## I. Lesson Overview

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|---|---|
| **Strand** | Numbers and Algebra |
| **Sub-Strand** | Quadratic Expressions and Equations 1 |
| **Specific Learning Outcome** | Solve quadratic equations by factorisation |
| **Key Inquiry Question** | How do we apply the concept of Quadratic equations? |
| **Grade Level** | Grade 10 |
| **Duration** | 40 minutes |
| **Materials & Resources** | CBC Grade 10 Mathematics Textbooks |

## II. Learning Objectives

By the end of this lesson, students will be able to:

| Category | Objective |
|---|---|
| **Know** | Understand that the roots of a quadratic equation are the values of x that satisfy the equation, and that factorisation converts $ax^2 + bx + c = 0$ into the form $(x + p)(x + q) = 0$. |
| **Do** | Factor quadratic expressions by finding two numbers whose product equals c (or ac) and whose sum equals b, then apply the Zero Product Property to solve for x. |
| **Apply** | Use factorisation to solve real-world problems involving quadratic equations, such as finding dimensions of a garden, determining time values, and forming equations from word problems. |

## III. Lesson Procedure

### Phase 1: Problem-Solving and Discovery (15 Minutes)

**Anchor Activity:** Group Factorisation Challenge

Divide students into groups of 4–5. Each group receives the following challenge:

Work in Groups. Find the roots of the equation using factorisation:

- (i) $x^2 - 5x + 6 = 0$
- (ii) $x^2 + 7x + 10 = 0$

After solving the equations, discuss the following:

- What steps did you follow in factorising the quadratic equation?
- How do the roots relate to the factors of the quadratic equation?
- Did any group use a different method to factorise the equation? Compare approaches.

Finally, each group will share their solved quadratic equations and the methods they used in front of the classroom.

**Teacher's Role:** Circulate among groups, listen to discussions, and ask probing questions:

- "What two numbers multiply to give the constant term?"
- "What do those same two numbers add up to?"
- "Once you have the factors, how do you find the values of x?"
- "Can you verify your answers by substituting back into the original equation?"

**Expected Student Discoveries:**

- (i) $x^2 - 5x + 6 = 0 \rightarrow (x - 2)(x - 3) = 0 \rightarrow x = 2$ or $x = 3$
- (ii) $x^2 + 7x + 10 = 0 \rightarrow (x + 2)(x + 5) = 0 \rightarrow x = -2$ or $x = -5$

## Phase 2: Structured Instruction (10 Minutes)

**Connecting Student Discoveries to Formal Concepts:**

The teacher builds on the group presentations to formalise the method:

**Key Takeaways:**

Numbers that satisfy an equation (its solutions) are called the roots of the equation.

Once you have factored the quadratic into the form:

**$(x + p)(x + q) = 0$**

Set each factor equal to zero and solve for x:

$(x + p) = 0$ or $(x + q) = 0$

Solving these will give the two solutions (roots) for x.

**The 4-Step Factorisation Method:**

1. Step 1: Write the equation in standard form: $ax^2 + bx + c = 0$
2. Step 2: Find two numbers m and n such that $m \times n = c$ (or ac when $a \neq 1$) and $m + n = b$
3. Step 3: Rewrite and factor: $(x + m)(x + n) = 0$
4. Step 4: Apply the Zero Product Property: set each factor = 0 and solve for x

**Common Misconception:** Students may forget that when they find factors like $(x + 2) = 0$, the root is $x = -2$ (not $x = 2$). Emphasise that solving $(x + p) = 0$ gives $x = -p$.

## Phase 3: Practice and Application (10 Minutes)

**Worked Example (Teacher-Led):**

Solve: $x^2 + 5x + 6 = 0$

- Step 1: Identify a = 1, b = 5, c = 6
- Step 2: Find two numbers that multiply to 6 and add to 5 → 2 and 3
- Step 3: Split the middle term: $x^2 + 2x + 3x + 6 = 0$
- Step 4: Factor by grouping: $x(x + 2) + 3(x + 2) = 0$
- Step 5: Factor out common binomial: $(x + 2)(x + 3) = 0$
- Step 6: Apply Zero Product Property:

$x + 2 = 0 \rightarrow x = -2$

$x + 3 = 0 \rightarrow x = -3$

**Therefore: $x = -2$ or $x = -3$**

## Phase 4: Assessment — Exit Ticket (5 Minutes)

Solve the following quadratic equations by factorisation:

5. (i) $x^2 + 7x + 10 = 0$
6. (ii) $x^2 - 5x + 6 = 0$
7. (iii) $x^2 + 3x - 4 = 0$

Word Problems:

8. (a) A car's speed is represented by a quadratic equation: $4x^2 - 16x + 15 = 0$. Find the possible values of x representing time.
9. (b) The sum of a number and its square is 42. Form a quadratic equation and solve it to find the number.
10. (c) Solve the quadratic equation: $3x^2 - 14x + 8 = 0$
11. (d) A garden's area is 56 square meters, and its length is 4 meters more than its width. Form and solve a quadratic equation to find the dimensions of the garden.

