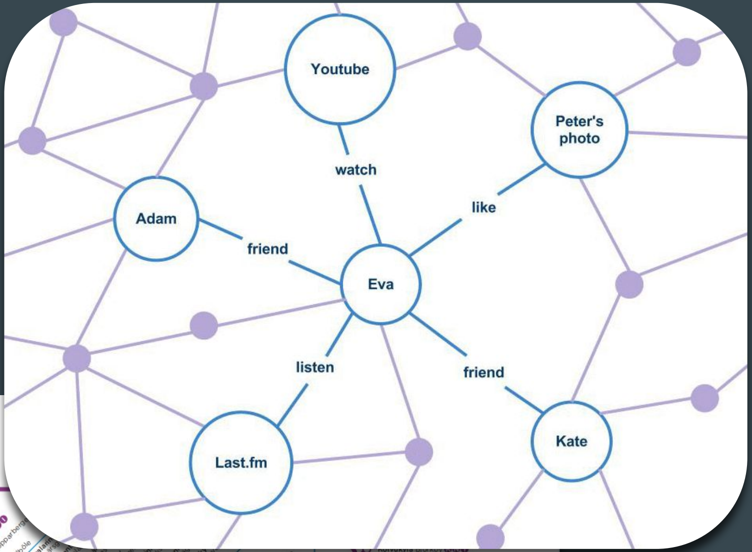
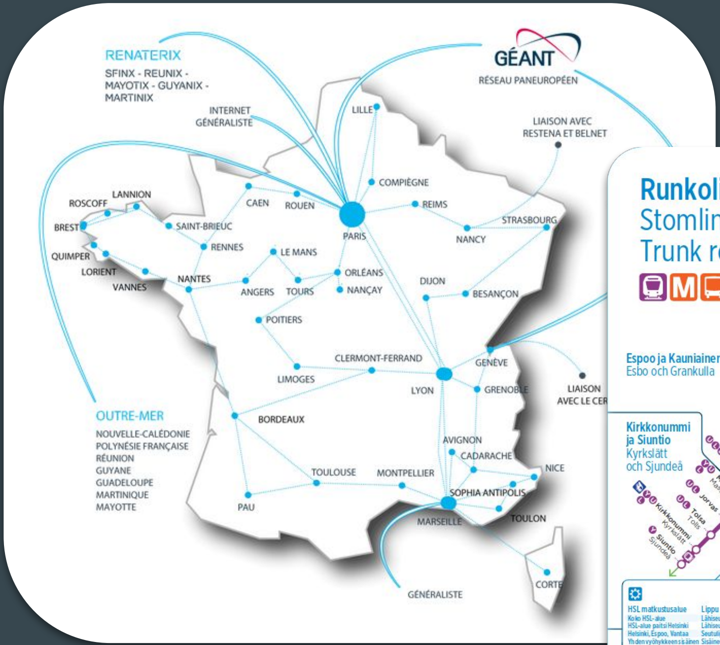
Graph Theory 1

