Grade 10 Mathematics Lesson Plan

Area of Heptagons

Strand:	Measurement and Geometry
Sub-Strand:	Area of Polygons: Area of Heptagon
Specific Learning Outcome:	Work out the area of regular heptagon and octagon. Explore the area of polygons as used in real-life situations.
Duration:	40 minutes
Key Inquiry Question:	How do we work out the area of polygons?
Learning Resources:	CBC Grade 10 textbooks, compass, ruler, protractor, pencil, eraser, graph paper, calculators

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Constructing a regular heptagon using geometry tools
Structured Instruction	10 minutes	Formalizing the properties and area formula
Practice and Application	10 minutes	Worked example using triangles to find area
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Constructing a Regular Heptagon

Objective: Learners are expected to know how to construct a regular heptagon (7-sided polygon) using a compass, ruler, and protractor.

Materials Needed:

- Compass
- Ruler
- Protractor

- Pencil
- Eraser
- Graph paper (optional)

Steps for the Activity:

1. Step 1: Draw a Circle. Place the compass pointer on the paper and draw a circle of any radius. Mark the center (O) of the circle.
2. Step 2: Draw a Horizontal Diameter. Use the ruler to draw a straight line measuring 10 cm through the center (O), creating a diameter (A0 to P1). Label the first point P1 on the circumference.
3. Step 3: Divide the Circle into Seven Equal Parts. Use a protractor to measure angles of 360 degrees divided by 7 = 51.43 degrees from point P1. Mark each 51.43 degree interval around the circle to get seven points.
4. Step 4: Connect the Points. Use the ruler to draw straight lines connecting the seven points in sequence. The heptagon is now complete!

Extended Activity:

- Try drawing a heptagon using only a compass (without a protractor)
- Shade the inside of the heptagon with different colors to make a pattern
- Find the sum of the interior angles of the heptagon. (Hint: (n minus 2) times 180 degrees)

Discussion Questions:

5. What is the sum of all interior angles of a heptagon?
6. How do we calculate the measure of one interior angle of a regular heptagon?
7. Can you find a heptagon in real life (architecture, logos, etc.)?

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand how to use the compass and protractor correctly.
- Ask probing questions: What do you notice about the angles? How many sides does your polygon have?
- For struggling groups: Let us draw the first angle together. Make sure it is 51.43 degrees.
- For early finishers: Can you find the sum of the interior angles? Can you find the measure of one interior angle?

- Guide students to articulate: A heptagon has seven sides, and the sum of interior angles is 900 degrees.
- Identify 2-3 groups with accurate constructions to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Properties and Area Formula

After students have completed the anchor activity and shared their findings, the teacher formalizes the properties of a regular heptagon and the area formula.

Key Takeaway: Properties of a Regular Heptagon

A heptagon is a seven-sided polygon. It has seven edges and seven vertices.

Sum of Interior Angles:

Sum of interior angles = (n minus 2) times 180 degrees, where n is the number of sides.

Example for a Heptagon (7-sided polygon):

(n minus 2) times 180 degrees

(7 minus 2) times 180 degrees = 900 degrees

Sum of Exterior Angles:

Sum of exterior angles of any polygon (regular or irregular) is always equal to 360 degrees for both regular and irregular polygons.

Each Interior and Exterior Angle (Regular Polygon Only):

Each Interior Angle (for a regular polygon):

Interior Angle = (n minus 2) times 180 degrees divided by n

Each Exterior Angle (for a regular polygon):

Exterior Angle = 360 degrees divided by n

Example for a Regular Heptagon:

Each Interior Angle = 900 degrees divided by 7 = 128.57 degrees

Each Exterior Angle = 360 degrees divided by 7 = 51.43 degrees

Finding the Area of a Regular Heptagon:

A regular heptagon can be divided into 7 congruent triangles by drawing lines from the center to each vertex.

To find the area, we calculate the area of one triangle and multiply by 7.

