I. Lesson Overview

Strand	Measurement and Geometry
Sub-Strand	Reflection and Congruence
Specific Learning Outcome	Draw an image given an object and a mirror line on a plane surface and Cartesian plane
Grade Level	Grade 10
Duration	40 minutes
Key Inquiry Question	How is reflection applied in day-to-day life? How can we accurately construct the reflected image of any shape given a mirror line?
Learning Resources	CBC Grade 10 Mathematics Textbooks, graph paper, rulers, set squares, protractors, tracing paper, small mirrors

II. Learning Objectives

Category	Objective
Know	Define reflection as a transformation that moves an object across a mirror line to the opposite side. State the three key properties of reflection: (1) the image is on the opposite side of the mirror line, (2) each point and its image are equidistant from the mirror line, and (3) the mirror line is the perpendicular bisector of the segment joining any point to its image. Identify the mirror line as the perpendicular bisector of the line connecting corresponding object and image points.
Do	Construct the reflected image of a triangle given a mirror line on a plane surface by drawing perpendiculars and measuring equal distances. Reflect polygons (pentagons, quadrilaterals) across vertical, horizontal, and diagonal mirror lines. Reflect shapes on the Cartesian plane across the x-axis, y-axis, and the line y = x. Find the coordinates of reflected vertices using reflection rules.
Apply	Apply reflection to solve problems involving coordinates on the Cartesian plane. Recognise reflection in real-world contexts such as mirrors, water reflections, and

	symmetrical designs. Use reflection to verify congruence between an object and its image.

III. Materials & Resources

- CBC Grade 10 Mathematics Textbooks
- Graph paper (squared/grid paper)
- Rulers and set squares
- Protractors
- Tracing paper (optional, for verification)
- Small flat mirrors (for demonstration)
- Coloured pencils or markers (to distinguish object from image)
- Printed handouts with the anchor activity (triangle on grid with mirror line M)

IV. Lesson Procedure

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: "Reflecting a Triangle on a Plane M"

Instructions (Work in groups):

Here is a step-by-step approach on reflection of a triangle on a plane M.

- Step 1: Draw a perpendicular line from vertex A to the mirror line M and measure the distance by counting the number of squares between vertex A and the mirror line M. Repeat this process for vertices B and C.
- Step 2: Determine the position of the reflected vertices. For vertex A, the perpendicular distance between the vertex and the mirror line is 2 squares. Count 2 squares from the mirror line to the opposite side of the mirror line and mark that point as A′, which is the reflected image of vertex A. Repeat the same procedure for the remaining vertices B and C.
- Step 3: Connect the reflected vertices A′, B′ and C′ to create the reflected image of the triangle ABC.

Follow-Up Discussion Questions:

- (a) Compare the original triangle ABC with its reflected image A′B′C′. What do you notice about the shape and size?
- (b) Measure the distance from each vertex to the mirror line, and from each reflected vertex to the mirror line. What pattern do you observe?
- (c) Draw the line segment connecting A to A′. What angle does this segment make with the mirror line M?
- (d) Where does the mirror line M intersect the segment AA′? What does this tell you about the mirror line?
- (e) If the mirror line were in a different position (e.g., horizontal or diagonal), would the same method work? Why or why not?
- (f) Can you think of a real-life example where you see reflection?

Teacher's Role During Discovery:

- Circulate among groups, ensuring students draw perpendicular lines correctly from each vertex to the mirror line.
- Ask probing questions: "How do you know your line is perpendicular to the mirror line?" "What tool can you use to check the 90° angle?"
- For struggling groups: Provide a small mirror placed along the mirror line M. Ask students to look at where each vertex appears in the mirror and mark that point.
- For early finishers: "What happens if a vertex is ON the mirror line? Where is its reflected image?" (Answer: it stays in the same position.)
- Guide students to discover the three key properties: (1) image is on the opposite side, (2) equidistant from mirror line, (3) connecting line is perpendicular to mirror line.
- Identify 2–3 groups with clear constructions and observations to share with the class.

