:mod: 'statistics' --- Mathematical statistics functions

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 1); backlink

Unknown interpreted text role "mod".

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 4)

Unknown directive type "module".

```
.. module:: statistics
:synopsis: Mathematical statistics functions
```

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 7)

Unknown directive type "moduleauthor".

.. moduleauthor:: Steven D'Aprano <steve+python@pearwood.info>

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Unknown directive type "sectionauthor".

.. sectionauthor:: Steven D'Aprano <steve+python@pearwood.info>

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Unknown directive type "versionadded".

.. versionadded:: 3.4

Source code: :source:`Lib/statistics.py`

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Unknown directive type "testsetup".

```
.. testsetup:: *
  from statistics import *
   __name__ = '<doctest>'
```

This module provides functions for calculating mathematical statistics of numeric (:class: ~numbers.Real'-valued) data.

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The module is not intended to be a competitor to third-party libraries such as NumPy, SciPy, or proprietary full-featured statistics packages aimed at professional statisticians such as Minitab, SAS and Matlab. It is aimed at the level of graphing and scientific

calculators.

Unless explicitly noted, these functions support 'class:'int', 'class:'float', 'class:'decimal.Decimal' and 'class:'fractions.Fraction'. Behaviour with other types (whether in the numeric tower or not) is currently unsupported. Collections with a mix of types are also undefined and implementation-dependent. If your input data consists of mixed types, you may be able to use :func:'map' to ensure a consistent result, for example: map(float, input data).

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Averages and measures of central location

These functions calculate an average or typical value from a population or sample.

:func:\mean\

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(cpython-main) (Doc)
(library) statistics.rst,
line 46); backlink

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Arithmetic mean ("average") of data.

:func:`fmean` **System Message:** ERROR/3 (D:\onboardingresources\sampleonboardingresources\cpython-Fast, floating point arithmetic mean, with optional weighting. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 47); backlink Unknown interpreted text role "func". :func:'geometric_mean' System Message: **ERROR/3** (D:\onboardingresources\sampleonboardingresources\cpython-Geometric mean of data. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 48); backlink Unknown interpreted text role "func". :func:`harmonic_mean` System Message: ERROR/3 (D:\onboardingresources\sampleonboardingresources\cpython-Harmonic mean of data. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 49); backlink Unknown interpreted text role "func". :func:\median\ System Message: **ERROR/3** (D:\onboardingresources\sampleonboardingresources\cpython-Median (middle value) of data. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 50); backlink Unknown interpreted text role "func".

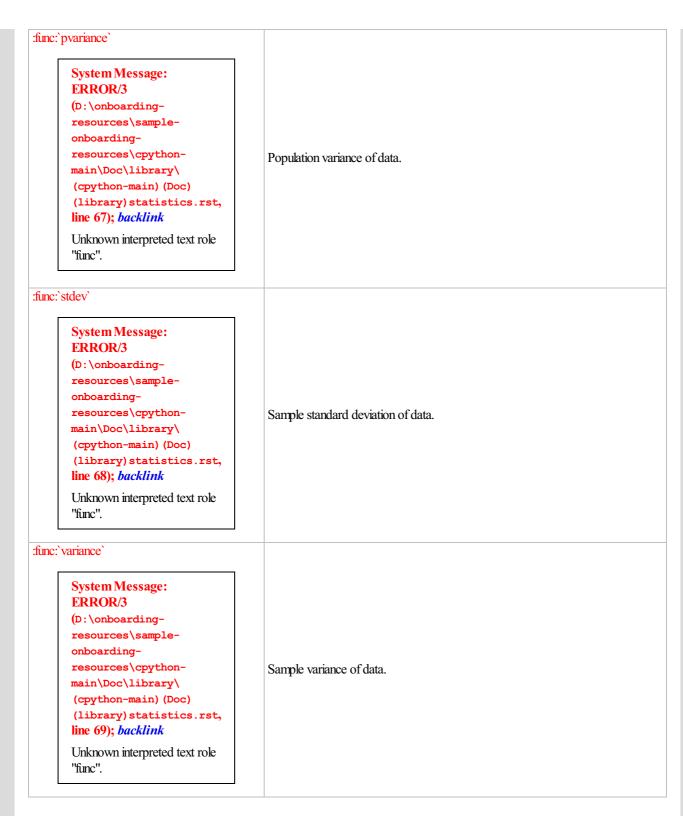
:func:\median_low\ System Message: ERROR/3 (D:\onboardingresources\sampleonboardingresources\cpython-Low median of data. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 51); backlink Unknown interpreted text role "func". :func:\median high\ System Message: ERROR/3 (D:\onboardingresources\sampleonboardingresources\cpython-High median of data. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 52); backlink Unknown interpreted text role "func". :func:\median_grouped\ System Message: ERROR/3 (D:\onboardingresources\sampleonboardingresources\cpython-Median, or 50th percentile, of grouped data. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 53); backlink Unknown interpreted text role "func". :func:\mode\ System Message: ERROR/3 (D:\onboardingresources\sampleonboardingresources\cpython-Single mode (most common value) of discrete or nominal data. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 54); backlink Unknown interpreted text role "func".