Introduction to graphs

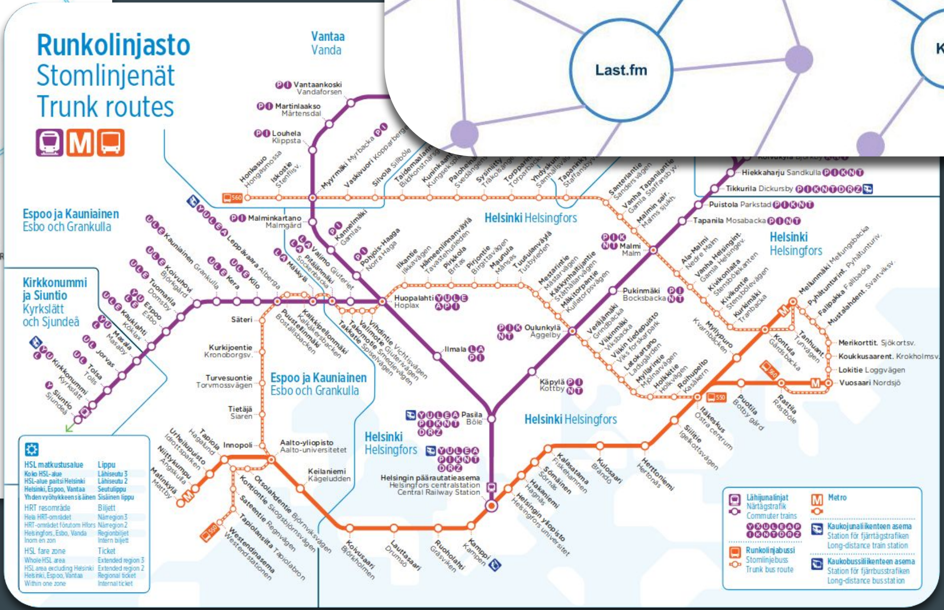
A world of graphs

Social interactions





Renater network

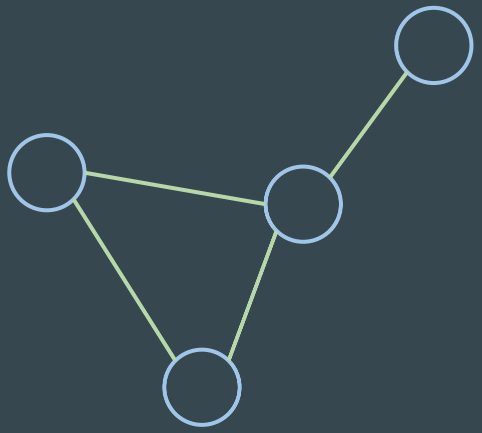


Helsinki trains map

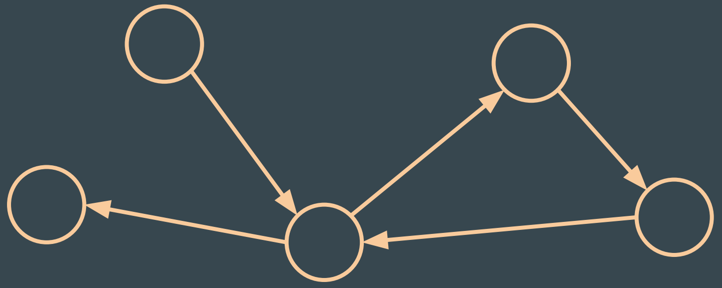
So what's a graph?

It's a tuple (V, E), where

- V is a **set of vertices**
- E is a **set of edges**



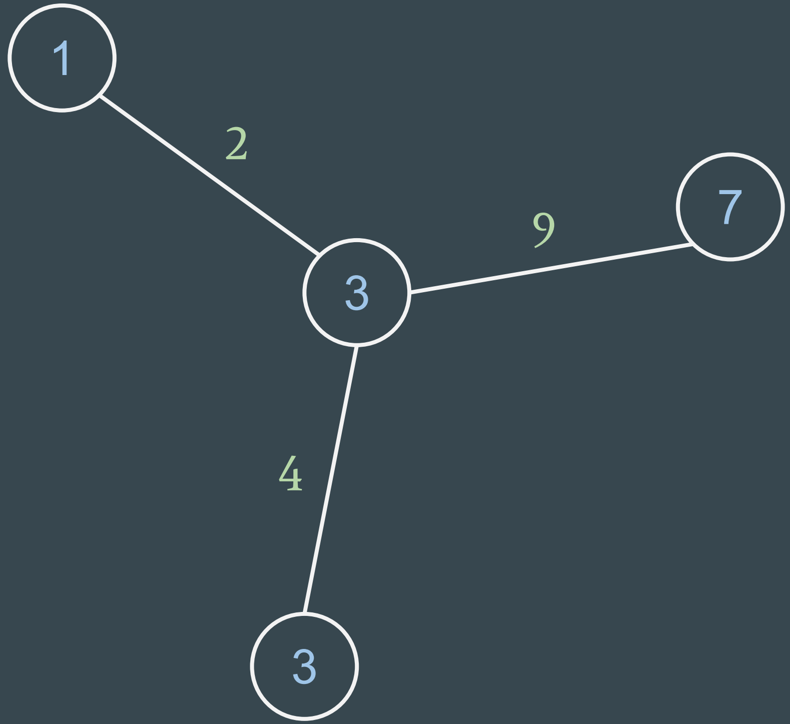
If the edges have an orientation, the graph is **directed**, otherwise it is **undirected**



So what’s a graph?

