Step by Step Guide: Equations of Mirror Lines

Pre-Class Preparation Checklist

- Prepare graph paper handouts with the anchor activity figure: an object and its reflected image on a Cartesian plane (reflected across the y-axis).
- Bring tracing paper or extra graph paper for the folding activity.
- Prepare a second figure for the worked example: object with vertices including D(0,0) and C(4,2), and image with D′(0,0) and C′(−2,−4).
- Write the five-step method on a chart or prepare to write on the board.
- Prepare the common mirror lines table ($x = 0$, $y = 0$, $y = x$, $y = -x$).
- Have rulers, set squares, and coloured pencils available.
- Have the digital textbook section open: Section 2.2.3 Equations of Mirror Lines.
- Prepare exit ticket handouts with 5 questions.

Phase 1: Problem-Solving and Discovery (15 minutes)

Opening and Connection to Prior Learning (3 minutes)

[SAY] "In our last lesson, we learned how to draw reflected images given a mirror line. Today, we're going to do the REVERSE — we're given the object AND its image, and we need to find the mirror line."

[SAY] "Think of it this way: if someone shows you a shape and its reflection, can you figure out where the mirror was?"

[ASK] "What do you remember about the properties of reflection? What is special about the mirror line?"

[WAIT] Expected: "The mirror line is the perpendicular bisector." "Points are equidistant from the mirror line."

[SAY] "Exactly! The mirror line is the perpendicular bisector of the segment joining any point to its image. Today, we'll use this property to find the EQUATION of the mirror line."

[SAY] "Let's start with a hands-on activity."

Anchor Activity Launch (2 minutes)

[DO] Distribute the handout with the object and its reflected image on a Cartesian plane.

[SAY] "On your handout, you can see a shape and its reflected image. Your task is to find the line of reflection."

[SAY] "Step 1: Copy the figure onto graph paper if you haven't already."

[SAY] "Step 2: Fold your graph paper so that the points of the object match with their respective images."

[SAY] "Step 3: Where does the fold line appear? Mark it."

[SAY] "Step 4: Can you write the equation of the fold line?"

[SAY] "Work in your groups. You have 8 minutes."

Student Work Time (8 minutes)

[DO] Circulate among groups.

[ASK] To a group folding the paper: "Did the points match up perfectly? Where is the fold line?"

[WAIT] Expected: "The fold line is on the y-axis."

[ASK] "What is the equation of the y-axis?"

[WAIT] Expected: "x = 0."

[SAY] "Good! Now try connecting each object point to its image point with a straight line. What do you notice about where the fold line crosses these connecting lines?"

[WAIT] Expected: "It crosses them in the middle." "It bisects them."

[ASK] "And what angle does the fold line make with each connecting line?"

[WAIT] Expected: "90 degrees." "It's perpendicular."

[SAY] "So the fold line is the perpendicular bisector of each connecting segment. This is the key property we'll use."

[DO] For struggling groups: "Try just one pair of points. Connect A to A′. Find the midpoint. The mirror line passes through that midpoint."

[ASK] To early finishers: "Can you find the mirror line WITHOUT folding? Just using the coordinates?"

Class Sharing (2 minutes)

[SAY] "Let's share. Where did the fold line appear?"

[WAIT] Expected: "On the y-axis."

[WRITE] On the board: "The fold line appears exactly on the y-axis."

[WRITE] "Therefore, the line of reflection is the y-axis."

[WRITE] "The equation of the line of reflection is x = 0."

[SAY] "A line of reflection can be defined with an equation. This is what we'll learn to find today — the equation of the mirror line."

Phase 2: Structured Instruction (10 minutes)

The Five-Step Method (5 minutes)

[SAY] "The folding method works, but we need an algebraic method that works for any mirror line. Here are the five steps."

[WRITE] "Step 1: Identify corresponding pairs of points (object and image)."

[SAY] "For example, A and A′, B and B′, and so on."

[WRITE] "Step 2: Check if any point maps to itself. If D = D′, the mirror line passes through D."

[SAY] "A point that maps to itself must be ON the mirror line."

[WRITE] "Step 3: Find the midpoint of the segment joining another pair of corresponding points."

[SAY] "Use the midpoint formula: M = $((x_1 + x_2)/2, (y_1 + y_2)/2)$. The mirror line passes through this midpoint."