Expected Student Discoveries:

Observation	Mathematical Property
The reflected triangle looks the same size and shape as the original.	Reflection preserves shape and size (congruence).
Each vertex and its image are the same distance from the mirror line.	A point and its reflection are equidistant from the mirror line.
The line from A to A′ crosses the mirror line at 90°.	The line connecting a point to its image is perpendicular to the mirror line.
The mirror line cuts the segment AA′ exactly in half.	The mirror line is the perpendicular bisector of the segment joining a point to its image.

Phase 2: Structured Instruction (10 minutes)

Key Takeaways:

Properties of Reflection:

1. Reflection moves the image of an object across the mirror line, that is, to the opposite side of the mirror line.

2. A point on the object is the same distance as its reflection from the mirror line.

3. The line connecting a point to its image is perpendicular to the mirror line. Therefore, the mirror line is the perpendicular bisector of the lines connecting the object points and the image points.

Reflection Rules on the Cartesian Plane:

Mirror Line	Rule	Example: A(3, 5)
x-axis (y = 0)	$(x, y) \to (x, -y)$	$A(3, 5) \to A'(3, -5)$
y-axis (x = 0)	$(x, y) \to (-x, y)$	$A(3, 5) \to A'(-3, 5)$
Line y = x	$(x, y) \to (y, x)$	$A(3, 5) \to A'(5, 3)$
Line y = −x	$(x, y) \to (-y, -x)$	$A(3, 5) \to A'(-5, -3)$

Reflection on a Vertical Line x = a:

For reflection on the line $x = a$, the rule is: $(x, y) \to (2a - x, y)$.

For example, reflecting $A(3, 5)$ on the line $x = -1$: $A'(2(-1) - 3, 5) = A'(-5, 5)$.

Connecting to Student Discoveries:

- Reference the anchor activity: "You discovered that the mirror line is the perpendicular bisector of the segment joining each point to its image. This is the fundamental property that makes all reflection rules work."
- Show how the perpendicular-and-measure method from the anchor activity translates to coordinate rules: "On the Cartesian plane, reflecting across the y-axis means each point's x-coordinate changes sign, because the y-axis is the perpendicular bisector."
- Address misconception: "Reflection does NOT mean just flipping the shape. You must flip it across a SPECIFIC mirror line. The same shape reflected across different mirror lines gives different images."
- Connect to real life: "When you look in a mirror, your left hand appears as the right hand in the image. This is reflection — the mirror is the mirror line, and your image is on the opposite side."

Phase 3: Practice and Application (10 minutes)

Problem 1: Reflecting a Pentagon on a Diagonal Mirror Line

Draw the image of the pentagon under the reflection on the diagonal mirror line M.

Solution:

To obtain the image A′ of A, draw a perpendicular line from A to the mirror line M, extend the line the same distance on the opposite side of the mirror line and mark the point as A′. Similarly, obtain the images B′, C′, D′, E′ — the images of vertices B, C, D, E respectively.

Connect the images of the vertices to form the reflection of the pentagon.

Key Observation:

The method works for ANY mirror line orientation — vertical, horizontal, or diagonal. The critical step is always drawing the perpendicular from each vertex to the mirror line and measuring equal distances on both sides.

Problem 2: Reflecting on the Cartesian Plane — y-axis

Reflect the object about the y-axis.

Solution:

Apply the reflection rule for the y-axis: $(x, y) \to (-x, y)$. Each x-coordinate changes sign while the y-coordinate remains the same. Plot the reflected vertices and connect them to form the image.

Problem 3: Successive Reflections

The vertices of a polygon are given as: $A(-5, 5)$, $B(-6, 3)$, $C(-5, 1)$, $D(-3, 0)$, $E(-2, 2)$ and $F(-3, 4)$. Find the image of the polygon under the following reflection lines:

(a) $y = x$ followed by $y = 0$

(b) $x = 0$

Solution:

(a) Step 1: Reflect on y = x. Rule: (x, y) ⟶ (y, x).