Some graphs contain more information such as:

- values on vertices
- values on edges



How to represent a graph? Solution #1 : edge list

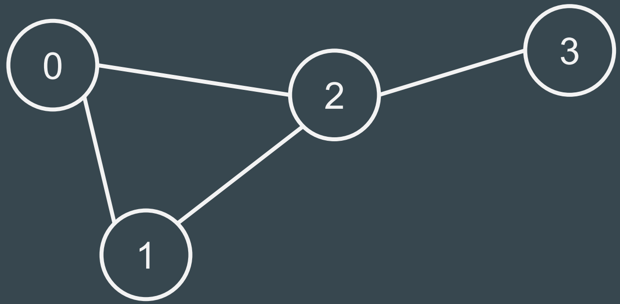
Let’s label the vertices from 0 to n-1

We need to find a data structure for the edges...

What about a list of couples?

Check if i and j are neighbors	O(|E|)
Find all neighbors of i	O(|E|)

→ awfully long for such simple operations

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[(0,1),(0,2),(1,2),
(2,3)]
```

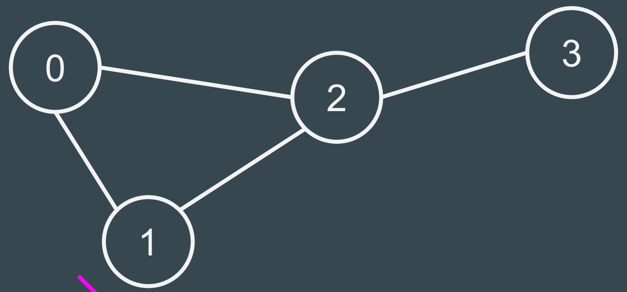
How to represent a graph? Solution #2 : adjacency matrix

Another idea is to store the relations as a matrix

→ boolean, or the value on the edge if the graph has some (and a dead value like None if there is no edge between two nodes)

Check if i and j are neighbors	O(1)
Find all neighbors of i	O(n)

→ but always takes $O(n^2)$ in memory (not suitable for big graphs)



0	1	1	0
1	0	1	0
1	1	0	1
0	0	1	0

Symmetry for a directed graph

How to represent a graph?

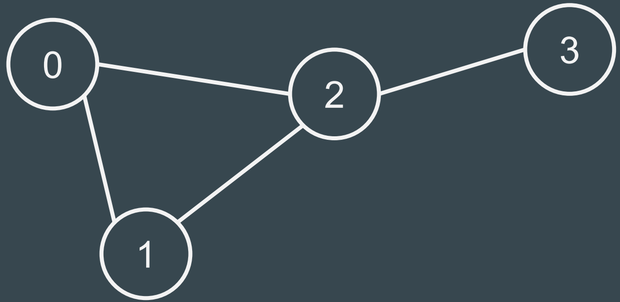
Solution #3 : Adjacency list

→ store all neighbors of each node

(there is a variant with sets, they take more memory but allow quicker adjacency check)

Check if i and j are neighbors	O(|E| / n) avg
Find all neighbors of i	O(|E| / n) avg

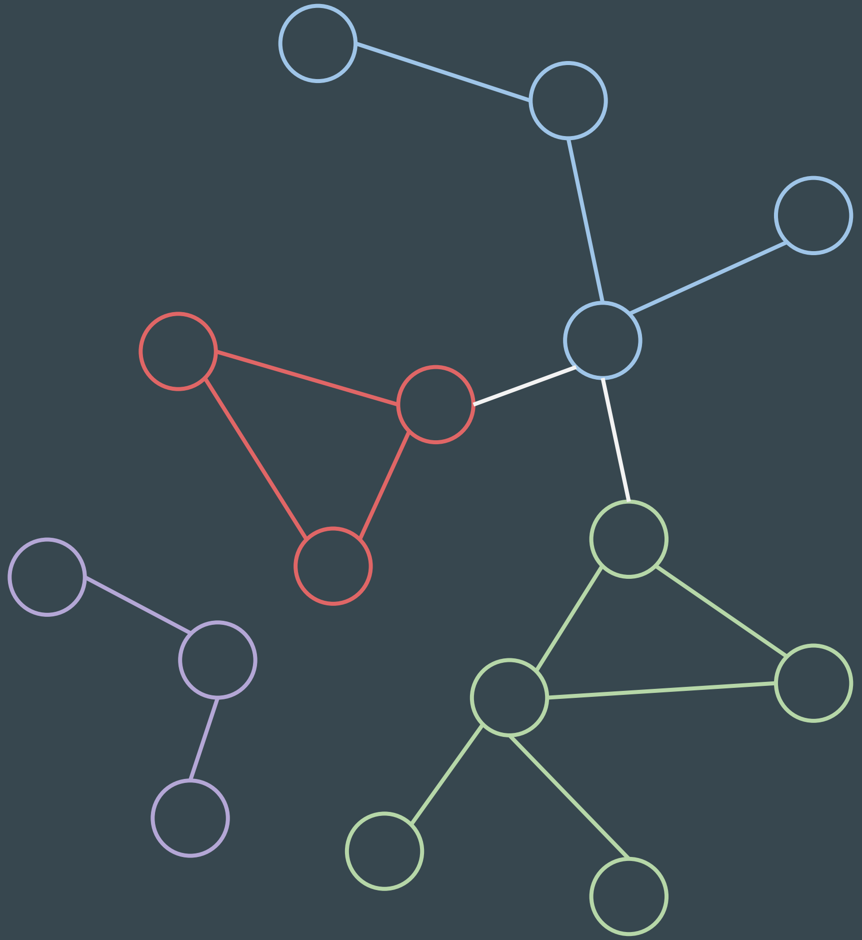
→ this representation is very often preferred as it is compact and efficient



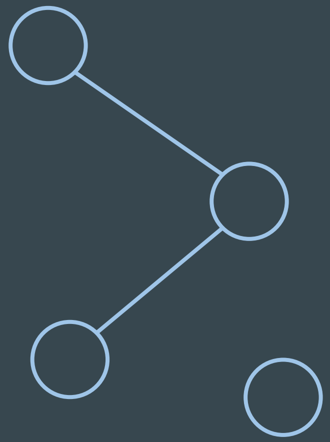

