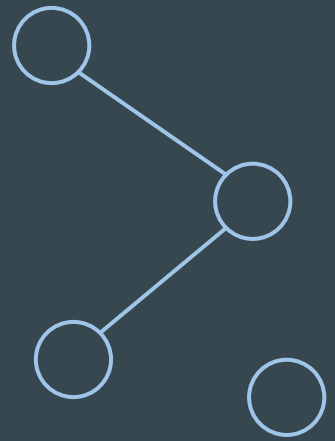
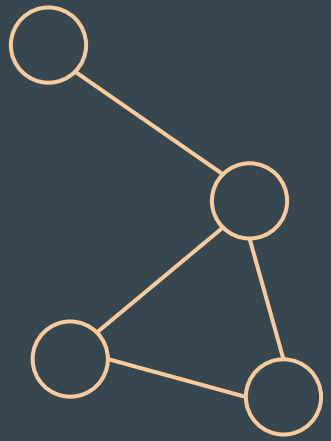

Segment Trees
Search trees on steroids

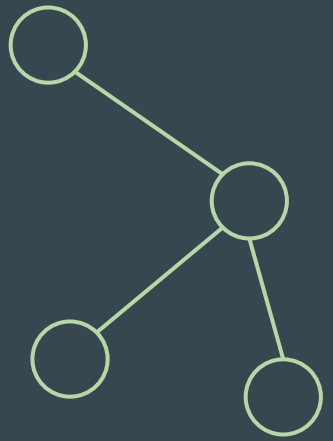
Reminder : what is a tree ?



acyclic graph



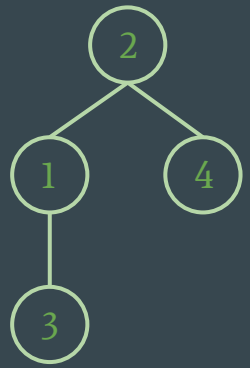
connected graph



tree = connected + acyclic

Properties of a segment tree?

- Similar to a binary tree:
  - It's a connected & acyclic graph
  - Finding a leaf in O(log(n))
  - Linear memory usage: 4n nodes needed for n elements
- But also different!
  - Find a subarray of element that verify of property in log(n)
  - Modification of one or multiple element in a subarray in log(n)



Sum of a subarray

We are given a list of elements and asked to answer k requests for the sum of a given subarray

→ “Moving sum” array:

We create a new array where each element is the sum of all the element before it.

1	5	7	6	7	9	3

0	1	6	13	19	26	35	38

Sum of a subarray

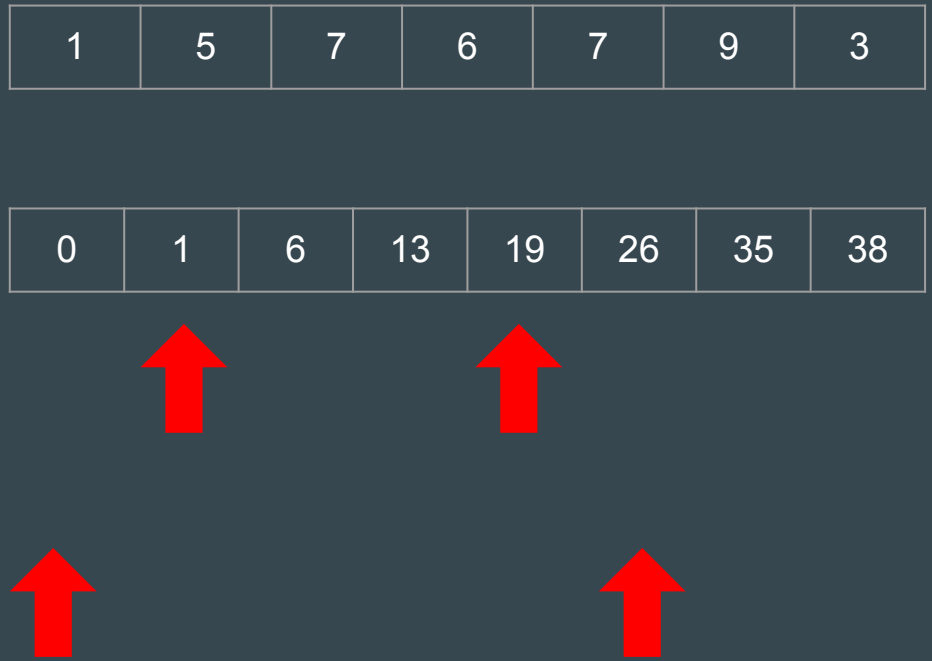
We are given a list of elements and asked to answer k requests for the sum of a given subarray

→ “Moving sum” array:

We create a new array where each element is the sum of all the element before it.