:func:\multimode\ System Message: ERROR/3 (D:\onboardingresources\sampleonboardingresources\cpython-List of modes (most common values) of discrete or nominal data. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 55); backlink Unknown interpreted text role "func". :func:'quantiles' System Message: ERROR/3 (D:\onboardingresources\sampleonboarding-Divide data into intervals with equal probability. resources\cpythonmain\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 56); backlink Unknown interpreted text role "func".

Measures of spread

These functions calculate a measure of how much the population or sample tends to deviate from the typical or average values.

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(cpython-main) (Doc)
(library) statistics.rst,
line 66); backlink
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"fimc".



Statistics for relations between two inputs

These functions calculate statistics regarding relations between two inputs.

:func:`covariance` System Message: ERROR/3 (D:\onboardingresources\sampleonboardingresources\cpython-Sample covariance for two variables. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 78); backlink Unknown interpreted text role "func". :func:`correlation` **System Message:** ERROR/3 (D:\onboardingresources\sampleonboardingresources\cpython-Pearson's correlation coefficient for two variables. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 79); backlink Unknown interpreted text role "func". :func:\linear_regression\ System Message: ERROR/3 (D:\onboardingresources\sampleonboardingresources\cpython-Slope and intercept for simple linear regression. main\Doc\library\ (cpython-main) (Doc) (library) statistics.rst, line 80); backlink Unknown interpreted text role "func".

Function details

the data.

Note: The functions do not require the data given to them to be sorted. However, for reading convenience, most of the examples show sorted sequences.

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 89)

Unknown directive type "function".

.. function:: mean(data)

Return the sample arithmetic mean of *data* which can be a sequence or iterable.

The arithmetic mean is the sum of the data divided by the number of data points. It is commonly called "the average", although it is only one of many different mathematical averages. It is a measure of the central location of
```

```
If *data* is empty, :exc:`StatisticsError` will be raised.
Some examples of use:
.. doctest::
  >>> mean([1, 2, 3, 4, 4])
  2.8
  >>> mean([-1.0, 2.5, 3.25, 5.75])
  2.625
  >>> from fractions import Fraction as F
   >>> mean([F(3, 7), F(1, 21), F(5, 3), F(1, 3)])
  Fraction(13, 21)
  >>> from decimal import Decimal as D
   >>> mean([D("0.5"), D("0.75"), D("0.625"), D("0.375")])
  Decimal('0.5625')
.. note::
  The mean is strongly affected by `outliers
  <a href="https://en.wikipedia.org/wiki/Outlier"> \_ and is not necessarily a
   typical example of the data points. For a more robust, although less
   efficient, measure of `central tendency
  <https://en.wikipedia.org/wiki/Central tendency>` , see :func:`median`.
  The sample mean gives an unbiased estimate of the true population mean,
   so that when taken on average over all the possible samples,
   ``mean(sample)`` converges on the true mean of the entire population. If
   *data* represents the entire population rather than a sample, then
   ``mean(data)`` is equivalent to calculating the true population mean μ.
```

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```
Unknown directive type "function".
   .. function:: fmean(data, weights=None)
      Convert *data* to floats and compute the arithmetic mean.
      This runs faster than the :func:`mean` function and it always returns a
      :class:`float`. The *data* may be a sequence or iterable. If the input
      dataset is empty, raises a :exc:`StatisticsError`.
      .. doctest::
         >>> fmean([3.5, 4.0, 5.25])
         4.25
      Optional weighting is supported. For example, a professor assigns a
      grade for a course by weighting quizzes at 20%, homework at 20%, a
      midterm exam at 30%, and a final exam at 30%:
      .. doctest::
         >>> grades = [85, 92, 83, 91]
         >>> weights = [0.20, 0.20, 0.30, 0.30]
         >>> fmean(grades, weights)
      If *weights* is supplied, it must be the same length as the *data* or
      a :exc: ValueError` will be raised.
      .. versionadded:: 3.8
      .. versionchanged:: 3.11
         Added support for *weights*.
```