[WRITE] "Step 4: Calculate the gradient of the mirror line using two known points on it."

[SAY] "Use the gradient formula: $m = (y_2 - y_1)/(x_2 - x_1)$."

[WRITE] "Step 5: Write the equation using point-slope form: $y - y_1 = m(x - x_1)$. Simplify."

[SAY] "That's it — five steps. Let me show you how it works with an example."

Common Mirror Lines Table (2 minutes)

[SAY] "Before we do the example, let's note the common mirror lines you should recognise."

[WRITE] Draw a table on the board:

[WRITE] "y-axis: x = 0 — x-coordinates change sign, y stays"

[WRITE] "x-axis: y = 0 — y-coordinates change sign, x stays"

[WRITE] "y = x — coordinates swap"

[WRITE] "y = −x — coordinates swap and both change sign"

[SAY] "If you spot one of these patterns, you can write the equation immediately without going through all five steps."

Addressing Misconceptions (3 minutes)

[SAY] "Common mistake number one: confusing the mirror line with the line connecting the object to its image. The mirror line is NOT the connecting line — it is the PERPENDICULAR BISECTOR of the connecting line."

[DO] Draw a diagram showing the connecting line and the mirror line crossing it at 90° at the midpoint.

[SAY] "Common mistake number two: forgetting that you need TWO points to determine a line. One midpoint gives you one point on the mirror line. You need a second point — either from another midpoint or from a point that maps to itself."

[SAY] "Common mistake number three: when the connecting segment is horizontal, the mirror line is vertical, and vertical lines have the form x = a, NOT y = a."

[ASK] "If the segment from O(0,0) to O′(2,0) is horizontal, what direction is the mirror line?"

[WAIT] Expected: "Vertical."

[SAY] "Correct. And the equation of a vertical line through x = 1 is simply x = 1."

Phase 3: Practice and Application (10 minutes)

Worked Example: Finding the Equation y = −x (3 minutes)

[SAY] "Let’s work through the textbook example together."

[SAY] "We’re given an object and its image. The coordinates include D and D′ both at (0, 0), and C at (4, 2) with C′ at (−2, −4)."

[SAY] "Step 1: Corresponding pairs are D↔D′ and C↔C′."

[SAY] "Step 2: D and D′ are both at (0, 0). So D maps to itself. The mirror line passes through (0, 0)."

[WRITE] "Mirror line passes through (0, 0)."

[SAY] "Step 3: Find the midpoint of CC′. C is (4, 2) and C′ is (−2, −4)."

[WRITE] "M = ((4 + (−2))/2, (2 + (−4))/2) = (2/2, −2/2) = (1, −1)"

[SAY] "The mirror line also passes through (1, −1)."

[SAY] "Step 4: Gradient using (0, 0) and (1, −1)."

[WRITE] "m = (−1 − 0)/(1 − 0) = −1"

[SAY] "Step 5: Equation using point (1, −1) and gradient −1."

[WRITE] "y − (−1) = −1(x − 1)"

[WRITE] "y + 1 = −x + 1"

[WRITE] "y = −x"

[SAY] "The mirror line is y = −x. This is one of our common mirror lines!"

Student Practice: Triangle on x-axis (3 minutes)

[SAY] "Now try this one on your own. Triangle A(1, 2), B(3, 4), C(5, 4). Image: A′(1, −2), B′(3, −4), C′(5, −4). Find the equation of the mirror line."

[SAY] "Use the five-step method. You have 2 minutes."

[DO] Circulate and check student work.

[WAIT] 2 minutes.

[SAY] "Let's check. Midpoint of AA′: ((1+1)/2, (2+(−2))/2) = (1, 0)."

[SAY] "Midpoint of BB′: ((3+3)/2, (4+(−4))/2) = (3, 0)."

[SAY] "Both midpoints have y = 0. The line through (1, 0) and (3, 0) has gradient 0."

[SAY] "The equation is y = 0 — the x-axis!"

[ASK] "Could you have spotted this without calculating? Look at the coordinates."

[WAIT] Expected: "x-coordinates stay the same, y-coordinates change sign."

[SAY] "Exactly! That's the pattern for reflection in the x-axis."

