# Graph Theory 3

Shortest Path

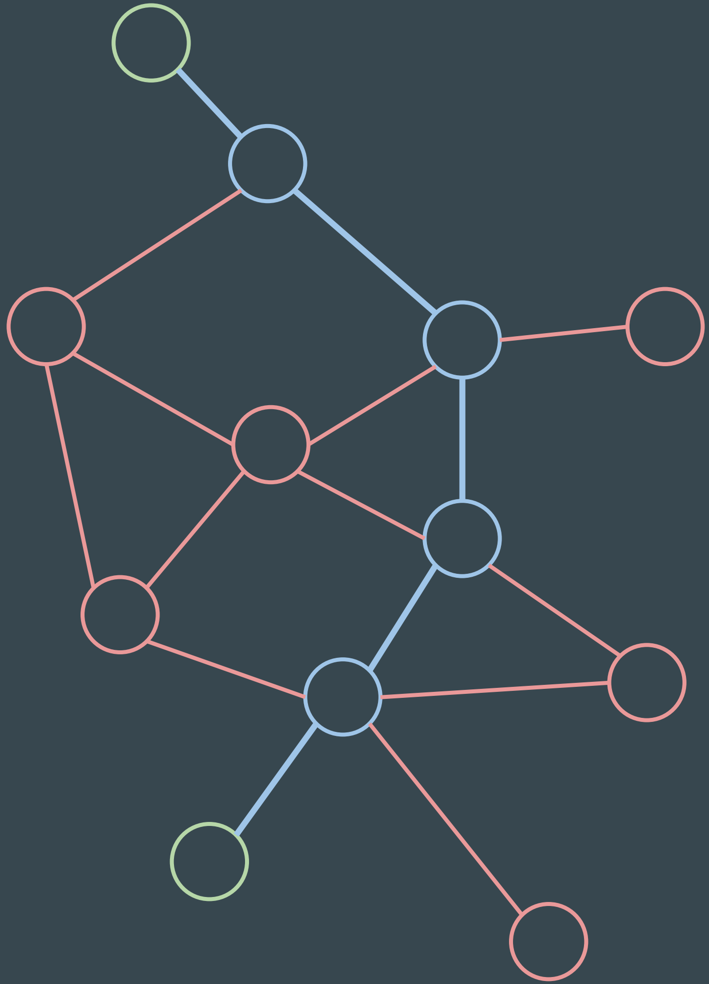
# Last time

We are given a connected graph which edges all have the same weight

→ find the shortest path that connects two given nodes

(on this graph, in blue - its length is 5)

/!\ Multiple shortest paths are possible



# Breadth-first search

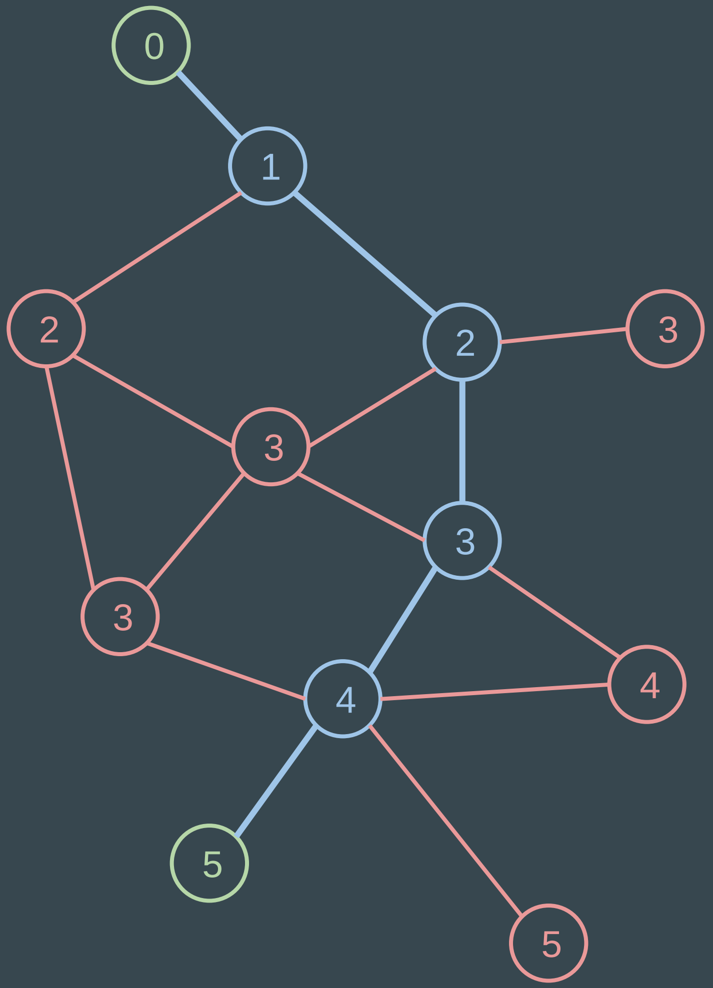
Put one end of the path in a queue and iterate:

