Common data structures

•••

Implementation and complexity

What’s a data structure?

(Wikipedia) “a collection of **data values**, the **relationships** among them, and the functions or **operations** that can be applied to the data”

- an **abstraction** of data that is easy for a programmer to work with
- contains more than data: the data is **organized** in a specific way
- well-defined **operations** can be applied to the data

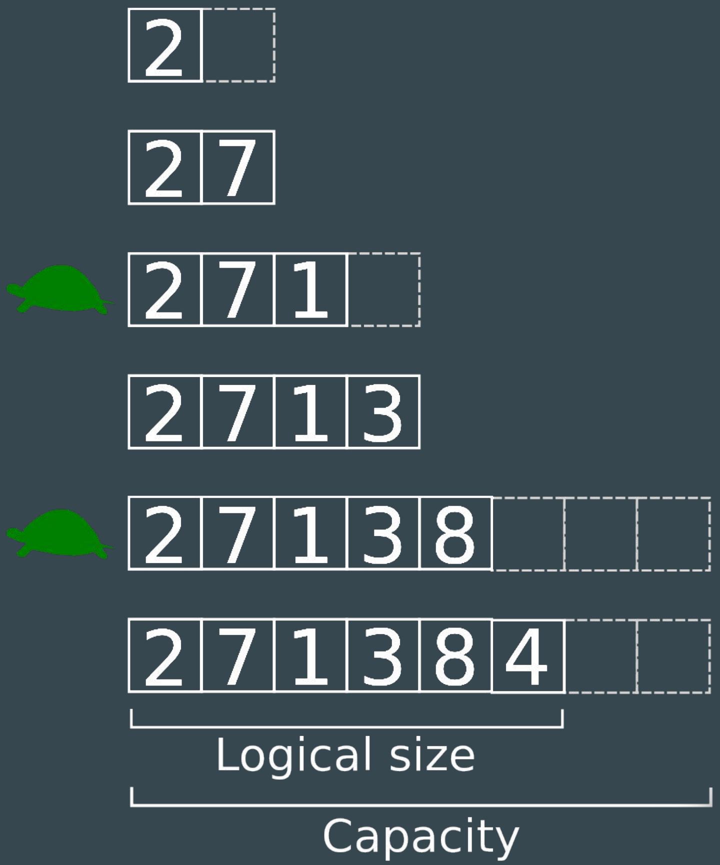
→ it is important to know what data structures exist and which operations can be applied on them

Dynamic arrays (Python lists)

Goal: quickly **access indexed items** in a container and **append new ones** (or remove the last one)

Implementation:

- the language uses more room than needed
- while there is room, appending costs nothing
- when there is no more room, create a new array with more room and copy everything



Dynamic arrays (Python lists)

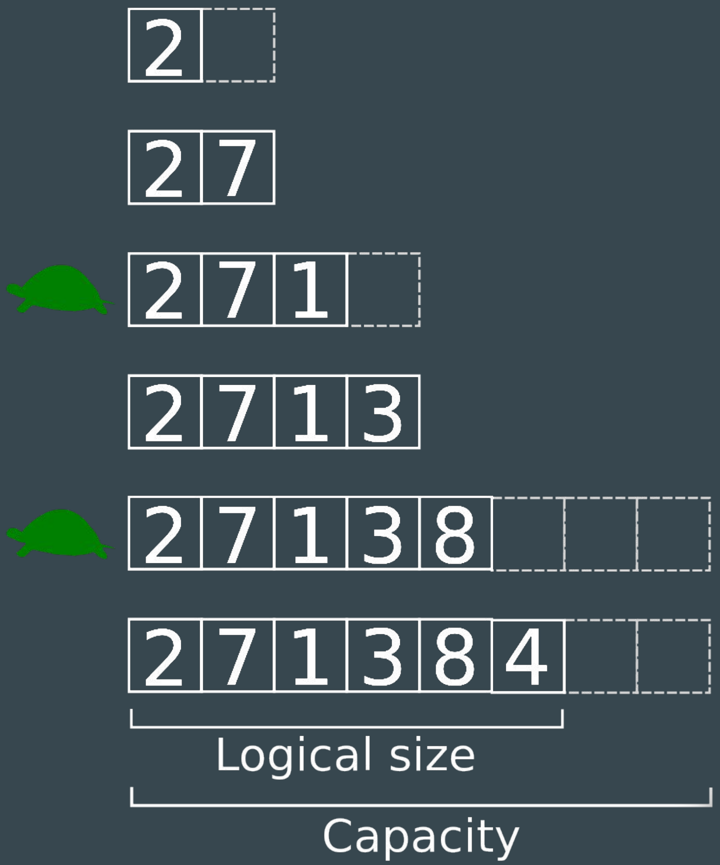
What's the complexity of adding a new item?

if you reach the capacity c_1 and extend the size to $c_2 = 2 * c_1$, the cost is c_1 only once but then the cost will be const for the next c_1 *append* operations

→ on average, the cost is O(1)

What's the complexity of accessing an item?

