?