Original Point	After y = x	After y = 0
A(−5, 5)	A′(5, −5)	A″(5, 5)
B(−6, 3)	B′(3, −6)	B″(3, 6)
C(−5, 1)	C′(1, −5)	C″(1, 5)
D(−3, 0)	D′(0, −3)	D″(0, 3)
E(−2, 2)	E′(2, −2)	E″(2, 2)
F(−3, 4)	F′(4, −3)	F″(4, 3)

(b) Reflect on x = 0 (y-axis). Rule: (x, y) ⟶ (−x, y).

Original Point	After x = 0
A(−5, 5)	A′(5, 5)
B(−6, 3)	B′(6, 3)
C(−5, 1)	C′(5, 1)
D(−3, 0)	D′(3, 0)
E(−2, 2)	E′(2, 2)
F(−3, 4)	F′(3, 4)

Problem 4: Finding Original Points from Image Points

The points A′(−4, 1), B′(−2, 4) and C′(−1, 3) are the images of points A, B and C respectively under a reflection on the line x = −1. Find the coordinates for points A, B and C.

Solution:

For reflection on x = −1, the rule is: (x, y) ⟶ (2(−1) − x, y) = (−2 − x, y).

Since the image points are given, we reverse the process: x_original = −2 − x_image.

Image Point	Calculation	Original Point
A′(−4, 1)	x = −2 − (−4) = 2, y = 1	A(2, 1)
B′(−2, 4)	x = −2 − (−2) = 0, y = 4	B(0, 4)
C′(−1, 3)	x = −2 − (−1) = −1, y = 3	C(−1, 3)

Note:

Point C′(−1, 3) lies ON the mirror line x = −1, so its original point C is also (−1, 3). A point on the mirror line is its own reflection.

Phase 4: Assessment — Exit Ticket (5 minutes)

Assessment Questions:

1. Copy the figure below and draw its image under the reflection on the mirror line M. (Provide a simple quadrilateral on grid paper with a vertical mirror line.)

2. The triangle PQR has vertices P(1, 4), Q(3, 1), and R(5, 3). Find the coordinates of the image of triangle PQR under reflection in:

(a) the x-axis

(b) the y-axis

(c) the line y = x

3. State three properties of reflection.

4. The point A′(6, −3) is the image of point A under reflection in the x-axis. Find the coordinates of A.

5. A shape is reflected in the line x = 2. The image of point B is B′(−1, 5). Find the coordinates of B.

Answer Key:

- 1. Students should draw perpendiculars from each vertex to the mirror line, measure equal distances on the other side, and connect the reflected vertices. The image should be congruent to the original and on the opposite side of the mirror line.
- 2. (a) x-axis: P′(1, −4), Q′(3, −1), R′(5, −3). (b) y-axis: P′(−1, 4), Q′(−3, 1), R′(−5, 3). (c) y = x: P′(4, 1), Q′(1, 3), R′(3, 5).
- 3. (i) Reflection moves the image to the opposite side of the mirror line. (ii) Each point and its image are equidistant from the mirror line. (iii) The mirror line is the perpendicular bisector of the segment joining any point to its image.
- 4. Reflection in x-axis: (x, y) → (x, −y). Since A′(6, −3), then A = (6, 3).
- 5. Reflection in x = 2: x_original = 2(2) − (−1) = 5, y = 5. So B = (5, 5).