Sum of the subarray [1:3]
→ 19-1=18

Sum of the subarray [0:5]
→ 26-0=26



Sum of a subarray

We are given a list of elements and asked to answer k requests for the sum of a given subarray

But what if we also need to change a value in the array?

→ We have to recompute everything!

1	10	7	6	7	9	3

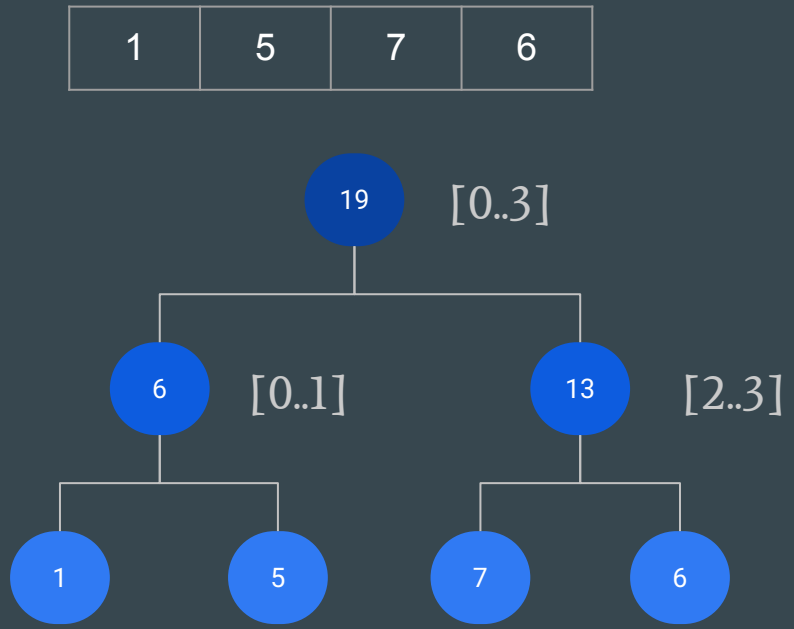
0	1	11	18	24	31	40	43

Sum of a subarray, with segment trees

Leaves are the array, in the same order. Each parent is the sum of his two children.

How to compute the sum in the subarray?

We are given a list of elements and asked to answer k requests to either update the array or to compute the sum of a subarray



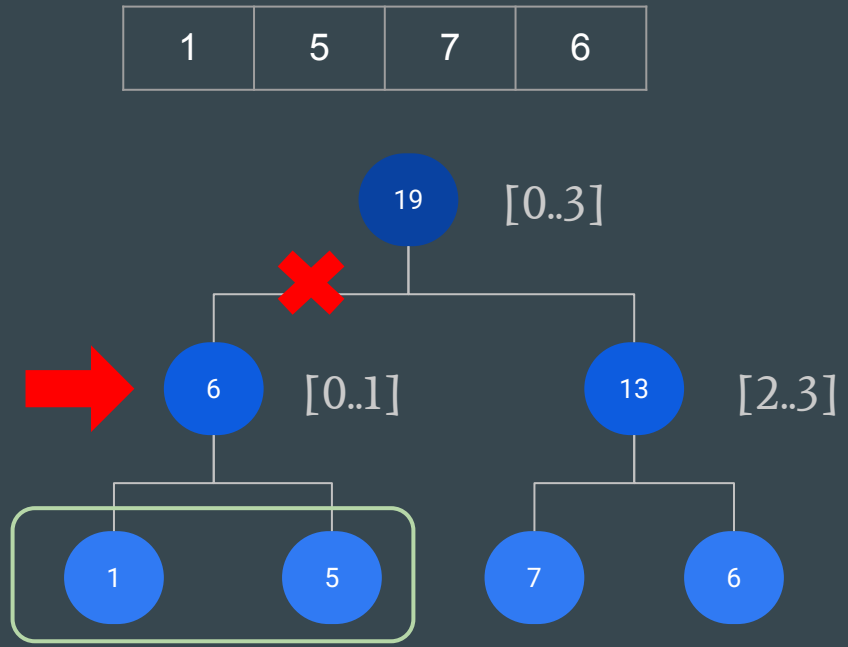
Sum of a subarray, with segment trees

Leaves are the array, in the same order. Each parent is the sum of his two children.