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```
Convert *data* to floats and compute the geometric mean.
      The geometric mean indicates the central tendency or typical value of the
      *data* using the product of the values (as opposed to the arithmetic mean
      which uses their sum).
      Raises a :exc:`StatisticsError` if the input dataset is empty,
      if it contains a zero, or if it contains a negative value.
      The *data* may be a sequence or iterable.
      No special efforts are made to achieve exact results.
      (However, this may change in the future.)
      .. doctest::
         >>> round(geometric mean([54, 24, 36]), 1)
         36.0
      .. versionadded:: 3.8
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-
main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 188)
Unknown directive type "function".
   .. function:: harmonic mean(data, weights=None)
      Return the harmonic mean of *data*, a sequence or iterable of
      real-valued numbers. If *weights* is omitted or *None*, then
      equal weighting is assumed.
      The harmonic mean is the reciprocal of the arithmetic :func:`mean` of the
      reciprocals of the data. For example, the harmonic mean of three values {}^{\star}a^{\star},
      *b* and *c* will be equivalent to ``3/(1/a + 1/b + 1/c)``. If one of the
      values is zero, the result will be zero.
      The harmonic mean is a type of average, a measure of the central
      location of the data. It is often appropriate when averaging
      ratios or rates, for example speeds.
      Suppose a car travels 10 km at 40 km/hr, then another 10 km at 60 km/hr.
      What is the average speed?
      .. doctest::
         >>> harmonic mean([40, 60])
         48.0
      Suppose a car travels 40 km/hr for 5 km, and when traffic clears,
      speeds-up to 60 km/hr for the remaining 30 km of the journey. What
      is the average speed?
      .. doctest::
         >>> harmonic mean([40, 60], weights=[5, 30])
         56.0
      :exc:`StatisticsError` is raised if *data* is empty, any element
      is less than zero, or if the weighted sum isn't positive.
      The current algorithm has an early-out when it encounters a zero
      in the input. This means that the subsequent inputs are not tested
      for validity. (This behavior may change in the future.)
      .. versionadded:: 3.6
      .. versionchanged:: 3.10
         Added support for *weights*.
```

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 232)

Unknown directive type "function".

.. function:: median(data)

.. function:: geometric mean(data)

Return the median (middle value) of numeric data, using the common "mean of middle two" method. If *data* is empty, :exc:`StatisticsError` is raised. *data* can be a sequence or iterable.

The median is a robust measure of central location and is less affected by the presence of outliers. When the number of data points is odd, the middle data point is returned:

```
.. doctest::

>>> median([1, 3, 5])
```

When the number of data points is even, the median is interpolated by taking the average of the two middle values:

.. doctest::

>>> median([1, 3, 5, 7])
4.0

This is suited for when your data is discrete, and you don't mind that the median may not be an actual data point.

If the data is ordinal (supports order operations) but not numeric (doesn't support addition), consider using :func:`median_low` or :func:`median_high` instead.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 262)

Unknown directive type "function".

.. function:: median_low(data)

Return the low median of numeric data. If *data* is empty, :exc:`StatisticsError` is raised. *data* can be a sequence or iterable.

The low median is always a member of the data set. When the number of data points is odd, the middle value is returned. When it is even, the smaller of the two middle values is returned.

.. doctest::

```
>>> median_low([1, 3, 5])
3
>>> median_low([1, 3, 5, 7])
3
```

Use the low median when your data are discrete and you prefer the median to be an actual data point rather than interpolated.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 282)

Unknown directive type "function".

.. function:: median_high(data)

Return the high median of data. If *data* is empty, :exc:`StatisticsError` is raised. *data* can be a sequence or iterable.

The high median is always a member of the data set. When the number of data points is odd, the middle value is returned. When it is even, the larger of the two middle values is returned.

.. doctest::

```
>>> median_high([1, 3, 5])
3
>>> median_high([1, 3, 5, 7])
5
```

Use the high median when your data are discrete and you prefer the median to be an actual data point rather than interpolated.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 302)

Unknown directive type "function".

.. function:: median grouped(data, interval=1)

Return the median of grouped continuous data, calculated as the 50th percentile, using interpolation. If *data* is empty, :exc:`StatisticsError` is raised. *data* can be a sequence or iterable.

.. doctest::

```
>>> median_grouped([52, 52, 53, 54])
52.5
```

In the following example, the data are rounded, so that each value represents the midpoint of data classes, e.g. 1 is the midpoint of the class 0.5--1.5, 2 is the midpoint of 1.5--2.5, 3 is the midpoint of 2.5--3.5, etc. With the data given, the middle value falls somewhere in the class 3.5--4.5, and interpolation is used to estimate it:

.. doctest::

