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CATHETER SYSTEM

$$\begin{cases} \dfrac{\partial A}{\partial t} + \dfrac{\partial}{\partial z} u(A-A_c) = 0 \\ \dfrac{\partial u(A-A_c)}{\partial t} + \dfrac{\partial \alpha u^2(A-A_c)}{\partial z} + \dfrac{A-A_c}{\rho}\dfrac{\partial P}{\partial z} = \dfrac{f}{\rho} \end{cases}$$

set

$$Q = u(A-A_c)$$

then

$$\begin{cases} \dfrac{\partial A}{\partial t} + \dfrac{\partial Q}{\partial z} = 0 \\ \dfrac{\partial Q}{\partial t} + \dfrac{\partial}{\partial z}\left(\alpha \dfrac{Q^2}{A-A_c}\right) + \dfrac{A-A_c}{\rho}\dfrac{\partial P}{\partial z} = \dfrac{f}{\rho} \end{cases}$$

with $P = \beta\left(\sqrt{\dfrac{A}{A_0}} - 1\right) = \dfrac{\beta}{\sqrt{A_0}}\sqrt{A} - \beta$

$$\frac{\partial P}{\partial A} = \frac{\beta}{2\sqrt{A_0}}\frac{1}{\sqrt{A}}$$

and

$$\frac{A-A_c}{\rho}\frac{\partial P}{\partial A} = \sqrt{A}\,\frac{\beta}{2\rho\sqrt{A_0}} - \frac{A_c}{\sqrt{A}}\,\frac{\beta}{2\rho\sqrt{A_0}}$$

with $\gamma = \dfrac{\beta}{3\rho\sqrt{A_0}}$

$$\frac{A-A_c}{\rho}\frac{\partial P}{\partial A} = \frac{3}{2}\gamma\sqrt{A} - \frac{A_c}{\sqrt{A}}\frac{3}{2}\gamma = \left(\gamma A^{\frac{3}{2}} - 3\gamma A_c\sqrt{A}\right)_z$$

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with $\alpha = 1$

$$U_t + F_z = S$$

$$F = \begin{pmatrix} Q \\ \dfrac{Q^2}{A-A_c} + \gamma A^{\frac{3}{2}} - 3\gamma A_c \sqrt{A} \end{pmatrix}$$

JACOBIAN AND INVARIANTS

$$H = \begin{pmatrix} 0 & 1 \\ \frac{3}{2}\gamma\sqrt{A} - \frac{3}{2}\gamma\frac{A_c}{\sqrt{A}} - \frac{Q^2}{(A-A_c)^2} & 2\frac{Q}{A-A_c} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{3}{2}\gamma\sqrt{A} - \frac{3}{2}\gamma\frac{A_c}{\sqrt{A}} - u^2 & 2u \end{pmatrix}$$

$$|H - \lambda \mathbb{1}| = -\lambda(2u - \lambda) - \frac{3}{2}\gamma\sqrt{A} + \frac{3}{2}\gamma\frac{A_c}{\sqrt{A}} + u^2 = 0$$

$$\frac{2u \pm \sqrt{4u^2 - 4\left(-\frac{3}{2}\gamma\sqrt{A} + \frac{3}{2}\gamma\frac{A_c}{\sqrt{A}} + u^2\right)}}{2} = \frac{2u \pm \sqrt{4\frac{3}{2}\gamma\sqrt{A} - 4\frac{3}{2}\gamma\frac{A_c}{\sqrt{A}}}}{2} =$$

$$= u \pm \sqrt{\frac{3}{2}\gamma}\sqrt{\sqrt{A} - \frac{A_c}{\sqrt{A}}} = u \pm \underbrace{\sqrt{\frac{3}{2}\gamma}\sqrt[4]{A}\sqrt{1 - \frac{A_c}{A}}}_{\text{sound speed for catheter system}}$$