How to compute the sum in the subarray?

Sum of the subarray [0:1]
→ 6

We are given a list of elements and asked to answer k requests to either update the array or to compute the sum of a subarray



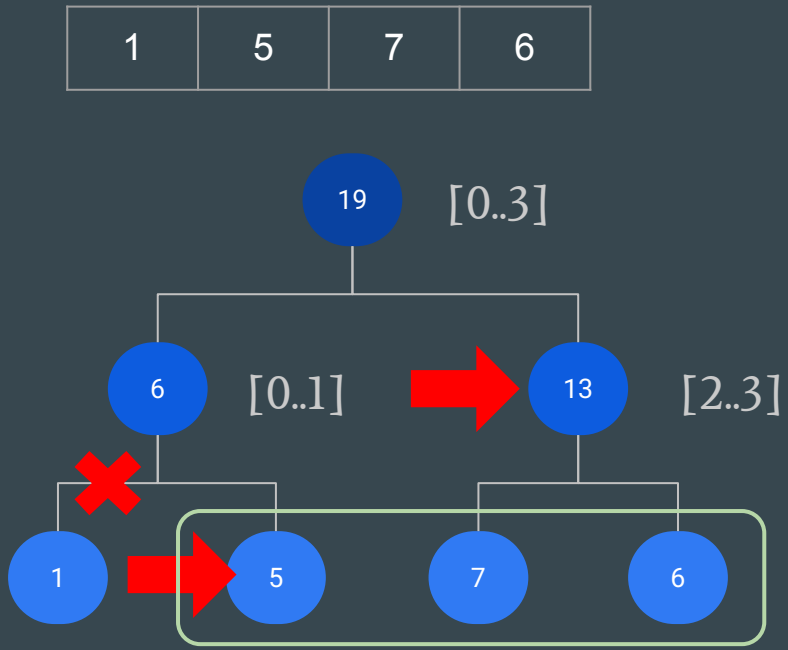
Sum of a subarray, with segment trees

Leaves are the array, in the same order. Each parent is the sum of his two children.

How to compute the sum in the subarray?

Sum of the subarray [1:3]
→ 5+13=18

We are given a list of elements and asked to answer k requests to either update the array or to compute the sum of a subarray



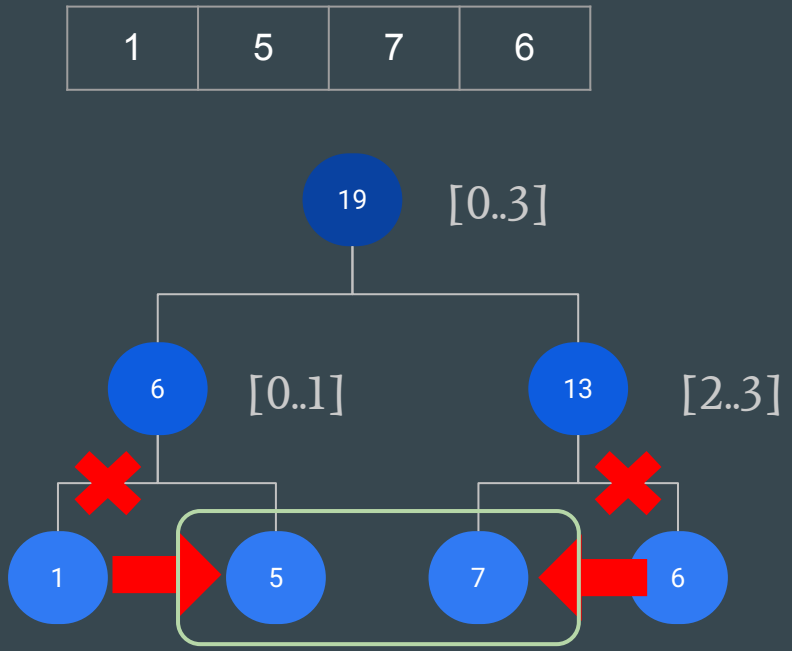
Sum of a subarray, with segment trees

Leaves are the array, in the same order. Each parent is the sum of his two children.