Student Practice: Circle Centre (2 minutes)

[SAY] "Last practice problem. O(0, 0) is the centre of a circle of radius 2 cm. O'(2, 0) is the reflection. Find the equation of the mirror line."

[SAY] "Try it. 1 minute."

[WAIT] 1 minute.

[SAY] "Midpoint of OO': ((0+2)/2, (0+0)/2) = (1, 0)."

[SAY] "Gradient of OO': (0−0)/(2−0) = 0. The segment is horizontal."

[SAY] "The mirror line is perpendicular to a horizontal line, so it's vertical."

[SAY] "A vertical line through (1, 0) has equation x = 1."

[WRITE] "Mirror line: x = 1"

[SAY] "Notice: when the connecting segment is horizontal, the mirror line is vertical. When the connecting segment is vertical, the mirror line is horizontal."

Quick Check: Recognising Patterns (2 minutes)

[SAY] "Before the exit ticket, let's do a quick pattern check."

[ASK] "If A(3, 5) maps to A'(−3, 5), what's the mirror line?"

[WAIT] Expected: "x = 0, the y-axis."

[ASK] "If P(2, 1) maps to P'(1, 2), what's the mirror line?"

[WAIT] Expected: "y = x, because the coordinates swapped."

[ASK] "If Q(4, −1) maps to Q'(4, 1), what's the mirror line?"

[WAIT] Expected: "y = 0, the x-axis."

[SAY] "Good! Recognising these patterns saves you time."

Phase 4: Assessment — Exit Ticket (5 minutes)

[SAY] "For our exit ticket, answer these five questions on a separate piece of paper. You have 5 minutes."

[SAY] "Question 1: Determine if the transformation shown is a reflection. If so, find the equation of the mirror line."

[SAY] "Question 2: The point A(3, 5) is reflected to A′(−3, 5). Find the equation of the mirror line."

[SAY] "Question 3: The point P(2, 1) is reflected to P′(1, 2). Find the equation of the mirror line."

[SAY] "Question 4: Triangle XYZ has vertices X(0, 3), Y(4, 3), Z(2, 6). Image: X′(0, −3), Y′(4, −3), Z′(2, −6). Find the equation of the mirror line."

[SAY] "Question 5: Describe the five-step method for finding the equation of a mirror line."

[DO] Collect exit tickets as students finish.

Answer Key:

- 1. Check equidistance and perpendicularity. If confirmed, find the equation using the five-step method.
- 2. Midpoint = (0, 5). Segment AA′ is horizontal (gradient 0). Mirror line is vertical: x = 0 (y-axis).
- 3. Midpoint = (1.5, 1.5). Gradient of PP′ = −1. Mirror line gradient = 1. Equation: y = x.
- 4. Midpoints all have y = 0. Mirror line: y = 0 (x-axis).
- 5. Five steps: (i) Identify corresponding pairs. (ii) Check for self-mapping points. (iii) Find midpoint. (iv) Calculate gradient. (v) Write equation using point-slope form.

Differentiation Notes

Struggling Learners:

Provide graph paper with objects and images pre-plotted. Allow physical folding before algebraic work. Start with axis reflections (x = 0, y = 0). Provide a five-step worksheet with blanks. Review midpoint and gradient formulas. Use colour coding for object, image, and mirror line.

On-Level Learners:

Complete all practice problems using the perpendicular bisector method. Verify answers by checking all midpoints lie on the mirror line. Recognise standard mirror lines from coordinate patterns. Solve the circle centre problem independently.

Advanced Learners:

Investigate: Given a point and a mirror line, find the image algebraically. Derive the general formula for reflection in $y = mx + c$. Create problems with non-standard mirror lines (e.g., $y = 2x + 1$). Explore the relationship between the gradient of the connecting segment and the gradient of the mirror line (they are negative reciprocals).

Post-Lesson Reflection

1. Did the folding activity effectively help students discover the mirror line?

2. Were students able to transition from folding to the algebraic five-step method?

3. Did students correctly apply the midpoint and gradient formulas?

4. Were students able to recognise standard mirror lines from coordinate patterns?

5. Did students understand why two points are needed to determine the mirror line?

6. What common errors arose (e.g., confusing the connecting line with the mirror line)?

7. Were students able to handle vertical mirror lines ($x = a$)?

8. What adjustments would improve the lesson for future delivery?