- dequeue one element
- enqueue all unvisited neighbors and mark the element as their parent

Then, from the other end of the path, follow the parent-child relationships

(the nodes are visited in order of distance from the starting node, as shown on the right)

→ Complexity : O(|E| + |V|)



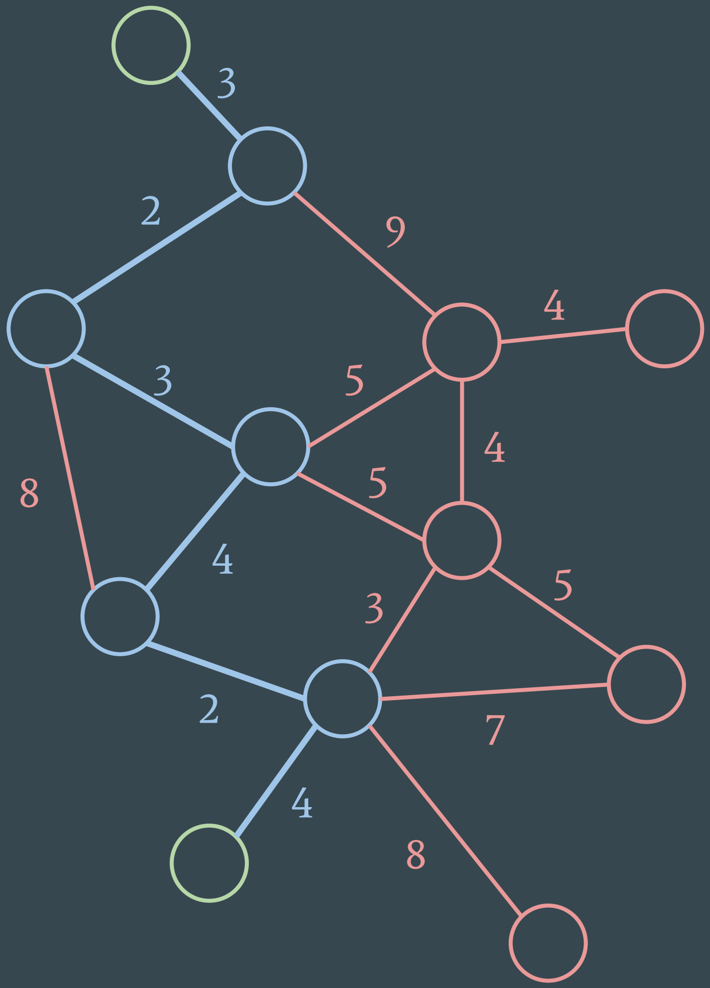
# Harder case: weighted edges

Now the edges have positive weights

→ we want to find the path which edges have the smallest sum, between two given nodes

(on this graph, this sum is 18)