O(1), position in RAM deduced from its index



Dynamic arrays: in Python and C++

Common operations:

	Python	C++	Complexity
Access the i-th item	`arr[i]`	`arr[i]`	`O(1)`
Add v at the end	`arr.append(v)`	`arr.push_back(v)`	`O(1) avg` `O(n) max`
Insert v at position i	`arr.insert(i, v)`	`arr.insert(i, v)`	`O(n)`
Find the position of v	`arr.index(v)`	`std::find(...)`	`O(n)`

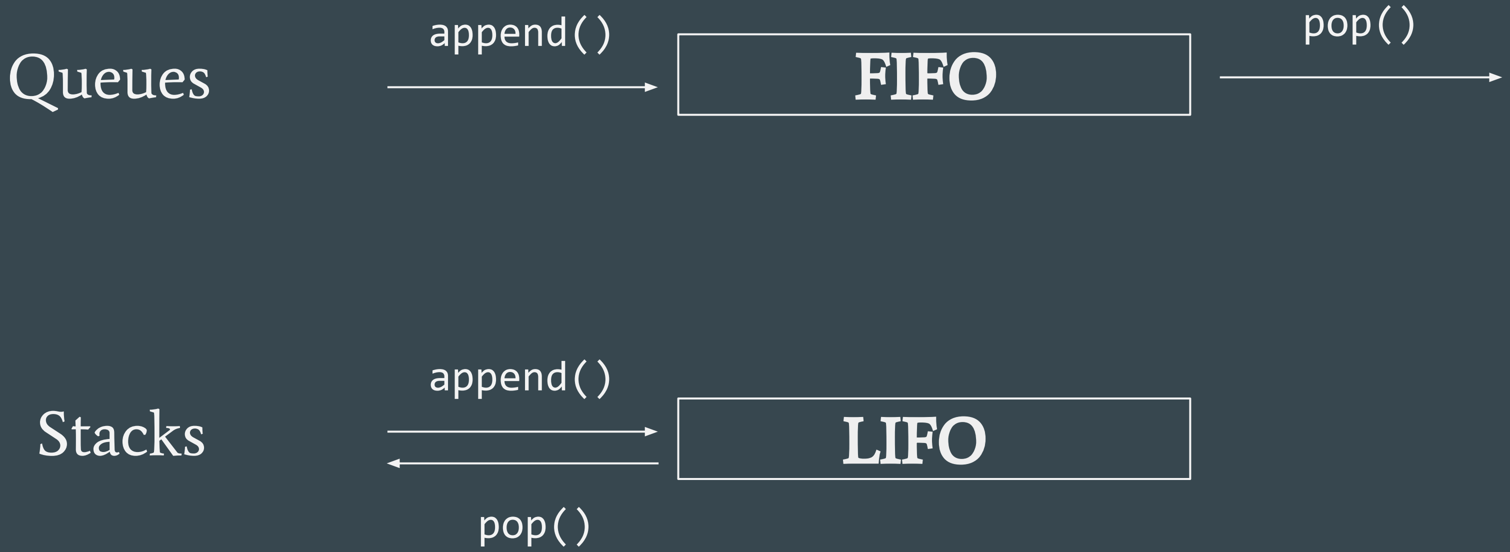
When to use:

There are all kinds of use cases. If you often need to perform operations that are not O(1), check if another data structure matches your needs

Quick implementation cheat sheet - Python array

	Instruction	Complexity
Create a new one	`L = list(),` `L = []`	`O(1)`
Access the i-th item	`arr[i]`	`O(1)`
Add v at the end	`arr.append(v)`	`O(1) avg` `O(n) max`
Insert v at position i	`arr.insert(i, v)`	`O(n)`
Find the position of v	`arr.index(v)`	`O(n)`

