Designing Service Regions for Bike Sharing Systems: Integration with Public Transport Networks

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Abstract

Designing effective service regions for bike-sharing systems (BSS) is critical for enhancing urban mobility and ensuring seamless integration with public transport (PT) networks. Existing studies on bike station location and operational cost optimization often overlook the integration of BSS with PT networks. In this study, we explicitly consider multimodal travel patterns and multiple path possibilities for each origin-destination (OD) pair. By incorporating k-order shortest paths, we propose an integer programming optimization model to define optimal service regions that maximize demand coverage under the constraints of construction and operational budgets. Our approach prioritizes optimal paths for users while strategically allowing slightly suboptimal paths to enhance coverage and reduce operational costs (e.g., rebalancing). To solve the model, we employ a mathematical solver, Gurobi, and conduct initial tests on synthesized data. Our preliminary results indicate that, compared to a baseline approach (where only the first-order shortest paths are allowed), our method can enhance demand coverage and improve operational efficiency. We will further validate our model using TPG data and real-world trip data from Geneva's bike-sharing system. These findings highlight the potential of our approach in designing efficient, multimodal-integrated BSS service regions while optimizing resource allocation within real-world multimodal networks.

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1 Mathematical model

1.1 Notations

$\mathcal{M}$ is categorized into three groups based on travel mode composition:
$\mathcal{M}^b$: Cycling-only mode

- (b) : Cycling-only mode

$\mathcal{M}^{pt}$: Cycling and public transport (PT) integrated modes

- (b, pt, b) : Cycling to PT access point, using PT, and cycling from the PT egress point
- (b, pt, w) : Cycling to PT access point, using PT, and walking from the PT egress point
- (w, pt, b) : Walking to PT access point, using PT, and cycling from the PT egress point

$\mathcal{M}^w$: Other Modes (Walking-Only and Walking-PT Combinations)

- (w) : Walking-only mode
- (w, pt, w) : Walking to PT access point, using PT, and walking from the PT egress point

1.2 Maximizing covered demand without rebalancing

Most of the literature assumes that the potential demand for bike usage is known without specifying detailed path choices or whether it involves integration with other modes [1, 2]. In contrast, we formulate the mathematical programming

Table 1: Notation.

Sets	**Description**
$\mathcal{K}$	Set of all origin-destination (OD) pairs
$\mathcal{I}$	Set of zones, each zone can serve as an origin or destination for trips, and may also potentially contain a bike-sharing station
$\mathcal{J}$	Set of public transport stops
$\mathcal{M}$	Set of all travel modes, defined as $\mathcal{M} = \mathcal{M}^b \cup \mathcal{M}^{pt} \cup \mathcal{M}^w$
$\mathcal{M}^b$	Set of bike only travel mode $\mathcal{M}^b = \{b\}$
$\mathcal{M}^{pt}$	Set of Bike-PT connected multi-modal travel modes, defined as $\mathcal{M}^{pt} = \{(b, pt, b), (b, pt, w), (w, pt, b)\}$
$\mathcal{M}^w$	Set of other travel modes, defined as $\mathcal{M}^w = \{(w), (w, pt, w)\}$
$\mathcal{P}_i^{walk}$	The set of PT stations reachable by walk from zone i (within the catchment area). Formally, $\mathcal{P}_i^{walk} = \{j \in \mathcal{J} : \text{distance}(i, j) \leq \tilde{d_w}$
$\mathcal{B}_i^{bike}$	The set of bike stations reachable by bike from zone i (within the catchment area). Formally, $\mathcal{B}_i^{bike} = \{j \in \mathcal{J} : \text{distance}(i, j) \leq \tilde{d_r}$
$\mathcal{B}_i^{walk}$	The set of bike sharing stations reachable by walk from zone i (within the catchment area). Formally, $\mathcal{B}_i^{walk} = \{j \in \mathcal{I} : \text{distance}(i, j) \leq \tilde{d_w}$
$\mathcal{R}_k^m$	The set of k-shortest-paths associated with mode m for OD pair k
$\mathcal{A}^b$	The set of all cycling arcs
$\mathcal{P}_{ij}^m$	The set of all feasible mode m paths that include the bike arc from station i to station j, where $m \in \mathcal{M}^b \cup \mathcal{M}^{pt}, (i, j) \in \mathcal{A}^b$
$\mathcal{P}_i^m$	The set of all feasible paths of mode m that pass through bike station i
$\mathcal{T}$	Set of operation periods