/!\ Multiple best paths are possible

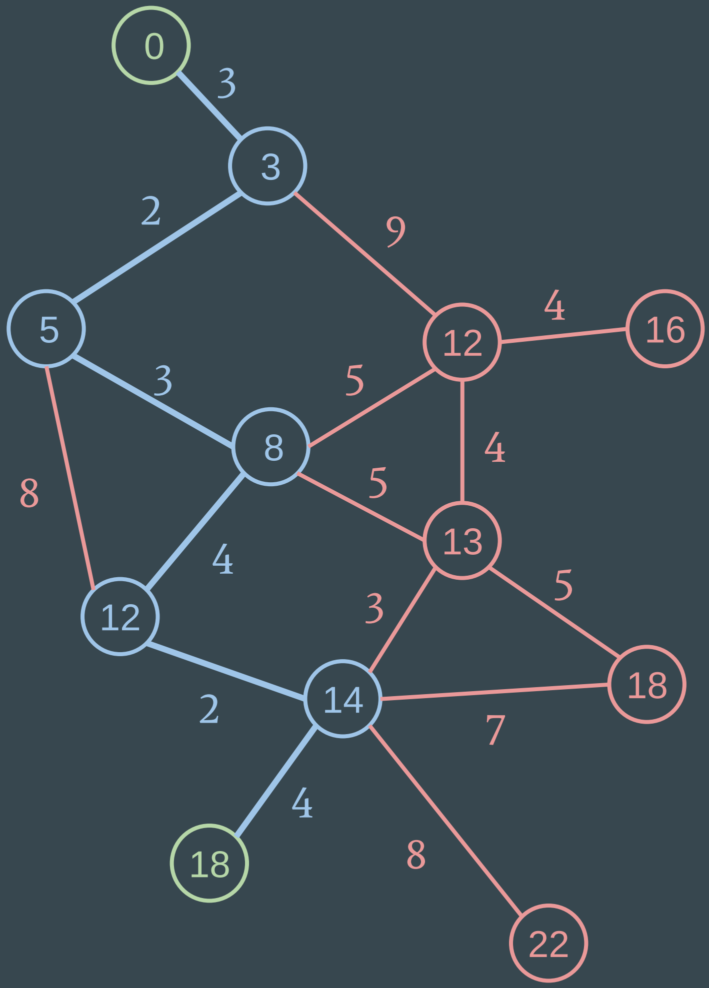


# Dijkstra's algorithm

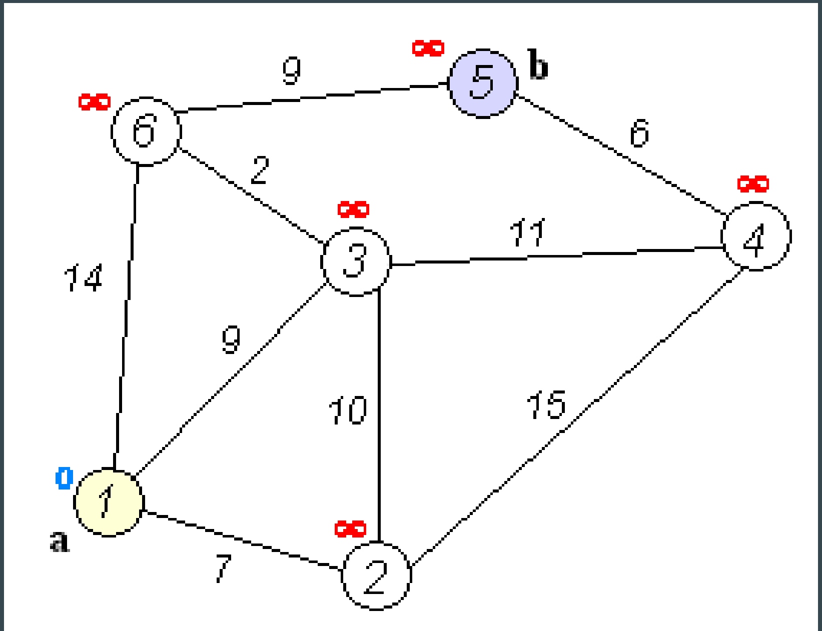
- Works only if the edges have positive weights
- It's a same idea than BFS
- This time the nodes are added to a heap queue so that you always check the closest to the starting node

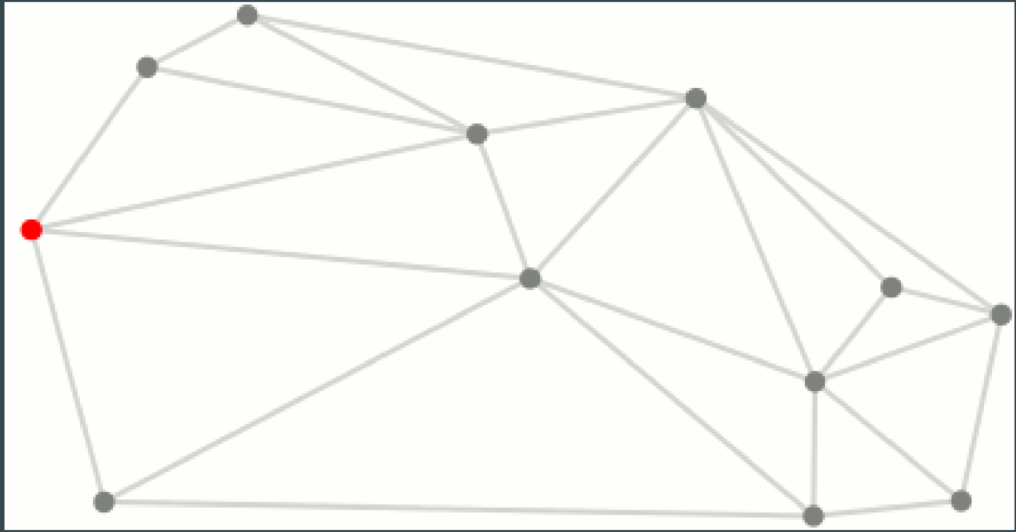
(on the right, the distance to the starting node is written on every node)