Area of one triangle = (1/2) times a times b times sin(angle)

where a and b are the two sides from the center to the vertices, and the angle is 51.43 degrees (360 degrees divided by 7).

Total Area = 7 times Area of one triangle

Scaffolding Strategies to Address Misconceptions:

- Misconception: All polygons have the same interior angle sum. Clarification: No, the sum depends on the number of sides. Use the formula (n minus 2) times 180 degrees.
- Misconception: I can just multiply the side length by 7 to get the area. Clarification: No, that gives you the perimeter, not the area. We need to use triangles.
- Misconception: The exterior angle is the same as the interior angle. Clarification: No, they are supplementary. Interior plus Exterior = 180 degrees.
- Misconception: A heptagon is the same as a hexagon. Clarification: No, a hexagon has 6 sides, while a heptagon has 7 sides.

Phase 3: Practice and Application (10 minutes)

Worked Example (Textbook Example 2.5.13):

A regular heptagon measures 10 cm, find its area given the sum of its interior angles is 900 degrees.

Solution:

There are 7 triangles since a heptagon has 7 sides.

Area of Triangle P0OP1 = (1/2) times a times b times sin 51.43 degrees

Triangle P0OP1 = (1/2) times 10 times 10 times sin 51.43 degrees

= (1/2) times 100 cm squared times sin 51.43 degrees

= (1/2) times 100 cm squared times 0.7818

= 39.0923 cm squared

Therefore the total area:

= 7 times 39.0923 cm squared

= 273.65 cm squared

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. What is the sum of all interior angles of a heptagon?

2. How do we calculate the measure of one interior angle of a regular heptagon?

3. A regular heptagon has a radius of 8 cm. Find the area of one triangle formed by the center and two adjacent vertices.

4. Can you find heptagons in real life (architecture, logos, etc.)?

Answer Key:

1. 900 degrees

2. Interior Angle = (n minus 2) times 180 degrees divided by n = 900 degrees divided by 7 = 128.57 degrees

3. Area of one triangle = (1/2) times 8 times 8 times sin 51.43 degrees = (1/2) times 64 times 0.7818 = 25.02 cm squared

4. Examples: British 50 pence coin, some architectural designs, logos, etc.

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn circles with center marked.
- Use simpler numbers for radius lengths.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators.

For On-Level Learners:

- Encourage students to draw their own heptagons from word problems.
- Ask students to explain which formula they chose and why.
- Provide mixed practice with both finding area and finding missing dimensions.

For Advanced Learners:

- Challenge students to derive the formula themselves using the concept of dividing into triangles.
- Explore real-world applications: architecture, coin design, logo design.
- Investigate the relationship between the area of a heptagon and other regular polygons.
- Apply trigonometry to find all angles and side lengths when given only the radius.

Extension Activity

Real-World Application: Designing Heptagon-Shaped Structures

Students work in groups to design a heptagon-shaped structure (coin, logo, architectural feature, garden) and calculate its area.

Materials: Graph paper, rulers, protractors, compasses, calculators, colored pencils

Tasks:

8. Choose a real-world application that uses heptagon shapes (coin, logo, architectural feature, garden, etc.).
9. Draw the heptagon on graph paper with appropriate dimensions.
10. Measure or specify the radius of the circumscribed circle.
11. Calculate the area using the triangle method.
12. If applicable, find the sum of interior angles and each interior angle.
13. Present your findings to the class, explaining your design choices and calculations.

Key Takeaway:

Students should understand how the area formula for heptagons is used in real-world professions such as engineering, architecture, and design to calculate areas of structures, coins, and logos.

Teacher Reflection Prompts

- Did students successfully construct the heptagon and identify the seven equal angles in the anchor activity?
- Were students able to discover the properties of a heptagon by measuring angles and sides?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand when to use the triangle method to find the area?
- What adjustments would improve this lesson for future classes?