Decision variables	**Description**
y_i	Binary variable indicating whether a bike-sharing station is installed in zone i
z_i	Number of docks in zone i
x_{ktr}^m	The flow assigned to path $r, r \in \mathcal{R}_k^m$ for OD pair k at time period t where mode $m \in \mathcal{M}^b \cup \mathcal{M}^{pt} \cup \mathcal{M}^w$
α_{ktr}^m	Binary variable indicating whether path r, where $r \in \mathcal{R}_k^m$, is selected for OD pair k at time period t, with mode $m \in \mathcal{M}^b \cup \mathcal{M}^{pt} \cup \mathcal{M}^w$
$v_{i,t}$	Number of bikes at the beginning of period t in station $i, i \in \mathcal{I}$
$f_{i,j,t}$	Quantity of bike flow from station i to station j in period t
$r_{i,j,t}$	Number of bikes rebalanced from station i to station j in period t

Parameters	**Description**
Q	The total available budget
Q_r	The total operational budget
c_s	The station setup cost
c_p	The cost for each additional dock
c_v	The unit purchase cost per bike
c_d	The cost per unit distance for bike rebalancing
q_i^{ub}	The site capacity for each zone $i \in \mathcal{I}$
$c_{i,j,t}$	Cost per bike of rebalancing from station i to station j in period t
$u_{k,t}$	Demand flow for OD pair $k = (i, j) \in \mathcal{K}$ in period t, with origin and destination $i, j \in \mathcal{I}$
$w_{k,r}$	Weight parameter to control the contribution of path r for OD pair k
$\pi_k(r)$	Rank of path r for flow k in K-shortest paths
δ_{ijr}	Equals 1 if path $r \in \mathcal{R}_k^m$ includes the bike arc from station i to station j, where mode $m \in \mathcal{M}^b \cup \mathcal{M}^{pt}$
T_r	Total travel time for the path $r \in \mathcal{R}_k^m$
D_r	Total travel distance for the path $r \in \mathcal{R}_k^m$
d_{ij}	Travel distance between nodes i and j
$\tilde{d}_r$	Maximum bike riding distance
$\tilde{d}_w$	Maximum walking distance
d_k^{car}, t_k^{car}	The minimum travel distance and time for each OD pair k using car mode at time period t
d_{ae}^{pt}, t_{ae}^{pt}	The minimum travel distance and time for public transport between the access point a and the egress point e
γ	A threshold for the total travel time ratio
β	A threshold for the detour ratio
M	A large constant

model as follows:

$$\text{Maximize} \quad \sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{r\in\mathcal{R}_k^m}(1-\lambda\cdot\pi_k(r))\cdot x_{ktr}^m \tag{1}$$

$$\text{Subject to:} \quad \sum_{m\in\mathcal{M}}\sum_{r\in\mathcal{R}_k^m} x_{ktr}^m \le u_{k,t}, \quad \forall k\in\mathcal{K}, t\in\mathcal{T} \tag{2}$$

$$x_{ktr}^m \le u_{k,t}\alpha_{ktr}^m, \quad \forall m\in\mathcal{M}, k\in\mathcal{K}, t\in\mathcal{T}, r\in\mathcal{R}_k^m \tag{3}$$

$$\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}^b\cup\mathcal{M}^{pt}}\sum_{r\in\mathcal{P}_i^m}\alpha_{ktr}^m \le M\cdot y_i, \quad \forall i\in\mathcal{I} \tag{4}$$

$$f_{i,j,t} = \sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}^b\cup\mathcal{M}^{pt}}\sum_{r\in\mathcal{P}_{ij}^m} x_{ktr}^m, \quad \forall (i,j)\in\mathcal{A}^b, t\in\mathcal{T} \tag{5}$$

$$v_{i,t} = v_{i,t-1} - \sum_{j\in\mathcal{I}} f_{i,j,t-1} + \sum_{j\in\mathcal{I}} f_{j,i,t-1}, \quad \forall i\in\mathcal{I}, t\in\mathcal{T}\setminus\{1\} \tag{6}$$

$$\sum_{i\in\mathcal{I}} v_{i,1} = \sum_{i\in\mathcal{I}} v_{i,T} \tag{7}$$

$$v_{i,t} \le z_i, \quad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{8}$$