→ Complexity : O(|E| + |V| log |V|)



# Dijkstra's algorithm : stolen gifs



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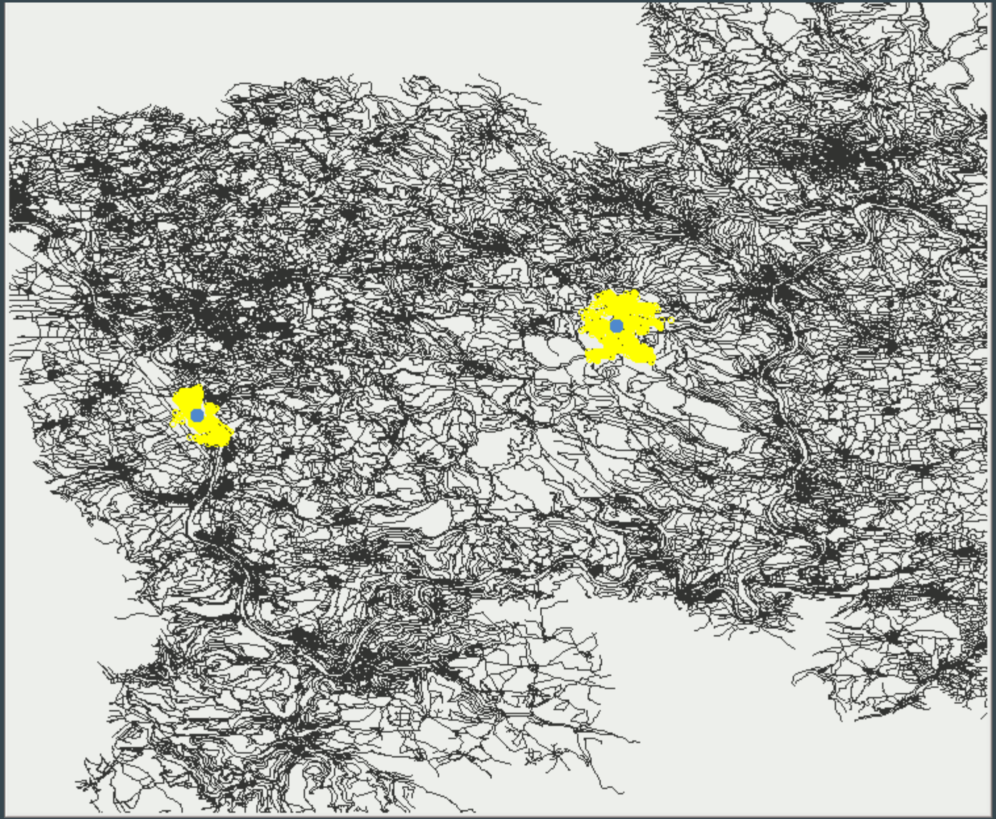


