Search space
and
brute force

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If all you have is a hammer, everything looks like a nail

Search Space

- Ensemble of all solutions we have to choose from to get the optimal answer.

- Often what is asked from us is not the solution but one of its properties.

Search Space on an example : minimum pair

- Find the minimum pair on the list:

  I.E. the pair with the minimum sum

0 5 8 6 -1 4 9 10 2

⇒ Search space : all the pairs

Make sense of your search space

Another representation:

Matrix where:

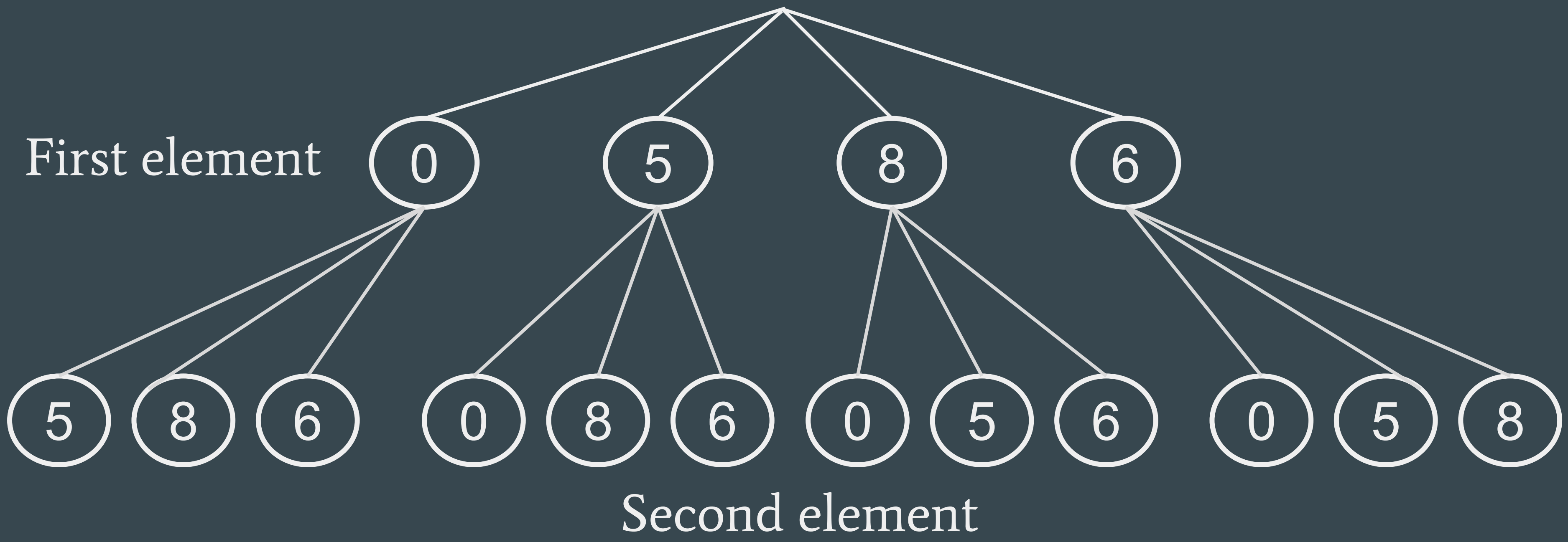
Line: first element of the pair

Column: second element

Implementation: two nested for loops

	0	5	8	6
0	------	5	8	6
5	5	------	13	11
8	8	13	------	14
6	6	11	14	------

Make sense of your search space



Brute force - introduction

Example: I want to find all subsets of a set which sum is 42.

I can:

- make a list of all subsets,
- for each subset, calculate the sum and check if it is 42.

