Greedy algorithms

cuz data tastes good

A simple coin change problem

You have 20$, 10$, 5$, 2$ and 1$ bills
→ you want to pay N$ with as few bills as possible

e.g 58$ = 2*20$ + 1*10$ + 1*5$ + 1*2$ + 1*1$ → 6 bills

Strategy: let's try to use the biggest bills as long as possible

A simple coin change problem

Algorithm:

```
n = int(input())
bills = [20, 10, 5, 2, 1]
nbill = []

for bill in bills:
    nbill.append(n // bill)
    n -= nbill[-1] * bill

print(" + ".join("%d*%d$" % (nbill[i], bills[i])
                 for i in range(len(bills))))
```

Complexity:

O(len(bills)) → const, O(1)

Why it works:

The dollar system is called a *canonical* coin system

→ cf Xuan Cai, https://arxiv.org/pdf/0809.0400.pdf

Non-canonical system: [9, 4, 1]

Optimal: 12 = 3*4

Greedy: 12 = 1*9 + 3*1

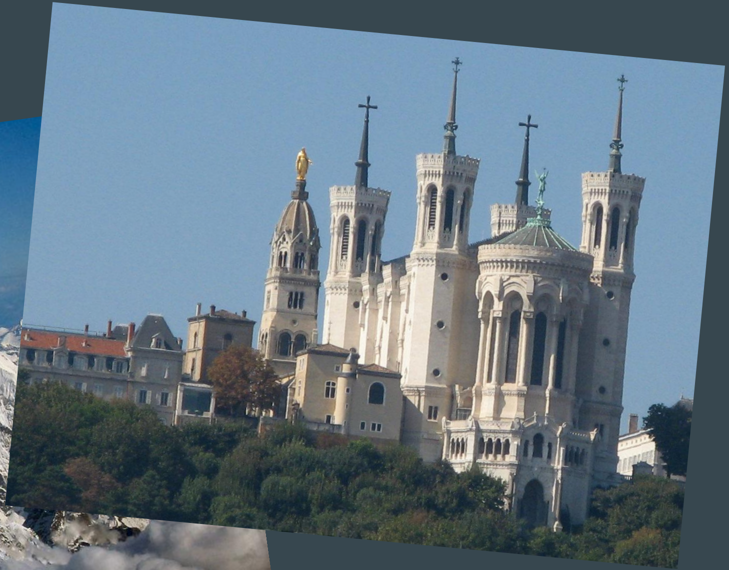
So what’s a greedy algorithm?

- it’s an algorithm that makes **locally-optimal choices** at each step
- the global solution might not be the optimal one
- but many well-known greedy algorithms are proven to find the optimal solution, e.g:
  - Kruskal’s and Prim’s algorithms (minimum spanning tree)
  - Dijkstra’s and A* algorithms (shortest path)

Greedy algorithms are usually **much faster** than their *dynamic programming* equivalents, but their correctness can be hard to prove...

Example of a non-optimal greedy algorithm

You want to climb on the highest mountain.
You think “if I keep going up, I will reach the highest point”.
You end up on the Fourvière hill.
Not quite as tall as Mount Everest.



The activity selection problem

You want to do as many activities as possible, but some of them are overlapping. Given their start and end dates, find a set of compatible activities of maximum size.

Greedy algorithm:

- sort the activities by increasing end date
- take the first activities
- iterate over the remaining activities
- if an activity is compatible with the previously taken activities, take it

The activity selection problem

```
n = int(input())
activities = [(lambda s: (s[0], int(s[1]),
               int(s[2])))(input().split())
              for _ in range(n)]
activities.sort(key=lambda x: x[2])

selected = [activities[0][0]]
latest_f = activities[0][2]
for name, s, f in activities[1:]:
     if s >= latest_f:
     selected.append(name)
     latest_f = f

print(" ".join(selected))
```

IN

```
5
Aquaponey 8 10
Beurk 12 14
Réu 11 13
INSAlgo 18 20
Concert 19 22
```

OUT

```
Aquaponey Réu INSAlgo
```

The activity selection problem

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activities = [(lambda s: (s[0], int(s[1]),
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for name, s, f in activities[1:]:
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     selected.append(name)
     latest_f = f

print(" ".join(selected))
```

Complexity:

O(n log n) as we sort the activities

Why it works:

You can check the proof here: https://en.wikipedia.org/wiki/Activity_selection_problem#Proof_of_optimality

Credits

Slides: Louis Sugy for INSAlgo

Pictures: Wikipedia

Picture of the cookie monster: fair use of the character from *Sesame Street*