The invariants can be computed the equivalent (A,u) conservative form with $\alpha = 1$ and

$$F = \begin{pmatrix} u(A-A_c) \\ \frac{u^2}{2} + \frac{p}{\rho} \end{pmatrix} \qquad H = \begin{pmatrix} u & A-A_c \\ \frac{1}{\rho}\frac{\partial p}{\partial A} & u \end{pmatrix}$$

$$\lambda_{1,2} = u \pm c \quad , \quad c = \sqrt{\frac{3}{2}\gamma}\,\sqrt[4]{A}\,\sqrt{1-\frac{A_c}{A}}$$

$$L = \begin{pmatrix} -\frac{c}{A-A_c} & 1 \\ \frac{c}{A-A_c} & 1 \end{pmatrix}$$

$$\frac{\partial W}{\partial U} = L \longrightarrow W_1 = u - \underbrace{\int \frac{c}{A-A_c}\, dA}$$

$$\boxed{\sqrt{\frac{3}{2}\gamma}\int \frac{\sqrt{A-A_c}}{\sqrt[4]{A}\,(A-A_c)}\, dA = 2\sqrt{A-A_c}\,\sqrt[4]{\frac{A}{A_c}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},1-\frac{A}{A_c}\right)}$$

$$= 4c\,\boxed{\frac{\sqrt[4]{\frac{A}{A_c}}\;{}_2F_1}{2}} \longrightarrow \text{correction factor to the Riemann invariants}$$

$$\boxed{W_2 = u + 4c\,\frac{\sqrt[4]{\frac{A}{A_c}}\;{}_2F_1}{2}}$$

VISCOUS TERM

$$V_z = c\,\mu\,\frac{(r-R)(kR_o - r)}{(R-kR_o)^2}$$

~~then~~

$$\cancel{\frac{f}{\rho} = \frac{2\mu}{\rho}\left(r\frac{\partial V_z}{\partial r}\right)\Big|_{kR_o}^{R} = \frac{2\mu}{\rho}\frac{c\,\mu}{(R-kR_o)^2}\left(R^2-(kR_o)^2\right)}$$

with this V_z

$$\alpha = \frac{1}{(R^2-k^2R_o^2)\mu^2}\int_{kR_o}^{R} 2r\,V_z^2\,dr = \frac{c^2}{30} \Rightarrow c=\sqrt{30}$$

for $\alpha=1$

then the viscous Term is

$$\frac{f}{\rho} = \frac{2\mu}{\rho}\left(r\frac{\partial V_z}{\partial r}\right)\Big|_{kR_o}^{R} = -\frac{2\mu}{\rho}\frac{\overbrace{R^2-(kR_o)^2}^{A-A_c}}{(R-kR_o)^2}\,c\,\mu\,\frac{\pi}{\pi} =$$

$$-\frac{2\mu}{\rho}\frac{\overbrace{(A-A_c)\,\mu}^{Q}\,c}{(R-kR_o)^2\pi} = -\frac{2\mu}{\rho}\,c\,\frac{Q}{(\sqrt{A}-\sqrt{A_c})^2}$$

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CONJUNCTION

$$
\begin{cases}
W_1 = u_1 + 4c_1 g_2 \\
W_2 = u_2 - 4c_2 g_2 \\
(A_1 - A_{c_1}) u_1 = (A_2 - A_{c_2}) u_2 \\
\beta_1 \left(\sqrt{\frac{A_1}{A_{0_1}}} - 1 \right) + \frac{1}{2} \rho u_1^2 = \beta_2 \left(\sqrt{\frac{A_2}{A_2}} - 1 \right) + \frac{1}{2} \rho u_2^2
\end{cases}
$$

g_i correction factor for the Riemann invariants of the i-th vessel

with

$$q = \left(u_1 \quad u_2 \quad A_1^{\frac{1}{4}} \quad A_2^{\frac{1}{4}} \right)$$