```
>>> median_grouped([1, 2, 2, 3, 4, 4, 4, 4, 4, 5])
3.7
```

Optional argument *interval* represents the class interval, and defaults to 1. Changing the class interval naturally will change the interpolation:

.. doctest::

```
>>> median_grouped([1, 3, 3, 5, 7], interval=1)
3.25
>>> median_grouped([1, 3, 3, 5, 7], interval=2)
3.5
```

This function does not check whether the data points are at least *interval* apart.

.. impl-detail::

Under some circumstances, :func:`median_grouped` may coerce data points to floats. This behaviour is likely to change in the future.

- .. seealso::
 - * "Statistics for the Behavioral Sciences", Frederick J Gravetter and Larry B Wallnau (8th Edition).
 - * The `SSMEDIAN
 https://help.gnome.org/users/gnumeric/stable/gnumeric.html#gnumeric-function-\$SMEDIAN>`function in the Gnome Gnumeric spreadsheet, including `this discussion https://mail.gnome.org/archives/gnumeric-list/2011-April/msg00018.html>`.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 353)

Unknown directive type "function".

.. function:: mode(data)

Return the single most common data point from discrete or nominal *data*. The mode (when it exists) is the most typical value and serves as a measure of central location.

If there are multiple modes with the same frequency, returns the first one encountered in the *data*. If the smallest or largest of those is desired instead, use ``min(multimode(data))`` or ``max(multimode(data))``. If the input *data* is empty, :exc:`StatisticsError` is raised.

``mode`` assumes discrete data and returns a single value. This is the standard treatment of the mode as commonly taught in schools:

.. doctest::

```
>>> mode([1, 1, 2, 3, 3, 3, 4])
```

The mode is unique in that it is the only statistic in this package that also applies to nominal (non-numeric) data:

.. doctest::

```
>>> mode(["red", "blue", "blue", "red", "green", "red", "red"])
'red'
```

.. versionchanged:: 3.8

Now handles multimodal datasets by returning the first mode encountered. Formerly, it raised :exc:`StatisticsError` when more than one mode was found.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 386)

Unknown directive type "function".

.. function:: multimode(data)

Return a list of the most frequently occurring values in the order they were first encountered in the *data*. Will return more than one result if there are multiple modes or an empty list if the *data* is empty:

.. doctest::

```
>>> multimode('aabbbbccddddeeffffgg')
['b', 'd', 'f']
>>> multimode('')
[]
```

.. versionadded:: 3.8

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 402)

Unknown directive type "function".

.. function:: pstdev(data, mu=None)

Return the population standard deviation (the square root of the population variance). See :func:`pvariance` for arguments and other details.

.. doctest::

```
>>> pstdev([1.5, 2.5, 2.5, 2.75, 3.25, 4.75])
0.986893273527251
```

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 413)

Unknown directive type "function".

.. function:: pvariance(data, mu=None)

Return the population variance of *data*, a non-empty sequence or iterable of real-valued numbers. Variance, or second moment about the mean, is a measure of the variability (spread or dispersion) of data. A large variance indicates that the data is spread out; a small variance indicates it is clustered closely around the mean.

If the optional second argument *mu* is given, it is typically the mean of the *data*. It can also be used to compute the second moment around a point that is not the mean. If it is missing or ``None`` (the default), the arithmetic mean is automatically calculated.

Use this function to calculate the variance from the entire population. To estimate the variance from a sample, the :func:`variance` function is usually a better choice.

```
Raises :exc:`StatisticsError` if *data* is empty.
Examples:
.. doctest::
   >>> data = [0.0, 0.25, 0.25, 1.25, 1.5, 1.75, 2.75, 3.25]
   >>> pvariance(data)
If you have already calculated the mean of your data, you can pass it as the
optional second argument *mu* to avoid recalculation:
.. doctest::
   >>> mu = mean(data)
  >>> pvariance(data, mu)
   1.25
Decimals and Fractions are supported:
.. doctest::
   >>> from decimal import Decimal as D
   >>> pvariance([D("27.5"), D("30.25"), D("30.25"), D("34.5"), D("41.75")])
  Decimal('24.815')
   >>> from fractions import Fraction as F
   >>> pvariance([F(1, 4), F(5, 4), F(1, 2)])
   Fraction(13, 72)
.. note::
   When called with the entire population, this gives the population variance
   \hat{I}f\hat{A}^2. When called on a sample instead, this is the biased sample variance
   s\hat{A}^{2}\text{,} also known as variance with N degrees of freedom.
   If you somehow know the true population mean 14, you may use this
   function to calculate the variance of a sample, giving the known
   population mean as the second argument. Provided the data points are a
   random sample of the population, the result will be an unbiased estimate
   of the population variance.
```

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 474)

Unknown directive type "function".