Stacks & Queues



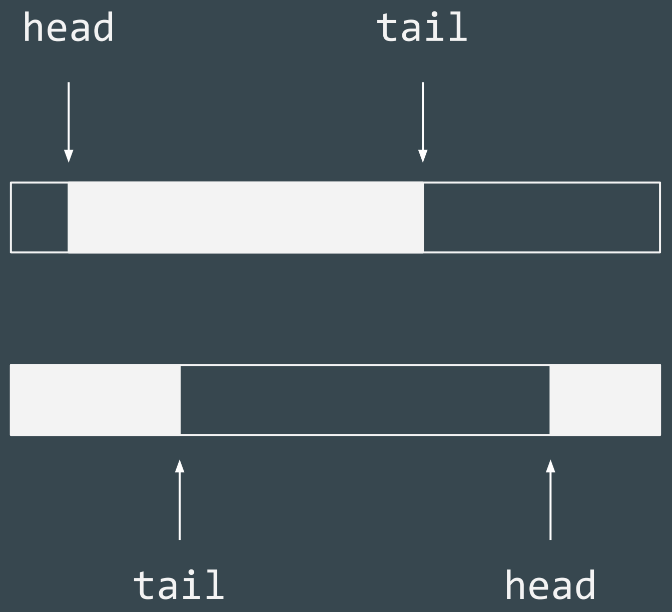
Double-ended queues

Goal: quickly access, remove and add items **on both ends**

Multiple implementations are used

Double-ended queues - with a ring buffer

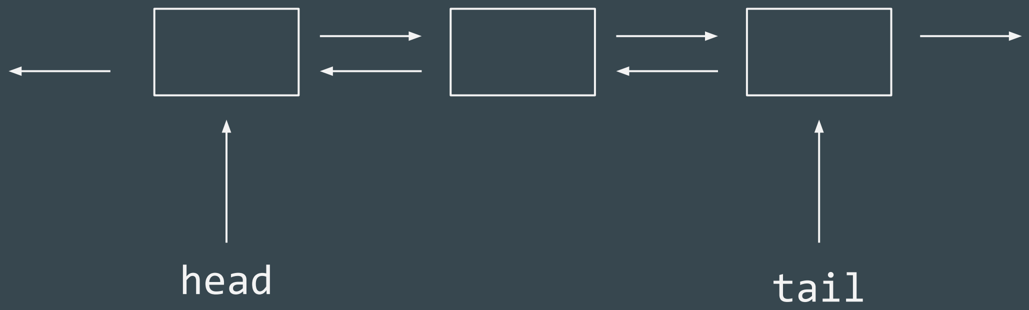
- the queue is stored in an array
- head and tail indices are updated when removing or adding an item
- when too big, the queue is copied in a bigger array



Double-ended queues - with a linked list

```
type Element
    value
    → previous
    → next

type LinkedList
    → head
    → tail
```



No direct access to other elements than head and tail but this is a very fast implementation

Quick implementation cheat sheet - Python deque

Python: `deque` in `collections`

	Python	Complexity
Create a deque	`dq = deque()`	O(1)
Access the i-th element	`dq[i]`	O(n)
Add v at the head	`dq.appendleft(v)`	O(1)
Remove the head and put it in v	`v = dq.popleft()`	O(1)
Add v at the tail	`dq.append(v)`	O(1)
Remove the tail and put it in v	`v = dq.pop()`	O(1)

Double-ended queues - usage

When to use:

whenever you need a **queue** (e.g in *breadth-first search* algorithm)

When not to use:

- if you want to access very often elements in the middle of the sequence
- you don't need it when inserting and removing elements only on one side (e.g a a **stack**). Just don't do that on the left side

Dictionaries / hash maps

Goals:

- associate **values** to **keys**
- retrieve the value associated to a key in $O(1)$ time

→ unlike real-life dictionaries, these ones are **not ordered**

(in real life, finding a word in a dictionary is $O(\log n)$
unless you have forgotten the alphabetical order)

Dictionaries / hash maps

The underlying structure is called a **hash table**

- a hash function turns the keys into an index
- the keys and values are stored in the data structure based on this index
- multiple strategies exist to manage collisions (cf next slides)

Example of hash function: sum of the ASCII codes for a string, modulo 42

“hello” → 28 “world” → 6 “INSAlgo” → 33

Dictionaries / hash maps - separate chaining

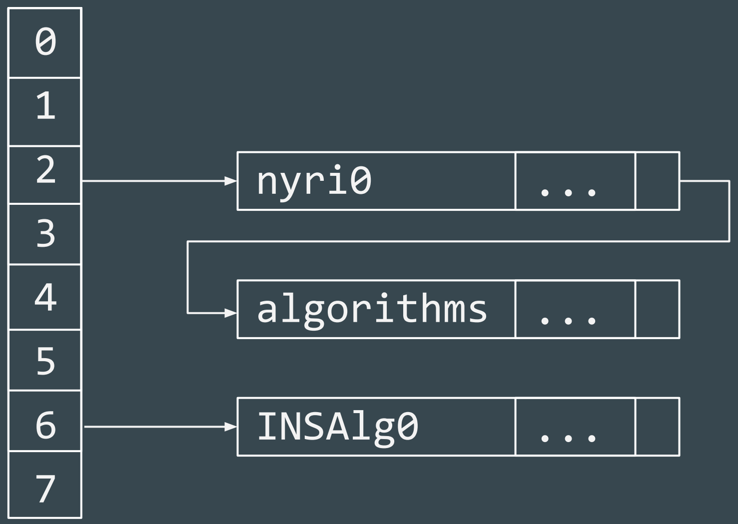
(key, value) couples where keys have the same hash are stored in a common data structure

