Step by step guide: Factorisation Of Quadratic Expressions

Grade 10 Mathematics | 40-Minute Lesson

Before Class Begins

Preparation Checklist:

- Write the key formula on the board: $x^2 + bx + c = (x + m)(x + n)$ where $m + n = b$ and $m \times n = c$
- Prepare factor pair reference cards for struggling learners
- Prepare exit tickets for distribution
- Set timer for phase transitions

[WRITE on board before class]:

"FACTORISING $x^2 + bx + c$:
Find m and n where:
• $m + n = b$
• $m \times n = c$
Then: $x^2 + bx + c = (x + m)(x + n)$"

PHASE 1: Problem-Solving and Discovery (15 Minutes)

Opening (2 minutes)

[SAY]:

"Good morning/afternoon, class! Last lesson we learned how to EXPAND brackets like $(x + 3)(x + 4)$. Today we're going to learn the REVERSE - how to FACTORISE expressions like $x^2 + 7x + 12$ back into brackets!"

[SAY]:

"Our key question today is: How do we apply the concept of Quadratic equations? Factorisation is a crucial skill for solving these equations."

Anchor Activity Introduction (3 minutes)

[SAY]:

"In your groups, I want you to explore the connection between expanding and factorising.

Part A: First, EXPAND these expressions:
- *$(x + 3)(x + 4)$*
- *$(x - 6)(x - 5)$*
- *$(x + 2)(x + 3)$*

Part B: Then complete the table I'm giving you and look for a PATTERN."

[DISTRIBUTE table worksheets]

Group Work (7 minutes)

[SAY]:

"After completing the table, try the REVERSE challenge:

Given $x^2 + 5x + 6$, can you find two numbers that:
- *ADD to give 5?*
- *MULTIPLY to give 6?*

You have 6 minutes. Begin!"

[DO]: Walk around the room, observing group discussions.

[ASK probing questions as you circulate]:

- "What do you notice about the sum of the constants in the factors?"
- "What do you notice about the product of the constants?"
- "For $x^2 + 5x + 6$, what two numbers add to 5 and multiply to 6?"
- "How does this help you write the factorised form?"

[TIME CHECK]: At 5 minutes, announce: "One more minute!"

Class Discussion (3 minutes)

[SAY]:

"Let's share discoveries. When you expanded $(x + 3)(x + 4)$, what did you get?"

[Expected answer]: "$x^2 + 7x + 12$"

[ASK]:

"What's $3 + 4$? And what's 3×4?"

[Expected answer]: "3 + 4 = 7 (the coefficient of x) and 3 × 4 = 12 (the constant term)!"

[ASK]:

"So for $x^2 + 5x + 6$, what two numbers add to 5 and multiply to 6?"

[Expected answer]: "2 and 3!"

[TRANSITION]:

"Excellent! You've discovered the key to factorisation. Let me formalize this."

PHASE 2: Structured Instruction (10 Minutes)

Definitions (3 minutes)

[SAY]:

"First, let's define our terms. A QUADRATIC EXPRESSION has the form $ax^2 + bx + c$ where:
- *a is the coefficient of x^2*
- *b is the coefficient of x*
- *c is the constant term"*

[SAY]:

"Today we focus on expressions where $a = 1$, so we have $x^2 + bx + c$."

The Factorisation Method (5 minutes)

[SAY]:

"Here's what you discovered:"

[WRITE on board]:

"To factorise $x^2 + bx + c$:

Find two numbers m and n where:
- *$m + n = b$ (the coefficient of x)*
- *$m \times n = c$ (the constant term)*

Then: $x^2 + bx + c = (x + m)(x + n)$"

[SAY]:

"Let's verify with our examples:

$(x + 3)(x + 4) = x^2 + 7x + 12$
Check: $3 + 4 = 7$ ✓ and $3 \times 4 = 12$ ✓

$(x - 6)(x - 5) = x^2 - 11x + 30$
Check: $(-6) + (-5) = -11$ ✓ and $(-6) \times (-5) = 30$ ✓ "

Misconception Alert (2 minutes)

[SAY - IMPORTANT]:

"CAUTION! When both numbers in the factors are NEGATIVE:
• The middle term is NEGATIVE (negative + negative = negative)
• But the constant term is POSITIVE (negative × negative = positive)

So $x^2 - 11x + 30$ has NEGATIVE numbers in its factors: $(x - 6)(x - 5)$"

[TRANSITION]:

"Now let's practice factorising!"

PHASE 3: Practice and Application (15 Minutes)

Worked Example (5 minutes)

[SAY]:

"Let's factorise $x^2 + 5x + 6$ step by step."