```
.. function:: stdev(data, xbar=None)
```

Return the sample standard deviation (the square root of the sample variance). See :func:`variance` for arguments and other details.

.. doctest::

```
>>> stdev([1.5, 2.5, 2.5, 2.75, 3.25, 4.75])
1.0810874155219827
```

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 485)

Unknown directive type "function".

.. function:: variance(data, xbar=None)

Return the sample variance of *data*, an iterable of at least two real-valued numbers. Variance, or second moment about the mean, is a measure of the variability (spread or dispersion) of data. A large variance indicates that the data is spread out; a small variance indicates it is clustered closely around the mean.

If the optional second argument *xbar* is given, it should be the mean of *data*. If it is missing or ``None`` (the default), the mean is automatically calculated.

Use this function when your data is a sample from a population. To calculate

the variance from the entire population, see :func:`pvariance`.

Raises :exc:`StatisticsError` if *data* has fewer than two values.

Examples:

.. doctest::

```
>>> data = [2.75, 1.75, 1.25, 0.25, 0.5, 1.25, 3.5] 
>>> variance(data) 
1.3720238095238095
```

If you have already calculated the mean of your data, you can pass it as the optional second argument *xbar* to avoid recalculation:

.. doctest::

```
>>> m = mean(data)
>>> variance(data, m)
1.3720238095238095
```

This function does not attempt to verify that you have passed the actual mean as *xbar*. Using arbitrary values for *xbar* can lead to invalid or impossible results.

Decimal and Fraction values are supported:

.. doctest::

```
>>> from decimal import Decimal as D
>>> variance([D("27.5"), D("30.25"), D("30.25"), D("34.5"), D("41.75")])
Decimal('31.01875')
>>> from fractions import Fraction as F
>>> variance([F(1, 6), F(1, 2), F(5, 3)])
Fraction(67, 108)
```

.. note::

This is the sample variance $s\hat{A}^2$ with Bessel's correction, also known as variance with N-1 degrees of freedom. Provided that the data points are representative (e.g. independent and identically distributed), the result should be an unbiased estimate of the true population variance.

If you somehow know the actual population mean $\hat{1}^{1}_{4}$ you should pass it to the :func:`pvariance` function as the *mu* parameter to get the variance of a sample.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 546)

Unknown directive type "function".

.. function:: quantiles(data, *, n=4, method='exclusive')

Divide *data* into *n* continuous intervals with equal probability. Returns a list of ``n - 1`` cut points separating the intervals.

Set *n* to 4 for quartiles (the default). Set *n* to 10 for deciles. Set *n* to 100 for percentiles which gives the 99 cuts points that separate *data* into 100 equal sized groups. Raises :exc:`StatisticsError` if *n* is not least 1.

The *data* can be any iterable containing sample data. For meaningful results, the number of data points in *data* should be larger than *n*. Raises :exc:`StatisticsError` if there are not at least two data points.

The cut points are linearly interpolated from the two nearest data points. For example, if a cut point falls one-third of the distance between two sample values, ``100`` and ``112``, the cut-point will evaluate to ``104``.

The *method* for computing quantiles can be varied depending on whether the *data* includes or excludes the lowest and highest possible values from the population.

The default *method* is "exclusive" and is used for data sampled from a population that can have more extreme values than found in the samples. The portion of the population falling below the *i-th* of *m* sorted data points is computed as ``i / (m + 1)``. Given nine

```
sample values, the method sorts them and assigns the following
percentiles: 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%.
Setting the *method* to "inclusive" is used for describing population
data or for samples that are known to include the most extreme values
from the population. The minimum value in *data* is treated as the Oth
percentile and the maximum value is treated as the 100th percentile.
The portion of the population falling below the *i-th* of *m* sorted data points is computed as ``(i - 1) / (m - 1)``. Given 11 sample
values, the method sorts them and assigns the following percentiles:
0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, 100%.
.. doctest::
     # Decile cut points for empirically sampled data
     >>> data = [105, 129, 87, 86, 111, 111, 89, 81, 108, 92, 110,
                   100, 75, 105, 103, 109, 76, 119, 99, 91, 103, 129, 106, 101, 84, 111, 74, 87, 86, 103, 103, 106, 86,
     . . .
                   111, 75, 87, 102, 121, 111, 88, 89, 101, 106, 95,
     . . .
                   103, 107, 101, 81, 109, 104]
     >>> [round(q, 1) for q in quantiles(data, n=10)]
     [81.0, 86.2, 89.0, 99.4, 102.5, 103.6, 106.0, 109.8, 111.0]
```

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 598)

Unknown directive type "function".

