Grade 10 Mathematics Lesson Plan

Surface Area of Cones

Strand:	**Measurement and Geometry**
Sub-Strand:	Surface Area of a Cone
Specific Learning Outcome:	Determine the surface area of prisms, pyramids, cones, frustums and spheres.
Duration:	40 minutes
Key Inquiry Question:	How do we determine the surface area and volume of solids? Why do we determine the surface area and volume of solids?
Learning Resources:	CBC Grade 10 textbooks, paper, pencils, rulers, calculators

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Deriving the cone surface area formula
Structured Instruction	10 minutes	Formalizing the cone surface area formula and addressing misconceptions
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Deriving the Surface Area Formula for Cones

Objective: Students will discover the formula for finding the surface area of a cone by identifying the faces that make up a cone and deriving the formula using geometric reasoning and the Pythagorean theorem.

Problem:

If a cone has a height of h and a base of radius r, show that the surface area is: π r squared + π r square root of (r squared + h squared)

Steps for the Activity:

1. Step 1: Sketch and Label the Cone. Draw a cone with height h, base radius r, and slant height l.
2. Step 2: Identify the Faces. Identify the faces that make up the cone. The cone has two faces: the base (a circle of radius r) and the curved surface (which can be opened out to a sector).
3. Step 3: Calculate the Curved Surface Area. The curved surface can be cut into many thin triangles with height close to l (slant height). The area is: Area = (1 / 2) times base times height = (1 / 2) times 2 π r times l = π r l.
4. Step 4: Calculate the Slant Height. Use the Pythagorean theorem: l = square root of (r squared + h squared).
5. Step 5: Calculate the Base Area. Area of circular base C1 = π r squared.
6. Step 6: Calculate the Curved Surface Area. Area of curved walls C2 = π r l = π r square root of (r squared + h squared).
7. Step 7: Find the Total Surface Area. A = C1 + C2 = π r squared + π r square root of (r squared + h squared) = π r (r + square root of (r squared + h squared)).

Discussion Questions:

8. What is a cone?
9. What faces make up a cone?
10. How is the curved surface related to a sector of a circle?
11. How do we find the slant height if we know the height and radius?
12. What is the formula for the surface area of a cone?

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand how to identify the faces of a cone.
- Ask probing questions: What shapes make up the cone? How can we find the area of the curved surface?
- For struggling groups: Let us start by finding the area of the base. Then think about how the curved surface can be unfolded.
- For early finishers: Can you derive the formula for a cone without a base (just the curved surface)?
- Guide students to articulate: The surface area of a cone equals the base area plus the curved surface area.
- Identify 2-3 groups with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Cone Surface Area Formula and Addressing Misconceptions

After students have completed the anchor activity and shared their findings, the teacher formalizes the formula for finding the surface area of a cone.

Key Takeaway: What is a Cone?

A cone is a three-dimensional geometric shape with a circular base and a curved surface that tapers to a point called the apex.

Net of a Cone:

The net of a cone is a two-dimensional representation of the three-dimensional shape. It consists of:

- A circular base
- A curved surface that can be laid out flat (sector of a circle)

Formula:

Surface Area of a Cone = π r squared + π r l

Where:

- r is the radius of the base
- l is the slant height
- l = square root of (r squared + h squared) using Pythagorean theorem

Alternative form:

Surface Area = π r squared + π r square root of (r squared + h squared)

= π r (r + square root of (r squared + h squared))

Scaffolding Strategies to Address Misconceptions:

- Misconception: A cone is the same as a cylinder. Clarification: No, a cone has a circular base and tapers to a point, while a cylinder has two parallel circular bases.
- Misconception: I only need to find the curved surface area. Clarification: No, you must add the base area to the curved surface area for the total surface area.
- Misconception: The height and slant height are the same. Clarification: No, the slant height is the distance along the curved surface from the apex to the base edge, while the height is the perpendicular distance from the apex to the base.
- Misconception: I can use the height directly in the formula. Clarification: No, you must use the slant height, which can be calculated using the Pythagorean theorem if you know the height and radius.