$$z_i \le q_i^{ub}, \quad \forall i\in\mathcal{I} \tag{9}$$

$$v_{i,t} \ge \sum_{j\in\mathcal{I}} f_{i,j,t}, \quad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{10}$$

$$\sum_{i\in\mathcal{I}} c_s y_i + \sum_{i\in\mathcal{I}} c_p z_i + \sum_{i\in\mathcal{I}} c_v v_{i1} \le Q \tag{11}$$

$$y_i \in \{0,1\}, \quad \forall i\in\mathcal{I} \tag{12}$$

$$z_i \in \mathbb{Z}, \quad \forall i\in\mathcal{I} \tag{13}$$

$$v_{i,t} \in \mathbb{Z}, \quad \forall i\in\mathcal{I}, t\in\mathcal{T} \tag{14}$$

$$\alpha_{ktr}^m \in \{0,1\}, \quad \forall k\in\mathcal{K}, t\in\mathcal{T}, m\in\mathcal{M}, r\in\mathcal{R}_k^m, \tag{15}$$

1.2.1 Objective Function

The objective function aims to maximize the total number of travelers using non-private car modes (bike, walk, and public transport) across all origin-destination (OD) pairs, time periods, and modes.

To increase the number of users utilizing optimal paths in the system, we introduce a mechanism for penalizing later-ranked paths, ensuring that the optimal solution prioritizes the best-ranked paths. We incorporate a penalty term to discourage the selection of lower-ranked paths.

$$\max \sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{r\in\mathcal{R}_k^m} x_{ktr}^m - \lambda\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}\sum_{r\in\mathcal{R}_k^m}\pi_k(r)\cdot x_{ktr}^m \tag{1}$$

where λ is a penalty coefficient that determines the impact of the ranking penalty. Higher-ranked paths contribute less to the penalty term, ensuring they are preferred over lower-ranked paths.

1.2.2 Demand and Flow Allocation Constraints

$$\sum_{m\in\mathcal{M}}\sum_{r\in\mathcal{R}_k^m} x_{ktr}^m \le u_{k,t}, \quad \forall k\in\mathcal{K}, t\in\mathcal{T} \tag{2}$$

$$x_{ktr}^m \le u_{k,t}\alpha_{ktr}^m, \quad \forall m\in\mathcal{M}, k\in\mathcal{K}, t\in\mathcal{T}, r\in\mathcal{R}_k^m \tag{3}$$

Constraints (2) ensure that the total flow assigned across all travel modes and paths for an OD pair k at any time period t does not exceed the available demand. Constraints (3) enforce logical consistency by allowing flow on a path only if it is selected ($\alpha_{ktr}^m = 1$); if a path is not chosen, no flow is assigned to it.

1.2.3 Bike Infrastructure and Flow Feasibility Constraints

$$\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}^b\cup\mathcal{M}^{pt}}\sum_{r\in\mathcal{P}_i^m} \alpha_{ktr}^m \le M\cdot y_i, \quad \forall i\in\mathcal{I} \tag{4}$$

Constraints (4) ensure that for each station i, if any feasible path utilizing a bike-sharing arc that passes through station i is used, then a bike-sharing station must be constructed(i.e., $y_i = 1$).

1.2.4 Flow Conservation Constraints at Bike Stations

$$f_{i,j,t} = \sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}^b\cup\mathcal{M}^{pt}}\sum_{r\in\mathcal{P}_{ij}^m} x_{ktr}^m, \quad \forall (i,j)\in\mathcal{A}^b, t\in\mathcal{T} \tag{5}$$

$$v_{i,t} = v_{i,t-1} - \sum_{j\in\mathcal{I}} f_{i,j,t-1} + \sum_{j\in\mathcal{I}} f_{j,i,t-1}, \quad \forall i\in\mathcal{I}, t\in\mathcal{T}\setminus\{1\} \tag{6}$$

$$\sum_{i\in\mathcal{I}} v_{i,1} = \sum_{i\in\mathcal{I}} v_{i,T} \tag{7}$$

Constraints (5) ensure that the total flow of bikes from the bike sharing station i to j at time t, denoted as $f_{i,j,t}$, is equal to the sum of all flows along the paths that include the bike arc (i,j). The flow accounts for both:

1. Direct bike-only trips ($\mathcal{M}^b$), where users travel solely by bike;
2. Multi-modal trips integrating bikes and public transport ($\mathcal{M}^{pt}$), which include a bike leg between i and j.

