I. Lesson Overview

Lesson Title	Using Reciprocals of Real Numbers in Mathematical Computations
Strand	Numbers and Algebra
Sub-Strand	Real Numbers
Grade Level	10
Estimated Duration	40 minutes
Key Inquiry Question	How do we use real numbers in day-to-day activities?
Materials & Resources	CBC Grade 10 Mathematics Learner's Book CBC Grade 10 Mathematics Teacher's Book

II. Learning Objectives

Know (Conceptual Understanding): Understand that the reciprocal of a number a is 1/a, that a number multiplied by its reciprocal equals 1, and that zero has no reciprocal.

Do (Procedural Skill): Convert division problems into multiplication by using reciprocals, and apply reciprocals to solve equations and simplify expressions involving fractions, decimals, and whole numbers.

Apply (Application/Problem-Solving): Use reciprocals to solve real-world problems involving rates, proportions, and resource allocation.

III. Lesson Procedure

Phase 1: Problem-Solving and Discovery (15 minutes)

Objective: Activate prior knowledge of reciprocals and explore their properties through hands-on group work.

Anchor Activity

Instructions: In groups, find an exercise book and a pen. Complete the following tasks:

1. Write different numbers (positive, negative, decimal numbers and fractions) on the exercise book. Example: 2, −4, 0.25, 5/8

2. Find the reciprocals of the numbers you have formed. Example: The number formed is 5/8, its reciprocal is given by $1 \div 5/8 = 1 \times 8/5 = 8/5$

3. Multiply the original number you formed by its corresponding reciprocal and observe the result. Example: $5/8 \times 8/5 = 1$

4. Discuss cases where reciprocals do not exist.

5. Identify a real-world problem where reciprocals are useful.

6. Share your work with fellow learners.

Teacher Circulation Questions

- "What happens when you multiply a number by its reciprocal?"
- "Can you find the reciprocal of zero? Why or why not?"
- "How did you find the reciprocal of a decimal number like 0.25?"
- "What pattern do you notice about the reciprocal of a fraction?"

Phase 2: Structured Instruction (10 minutes)

Objective: Formalize the three key applications of reciprocals in mathematical computations.

Key Takeaway 1: Converting Division into Multiplication

Rule: Instead of dividing a number, we multiply by the reciprocal: $a \div b = a \times 1/b$

Example with whole numbers: $8 \div 2 = 8 \times 1/2 = 4$

Example with fractions: $5/6 \div 2/3 = 5/6 \times 3/2 = 15/12 = 5/4$

Key Takeaway 2: Solving Equations

When solving equations where a variable is multiplied by a number, we use the reciprocal to isolate the variable.

Example: Solve for x in the equation $3x = 12$

To solve for x, multiply both sides by the reciprocal of 3.

Therefore, $x = 12 \times 1/3 = 4$

Key Takeaway 3: Working with Proportions and Ratios

Example: If a recipe uses 2/3 of a cup of sugar per serving, how many servings can you make with 4 cups?

To find the number of servings, compute: $4 \div 2/3 = 4 \times 3/2 = 12/2 = 6$ servings

Addressing Misconceptions

- The reciprocal of 0 does NOT exist (division by zero is undefined)
- The reciprocal of a negative number is also negative (reciprocal of -4 is $-1/4$)
- To find the reciprocal of a decimal, first convert to a fraction (0.25 = 1/4, reciprocal = 4)

Phase 3: Practice and Application (15 minutes)

Objective: Apply reciprocals to solve varied real-world problems.

Worked Examples

Problem 1: A printing machine can print 5/6 of a book page per second. How long will it take to print 20 pages?

If 1 second = 5/6 of a page, then for 20 pages:

Time = 20 ÷ 5/6 = 20 × 6/5 = 120/5 = 24 seconds

Answer: The printing machine will take 24 seconds to print 20 pages.

Problem 2: If a car travels at 80 km/h, find the time taken per km using reciprocals.

Time per km = 1/80 hours per km

Answer: The car takes 1/80 hours (or 0.75 minutes) per km.

Problem 3: If a factory produces 300 items in 5 hours, determine the time needed per item.

Production rate = 300/5 = 60 items per hour

Time per item = reciprocal of 60 = 1/60 hours per item

Answer: The factory produces 1 item every 1/60 hours (or 1 minute).

Problem 4: Kerich is a farmer and has 2/3 of an acre of land and wants to divide it into plots of 1/6 acre each. How many plots can Kerich make?

Divide 2/3 by 1/6 and solve by multiplying 2/3 by the reciprocal of 1/6:

2/3 ÷ 1/6 = 2/3 × 6/1 = 12/3 = 4

Answer: Kerich can make 4 plots.

Problem 5: Njoki, a business woman, invests Ksh 5,000 in a business that promises to double her investment every 1.5 years. What fraction of her initial investment will she have after 4.5 years?

Number of doubling periods: 4.5 ÷ 1.5 = 4.5 × 1/1.5 = 3

Her investment will double 3 times:

5,000 × 2 × 2 × 2 = 5,000 × 8 = 40,000

Answer: After 4.5 years, Njoki will have Ksh 40,000 (8 times her initial investment).