How to compute the sum in the subarray?

Sum of the subarray [1:2]
→ 5+7=12

We are given a list of elements and asked to answer k requests to either update the array or to compute the sum of a subarray



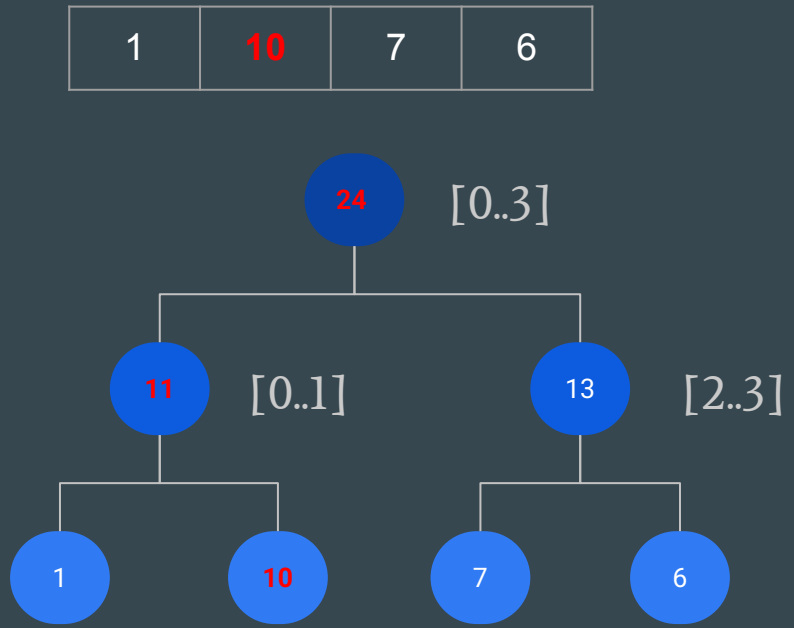
Sum of a subarray, with segment trees

And what if we need to update a value?

→ Update the leaf and all of its parents

O(log(n)) operations, this is much better than the previous O(n)

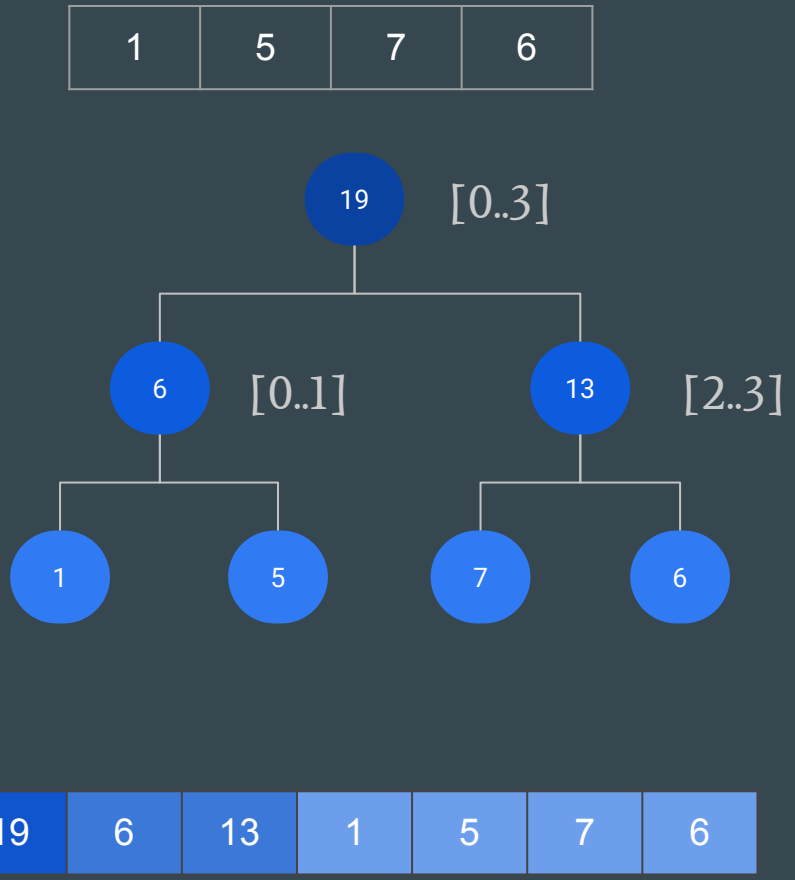
We are given a list of elements and asked to answer k requests to either update the array or to compute the sum of a subarray