```
.. function:: covariance(x, y, /)
```

.. versionadded:: 3.8

Return the sample covariance of two inputs *x* and *y*. Covariance is a measure of the joint variability of two inputs.

Both inputs must be of the same length (no less than two), otherwise :exc:`StatisticsError` is raised.

Examples:

.. doctest::

```
>>> x = [1, 2, 3, 4, 5, 6, 7, 8, 9]

>>> y = [1, 2, 3, 1, 2, 3, 1, 2, 3]

>>> covariance(x, y)

0.75

>>> z = [9, 8, 7, 6, 5, 4, 3, 2, 1]

>>> covariance(x, z)

-7.5

>>> covariance(z, x)
```

.. versionadded:: 3.10

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 622)

Unknown directive type "function".

```
.. function:: correlation(x, y, /)
```

Return the `Pearson's correlation coefficient
<https://en.wikipedia.org/wiki/Pearson_correlation_coefficient>`_
for two inputs. Pearson's correlation coefficient *r* takes values
between -1 and +1. It measures the strength and direction of the linear
relationship, where +1 means very strong, positive linear relationship,
-1 very strong, negative linear relationship, and 0 no linear relationship.

Both inputs must be of the same length (no less than two), and need not to be constant, otherwise :exc:`StatisticsError` is raised.

Examples:

.. doctest::

```
>>> x = [1, 2, 3, 4, 5, 6, 7, 8, 9]
>>> y = [9, 8, 7, 6, 5, 4, 3, 2, 1]
>>> correlation(x, x)
```

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-
main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 647)
Unknown directive type "function".
   .. function:: linear_regression(x, y, /, *, proportional=False)
      Return the slope and intercept of `simple linear regression
      <https://en.wikipedia.org/wiki/Simple_linear_regression>`
      parameters estimated using ordinary least squares. Simple linear
      regression describes the relationship between an independent variable *x* and
      a dependent variable *y* in terms of this linear function:
          where ``slope`` and ``intercept`` are the regression parameters that are estimated, and ``noise`` represents the
      variability of the data that was not explained by the linear regression
      (it is equal to the difference between predicted and actual values
      of the dependent variable).
      Both inputs must be of the same length (no less than two), and
      the independent variable *x* cannot be constant;
      otherwise a :exc:`StatisticsError` is raised.
      For example, we can use the `release dates of the Monty
      Python films <a href="https://en.wikipedia.org/wiki/Monty">https://en.wikipedia.org/wiki/Monty</a> Python#Films>`
      to predict the cumulative number of Monty Python films
      that would have been produced by 2019
      assuming that they had kept the pace.
      .. doctest::
         >>> year = [1971, 1975, 1979, 1982, 1983]
         >>> \overline{\text{films total}} = [1, 2, 3, 4, 5]
         >>> slope, intercept = linear regression(year, films total)
         >>> round(slope * 2019 + intercept)
      If *proportional* is true, the independent variable *x* and the
      dependent variable *y* are assumed to be directly proportional.
      The data is fit to a line passing through the origin.
      Since the *intercept* will always be 0.0, the underlying linear \left( \frac{1}{2} \right)
      function simplifies to:
          *y = slope \  \  \  \  \  \  \  \   + noise*
      .. versionadded:: 3.10
      .. versionchanged:: 3.11
         Added support for *proportional*.
```

Exceptions

A single exception is defined:

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 699)
```

Unknown directive type "exception".

1.0

-1.0

>>> correlation(x, y)

.. versionadded:: 3.10

```
.. exception:: StatisticsError

Subclass of :exc:`ValueError` for statistics-related exceptions.
```

:class:'NormalDist' objects

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 704); backlink

Unknown interpreted text role "class".

class: 'NormalDist' is a tool for creating and manipulating normal distributions of a random variable. It is a class that treats the mean and standard deviation of data measurements as a single entity.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 707); backlink

Unknown interpreted text role "class".

Normal distributions arise from the Central Limit Theorem and have a wide range of applications in statistics.

Returns a new NormalDist object where mu represents the arithmetic mean and sigma represents the standard deviation.

If *sigma* is negative, raises :exc: StatisticsError.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 724); backlink

Unknown interpreted text role "exc".

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 726)

Unknown directive type "attribute".

distribution.