Phase 4: Assessment (Exit Ticket)

Objective: Assess individual understanding of reciprocals in mathematical computations.

Assessment Questions

1. Solve the following using reciprocals:
   a) 6 ÷ 2/3
   b) 4x = 12
   c) 3/5 × x = 9
   d) 0.2 ÷ 4.5

2. Solve: (4/7 ÷ 2/5) × (3/8 ÷ 9/16)

3. If x × 3/8 = 5/12, find the value of x using reciprocals.

4. A rope of length 18 m is cut into pieces each measuring 3/4 m. How many pieces are obtained?

5. A hiker is walking 24 km and takes breaks every 2/3 km. How many breaks will he take by the end of the journey? If he changes his break interval to 1 1/3 km, how does the number of breaks change?

6. A road is 12.6 km long and is divided into equal segments of 0.35 km for maintenance. How many segments are there?

Answer Key

1a) 6 ÷ 2/3 = 6 × 3/2 = 18/2 = 9

1b) 4x = 12 ⟶ x = 12 × 1/4 = 3

1c) 3/5 × x = 9 ⟶ x = 9 × 5/3 = 45/3 = 15

1d) 0.2 ÷ 4.5 = 1/5 ÷ 9/2 = 1/5 × 2/9 = 2/45

2. (4/7 ÷ 2/5) × (3/8 ÷ 9/16)

   = (4/7 × 5/2) × (3/8 × 16/9)

   = (20/14) × (48/72)

   = (10/7) × (2/3)

= 20/21

3. $x \times 3/8 = 5/12 \longrightarrow x = 5/12 \times 8/3 = 40/36 = 10/9$

4. $18 \div 3/4 = 18 \times 4/3 = 72/3 = 24$ pieces

5. Breaks at 2/3 km intervals: $24 \div 2/3 = 24 \times 3/2 = 36$ breaks

   Breaks at 1 1/3 km intervals: $24 \div 4/3 = 24 \times 3/4 = 18$ breaks

   The number of breaks is halved (from 36 to 18) when the interval doubles.

6. $12.6 \div 0.35 = 126/10 \div 35/100 = 126/10 \times 100/35 = 12600/350 = 36$ segments

IV. Differentiation Strategies

Student Group	Strategy
Struggling Learners	Provide a step-by-step reference card showing how to find reciprocals of whole numbers, fractions, and decimals. Begin with simple whole number divisions before progressing to fractions. Use visual fraction models to illustrate division by a fraction.
On-Level Learners	Complete all practice problems as designed. Encourage students to verify answers by multiplying the result by the divisor. Pair with peers for collaborative problem-solving.
Advanced Learners	Extend to the challenge problems in the Extension Activity below. Ask students to create their own real-world problems involving reciprocals and present them to the class.

Extension Activity

Challenge Problem 1: A water tank fills at a rate of 3/4 litres per minute. A second tank fills at 5/8 litres per minute. If both tanks need to hold 30 litres, how much longer will the second tank take to fill compared to the first? Use reciprocals in your solution.

Challenge Problem 2: A tailor has 15 metres of fabric. She needs 2/5 of a metre for a small bag and 7/8 of a metre for a large bag. How many of each type can she make if she wants to make equal numbers of both? Use reciprocals to determine the maximum number of sets.

Challenge Problem 3: Worker A can complete a job in 6 hours and Worker B can complete the same job in 8 hours. Using reciprocals, find their combined work rate and determine how long it takes them to complete the job together.

V. Assessment Methods

Phase	Method	Purpose
Phase 1	Observation of group work	Gauge prior knowledge and collaboration skills
Phase 2	Questioning during instruction	Check conceptual understanding of reciprocal rules
Phase 3	Monitoring practice work	Identify procedural errors and misconceptions
Phase 4	Exit ticket (6 questions)	Evaluate individual mastery of reciprocal computations

Checkpoint Integration

Checkpoint protocol for Learners:

1. Click “Show new example question” to load the problem
2. Solve the displayed question
3. Click “submit” to check your answer
4. If incorrect, carefully read the feedback and analyse the error before trying a new question. The immediate feedback from checkpoint submissions allows Learners to identify and correct errors in real-time.
5. Complete at least 5 questions before rotating
6. Pair Learners strategically so stronger learners can explain reasoning to peers.

Teacher's Role: Collect and review the exit tickets to gauge Learner understanding and identify any common misconceptions that need to be addressed in the next lesson.

VI. Teacher Reflection

To be completed after the lesson.

1. Did students successfully connect the anchor activity to the formal rules for using reciprocals?

2. Which application of reciprocals (division, equations, proportions) did students find most challenging?
3. Were the real-world problems effective in demonstrating the practical value of reciprocals?
4. How well did the differentiation strategies support diverse learners?
5. Was the 40-minute timeframe sufficient for all four phases?
6. Which students need additional support or extension in the next lesson?
7. What modifications would improve this lesson for future delivery?