Implementation of a segment tree

Similar to most binary trees.

For a node at the index i, its children are stored at the indexes 2*i and 2*i+1.



Implementation of a segment tree

Can be generalized to higher dimensions!

With a 2D matrix, we first construct the segment tree on each row (instead of each cell/value)

Then, we construct a segment tree for each node of the previous tree, on the values of each row

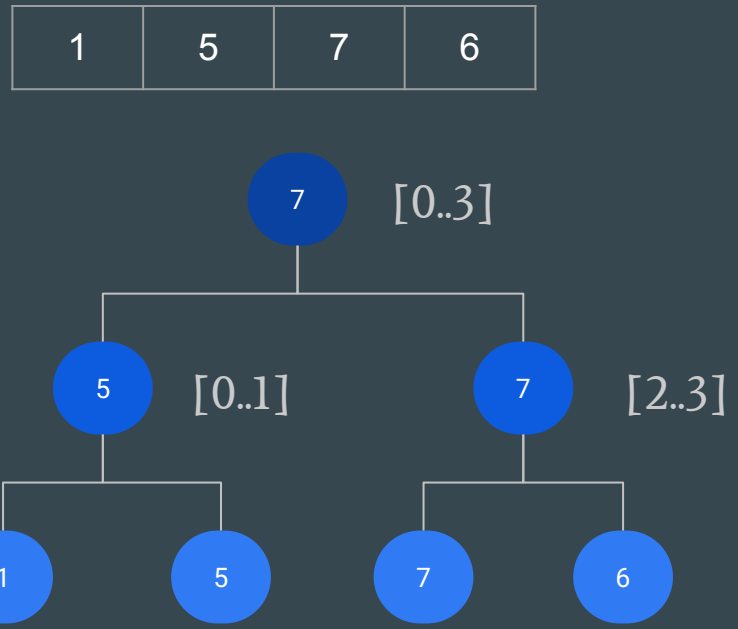
1	5	3	7
13	9	6	7
2	12	4	3
3	6	3	2

86	51	35	19	32	16	19
51	28	23	14	14	9	14
35	23	12	5	18	7	5
16	6	10	1	5	3	7
35	22	13	13	9	6	7
21	14	7	2	12	4	3
14	9	5	3	6	3	2

Generalization

The heuristic can be adapted to solve a wide range of problems.