Search space:	all subsets of the input array	A *brute force search* has a *search space* and a *test.*
Test:	the sum is 42	

Brute force - construct the search space

The python module `itertools` provides useful tools to construct the search space.

```
>>> list(product("ab", "cd"))
[('a', 'c'), ('a', 'd'), ('b', 'c'), ('b', 'd')]
>>> list(permutations("abc", 2))
[('a', 'b'), ('a', 'c'), ('b', 'a'), ('b', 'c'), ('c', 'a'), ('c', 'b')]
>>> list(combinations("abc", 2))
[('a', 'b'), ('a', 'c'), ('b', 'c')]
>>> list(combinations_with_replacement("abc", 2))
[('a', 'a'), ('a', 'b'), ('a', 'c'), ('b', 'b'), ('b', 'c'), ('c', 'c')]
```

Note: we use `list()` because the return values are *lazy-evaluated* iterables.

→ these are very useful to save memory

Quick tour of theses functions

product(q_1,q_2,...,[repeat=1])

`product('ABCD', repeat=2)`	AA AB AC AD BA BB BC BD CA CB CC CD DA DB DC DD

Can take multiple iterables and output the cartesian product between them.

The repeat field indicate how many time do we take each iterable.

Complexity : $O((\prod q_i)^{repeat})$ With repeat=2, the complexity is $(q_1 * q_2)^2$

Where q_i is the size of each iterable.

Quick tour of theses functions

permutation(p, [r=len(p)])

`permutations('ABCD', 2)`	`AB AC AD BA BC BD CA CB CD DA DB DC`

Take an iterable and create every permutation of length r.

A permutation is an ordered (order is important) of element without repetition.

Complexity : $O(\frac{p!}{(p-r)!})$ For example with r=2, the complexity is $p * (p-1)$

Where p is the size of the iterable p.

Quick tour of theses functions

combinations(p, r)

```
combinations('ABCD', 2)
```

```
AB AC AD BC BD CD
```

Take an iterable and create every combination of length r.

A combination is a non ordered of element without repetition, since it is non ordered, AB and BA are the same.

Complexity : $O(\binom{p}{r}) = O(\frac{p!}{r!(p-r)!})$ Here, it's $\frac{p*(p-1)}{2}$

Where p is used as the size of the iterable p

Quick tour of theses functions

combinations_with_replacement(p, r)

`combinations_with_replacement('ABCD', 2)`	`AA AB AC AD BB BC BD CC CD DD`

Take an iterable and create every combination of length r with replacement.

Same as combination, but we can repeat the same element multiple times

Complexity : $O(\binom{p+r-1}{r}) = O(\frac{(p+r-1)!}{r!(p-1)!})$ Here, it's $\frac{(p+1)*p}{2}$

Where p is used as the size of the iterable p

Brute force - let’s solve our example

```
from itertools import combinations

arr = [int(n) for n in input().split()]

gen = (sub for size in range(len(arr))
       for sub in combinations(arr, size))

for sub in gen:
    if sum(sub) == 42:
        print(sub)
```

```
IN:
10 12 25 30 17 8 14 9 6

OUT:
(12, 30)
(25, 17)
(25, 8, 9)
(10, 12, 14, 6)
(10, 17, 9, 6)
```

`gen` is Python magic called a *generator expression*.
Ask me if you want to know more.

Why not to use brute force

There is a problem called combinatorial explosion.
TL;DR: the search space is often a lot bigger than the input size (e.g. exponential).

Previous example: for an array of size n, there are 2^n - 1 non-empty subsets...

For an algorithm in O(n!) with n=30, your algorithm would still be running after the "end" of the universe.

$$10^{20} years < \frac{30!}{365 * 24 * 3600 * 10^9}$$

Why ~~not~~ to use brute force

- Brute force is a good way to **test** if more complicated algorithms are correct. (you compare their results on small sets)
- Sometimes you just don't know a better algorithm. (password decryption...)

But, can't we reduce the search space a little bit?

→ often you can use *backtracking* (see you next week)

Credits

Slides: Louis Sugy, Arthur Tondereau, William Michaud