## Answer Key

- (i) $x^2 + 7x + 10 = 0 \rightarrow (x + 2)(x + 5) = 0 \rightarrow x = -2$ or $x = -5$
- (ii) $x^2 - 5x + 6 = 0 \rightarrow (x - 2)(x - 3) = 0 \rightarrow x = 2$ or $x = 3$
- (iii) $x^2 + 3x - 4 = 0 \rightarrow (x + 4)(x - 1) = 0 \rightarrow x = -4$ or $x = 1$

(a) $4x^2 - 16x + 15 = 0$

$ac = 4 \times 15 = 60$. Find m, n: $m \times n = 60$ and $m + n = -16 \rightarrow m = -10, n = -6$

$4x^2 - 10x - 6x + 15 = 0 \rightarrow 2x(2x - 5) - 3(2x - 5) = 0$

$(2x - 5)(2x - 3) = 0 \rightarrow x = 5/2 = 2.5$ or $x = 3/2 = 1.5$

(b) Let the number be x. Then: $x + x^2 = 42 \rightarrow x^2 + x - 42 = 0$

$(x + 7)(x - 6) = 0 \rightarrow x = -7$ or $x = 6$

(c) $3x^2 - 14x + 8 = 0$

$ac = 3 \times 8 = 24$. Find m, n: $m \times n = 24$ and $m + n = -14 \rightarrow m = -12, n = -2$

$3x^2 - 12x - 2x + 8 = 0 \rightarrow 3x(x - 4) - 2(x - 4) = 0$

$(3x - 2)(x - 4) = 0 \rightarrow x = 2/3$ or $x = 4$

(d) Let width = w. Then length = w + 4.

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Area: $w(w + 4) = 56$

$w^2 + 4w - 56 = 0$

Using the quadratic formula: $w = (-4 \pm \sqrt{(16 + 224)}) / 2 = (-4 \pm \sqrt{240}) / 2$

$w = (-4 \pm 4\sqrt{15}) / 2 = -2 \pm 2\sqrt{15}$

Since width must be positive: $w = -2 + 2\sqrt{15} \approx 5.75$ metres

Length = $w + 4 \approx 9.75$ metres

The garden is approximately 5.75 m wide and 9.75 m long.

## IV. Differentiation Strategies

| Learner Level | Strategy |
|---|---|
| **Struggling Learners** | Provide a factor-pair reference sheet listing common products and sums. Use colour-coded steps: Step 1 (blue), Step 2 (green), Step 3 (red). Start with simpler equations where a = 1 and c is positive. Allow use of multiplication tables. |
| **On-Level Learners** | Complete all anchor activity and assessment problems independently. Verify solutions by substituting roots back into the original equation. Work in pairs to compare methods and check each other's work. |
| **Advanced Learners** | Solve equations where $a \neq 1$ (e.g., $3x^2 - 14x + 8 = 0$). Create their own word problems that lead to quadratic equations. Explore the relationship between roots and coefficients (sum of roots = $-b/a$, product of roots = $c/a$). |

### Extension Activity

A rectangular swimming pool has an area of 120 square metres. The length of the pool is 2 metres more than twice its width.

- (a) Form a quadratic equation to represent this situation.
- (b) Solve the equation by factorisation to find the dimensions of the pool.
- (c) If a 1.5-metre-wide path is built around the pool, find the total area including the path.

**Extension Answer Key:**

A rectangular swimming pool has an area of 60 square metres. The length of the pool is 4 metres more than its width.

- (a) Form a quadratic equation to represent this situation.
- (b) Solve the equation by factorisation to find the dimensions of the pool.
- (c) If a 2-metre-wide path is built around the pool, find the total area including the path.

**Extension Answer Key:**

Let width = w. Length = w + 4.

$w(w + 4) = 60 \rightarrow w^2 + 4w - 60 = 0$

Find two numbers: product = −60, sum = 4 → 10 and −6

$(w + 10)(w - 6) = 0 \rightarrow w = 6$ (reject $w = -10$)

Width = 6 m, Length = 10 m

(c) With 2 m path: total width = 6 + 2(2) = 10 m, total length = 10 + 2(2) = 14 m

Total area = $10 \times 14 = 140\ m^2$

## V. Assessment Methods

| Type | Description |
|---|---|
| **Formative (Ongoing)** | Observation during group work; questioning during circulation; monitoring factorisation steps on mini-whiteboards. |
| **Peer Assessment** | Groups compare methods and verify each other's solutions by substituting roots back into original equations. |
| **Summative (Exit Ticket)** | Seven assessment questions testing factorisation skills, word problem formulation, and real-world application. |

### Checkpoint Integration

**Checkpoint protocol for Learners:**

1. Click "Show new example question" to load the problem
2. Solve the displayed question
3. Click "submit" to check your answer
4. If incorrect, carefully read the feedback and analyse the error before trying a new question. The immediate feedback from checkpoint submissions allows Learners to identify and correct errors in real-time.
5. Complete at least 5 questions before rotating
6. Pair Learners strategically so stronger learners can explain reasoning to peers.

## VI. Teacher Reflection (Post-Lesson)

- Did students successfully identify the factor pairs needed for factorisation?
- Were students able to apply the Zero Product Property correctly to find roots?
- How effectively did groups collaborate and compare different factorisation approaches?
- Did the worked example adequately prepare students for the assessment problems?
- Were struggling learners able to access the content with the scaffolds provided?
- Did advanced learners engage meaningfully with equations where $a \neq 1$?
- What adjustments should I make for the next lesson on quadratic equations?