For each mode m, the flow contributions are aggregated over all paths $\mathcal{P}_{ij}^m$ that traverse the bike arc (i,j). Constraints (6) implement flow conservation by ensuring that the bike inventory at each station is updated based on inflows and outflows during each time period. Constraint (7) enforces periodicity, ensuring that the total inventory at the end of the day equals the total inventory at the beginning.

Flow Conservation Constraints with Rebalancing

To account for the rebalancing operation in the bike-sharing system, we modify the flow conservation constraints to integrate rebalancing flows explicitly (as a transshipment problem).

$$f_{i,j,t} = \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}^b \cup \mathcal{M}^{pt}} \sum_{r \in \mathcal{P}_{ij}^m} x_{ktr}^m + r_{i,j,t} - r_{j,i,t}, \quad \forall (i,j) \in \mathcal{A}^b, t \in \mathcal{T} \tag{5*}$$

1.2.5 Station Inventory and Capacity Constraints

$$v_{i,t} \leq z_i, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{8}$$

$$z_i \leq q_i^{ub}, \quad \forall i \in \mathcal{I} \tag{9}$$

$$v_{i,t} \geq \sum_{j \in \mathcal{I}} f_{i,j,t}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{10}$$

Constraints (8) ensure that the bike inventory at any station does not exceed the station's capacity (number of docks). Constraints (9) enforce an upper limit on station capacity based on characteristics and limitations of the zone. Constraints (10) ensure sufficient inventory at each station to meet the outgoing demand during each time period.

1.2.6 Budget Constraints

$$\sum_{i \in \mathcal{I}} c_s y_i + \sum_{i \in \mathcal{I}} c_p z_i + \sum_{i \in \mathcal{I}} c_v v_{i1} \leq Q \tag{11}$$

This constraint ensures that the total cost, including the fixed setup cost for each station, the cost for each additional dock, and the bike procurement cost, does not exceed the available budget.

$$\sum_{(i,j) \in \mathcal{A}^b} \sum_{t \in \mathcal{T}} r_{i,j,t} \cdot d_{ij} \cdot c_d \leq Q_r \tag{12}$$

This constraint ensures that the total rebalancing cost, defined as the summation of rebalancing volume multiplied by rebalancing distance and the cost per unit distance moved, remains within the predefined budget limit.

1.3 Prelimiary results

Our preliminary results demonstrate the impact of incorporating rebalancing operations and varying the number of paths considered (k-shortest paths) on demand coverage. The key findings are summarized as follows:

$k \setminus Q_r$	**0**	**50**	**200**	**500**	**1000**	**2000**
1	2052	2111	2193	2260	2285	2285
2	2087(1.71%)	2141 (1.42%)	2216(1.05%)	2273	2292	2292
3	2090(1.85%)	2143 (1.52%)	2217(1.10%)	2274	-	-

Table 2: Bike Flow under Different Rebalancing Budgets Q_r and Shortest Paths k under a Moderate Total Budget

$k \setminus Q_r$	**0**	**100**	**500**
1	2855	2952	3093
2	2932 (2.70%)	3029 (2.61%)	3174 (2.62%)
3	2943 (3.08%)	3040 (2.98%)	3184 (2.94%)

Table 3: Bike Flow under Different Rebalancing Budgets Q_r and Shortest Paths k under a High Total Budget