```
.. attribute:: mean
A read-only property for the `arithmetic mean
    <a href="https://en.wikipedia.org/wiki/Arithmetic_mean">https://en.wikipedia.org/wiki/Arithmetic_mean</a> of a normal
```

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 732)

Unknown directive type "attribute".

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 738)

Unknown directive type "attribute".

```
.. attribute:: mode
   A read-only property for the `mode
   <https://en.wikipedia.org/wiki/Mode_(statistics)>`_ of a normal
   distribution.
```

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 744)

Unknown directive type "attribute".

main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 750)

Unknown directive type "attribute".

.. attribute:: variance

A read-only property for the `variance https://en.wikipedia.org/wiki/Variance of a normal distribution. Equal to the square of the standard deviation.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 756)

Unknown directive type "classmethod".

.. classmethod:: NormalDist.from samples(data)

Makes a normal distribution instance with *mu* and *sigma* parameters estimated from the *data* using :func:`fmean` and :func:`stdev`.

The *data* can be any :term:`iterable` and should consist of values that can be converted to type :class:`float`. If *data* does not contain at least two elements, raises :exc:`StatisticsError` because it takes at least one point to estimate a central value and at least two points to estimate dispersion.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 767)

Unknown directive type "method".

.. method:: NormalDist.samples(n, *, seed=None)

Generates *n* random samples for a given mean and standard deviation. Returns a :class:`list` of :class:`float` values.

If *seed* is given, creates a new instance of the underlying random number generator. This is useful for creating reproducible results, even in a multi-threading context.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 776)

Unknown directive type "method".

.. method:: NormalDist.pdf(x)

Using a `probability density function (pdf) `_, compute the relative likelihood that a random variable *X* will be near the given value *x*. Mathematically, it is the limit of the ratio ``P(x <= X < x+dx) / dx`` as *dx* approaches zero.

The relative likelihood is computed as the probability of a sample occurring in a narrow range divided by the width of the range (hence the word "density"). Since the likelihood is relative to other points, its value can be greater than `1.0`.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 789)

Unknown directive type "method".

.. method:: NormalDist.cdf(x)

Using a `cumulative distribution function (cdf) `_, compute the probability that a random variable *X* will be less than or equal to *x*. Mathematically, it is written ``P(X <= x)``.

main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 796)

Unknown directive type "method".

```
.. method:: NormalDist.inv_cdf(p)

Compute the inverse cumulative distribution function, also known as the 
  `quantile function <a href="https://en.wikipedia.org/wiki/Quantile_function">
    or the `percent-point
  <a href="https://www.statisticshowto.datasciencecentral.com/inverse-distribution-function/">
  function. Mathematically, it is written ``x : P(X <= x) = p``.

Finds the value *x* of the random variable *X* such that the 
  probability of the variable being less than or equal to that value 
  equals the given probability *p*.</pre>
```

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 808)

Unknown directive type "method".

```
.. method:: NormalDist.overlap(other)
```

Measures the agreement between two normal probability distributions. Returns a value between 0.0 and 1.0 giving `the overlapping area for the two probability density functions $< \frac{\text{https://www.rasch.org/rmt/rmt101r.htm}}{}$.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 815)

Unknown directive type "method".

```
.. method:: NormalDist.quantiles(n=4)
```

Divide the normal distribution into *n* continuous intervals with equal probability. Returns a list of (n-1) cut points separating the intervals.

Set *n* to 4 for quartiles (the default). Set *n* to 10 for deciles. Set *n* to 100 for percentiles which gives the 99 cuts points that separate the normal distribution into 100 equal sized groups.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 825)

Unknown directive type "method".

```
.. method:: NormalDist.zscore(x)

Compute the
   `Standard Score <a href="https://www.statisticshowto.com/probability-and-statistics/z-score/">https://www.statisticshowto.com/probability-and-statistics/z-score/</a>
   describing *x* in terms of the number of standard deviations
   above or below the mean of the normal distribution:
   ``(x - mean) / stdev``.
   .. versionadded:: 3.9
```

Instances of class: NormalDist' support addition, subtraction, multiplication and division by a constant. These operations are used for translation and scaling. For example:

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 835); backlink

Unknown interpreted text role "class".

 $System\,Message: ERROR/3~(\texttt{D:\onboarding-resources\sample-onboarding-resources\cpython-main\scalebox{\colored})} \label{eq:conboarding-resources\sample-onboarding-resources\cpython-main\scalebox{\colored})} In the extension of the state of the state$

Unknown directive type "doctest".

.. doctest::

```
>>> temperature_february = NormalDist(5, 2.5)  # Celsius
>>> temperature_february * (9/5) + 32  # Fahrenheit
NormalDist(mu=41.0, sigma=4.5)
```

Dividing a constant by an instance of class: NormalDist' is not supported because the result wouldn't be normally distributed.

