Apply Quadratic Identities in Numerical Cases

I. Lesson Overview

Strand	Numbers and Algebra
Sub-Strand	Quadratic Expressions and Equations 1
Specific Learning Outcome	Apply quadratic identities in numerical cases
Grade Level	Grade 10
Duration	40 minutes
Key Inquiry Questions	How can quadratic identities help us solve real-world problems? Where do we encounter quadratic relationships in everyday life?

II. Learning Objectives

Category	Objective
Know	Recall the three quadratic identities: $(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$ $(a + b)(a - b) = a^2 - b^2$
Do	Apply quadratic identities to expand, simplify, and solve numerical problems including area calculations
Apply	Use quadratic identities to model and solve real-world problems such as garden dimensions, construction areas, and financial calculations

III. Materials & Resources

• CBC Grade 10 Mathematics Textbooks

IV. Lesson Procedure

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Gardening with Quadratics

Scenario: Imagine you are planning a new garden in your backyard. You want to create a rectangular vegetable patch that has an area of 48 square feet. You decide to set the length of the garden to be twice the width.

Group Task:

1. Let w represent the width of the garden. If the length $l = 2w$, write an equation for the area $A = l \times w$.

2. Substitute $l = 2w$ into the area equation to get: $48 = 2w \times w$, which simplifies to $2w^2 = 48$.

3. Solve for w to find the width, then calculate the length.

4. Verify your answer by checking that $l \times w = 48$.

5. Extension: How would changing the area affect the dimensions? Try with an area of 72 square feet.

Solution to Anchor Activity:

$2w^2 = 48$

$w^2 = 24$

$w = \sqrt{24} = 2\sqrt{6} \approx 4.9$ feet

$l = 2w = 2 \times 2\sqrt{6} = 4\sqrt{6} \approx 9.8$ feet

Check: $4.9 \times 9.8 \approx 48$ ✓

Guiding Question:

How would changing the area you want for your vegetable garden affect the dimensions of your garden? Can you think of a different area and apply the quadratic identity to find new dimensions?

Teacher Role:

• Circulate among groups, asking probing questions: "What happens to w when you double the area?" "Can you express this using a quadratic identity?"

• Identify groups that connect the problem to $(a + b)^2$ or difference of squares patterns.

• Select 2–3 groups to share their approaches, progressing from concrete to abstract solutions.

Phase 2: Structured Instruction (10 minutes)

Connecting Discovery to Formal Concepts:

The teacher connects the garden problem to the three quadratic identities and demonstrates how they apply to numerical cases:

The Three Quadratic Identities:

Identity	Formula	Numerical Example
Perfect Square (Sum)	$(a + b)^2 = a^2 + 2ab + b^2$	$51^2 = (50 + 1)^2 = 2500 + 100 + 1 = 2601$
Perfect Square (Difference)	$(a - b)^2 = a^2 - 2ab + b^2$	$49^2 = (50 - 1)^2 = 2500 - 100 + 1 = 2401$
Difference of Squares	$(a + b)(a - b) = a^2 - b^2$	$51 \times 49 = (50 + 1)(50 - 1) = 2500 - 1 = 2499$

Worked Example: Applying Identities to the Garden Problem

Suppose a square garden has a side length of $(x + 3)$ metres. Find the area using a quadratic identity.

Solution:

Area = $(x + 3)^2 = x^2 + 2(x)(3) + 3^2 = x^2 + 6x + 9$

If $x = 5$: Area = $25 + 30 + 9 = 64$ square metres

Check: $(5 + 3)^2 = 8^2 = 64$ ✓

Addressing Misconceptions:

• Common Error: $(a + b)^2 = a^2 + b^2$ (forgetting the middle term 2ab)

• Correction: Always expand fully. Numerical proof: $(3 + 2)^2 = 25$, but $3^2 + 2^2 = 13$. The missing piece is $2(3)(2) = 12$.

• Common Error: Confusing $(a - b)^2$ with $a^2 - b^2$

• Correction: $(a - b)^2 = a^2 - 2ab + b^2$ has three terms; $a^2 - b^2 = (a+b)(a-b)$ has two terms.