[WRITE step by step]:

"Step 1: Identify the coefficients
Coefficient of $x^2 = 1$ ✓
Coefficient of x (b) = 5
Constant term (c) = 6

Step 2: Find m and n where $m + n = 5$ and $m \times n = 6$
List factor pairs of 6:
$1 \times 6 = 6$, but $1 + 6 = 7$ ✗
$2 \times 3 = 6$, and $2 + 3 = 5$ ✓

Step 3: Write the factorised form
$x^2 + 5x + 6 = (x + 2)(x + 3)$"

[SAY]:

"Always VERIFY by expanding:
$(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$ ✓ "

Guided Practice (8 minutes)

[SAY]:

"Try these with your partner:
1. Factorise $x^2 + 8x + 15$
2. Factorise $x^2 - 7x + 12$"

[GIVE 5 minutes, then review]:

"1. $x^2 + 8x + 15$:
Need: $m + n = 8$, $m \times n = 15$
Numbers: 3 and 5 ($3 + 5 = 8$, $3 \times 5 = 15$)
Answer: $(x + 3)(x + 5)$

2. $x^2 - 7x + 12$:
Need: $m + n = -7$, $m \times n = 12$
Numbers: -3 and -4 ($-3 + (-4) = -7$, $(-3)(-4) = 12$)
Answer: $(x - 3)(x - 4)$"

[TRANSITION]:

"Now I want to see what each of you has learned."

PHASE 4: Assessment / Checkpoint (8 Minutes)

Checkpoint exploration (5 minutes)

[DO] Project the digital textbook on the screen. Navigate to the "Checkpoint" section.

[SAY] "This is our digital mathematics textbook. It has something special called checkpoints. Watch what happens when I click this button..."

[DO] Click "Show new example question" on Checkpoint

[SAY] "See? A new number appeared! And if I click again..."

[DO] Click the button again to show randomization

[SAY] "A different number! This means you can practice with hundreds of different examples. The computer never runs out of problems to give you."

[SAY] "Now it's your turn. With your partner, open the digital textbook and find the checkpoint.

[SAY] Click “Show new example question” to load the problem

[SAY] Solve the displayed question

[SAY] Click “submit” to check your answer

[SAY] If incorrect, carefully read the feedback and analyse the error before trying a new question. The immediate feedback from checkpoint submissions allows students to identify and correct errors in real-time.

[SAY] Complete at least 5 questions

[DO] Circulate among pairs. Ask probing questions, for example, what patterns do you notice?

Independent Work (5 minutes)

[DISPLAY questions]:

"Factorise:
1. $x^2 + 4x + 4$
2. $x^2 + 3x + 2$"

[SAY]:

"You have 5 minutes. Show your working. Begin."

Collection and Closure (2 minutes)

[SAY]:

"Time's up. Please pass your exit tickets forward."

[COLLECT all tickets]

[SAY]:

"Today you learned how to FACTORISE quadratic expressions:

- *For $x^2 + bx + c$, find m and n where $m + n = b$ and $m \times n = c$*
- *Then write $(x + m)(x + n)$*
- *Always verify by expanding!*

Remember: The SUM gives the middle coefficient, the PRODUCT gives the constant.

Great work today!"

Differentiation Notes

For Struggling Learners:

- Provide factor pair tables for common numbers (6, 12, 15, etc.)
- Use visual diagrams showing the connection
- Start with expressions where both numbers are positive
- Allow verification by expansion

For Advanced Learners:

[GIVE these extensions]:

- Factorise $x^2 - x - 12 \rightarrow (x + 3)(x - 4)$
- Factorise $x^2 + 2x - 15 \rightarrow (x + 5)(x - 3)$
- Solve $x^2 + 5x + 6 = 0 \rightarrow x = -2$ or $x = -3$

Answer Key

Exit Ticket Answers:

(a) $x^2 + 4x + 4$:

$m + n = 4$, $m \times n = 4$

Numbers: 2 and 2

Answer: $(x + 2)^2$

(b) $x^2 + 8x + 15$:

$m + n = 8$, $m \times n = 15$

Numbers: 3 and 5

Answer: $(x + 3)(x + 5)$

(c) $x^2 - 7x + 12$:

$m + n = -7$, $m \times n = 12$

Numbers: -3 and -4

Answer: $(x - 3)(x - 4)$

(d) $x^2 - 6x + 9$:

$m + n = -6$, $m \times n = 9$

Numbers: -3 and -3

Answer: $(x - 3)^2$

(e) $x^2 + 3x + 2$:

$m + n = 3$, $m \times n = 2$

Numbers: 1 and 2

Answer: $(x + 1)(x + 2)$

Post-Lesson Reflection Prompts

1. What went well? Did students discover the sum/product relationship?

2. What would I change? Was the anchor activity effective?

3. Student Understanding: Could students handle negative numbers in factors?

4. Next Steps: Which students need more practice?