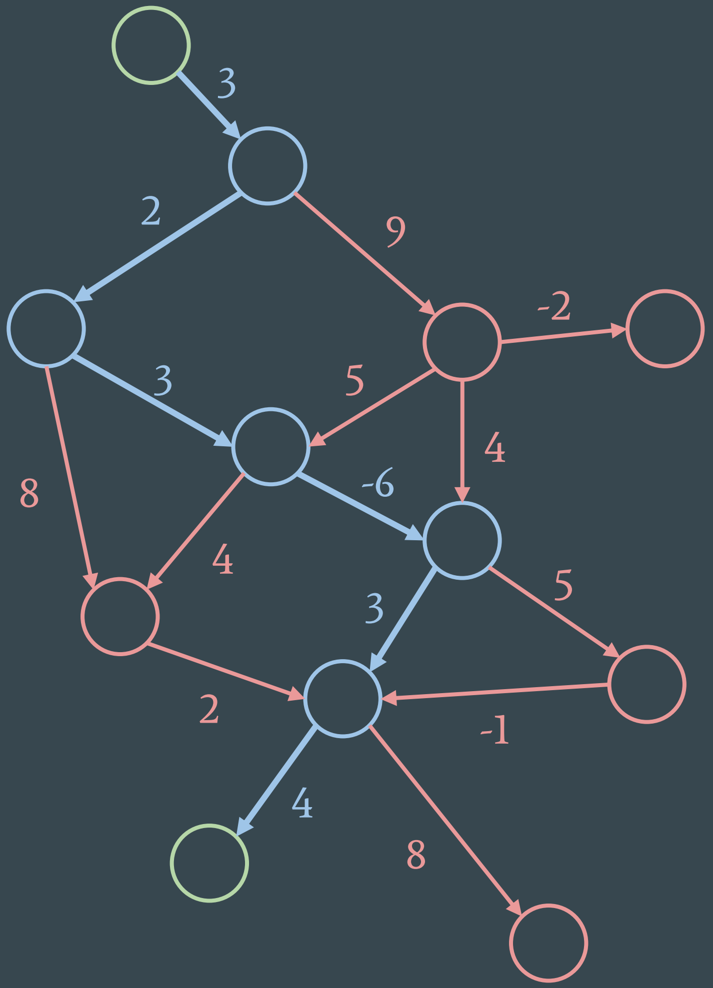
# Harder case: with negative edges

Now the edges can have positive or negative weights

This only works on directed graphs, because a negative edge on an undirected graph means a negative cycle and there is no solution

(on this graph, this sum is 9)

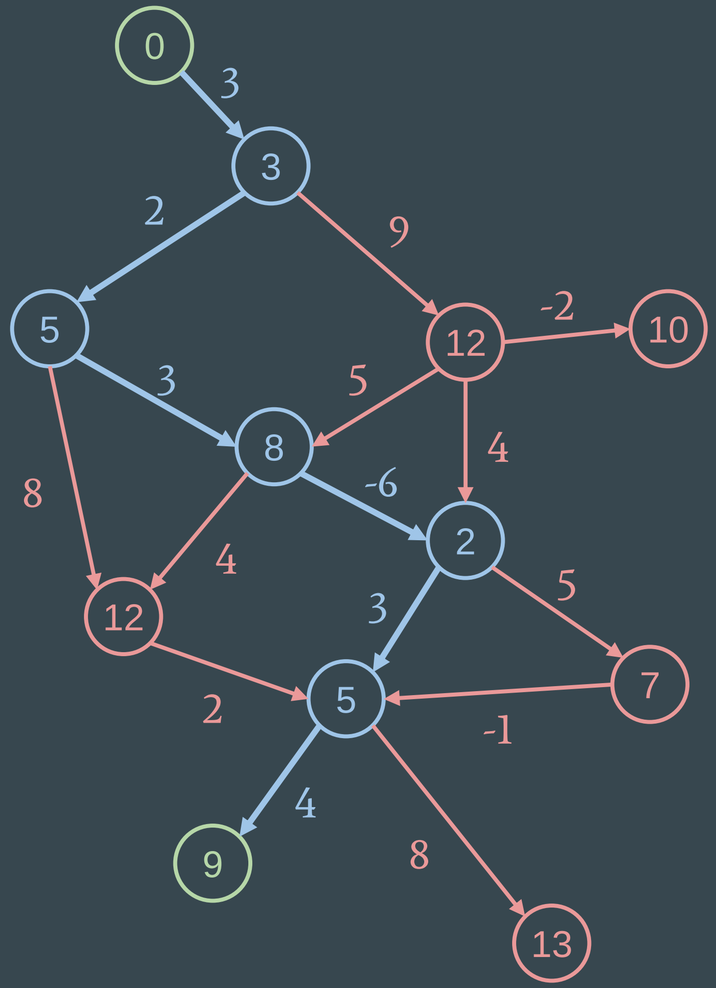
/!\ Multiple best paths are possible



# Bellman-Ford algorithm

- This algorithm marks all the nodes as infinitely far from the starting node
- It then executes |V| *relaxations*
- A *relaxation* browses each edge to check if it can improve the current distance of one of its ends to the center
- If at the end, progress can still be made, the graph contains a negative cycle

→ Complexity : O(|E| * |V|)



# More algorithms

- Depth-first search (in a tree): https://stackoverflow.com/questions/4977112/how-to-find-the-shortest-simple-path-in-a-tree-in-a-linear-time
- Floyd-Warshall (shortest path between any pair of nodes): https://en.wikipedia.org/wiki/Floyd_Warshall
- A* (extension of Dijkstra with heuristics): https://en.wikipedia.org/wiki/A*_search_algorithm - we will probably talk about it later

# Credits

Slides: Louis Sugy for INSAlgo
GIFs: Lecorché Adriaan for INSAlgo