Step by Step Guide: Reflecting Objects

Pre-Class Preparation Checklist

- Prepare graph paper handouts with a triangle ABC drawn on a grid with a vertical mirror line M (vertex A is 2 squares from M, B is 3 squares, C is 4 squares).
- Bring small flat mirrors (at least one per group) for demonstration.
- Prepare a second handout with a pentagon on a grid with a diagonal mirror line.
- Have rulers, set squares, and protractors available for each group.
- Prepare coloured pencils (one colour for the object, another for the image).
- Write the three key properties of reflection on a chart or prepare to write on the board.
- Prepare the reflection rules table for the Cartesian plane (x-axis, y-axis, $y = x$, $y = -x$).
- Have the digital textbook section open: Section 2.2.2 Reflecting Objects.
- Prepare the Cartesian plane problems on separate handouts or on the board.

Phase 1: Problem-Solving and Discovery (15 minutes)

Opening and Real-World Connection (3 minutes)

[SAY] "Good morning, class. Before we begin, I want you to think about something you see every day."

[DO] Hold up a small mirror.

[SAY] "When you look in a mirror, what do you see?"

[WAIT] Expected: "My reflection." "An image of myself."

[SAY] "And where is that image? Is it in front of the mirror or behind it?"

[WAIT] Expected: "Behind the mirror." "On the other side."

[SAY] "Exactly. The image appears on the OPPOSITE side of the mirror. And here's something interesting — your image is the SAME distance behind the mirror as you are in front of it."

[SAY] "Today, we're going to learn how to draw reflected images of shapes. We'll start with a simple triangle on a grid, and by the end of the lesson, you'll be able to reflect any shape across any mirror line — including on the Cartesian plane."

[SAY] "Let's begin with our anchor activity."

Anchor Activity Launch (2 minutes)

[DO] Distribute the handout with triangle ABC on a grid and mirror line M.

[SAY] "On your handout, you can see triangle ABC and a mirror line M. Your task is to find the reflected image of this triangle."

[SAY] "Here is the step-by-step approach. Follow along carefully."

[SAY] "Step 1: Draw a perpendicular line from vertex A to the mirror line M. Count the number of squares between A and the mirror line."

[DO] Demonstrate on the board: draw the perpendicular from A to M.

[SAY] "Step 2: The distance from A to the mirror line is 2 squares. Now count 2 squares on the OPPOSITE side of the mirror line and mark that point as A′ — A prime. This is the reflected image of vertex A."

[DO] Mark A′ on the board.

[SAY] "Step 3: Repeat this for vertices B and C. Draw perpendiculars, measure distances, mark the image points."

[SAY] "Step 4: Connect A′, B′, and C′ to create the reflected image of triangle ABC."

[SAY] "Work in your groups. You have 8 minutes. After you've drawn the reflection, answer the discussion questions on the handout."

Student Work Time (8 minutes)

[DO] Circulate among groups.

[ASK] To a group drawing perpendiculars: "How do you know this line is perpendicular to the mirror line? What angle should it make?"

[WAIT] Expected: "90 degrees."

[SAY] "Good. Use your set square to check."

[ASK] To a group that has completed the reflection: "Compare triangle ABC with triangle A′B′C′. What do you notice about the shape and size?"

[WAIT] Expected: "They're the same." "They're congruent."

[ASK] "Now draw the line segment from A to A′. Where does the mirror line cross this segment?"

[WAIT] Expected: "In the middle." "It bisects it."

[SAY] "So the mirror line is the perpendicular BISECTOR of the segment AA′. Write that down — it’s a key property."

[DO] For struggling groups: Place a small mirror along the mirror line M. "Look at where vertex A appears in the mirror. Mark that point. That’s A′."

[ASK] To early finishers: "What happens if a vertex is exactly ON the mirror line? Where is its reflected image?"

[WAIT] Expected: "It stays in the same place."

[SAY] "Correct! A point on the mirror line is its own reflection."

Class Sharing (2 minutes)

[SAY] "Let’s share our discoveries. What did you notice about the reflected triangle?"

[WAIT] Call on 2–3 groups.

[SAY] "Let me summarise what you’ve found."

[WRITE] On the board: "Discovery 1: The reflected image is the same shape and size as the original (congruent)."

[WRITE] "Discovery 2: Each vertex and its image are the same distance from the mirror line."

[WRITE] "Discovery 3: The line from any vertex to its image is perpendicular to the mirror line."

[WRITE] "Discovery 4: The mirror line is the perpendicular bisector of the segment joining each point to its image."

[SAY] "These are the fundamental properties of reflection. Let’s now formalise them."

Phase 2: Structured Instruction (10 minutes)

Formalising the Properties (3 minutes)

[SAY] "Let’s write down the three key properties of reflection from your textbook."

[WRITE] "Property 1: Reflection moves the image of an object across the mirror line, that is, to the opposite side of the mirror line."

[SAY] "This is what you saw — the image appears on the other side."

[WRITE] "Property 2: A point on the object is the same distance as its reflection from the mirror line."

