Grade 10 Mathematics Lesson Plan

Area of an Annulus

Strand:	Measurement and Geometry
Sub-Strand:	Area of a Part of a Circle: Area of an Annulus
Specific Learning Outcome:	Determine the area of an annulus in different situations. Apply the area of a part of a circle in real-life situations. Explore the use of the area of a part of a circle in real-life situations.
Duration:	40 minutes
Key Inquiry Question:	How do we use the concept of the area of a part of a circle in real life?
Learning Resources:	CBC Grade 10 textbooks, circular objects (cups, lids, rings), ruler or measuring tape, pen and paper, calculators

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Discovering the annulus formula using circular objects
Structured Instruction	10 minutes	Formalizing the annulus formula and addressing misconceptions
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Discovering the Annulus Formula

Objective: Students will discover the formula for finding the area of an annulus (ring shape) by measuring circular objects and exploring the relationship between the outer and inner circles.

Materials Needed:

- Circular objects (e.g., two different-sized cups, lids, or rings)
- Ruler or measuring tape
- Pen and paper (or a calculator)

Steps for the Activity:

1. Step 1: Find Two Circular Objects. Look around your surroundings and find two circular objects that can fit inside each other (e.g., two different-sized bowls, two bottle caps, or two CDs).
2. Step 2: Visualize the Annulus. Place the smaller object inside the larger one to visualize the annulus (ring shape).
3. Step 3: Record the Values. Measure the radius of the larger circle (R) from its center to the edge. Measure the radius of the smaller circle (r) in the same way.
4. Step 4: Calculate. Square both radii and record your result. Subtract the squared radius of the smaller circle from the squared radius of the larger circle. Multiply the result by 22/7 or 3.142.
5. Step 5: Discuss with your group how to calculate the area of an annulus.
6. Step 6: Try this activity with different circular objects and compare your results.

Discussion Questions:

7. What shape is created when you place a smaller circle inside a larger circle?
8. How did you calculate the area of the ring?
9. What pattern did you notice when calculating the area?
10. Did different circular objects give you the same formula?

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand how to measure the radii accurately.
- Ask probing questions: What is the relationship between the two circles? How do we find the area of the ring?
- For struggling groups: Let us start by finding the area of the larger circle first. Then find the area of the smaller circle. What do we do next?
- For early finishers: Can you write a general formula for finding the area of any annulus?
- Guide students to articulate: The area of an annulus equals the area of the outer circle minus the area of the inner circle.
- Identify 2-3 groups with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Annulus Formula and Addressing Misconceptions

After students have completed the anchor activity and shared their findings, the teacher formalizes the formula for finding the area of an annulus.

Key Takeaway: What is an Annulus?

An annulus is the region between two concentric circles that share the same center but have different radii.

The area of an annulus (a ring-shaped object) is found by subtracting the area of the smaller, inner circle from the area of the larger, outer circle.

Formula:

A annulus = A outer circle minus A inner circle

= π R squared minus π r squared

= π (R squared minus r squared)

Where:

- R is the radius of the outer circle
- r is the radius of the inner circle

Scaffolding Strategies to Address Misconceptions:

- Misconception: I can use the diameter directly in the formula. Clarification: No, the formula uses radius, not diameter. If given diameter, divide by 2 first.
- Misconception: I subtract the radii first, then square. Clarification: No, you must square each radius first, then subtract.
- Misconception: The two circles must touch. Clarification: No, the circles are concentric (share the same center) but the inner circle is smaller.
- Misconception: I can use different centers for the two circles. Clarification: No, both circles must share the same center to form an annulus.

Phase 3: Practice and Application (10 minutes)

Worked Example 1:

Find the area of an annulus where R = 10cm and r = 6cm.

Solution:

A = π (R squared minus r squared)

= π (10 squared minus 6 squared)

= π (100 minus 36)

= 64π

= 64 times 22/7

= 201.06 cm squared

Worked Example 2:

A wheel has an outer radius of 40 cm, and its inner hub has a radius of 10 cm. Find the area of the wheel annular region.

Solution:

The outer radius of the wheel = 40 cm

Inner hub radius = 10 cm

A = π (R squared minus r squared)

= 22/7 (40 squared minus 10 squared)

= 22/7 (1600 minus 100)

= 22/7 times 1500

= 4712.39 cm squared

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. A ring-shaped garden has an outer radius of 12 meters and an inner radius of 7 meters. Find the area of the garden.

2. A circular tabletop has a hole in the middle for an umbrella. The outer radius of the table is 1.5m, and the hole has a radius of 0.5m. Find the area of the tabletop.

3. A circular swimming pool has an outer radius of 8 meters, and a smaller circular island is in the center with a radius of 2 meters. Find the area of the water surface.

4. A steel pipe has an outer diameter of 80 units and an inner diameter of 60 units, what is the area of the cross-section?

Answer Key:

1. A = π (12 squared minus 7 squared) = π (144 minus 49) = 95π = 95 times 22/7 = 298.45 m squared

2. A = π (1.5 squared minus 0.5 squared) = π (2.25 minus 0.25) = 2π = 2 times 22/7 = 6.29 m squared

3. A = π (8 squared minus 2 squared) = π (64 minus 4) = 60π = 60 times 22/7 = 188.57 m squared

4. First convert diameter to radius: R = 40 units, r = 30 units. A = π (40 squared minus 30 squared) = π (1600 minus 900) = 700π = 700 times 22/7 = 2200 units squared

Differentiation Strategies

For Struggling Learners:

- Provide pre-measured circular objects with radii already labeled.

- Use simpler numbers (e.g., R = 10, r = 5) for initial practice.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators.
- Break down the formula into steps: Find area of outer circle, find area of inner circle, subtract.

For On-Level Learners:

- Encourage students to verify their formula with different circular objects.
- Ask students to explain why we subtract the areas rather than the radii.
- Provide mixed practice with different types of annulus problems.
- Challenge students to solve problems where diameter is given instead of radius.

For Advanced Learners:

- Challenge students to derive the formula algebraically.
- Explore real-world applications: running tracks, pipes, wheels, washers.
- Investigate the relationship between the width of the annulus and its area.
- Apply the concept to 3D shapes (e.g., hollow cylinders).
- Solve optimization problems: Given a fixed outer radius, what inner radius maximizes or minimizes the annulus area?

Extension Activity

Real-World Application: Running Track Project

Individual work

Situation: Imagine a running track built around a circular field. The track has an inner boundary (smaller circle) and an outer boundary (larger circle). The track itself forms an annulus.

A school is planning to paint the running track.

- The inner radius of the track is 30 meters, and the outer radius is 35 meters.
- The cost of painting is Ksh 5 per square meter.

Calculate:

11. The area of the track
12. The total cost of painting the track

Solution:

Step 1: Find the area of the track

A = π (R squared minus r squared)

= π (35 squared minus 30 squared)

= π (1225 minus 900)

= 325π

= 325 times 22/7

= 1021.43 m squared

Step 2: Find the total cost

Total cost = Area times Cost per square meter

= 1021.43 times 5

= Ksh 5107.14

Key Takeaway:

Students should understand how the area of an annulus is used in real-world professions such as construction, engineering, and urban planning to calculate areas of running tracks, pipes, wheels, and other ring-shaped objects.

Teacher Reflection Prompts

- Did students successfully discover the annulus formula through the anchor activity?
- Were students able to measure the radii accurately and calculate the area?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand the difference between radius and diameter?
- What adjustments would improve this lesson for future classes?