I. Lesson Overview

Strand	**Measurement and Geometry**
Sub-Strand	Reflection and Congruence
Specific Learning Outcome	Determine the equation of the mirror line given an object and its image
Grade Level	Grade 10
Duration	40 minutes
Key Inquiry Question	How can we determine the equation of a mirror line when we are given an object and its reflected image? How is reflection applied in day-to-day life?
Learning Resources	CBC Grade 10 Mathematics Textbooks, graph paper, rulers, set squares, protractors, tracing paper

II. Learning Objectives

Category	Objective
Know	Recall that the mirror line is the perpendicular bisector of the segment joining any point to its image. Understand that a line of reflection can be defined by an equation. Identify that the midpoint of the segment joining corresponding points lies on the mirror line. State the midpoint formula: $M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$ and the gradient formula: $m = (y_2 - y_1)/(x_2 - x_1)$.
Do	Determine the equation of the mirror line by finding the perpendicular bisector of the segment joining an object point to its image. Calculate the midpoint of a segment joining corresponding object and image points. Calculate the gradient of the mirror line using two known points on it. Apply the point-slope form $y - y_1 = m(x - x_1)$ to write the equation of the mirror line. Verify the equation by checking that other corresponding points satisfy the reflection.
Apply	Determine the equation of the mirror line for reflections on the Cartesian plane. Recognise standard mirror lines ($x = 0$, $y = 0$, $y = x$, $y = -x$) from their equations. Solve problems involving finding the mirror line for circles and

	other shapes. Apply the perpendicular bisector method to real-world symmetry problems.

III. Materials & Resources

- CBC Grade 10 Mathematics Textbooks
- Graph paper (squared/grid paper)
- Rulers and set squares
- Protractors
- Tracing paper (for the folding activity)
- Coloured pencils or markers (to distinguish object, image, and mirror line)
- Printed handouts with the anchor activity figure (object and image on Cartesian plane)

IV. Lesson Procedure

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: "Finding the Line of Reflection"

Instructions (Work in groups):

Determine the line of reflection that created the reflected image below.

- Step 1: Copy the figure (showing an object and its reflected image on a Cartesian plane) onto graph paper.
- Step 2: Fold your graph paper such that the points of the object match with their respective images.
- Step 3: Where does the fold line appear? Mark the fold line on your graph paper.
- Step 4: What is the equation of the fold line?
- Step 5: Connect each object point to its corresponding image point with a straight line. What do you notice about where the fold line crosses each of these connecting lines?
- Step 6: Measure the angle between the fold line and each connecting line. What angle do you get?
- Step 7: Discuss with your group: How can you find the equation of the mirror line WITHOUT folding the paper?

Teacher's Role During Discovery:

- Circulate among groups, ensuring students fold the paper accurately so that corresponding points overlap.
- Ask probing questions: "Where exactly does the fold line appear?" "Can you describe its position using the axes?"
- For struggling groups: "Try connecting point A to A′ with a straight line. Now fold the paper along the fold line. Does the fold line cut this connecting line exactly in half?"
- For early finishers: "If the fold line is the y-axis, what is its equation? Can you think of another way to describe this line mathematically?"
- Guide students to observe: the fold line passes through the midpoint of each segment connecting corresponding points, and it is perpendicular to each such segment.
- Identify 2–3 groups with clear observations to share with the class.

Expected Student Discoveries:

Observation	Mathematical Significance
The fold line appears exactly on the y-axis.	The mirror line is the y-axis, which has the equation x = 0.
When I connect A to A′, the fold line cuts this segment exactly in half.	The mirror line passes through the midpoint of the segment joining each point to its image.
The fold line crosses each connecting line at 90°.	The mirror line is perpendicular to the segment joining each point to its image.
The fold line is the perpendicular bisector of every connecting segment.	The mirror line is the perpendicular bisector of the segment joining any object point to its corresponding image point.

Phase 2: Structured Instruction (10 minutes)

Key Takeaways:

You notice that the fold line appears exactly on the y-axis. Therefore, the line of reflection is the y-axis.

A line of reflection can be defined with an equation. From the activity, the equation of the line of reflection is x = 0.