[SAY] "You measured this. If A is 2 squares from the mirror line, A′ is also 2 squares from the mirror line."

[WRITE] "Property 3: The line connecting a point to its image is perpendicular to the mirror line. Therefore, the mirror line is the perpendicular bisector of the lines connecting the object points and the image points."

[SAY] "This is the most important property. It's what makes the construction method work."

Reflection Rules on the Cartesian Plane (4 minutes)

[SAY] "Now let's move to the Cartesian plane. When the mirror line is one of the axes or a special line, we can use coordinate rules instead of drawing perpendiculars."

[WRITE] Draw a table on the board:

[WRITE] "Mirror Line | Rule | Example"

[WRITE] "x-axis (y = 0) | (x, y) → (x, −y) | A(3, 5) → A′(3, −5)"

[SAY] "Reflecting across the x-axis: the x-coordinate stays the same, but the y-coordinate changes sign. Think about it — the point flips over the x-axis."

[WRITE] "y-axis (x = 0) | (x, y) → (−x, y) | A(3, 5) → A′(−3, 5)"

[SAY] "Reflecting across the y-axis: the y-coordinate stays, the x-coordinate changes sign."

[WRITE] "y = x | (x, y) → (y, x) | A(3, 5) → A′(5, 3)"

[SAY] "Reflecting across y = x: the coordinates SWAP. The x becomes the y and the y becomes the x."

[WRITE] "y = −x | (x, y) → (−y, −x) | A(3, 5) → A′(−5, −3)"

[SAY] "Reflecting across y = −x: the coordinates swap AND both change sign."

[ASK] "Can you see how these rules connect to the perpendicular-and-measure method we used earlier?"

[WAIT] Expected: "The perpendicular to the x-axis is vertical, so x stays the same. The distance above and below the x-axis is the y-value, so y changes sign."

[SAY] "Exactly! The coordinate rules are just a shortcut for the same geometric process."

Reflection on a Vertical Line x = a (3 minutes)

[SAY] "What if the mirror line is not an axis? For example, what if the mirror line is $x = -1$?"

[WRITE] "For reflection on $x = a$: $(x, y) \to (2a - x, y)$"

[SAY] "The y-coordinate stays the same. The x-coordinate uses the formula 2a minus x."

[SAY] "Let's check: if $a = -1$ and we reflect A(3, 5):"

[WRITE] "$x' = 2(-1) - 3 = -2 - 3 = -5$, $y' = 5$"

[WRITE] "A′(−5, 5)"

[SAY] "Let's verify: the distance from A(3) to the mirror line $x = -1$ is $3 - (-1) = 4$ units. The distance from A′(−5) to $x = -1$ is $-1 - (-5) = 4$ units. Equal distances — correct!"

[SAY] "This formula works for any vertical mirror line. You'll need it for Problem 4 in our practice."

Phase 3: Practice and Application (10 minutes)

Worked Example 1: Pentagon on Diagonal Mirror Line (2 minutes)

[SAY] "Problem 1: Draw the image of a pentagon under reflection on a diagonal mirror line M."

[DO] Draw the pentagon and diagonal mirror line on the board.

[SAY] "The method is exactly the same as with the triangle. For each vertex, draw a perpendicular to the mirror line, measure the distance, and mark the same distance on the other side."

[SAY] "For vertex A: draw a perpendicular from A to M. Measure the distance. Extend the same distance on the other side. Mark A′."

[SAY] "Repeat for B, C, D, E to get B′, C′, D′, E′."

[SAY] "Connect all the image vertices to form the reflected pentagon."

[SAY] "Notice: the method works for ANY mirror line orientation — vertical, horizontal, or diagonal. The key is always the perpendicular."

Worked Example 2: Reflecting on the y-axis (2 minutes)

[SAY] "Problem 2: Reflect the object about the y-axis."

[SAY] "We use the rule: (x, y) ⟶ (−x, y). Each x-coordinate changes sign."

[DO] Demonstrate with a specific shape on the Cartesian plane.

[SAY] "If a vertex is at (3, 4), its image is at (−3, 4). If a vertex is at (1, −2), its image is at (−1, −2)."

[SAY] "Plot all the reflected vertices and connect them. The image should be a mirror image of the original across the y-axis."

Worked Example 3: Successive Reflections (3 minutes)

[SAY] "Problem 3: The vertices of a polygon are A(−5, 5), B(−6, 3), C(−5, 1), D(−3, 0), E(−2, 2), F(−3, 4)."

[SAY] "Part (a): Reflect in y = x, then reflect the result in y = 0."

[SAY] "Step 1: Reflect in y = x. The rule is (x, y) ⟶ (y, x). Swap the coordinates."

[WRITE] "A(−5, 5) ⟶ A′(5, −5)"

[WRITE] "B(−6, 3) ⟶ B′(3, −6)"

[SAY] "And so on for all vertices."

[SAY] "Step 2: Now reflect A′, B′, etc. in y = 0 (the x-axis). Rule: (x, y) ⟶ (x, −y)."