These structures are kept small enough to have O(1) time access. When too filled, the table is re-created bigger

Often used:

- linked lists
- trees

With sum of ASCII codes modulo 8, and using linked lists:



Dictionaries / hash maps - Open adressing

- No other data structure behind the array
- Collisions are solved with *probing*
- When the fill rate becomes too big, copy the data in a bigger hash table

Linear probing = put at next cell available

Randomized probing: follow a random sequence which seed is given by the hash

→ used in Python

0			
1			
2	2	nyri0	...
3	2	algorithms	...
4			
5			
6	6	INSAlg0	...
7			

(sum of ASCII codes modulo 8, open adressing with linear probing)

Dictionaries / hash maps - in Python and C++

Python: open adressing with randomized probing (congruential RNG)
initial size 8, resized when ⅔ full

C++: `unordered_map` is a hash table, `map` a red-black tree
(we will talk about trees later in the year)

Common operations:

	Python	C++ `unordered_map`	Complexity
Find or set the value associated to `key`	`dic[key]`	`dic[key]`	`O(1) avg` `O(n) max`
Check if `key` exists in the dictionary	`key in dic`	`dic.find(key)`	`O(1) avg` `O(n) max`

Dictionaries / hash maps - in Python and C++

When to use:

Whenever you need to associate a value to a key.

In Python, the ease of use and high performance make the dictionaries a **very powerful tool**.

When not to use:

When your keys are 0, 1, ..., n
You're better than that.

Quick implementation cheat sheet - Python dict

Python: `dict, built in.`

`Interesting variation : defaultdict in collections`

	Python	Complexity
Create a deque	`d = dict(),` `d = {}`	O(1)
Access an element	`d[key]`	Pseudo O(1)
Put an element	`d[key] = value`	Pseudo O(1)
Find if a key is in dict	`key in d`	Pseudo O(1)
Remove a key	`del d[key]`	Pseudo O(1)

Sets

Goals:

- store unique values
- quickly check if a value is in the set or not

Two common implementations:

- tree-based sets are ordered
- hash sets are faster but unordered
  (they're basically hash tables without values)

Sets - in Python and C++

Python: `set` is a hash set
C++: `unordered_set` is a hash table, `set` a tree

Common operations:

	Python	C++ `set` and `unordered_set`	Complexity	Complexity - C++ `set`
Add the value v to the set	`s.add(v)`	`s.emplace(v)`	`O(1) avg` `O(n) max`	`O(log n)`
Remove the value v from the set	`s.remove(v)`	`s.erase(v)`	`O(1) avg` `O(n) max`	`O(log n)`
Check if v is in the set	`v in s`	`s.find(v)`	`O(1) avg` `O(n) max`	`O(log n)`

Quick implementation cheat sheet - Python set

Python: `set, built in`

	Python	Complexity
Create a set	`S = set()`	O(1)
Access a specific element	XXXXXXX	
Put an element	`s.add(element)`	Pseudo O(1)
Find if an element is in a dict	`Element in s`	Pseudo O(1)
Remove an element	`s.remove(element)`	Pseudo O(1)

Set arithmetics in Python

<table>
<tr><th></th><th></th><th>Average complexity</th></tr>
<tr><td>s1 <= s2, s1 < s2</td><td>check if s1 is a [proper] subset of s2</td><td>O(n1)</td></tr>
<tr><td>s1 >= s2, s1 > s2</td><td>check if s1 is a [proper] superset of s2</td><td>O(n2)</td></tr>
<tr><td>s1 | s2 | … | sk</td><td>union of s1, s2, …, sk</td><td>O(n1 + n2 + … + sk)</td></tr>
<tr><td>s1 & s2 & … & sk</td><td>intersection of s1, s2, …, sk</td><td>O(min(n1, n2, …, sk))</td></tr>
<tr><td>s1 - s2</td><td>all elements of s1 that are not in s2</td><td>O(n1)</td></tr>
<tr><td>s1 ^ s2</td><td>all elements of s1 or s2 but not both</td><td>O(n1 + n2)</td></tr>
</table>

Sets too are cool :)

Sets - in Python and C++

When to use:

- to mark values already seen
- to keep a collection of unordered unique elements

When not to use:

- to make coffee (you just can’t)

To be continued...

You probably want to hear about red-black trees, heaps (priority queues), etc...

Well, see you in 2019! ... this was last year, see you in 2020 perhaps ;)

Slides: Louis Sugy, Arthur Tondereau for INSAlgo
Schema of dynamic arrays: Wikipedia