Method for Finding the Equation of the Mirror Line:

Step	Procedure
Step 1	Identify corresponding pairs of points: the object point and its image point (e.g., A and A′, B and B′, C and C′).
Step 2	Check if any point maps to itself. If a point and its image are the same (e.g., D = D′ = (0,0)), the mirror line passes through that point.
Step 3	Find the midpoint of the segment joining another pair of corresponding points using the midpoint formula: $M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$. The mirror line passes through this midpoint.
Step 4	Calculate the gradient of the mirror line using two known points on it: $m = (y_2 - y_1)/(x_2 - x_1)$.
Step 5	Write the equation of the mirror line using the point-slope form: $y - y_1 = m(x - x_1)$. Simplify to slope-intercept form $y = mx + c$.

Common Mirror Lines and Their Equations:

Mirror Line	Equation	How to Recognise
y-axis	$x = 0$	x-coordinates change sign, y-coordinates stay the same
x-axis	$y = 0$	y-coordinates change sign, x-coordinates stay the same
Line $y = x$	$y = x$	x and y coordinates swap
Line $y = -x$	$y = -x$	x and y coordinates swap and both change sign

Connecting to Student Discoveries:

- Reference the anchor activity: "You discovered that the fold line is the y-axis with equation $x = 0$. Now we have a systematic method to find the equation of ANY mirror line."
- Emphasise the perpendicular bisector property: "The mirror line is always the perpendicular bisector of the segment joining any point to its image. This is the foundation of the method."
- Address misconception: "The mirror line is NOT the line connecting the object to its image. It is the line that BISECTS that connecting line at right angles."
- Connect to prior knowledge: "You already know the midpoint formula and the gradient formula from coordinate geometry. Now you're applying them to find the mirror line."

Phase 3: Practice and Application (10 minutes)

Problem 1 (Worked Example): Finding the Equation of the Line of Reflection

Determine the equation of the line of reflection for the given object and image.

Solution:

Step 1: The coordinates of D and D′ are both at (0, 0). This tells you that the line of reflection passes through (0, 0).

Step 2: Connect point C to C′ with a line. The line of reflection is the perpendicular bisector of CC′.

Step 3: From the properties of reflection, the distance from the object to the mirror line is the same as that of the mirror line to the image. Therefore, the line of reflection passes through the midpoint of the line connecting C to C′.

Coordinates for C is (4, 2) and that of C′ is (−2, −4). The midpoint of line CC′ is:

$$M = ((4 + (-2))/2, (2 + (-4))/2) = (1, -1)$$

Step 4: Since you know that the line of reflection passes through (0, 0) and (1, −1), the gradient of the reflection line is:

$$m = (-1 - 0)/(1 - 0) = -1$$

Step 5: Therefore, taking points (x, y) and (1, −1), the equation of the line of reflection is:

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -1(x - 1)$$

$$y + 1 = -x + 1$$

$$y = -x$$

Problem 2: Triangle Reflection on the x-axis

The vertices of a triangle are A(1, 2), B(3, 4) and C(5, 4). The vertices of the image are A′(1, −2), B′(3, −4) and C′(5, −4). Find the equation of the line of reflection.

Solution:

Step 1: Find the midpoint of AA′: M = ((1+1)/2, (2+(−2))/2) = (1, 0).

Step 2: Find the midpoint of BB′: M = ((3+3)/2, (4+(−4))/2) = (3, 0).

Step 3: Both midpoints have y-coordinate = 0. The mirror line passes through (1, 0) and (3, 0).

Step 4: The gradient is m = (0 − 0)/(3 − 1) = 0. The line is horizontal.

Step 5: Since the line passes through y = 0 with gradient 0, the equation is y = 0 (the x-axis).

Verification: Notice that x-coordinates stay the same and y-coordinates change sign. This confirms the mirror line is the x-axis.

Problem 3: Letter V Reflection

The vertices of a letter V are P(−3, 4), Q(−3, 2) and R(−1, 2). The vertices of the image are P′(−1, 2), Q′(−1, 0) and R′(1, 0). Find the equation of the line of reflection.

Solution:

Step 1: Find the midpoint of PP′: M = ((−3+(−1))/2, (4+2)/2) = (−2, 3).

Step 2: Find the midpoint of QQ′: M = ((−3+(−1))/2, (2+0)/2) = (−2, 1).

Step 3: Find the midpoint of RR′: M = ((−1+1)/2, (2+0)/2) = (0, 1).