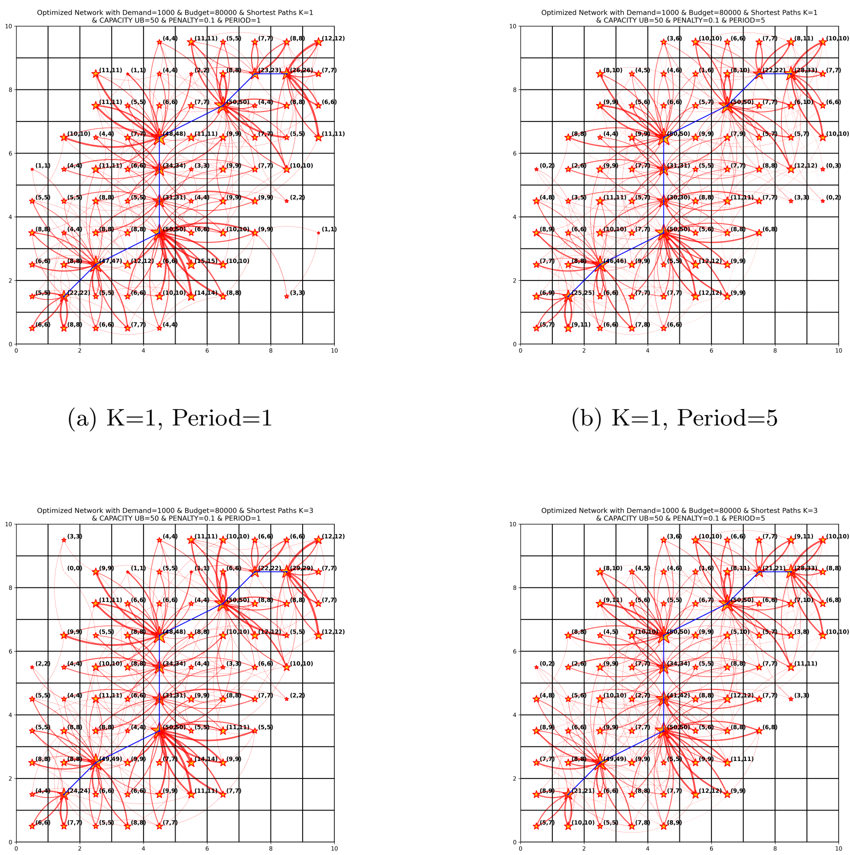
1. **Impact of Rebalancing on Demand Coverage:** Incorporating rebalancing operations improves the proportion of demand that can be covered. As the rebalancing cost increases, the improvement in coverage initially rises but then stabilizes. This suggests that the degree of improvement is highly dependent on the network configuration, and beyond a certain cost threshold, the optimization potential reaches a saturation point.

2. **Effect of Increasing the Number of Shortest Paths (k-shortest paths):** Increasing the value of k (e.g., from $k = 1$ to $k = 2$) leads to a higher number of covered demand trips for the same rebalancing cost. However, as the rebalancing budget increases, the gap in demand coverage improvement between different k values diminishes. Furthermore, when increasing from $k = 2$ to $k = 3$, the additional demand coverage gain becomes negligible, particularly as rebalancing costs continue to rise.

3. **Effect of Increasing the Total Construction Budget:** One possible reason for the currently limited optimization improvement is that the network is not yet dense enough. For instance, stations are primarily concentrated around public transport stops, and the number of such PT stops is limited. Our subsequent numerical experiments indicate that when the total budget is increased, allowing for a denser station deployment within the network, the optimization potential of higher-order shortest paths further improves (i.e., when increasing k from 1 to 3).

Visualization of Optimized Bike Flow Network

To provide an intuitive understanding of the optimized bike-sharing network, we present a visualization using synthetic data in Figure **??**.



(a) K=1, Period=1

(b) K=1, Period=5

(c) K=3, Period=1

(d) K=3, Period=5

Figure 1: Optimized bike-sharing network under different combinations of K and planning period

- **Red stars:** Each red star represents a bike-sharing station.
- **Tuple labels:** The numbers next to each station $(v_{i,1}, c_i)$ denote the *initial inventory* and *station capacity*, respectively.
- **Blue lines:** These represent predefined public transport (PT) routes that are integrated into the system.
- **Arcs:** The directed arcs illustrate the average bike flow between stations over the operational period.

We incorporated a certain level of rebalancing budget into our experiment to better reflect operational realities. Based on the observations, there are following patterns:

When the planning horizon (period) increases, the number of critical nodes tends to decrease, and the thick edges connecting these nodes become fewer. This suggests that the system optimizes from a more global perspective, leveraging the natural flow of users between stations over time. In contrast, single-period planning is more conservative and short-sighted, which may lead to resource fragmentation across multiple nodes and less efficient use of capacity.

On the other hand, increasing the number of shortest paths considered (K) introduces more path flexibility. Surprisingly, this may result in a more centralized network, where certain nodes take on larger capacities to accommodate fluctuations in supply and demand over multiple periods. In theory, increasing K improves redundancy and resilience (a more detailed quantitative analysis is needed to confirm these effects consistently across different scenarios).

Note that the current visualizations only display bike flow; integrating public transport (PT) information could provide additional insights into multimodal network flow dynamics and further validate the effectiveness of the proposed optimization strategies. We argue that (in theory, needs to be validated) the combination of multiple periods and higher K values results in the most steady network structure, characterized by broad yet structured path distributions and balanced load across nodes (with proper rebalancing).