then the equations of the junction are

$$
\begin{cases}
q_1 + 4K_1 g_1 q_3 - W_1 = 0 \\
q_2 - 4K_2 g_2 q_4 - W_2 = 0 \\
(q_3^4 - A_{c_1}) q_1 - (q_4^4 - A_{c_2}) q_2 = 0 \\
\beta_1 \left(\frac{q_3^2}{\sqrt{A_{0_1}}} - 1 \right) + \frac{1}{2} \rho q_1^2 - \beta_2 \left(\frac{q_4^2}{\sqrt{A_{0_2}}} - 1 \right) - \frac{1}{2} \rho q_2^2 = 0
\end{cases}
$$

with $K_i = \sqrt{\frac{3}{2} \gamma_i}$ $\quad g_i = \sqrt{1 - \frac{A_c}{A_i}}$

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for the gradient note that

$$4K_1 \frac{\partial g_1 q_3}{\partial q_3} = 4K_1\left(g_1' q_3 + g_1\right)$$

$$g_1' = \frac{A_c}{2A_i^2\sqrt{1-\frac{A_c}{A}}}$$

therefore the gradient at the junction is

$$J_c = \begin{pmatrix} 1 & 0 & 4K_1\left(\frac{A_{c_1}}{2A_1\sqrt{1-\frac{A_{c_1}}{A}}} + \sqrt{1-\frac{A_{c_1}}{A}}\right) & 0 \\ & & & \\ & \cancel{-q_4^4 + A_{c_2}} & 0 & -4K_2\left(\frac{A_{c_2}}{2A_2 g_2} + g_2\right) \\ q_3^4 - A_{c_1} & -q_4^4 + A_{c_2} & \text{identical to A 3.93} & \end{pmatrix}$$

$\boxed{\text{identical to 3.93}}$

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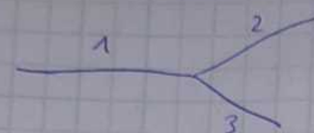
BIFURCATION

- characteristics 3
- continuity mass 1
- total pressure 2



$$q = \left(u_1 \; u_2 \; u_3 \; A_1^{\frac{1}{4}} \; A_2^{\frac{1}{4}} \; A_3^{\frac{1}{4}}\right) \qquad c_i = K_i q_{i+3}$$

the equations are

$$\begin{cases} q_1 - 4K_1 q_4 g_1 - W_1 = 0 \\ q_2 - 4K_2 q_5 g_2 - W_2 = 0 \\ q_3 - 4K_3 q_6 g_3 - W_3 = 0 \\ (q_4^4 - A_{c_1}) q_1 - (q_5^4 - A_{c_2}) q_2 - (q_6^4 - A_{c_3}) q_3 = 0 \\ \beta_1 \left(\frac{q_4^2}{\sqrt{A_{0,1}}} - 1\right) - \beta_2 \left(\frac{q_5^2}{\sqrt{A_{0,2}}} - 1\right) = 0 \\ \beta_1 \left(\frac{q_4^2}{\sqrt{A_{0,1}}} - 1\right) - \beta_3 \left(\frac{q_6^2}{\sqrt{A_{0,3}}} - 1\right) = 0 \end{cases}$$

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with gradient

$$J_c = \begin{pmatrix} 1 & 0 & 0 & 4K_1\left(\frac{A_{c1}}{2q_4 q_1} + q_1\right) & 0 & 0 \\ 0 & 1 & 0 & 0 & -4K_2\left(\frac{A_{c2}}{2q_5 q_2} + q_2\right) & 0 \\ 0 & 0 & 1 & 0 & 0 & -4K_3\left(\frac{A_{c3}}{2q_6 q_3} + q_3\right) \\ q_4^4 - A_{c1} & -q_5^4 + A_{c2} & -q_6^4 + A_{c3} & 4q_4^3 q_1 & -4q_5^3 q_2 & -4q_6^3 q_3 \end{pmatrix}$$

identical to 3.96

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