```
{0: [1, 2],
 1: [0, 2],
 2: [0, 1, 3],
 3: [2]}
```

Just a few terms

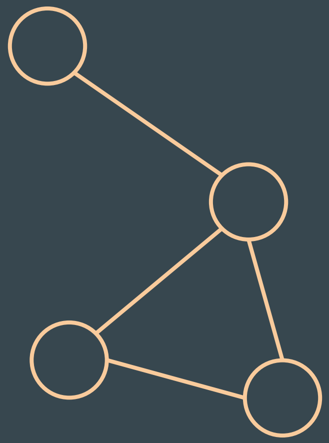
- a **path** is a sequence of 2-by-2 adjacent nodes and edges
- a **simple path** is a path that has no repeated vertices and edges
- a **cycle** is a path starting and ending at the same node
- a **connected component** is a subgraph in which there exists a path between any two nodes but not with any external node



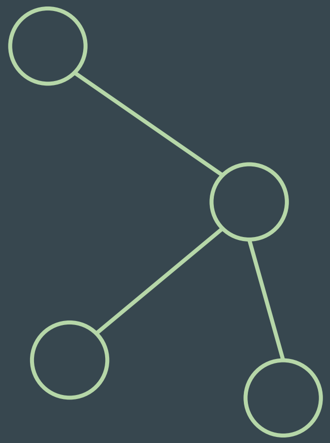
Special graphs



acyclic graph



connected graph



tree = connected + acyclic

Note on directed graphs

- their matrix isn't symmetrical
- being acyclic doesn't mean that the undirected counterpart is acyclic too
- there are two notions of connection for directed graphs:
  - weakly connected = the corresponding undirected graph is connected
  - strongly connected = there is a path in each direction for each pair of vertices

Some properties of trees...

- A tree has N-1 edges for N vertices
- A tree is a connected & acyclic graph
- Remove any edge from the tree and it disconnects the graph
- In the general case vertice can have any degree
  - Some trees are more specific : binary trees, k-trees

Slides: Louis Sugy for INSAlgo, revised in 2020 by Louis Gombert

Helsinki trains map:	HSL
The Internet in 2015:	The Opte Project
Renater network:	Renater