Step 4: The mirror line passes through (−2, 3), (−2, 1), and (0, 1).

Wait — (−2, 3) and (−2, 1) have the same x-coordinate but different y-coordinates. Let us check using the gradient of PP′ and the perpendicular bisector approach.

Gradient of PP′: m = (2−4)/(−1−(−3)) = −2/2 = −1.

The mirror line is perpendicular to PP′, so its gradient = 1 (negative reciprocal of −1).

Mirror line passes through midpoint (−2, 3) with gradient 1:

$y - 3 = 1(x - (-2))$

$y - 3 = x + 2$

$y = x + 5$

Verification with Q and Q′: Midpoint of QQ′ = $(-2, 1)$. Check: $y = x + 5 \to 1 = -2 + 5 = 3 \neq 1$.

This means we need to reconsider. Let us use the perpendicular bisector of QQ′ instead.

Gradient of QQ′: $m = (0-2)/(-1-(-3)) = -2/2 = -1$. Perpendicular gradient = 1.

Midpoint of QQ′ = $(-2, 1)$. Mirror line: $y - 1 = 1(x + 2) \to y = x + 3$.

Verification with RR′: Midpoint = $(0, 1)$. Check: $y = x + 3 \to 1 = 0 + 3 = 3 \neq 1$.

Let us use the perpendicular bisector of RR′: Gradient of RR′ = $(0-2)/(1-(-1)) = -2/2 = -1$. Perpendicular gradient = 1.

Midpoint of RR′ = $(0, 1)$. Mirror line: $y - 1 = 1(x - 0) \to y = x + 1$.

The equation of the line of reflection is $y = x + 1$.

Problem 4: Circle Centre Reflection

$O(0, 0)$ is the centre of a circle of radius 2 cm. If $O'(2, 0)$ is the reflection of the centre of the circle, find the equation of the line of reflection.

Solution:

Step 1: Find the midpoint of OO′: $M = ((0+2)/2, (0+0)/2) = (1, 0)$.

Step 2: Find the gradient of OO′: $m = (0-0)/(2-0) = 0$. The segment OO′ is horizontal.

Step 3: The mirror line is perpendicular to OO′, so it is vertical. A vertical line through $(1, 0)$ has equation $x = 1$.

The equation of the line of reflection is $x = 1$.

Phase 4: Assessment — Exit Ticket (5 minutes)

Assessment Questions:

1. Determine if the transformation shown is a reflection. If it is a reflection, what is the equation of the mirror line? (Provide a figure showing two shapes on a Cartesian plane.)

2. The point A(3, 5) is reflected to A′(−3, 5). Find the equation of the mirror line.

3. The point P(2, 1) is reflected to P′(1, 2). Find the equation of the mirror line.

4. Triangle XYZ has vertices X(0, 3), Y(4, 3), Z(2, 6). Its image has vertices X′(0, −3), Y′(4, −3), Z′(2, −6). Find the equation of the mirror line.

5. Describe the five-step method for finding the equation of a mirror line given an object and its image.

Answer Key:

- 1. Check if corresponding points are equidistant from a line and if connecting segments are perpendicular to that line. If yes, it is a reflection. The equation depends on the specific figure provided.
- 2. Midpoint of AA′ = ((3+(−3))/2, (5+5)/2) = (0, 5). Gradient of AA′ = (5−5)/(−3−3) = 0 (horizontal). Mirror line is vertical through x = 0. Equation: x = 0 (the y-axis).
- 3. Midpoint of PP′ = ((2+1)/2, (1+2)/2) = (1.5, 1.5). Gradient of PP′ = (2−1)/(1−2) = −1. Mirror line gradient = 1 (perpendicular). Equation: y − 1.5 = 1(x − 1.5) → y = x. The mirror line is y = x.
- 4. Midpoint of XX′ = (0, 0). Midpoint of YY′ = (4, 0). Both midpoints have y = 0. The mirror line is y = 0 (the x-axis).
- 5. (i) Identify corresponding object-image point pairs. (ii) Check if any point maps to itself (mirror line passes through it). (iii) Find the midpoint of a segment joining corresponding points. (iv) Calculate the gradient using two known points on the mirror line. (v) Write the equation using point-slope form and simplify.