V. Differentiation Strategies

Learner Level	Strategy
Struggling Learners	Provide grid paper with the mirror line and object already drawn. Allow students to use a small mirror placed along the mirror line to see where the image should appear before drawing it. Start with vertical and horizontal mirror lines before introducing diagonal ones. Provide a step-by-step checklist: (1) Pick a vertex. (2) Draw a perpendicular to the mirror line. (3) Measure the distance. (4) Mark the same distance on the other side. (5) Repeat for all vertices. (6) Connect the image vertices. Use tracing paper: trace the object, fold along the mirror line, and the traced shape shows the reflected image.
On-Level Learners	Complete all practice problems including the successive reflections problem. Apply coordinate reflection rules without grid paper (using the algebraic rules). Verify their constructions by checking that the mirror line is the perpendicular bisector of each point-to-image segment. Attempt the reverse problem: given the image, find the original points.
Advanced Learners	Investigate: "What single transformation is equivalent to reflecting first in $y = x$ and then in $y = 0$?" (Answer: a rotation of 90° clockwise about the origin.) Explore reflections in lines of the form $y = mx + c$. Prove algebraically that reflection preserves distances (i.e., show that $\|AB\| = \|A'B'\|$ using the distance formula). Design a symmetric pattern using multiple reflections across different mirror lines.

VI. Extension Activity

Activity: Reflection Patterns and Compositions

1. Draw triangle DEF with vertices D(1, 1), E(4, 1), and F(2, 4) on a Cartesian plane.

(a) Reflect triangle DEF in the y-axis to get triangle D′E′F′.

(b) Reflect triangle D′E′F′ in the x-axis to get triangle D″E″F″.

(c) What single transformation maps triangle DEF directly to triangle D″E″F″?

2. A designer wants to create a symmetric logo. Starting with a shape in the first quadrant, the designer reflects it in the y-axis, then reflects both shapes in the x-axis. How many copies of the original shape are in the final design? Sketch the result.

3. Investigate: If you reflect a shape in two parallel mirror lines, what single transformation is the result equivalent to? Test with a specific example.

4. Real-world application: A swimming pool is 20 m long. A swimmer at position (3, 2) looks at their reflection in the still water surface (the x-axis). What are the coordinates of the reflection? If the pool has a mirror on the wall at x = 20, what are the coordinates of the swimmer's image in that mirror?

Extension Answer Key:

- 1. (a) D′(−1, 1), E′(−4, 1), F′(−2, 4). (b) D″(−1, −1), E″(−4, −1), F″(−2, −4). (c) A rotation of 180° about the origin (or equivalently, a reflection through the origin).
- 2. There are 4 copies of the original shape — one in each quadrant. The design has both vertical and horizontal symmetry.
- 3. Reflecting in two parallel mirror lines is equivalent to a translation. The translation distance is twice the distance between the two mirror lines, in the direction perpendicular to the lines.
- 4. Reflection in x-axis (water surface): (3, −2). Reflection in x = 20 (wall mirror): (2(20) − 3, 2) = (37, 2).

VII. Assessment Methods

Type	Method
Formative	Observation during group work: Can students draw accurate perpendiculars to the mirror line? Do they measure equal distances on both sides? Questioning: "What happens to a point that is ON the mirror line?" "If I move the mirror line, how does the image change?" "Why must the connecting line be perpendicular to the mirror line?" Monitoring

	constructions: Check that reflected images are congruent and correctly positioned.
Summative	Exit ticket with 5 questions covering: construction on a plane surface, coordinate reflections across x-axis, y-axis, and y = x, stating properties of reflection, finding original points from image points, and reflection across a vertical line x = a. Complete answer key provided for marking.

VIII. Teacher Reflection

1. Were students able to accurately construct reflected images using the perpendicular-and-measure method?

2. Did students discover the three key properties of reflection through the anchor activity, or did they need significant guidance?

3. How well did students transition from constructing reflections on plain/grid paper to applying coordinate rules on the Cartesian plane?

4. Were students able to apply the reflection rules for x-axis, y-axis, and y = x correctly?

5. Did students understand the reverse problem (finding original points from image points)?

6. What common errors arose (e.g., measuring along the mirror line instead of perpendicular to it, forgetting to change sign)?

7. Did the use of physical mirrors help students visualise the concept?

8. What adjustments would improve the lesson for future delivery?