Phase 3: Practice and Application (15 minutes)

Practice Problems:

1. Use a quadratic identity to calculate 103^2 without a calculator.

2. Use a quadratic identity to calculate 97^2 without a calculator.

3. Use the difference of squares identity to calculate 63×57.

4. A rectangular field has dimensions $(x + 5)$ metres by $(x - 5)$ metres. Express the area using a quadratic identity. If $x = 12$, find the area.

5. A farmer wants to build a square pen with side length $(2n + 3)$ metres. Find the area as a quadratic expression. If $n = 4$, what is the area?

Answer Key:

1. $103^2 = (100 + 3)^2 = 10000 + 600 + 9 = 10{,}609$

2. $97^2 = (100 - 3)^2 = 10000 - 600 + 9 = 9{,}409$

3. $63 \times 57 = (60 + 3)(60 - 3) = 3600 - 9 = 3{,}591$

4. Area = $(x + 5)(x - 5) = x^2 - 25$. When $x = 12$: Area = $144 - 25 = 119$ m²

5. Area = $(2n + 3)^2 = 4n^2 + 12n + 9$. When $n = 4$: Area = $64 + 48 + 9 = 121$ m²

Phase 4: Assessment — Exit Ticket (5 minutes)

Answer the following questions independently:

1. Use a quadratic identity to evaluate 202^2.

2. Calculate 48×52 using the difference of squares identity.

3. A square playground has side length (y + 7) metres. Write the area as a quadratic expression and find the area when y = 3.

Exit Ticket Answer Key:

1. $202^2 = (200 + 2)^2 = 40000 + 800 + 4 = 40,804$

2. $48 \times 52 = (50 - 2)(50 + 2) = 2500 - 4 = 2,496$

3. Area $= (y + 7)^2 = y^2 + 14y + 49$. When $y = 3$: Area $= 9 + 42 + 49 = 100$ m^2

V. Differentiation Strategies

Learner Level	Strategy
Struggling Learners	• Provide a reference card with the three identities and numerical examples • Start with simpler numbers: 11^2, 19^2, 21×19 • Use colour-coded steps to show how a and b map into each identity • Pair with a peer mentor for guided practice
On-Level Learners	• Complete all practice problems independently • Verify answers using long multiplication to build confidence • Create their own real-world word problems using identities
Advanced Learners	• Extension: Use identities to simplify $(2x + 3y)^2 - (2x - 3y)^2$ • Challenge: A farmer has 100m of fencing. Using the identity $(a+b)^2 = a^2 + 2ab + b^2$, find the dimensions that maximise the enclosed area • Investigate: Can you use quadratic identities to prove that the product of two consecutive odd numbers is always one less than a perfect square?

VI. Assessment Methods

Type	Method
Formative	• Observation during group anchor activity • Targeted questioning during circulation • Monitoring whiteboard work during practice • Peer discussion quality assessment
Summative	• Exit ticket (3 questions) • Correct identity selection and application • Accuracy of numerical computation • Ability to connect identity to real-world context

Checkpoint Integration

Pre-class Preparation list:

1. Test internet connectivity and access to https://innodems.github.io/CBC-Grade-10-Maths/

2. Ensure all student devices can access the digital textbook
3. Pre-load the checkpoint page on the teacher's display device
4. Have backup printed worksheets in case of technical issues
5. Arrange seating for pair work and station rotations

Checkpoint protocol for Learners:

1. Click “Show new example question” to load the problem
2. Solve the displayed question
3. Click “submit” to check your answer
4. If incorrect, carefully read the feedback and analyse the error before trying a new question. The immediate feedback from checkpoint submissions allows students to identify and correct errors in real-time.
5. Complete at least 5 questions before rotating
6. Pair students strategically so stronger learners can explain reasoning to peers.

VII. Teacher Reflection

1. Did students successfully connect the garden scenario to quadratic identities?

2. Which identity did students find most challenging to apply numerically?

3. Were students able to distinguish between $(a + b)^2$ and $(a + b)(a - b)$?

4. How effectively did the real-world context motivate student engagement?

5. Did the differentiation strategies adequately support all learner levels?

6. What adjustments would improve the transition from discovery to formal instruction?

7. How can I better scaffold the connection between algebraic identities and numerical computation?