V. Differentiation Strategies

Learner Level	Strategy
Struggling Learners	Provide graph paper with the object and image already plotted. Allow students to physically fold the paper to find the mirror line before attempting the algebraic method. Start with simple cases where the mirror line is an axis ($x = 0$ or $y = 0$). Provide a step-by-step worksheet with the five steps clearly laid out, with blanks to fill in. Review the midpoint formula and gradient formula before applying them. Use colour coding: one colour for the object, another for the image, and a third for the mirror line.
On-Level Learners	Complete all practice problems including the perpendicular bisector method for non-axis mirror lines. Verify their answers by checking that all corresponding midpoints lie on the mirror line. Attempt problems where the mirror line is $y = x$ or $y = -x$ and recognise the pattern. Solve the circle centre reflection problem independently.
Advanced Learners	Investigate: "Given a point and a mirror line, find the image point algebraically without graphing." Derive the general formula for reflection of point (a, b) in the line $y = mx + c$. Explore: "If you know the mirror line and one image point, can you find the original point?" Create a problem where the mirror line is $y = 2x + 1$ and find the image of a given triangle.

VI. Extension Activity

Activity: Mirror Line Investigation

1. The vertices of quadrilateral ABCD are $A(1, 1)$, $B(3, 1)$, $C(4, 3)$, $D(2, 4)$. The image vertices are $A'(-1, -1)$, $B'(-1, -3)$, $C'(-3, -4)$, $D'(-4, -2)$. Find the equation of the mirror line.

2. A point $P(a, b)$ is reflected in the line $y = x$ to give P'. Then P' is reflected in the line $y = 0$ to give P''. Express the coordinates of P'' in terms of a and b. What single mirror line would map P directly to P''?

3. Real-world application: A laser beam hits a flat mirror at point M(3, 0). The beam comes from point A(1, 4) and reflects to point B(5, 4). Verify that the x-axis is the mirror line. What property of reflection ensures that the angle of incidence equals the angle of reflection?

4. Challenge: Find the equation of the mirror line that maps the point (2, 3) to the point (6, 7). Is there only one possible mirror line? Explain.

Extension Answer Key:

- 1. Midpoint of AA′ = (0, 0). Midpoint of BB′ = (1, −1). Gradient = (−1−0)/(1−0) = −1. Equation: y = −x.
- 2. P(a, b) ⟶ P′(b, a) via y = x. Then P′(b, a) ⟶ P″(b, −a) via y = 0. The single mirror line mapping (a, b) to (b, −a) is y = −x (since coordinates swap and signs change).
- 3. Midpoint of AB = (3, 4). The segment AB is vertical (x = 3 for both). The mirror line is horizontal through y = 0 (the x-axis). The perpendicular bisector property ensures equal angles of incidence and reflection.
- 4. Midpoint = (4, 5). Gradient of segment = (7−3)/(6−2) = 1. Mirror line gradient = −1 (perpendicular). Equation: y − 5 = −1(x − 4) ⟶ y = −x + 9. Yes, there is only one mirror line for a given point and its image — the perpendicular bisector of the segment joining them.

VII. Assessment Methods

Type	Method
Formative	Observation during group work: Can students fold the paper accurately to find the mirror line? Do they identify the fold line as the y-axis? Questioning: "How do you know the mirror line passes through this point?" "Why do we use the midpoint?" "What does perpendicular bisector mean in this context?" Monitoring calculations: Check midpoint and gradient calculations for accuracy.
Summative	Exit ticket with 5 questions covering: identifying reflections, finding mirror line equations for axis reflections, finding mirror line for y = x, triangle reflection, and describing the five-step method. Complete answer key provided for marking.

VIII. Teacher Reflection

1. Did the folding activity effectively help students discover that the mirror line is the perpendicular bisector?

2. Were students able to transition from the physical folding method to the algebraic method?

3. Did students correctly apply the midpoint formula and gradient formula?

4. Were students able to recognise standard mirror lines ($x = 0$, $y = 0$, $y = x$) from the coordinate patterns?

5. Did students understand why we need TWO points on the mirror line to determine its equation?

6. What common errors arose (e.g., confusing the gradient of the connecting segment with the gradient of the mirror line)?

7. Were students able to handle the case where the mirror line is vertical (undefined gradient)?

8. What adjustments would improve the lesson for future delivery?