[WRITE] "A′(5, −5) ⟶ A″(5, 5)"

[WRITE] "B′(3, −6) ⟶ B″(3, 6)"

[SAY] "Complete the table for all vertices."

[SAY] "Part (b): Reflect in x = 0 (y-axis). Rule: (x, y) ⟶ (−x, y)."

[WRITE] "A(−5, 5) ⟶ A′(5, 5)"

[WRITE] "B(−6, 3) ⟶ B′(6, 3)"

[SAY] "Notice something interesting: the final result of part (a) and part (b) give the same image for vertex A! This is because reflecting in y = x then y = 0 is equivalent to reflecting in the y-axis for certain configurations."

Worked Example 4: Finding Original Points (3 minutes)

[SAY] "Problem 4: The points A′(−4, 1), B′(−2, 4), C′(−1, 3) are images under reflection on x = −1. Find the original points."

[SAY] "This is the REVERSE problem. We know the images and need to find the originals."

[SAY] "For reflection on x = −1, the rule is (x, y) → (2(−1) − x, y) = (−2 − x, y)."

[SAY] "But since reflection is its own inverse — reflecting twice gives you back the original — we apply the SAME rule to the image points."

[WRITE] "A′(−4, 1): x = −2 − (−4) = −2 + 4 = 2, y = 1. So A = (2, 1)."

[WRITE] "B′(−2, 4): x = −2 − (−2) = 0, y = 4. So B = (0, 4)."

[WRITE] "C′(−1, 3): x = −2 − (−1) = −1, y = 3. So C = (−1, 3)."

[ASK] "Notice anything special about point C?"

[WAIT] Expected: "C and C′ are the same point!"

[SAY] "Yes! C′(−1, 3) is ON the mirror line x = −1, so it maps to itself. A point on the mirror line is its own reflection."

Phase 4: Assessment — Exit Ticket (5 minutes)

[SAY] "For our exit ticket, answer these five questions on a separate piece of paper. You have 5 minutes."

[SAY] "Question 1: Copy the figure on your handout and draw its image under reflection on the mirror line M."

[SAY] "Question 2: Triangle PQR has vertices P(1, 4), Q(3, 1), R(5, 3). Find the image under reflection in (a) the x-axis, (b) the y-axis, (c) the line y = x."

[SAY] "Question 3: State three properties of reflection."

[SAY] "Question 4: The point A′(6, −3) is the image of A under reflection in the x-axis. Find the coordinates of A."

[SAY] "Question 5: A shape is reflected in the line x = 2. The image of point B is B′(−1, 5). Find the coordinates of B."

[DO] Collect exit tickets as students finish.

Answer Key:

- 1. Students draw perpendiculars from each vertex to M, measure equal distances on the other side, and connect reflected vertices. Image should be congruent and on the opposite side.
- 2. (a) x-axis: $P'(1, -4)$, $Q'(3, -1)$, $R'(5, -3)$. (b) y-axis: $P'(-1, 4)$, $Q'(-3, 1)$, $R'(-5, 3)$. (c) $y = x$: $P'(4, 1)$, $Q'(1, 3)$, $R'(3, 5)$.
- 3. (i) Reflection moves the image to the opposite side of the mirror line. (ii) Each point and its image are equidistant from the mirror line. (iii) The mirror line is the perpendicular bisector of the segment joining any point to its image.
- 4. x-axis reflection: $(x, y) \to (x, -y)$. Since $A' = (6, -3)$, then $A = (6, 3)$.
- 5. Reflection in $x = 2$: $x\_original = 2(2) - (-1) = 5$. So $B = (5, 5)$.

Differentiation Notes

Struggling Learners:

Provide grid paper with the mirror line and object pre-drawn. Allow use of small mirrors to visualise reflections before drawing. Start with vertical and horizontal mirror lines before diagonal ones. Provide a step-by-step checklist for the construction method. Use tracing paper: trace the object, fold along the mirror line, and the traced shape shows the reflected image.

On-Level Learners:

Complete all practice problems including successive reflections. Apply coordinate rules without grid paper. Verify constructions by checking perpendicular bisector property. Attempt reverse problems: given image, find original points.

Advanced Learners:

Investigate: What single transformation equals two successive reflections? Explore reflections in lines $y = mx + c$. Prove algebraically that reflection preserves distances. Design a symmetric pattern using multiple reflections across different mirror lines.

Post-Lesson Reflection

1. Were students able to accurately construct reflected images using the perpendicular-and-measure method?

2. Did students discover the three key properties through the anchor activity?

3. How well did students transition from grid-based constructions to Cartesian plane coordinate rules?

4. Were students able to apply reflection rules for x-axis, y-axis, and $y = x$ correctly?

5. Did students understand the reverse problem (finding originals from images)?

6. What common errors arose (e.g., measuring along the mirror line instead of perpendicular to it)?

7. Did the physical mirrors help students visualise the concept?

8. What adjustments would improve the lesson for future delivery?