Phase 3: Practice and Application (10 minutes)

Worked Example:

Given a cone with radius r = 14 cm and an angle of 60 degrees. Find the surface area of the cone.

Solution:

Step 1: Find the area of the sector (curved surface).

Area of sector A = (θ / 360 degrees) times π r squared

= (60 / 360) times (22 / 7) times 14 times 14

= 102.67 cm squared

Step 2: Find the area of the circular base.

Area of circle B = π r squared

= (22 / 7) times 14 times 14

= 616 cm squared

Step 3: Find the total surface area.

Surface area = 102.67 + 616

= 718.67 cm squared

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. A circular cone has a base radius of 5 cm and a slant height of 12 cm. Calculate the total surface area of the cone, including both the curved surface and the circular base.

2. A cone is constructed with a base diameter of 16 cm and a height of 15 cm. Before finding the total surface area, determine the slant height of the cone using the Pythagorean theorem. Then, calculate the complete surface area.

3. A conical container, open at the top, is made of metal and has a base radius of 10 cm and a slant height of 18 cm. Determine the total metal sheet required to construct this container, excluding the base.

4. A conical tent made of waterproof fabric has a radius of 4.2 m and a slant height of 7.5 m. If the tent does not have a base, calculate the area of fabric required to cover the tent completely.

Answer Key:

1. Base area = π times 5 squared = 78.54 cm squared. Curved surface area = π times 5 times 12 = 188.50 cm squared. Total surface area = 78.54 + 188.50 = 267.04 cm squared.

2. Radius = 16 / 2 = 8 cm. Slant height = square root of (8 squared + 15 squared) = square root of 289 = 17 cm. Base area = π times 8 squared = 201.06 cm squared. Curved surface area = π times 8 times 17 = 427.26 cm squared. Total surface area = 201.06 + 427.26 = 628.32 cm squared.

3. Curved surface area only (no base) = π times 10 times 18 = 565.49 cm squared.

4. Curved surface area only (no base) = π times 4.2 times 7.5 = 99.00 m squared.

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn cone diagrams with labeled dimensions.
- Use cones with simple dimensions for initial practice.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators for calculations.
- Break down the formula into steps: Find base area, find slant height, find curved surface area, add.

For On-Level Learners:

- Encourage students to verify their formula with different cone dimensions.
- Ask students to explain the difference between a cone and a cylinder.
- Provide mixed practice with cones with and without bases.
- Challenge students to find the surface area when only height and radius are given.

For Advanced Learners:

- Challenge students to derive the formula for the curved surface area using calculus or geometric reasoning.
- Explore real-world applications: ice cream cones, traffic cones, conical tents, funnels, party hats.
- Investigate problems where the slant height must be calculated using the Pythagorean theorem.
- Apply the concept to composite solids involving cones.
- Solve optimization problems: Given a fixed surface area, what dimensions maximize the volume?

Extension Activity

Real-World Application: Conical Objects Investigation

Work in groups

Situation: Identify cone-shaped objects around school or home (e.g., ice cream cones, traffic cones, conical tents, funnels, party hats, etc.).

Tasks:

13. Identify at least three cone-shaped objects in your environment.
14. Measure the dimensions of each object (radius, height, or slant height) or use estimated values.
15. Calculate the surface area of each object using the cone formula.
16. For objects without a base (like tents or party hats), calculate only the curved surface area.
17. Compare your results with classmates and discuss any differences.
18. Present your findings with diagrams, measurements, and calculations.

Key Takeaway:

Students should understand how the surface area of cones is used in real-world contexts such as food packaging (ice cream cones), safety equipment (traffic cones), camping gear (conical tents), kitchen utensils (funnels), and party supplies (party hats).

Teacher Reflection Prompts

- Did students successfully derive the cone surface area formula through the anchor activity?
- Were students able to identify the faces of a cone and understand the net?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand the difference between height and slant height?
- What adjustments would improve this lesson for future classes?