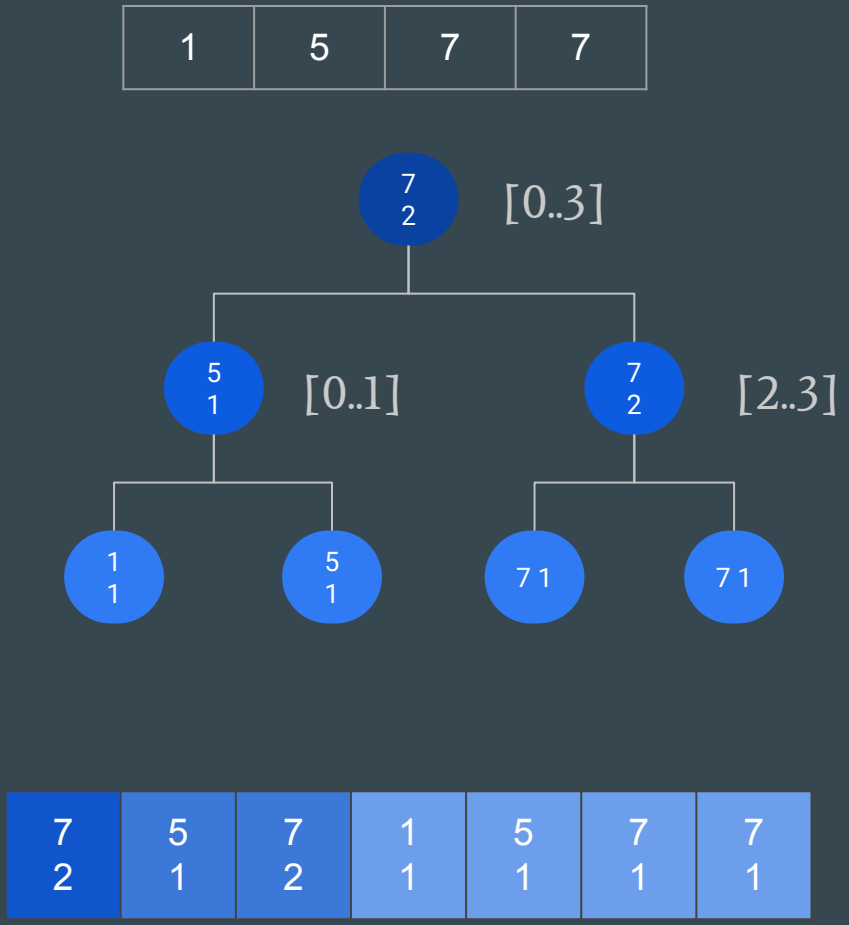
→ Search of the local maximum



Generalization

The heuristic can be adapted to solve a wide range of problems.

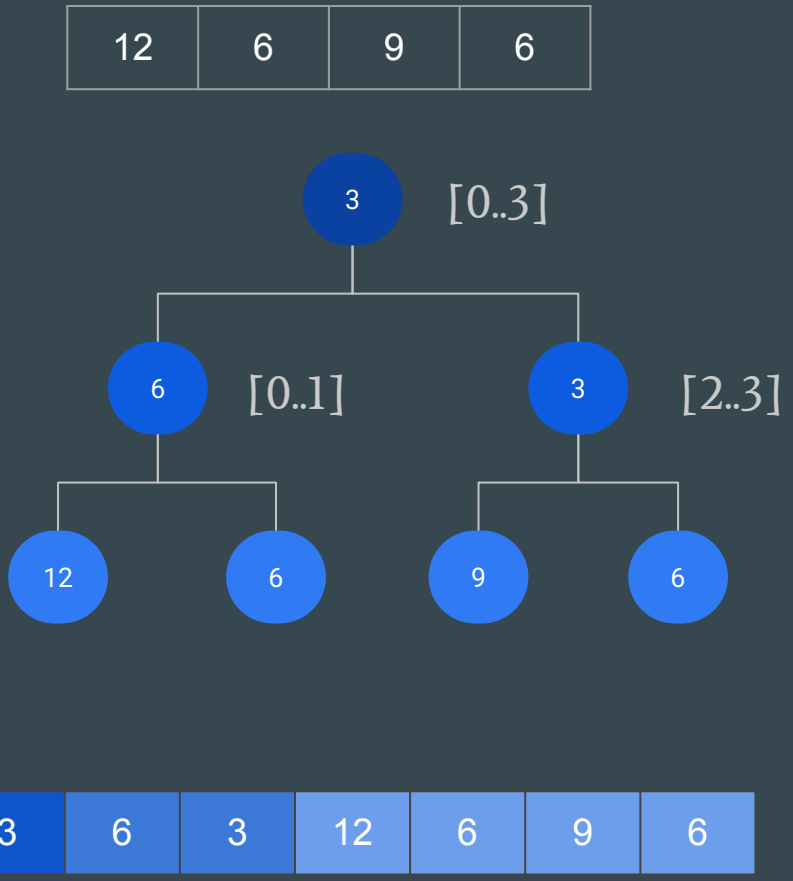
→ Search of the local maximum, and how many times it appears



Generalization

The heuristic can be adapted to solve a wide range of problems.

→ Computing the GCD (or LCM) of a subarray



Credits

Slides: William Michaud for INSAlgo

Sources:

- https://cp-algorithms.com/data_structures/segment_tree.html#advanced-versions-of-segment-trees
- https://en.wikipedia.org/wiki/Segment_tree