```
System\,Message: ERROR/3~(\texttt{D:\noboarding-resources\scample-onboarding-resources\cpython-main\noc\library\cpython-main)}~(\texttt{Doc})~(\texttt{library})~statistics.rst, line~845); \\ \textit{backlink}
```

Unknown interpreted text role "class".

Since normal distributions arise from additive effects of independent variables, it is possible to add and subtract two independent normally distributed random variables represented as instances of class: NormalDist'. For example:

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 848); backlink

Unknown interpreted text role "class".

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 854)

Unknown directive type "doctest".

```
.. doctest::

>>> birth_weights = NormalDist.from_samples([2.5, 3.1, 2.1, 2.4, 2.7, 3.5])
>>> drug_effects = NormalDist(0.4, 0.15)
>>> combined = birth_weights + drug_effects
>>> round(combined.mean, 1)
3.1
>>> round(combined.stdev, 1)
0.5
```

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 864)

Unknown directive type "versionadded".

```
.. versionadded:: 3.8
```

:class:'NormalDist' Examples and Recipes

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 867); backlink

Unknown interpreted text role "class".

:class: NormalDist' readily solves classic probability problems.

System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 870); backlink

Unknown interpreted text role "class".

For example, given historical data for SAT exams showing that scores are normally distributed with a mean of 1060 and a standard deviation of 195, determine the percentage of students with test scores between 1100 and 1200, after rounding to the nearest whole number:

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 878)
```

Unknown directive type "doctest".

```
.. doctest::

>>> sat = NormalDist(1060, 195)
```

```
>>> fraction = sat.cdf(1200 + 0.5) - sat.cdf(1100 - 0.5)
>>> round(fraction * 100.0, 1)
18.4
```

Find the quartiles and deciles for the SAT scores:

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-
main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 888)

Unknown directive type "doctest".

.. doctest::

>>> list(map(round, sat.quantiles()))
[928, 1060, 1192]
>>> list(map(round, sat.quantiles(n=10)))
[810, 896, 958, 1011, 1060, 1109, 1162, 1224, 1310]
```

To estimate the distribution for a model than isn't easy to solve analytically, "class: NormalDist" can generate input samples for a Monte Carlo simulation:

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 895); backlink
Unknown interpreted text role "class".
```

Normal distributions can be used to approximate Binomial distributions when the sample size is large and when the probability of a successful trial is near 50%.

For example, an open source conference has 750 attendees and two rooms with a 500 person capacity. There is a talk about Python and another about Ruby. In previous conferences, 65% of the attendees preferred to listen to Python talks. Assuming the population preferences haven't changed, what is the probability that the Python room will stay within its capacity limits?

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-
main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 922)
Unknown directive type "doctest".
   .. doctest::
       >>> n = 750
                               # Sample size
      p = 0.65
                               # Preference for Python
       >>> q = 1.0 - p
                              # Preference for Rubv
      >>> k = 500
                               # Room capacity
       >>> # Approximation using the cumulative normal distribution
       >>> from math import sqrt
       >>> round(NormalDist(mu=n*p, sigma=sqrt(n*p*q)).cdf(k + 0.5), 4)
       0.8402
       >>> # Solution using the cumulative binomial distribution
       >>> from math import comb, fsum
       >>> round(fsum(comb(n, r) * p**r * q**(n-r) for r in range(k+1)), 4)
       0.8402
       >>> # Approximation using a simulation
```

```
>>> from random import seed, choices
>>> seed(8675309)
>>> def trial():
... return choices(('Python', 'Ruby'), (p, q), k=n).count('Python')
>>> mean(trial() <= k for i in range(10_000))
0.8398</pre>
```

Normal distributions commonly arise in machine learning problems.

Wikipedia has a nice example of a Naive Bayesian Classifier. The challenge is to predict a person's gender from measurements of normally distributed features including height, weight, and foot size.

We're given a training dataset with measurements for eight people. The measurements are assumed to be normally distributed, so we summarize the data with class: 'NormalDist':

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 954); backlink
Unknown interpreted text role "class".
```

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-
main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 958)

Unknown directive type "doctest".

.. doctest::

>>> height_male = NormalDist.from_samples([6, 5.92, 5.58, 5.92])
>>> height_female = NormalDist.from_samples([5, 5.5, 5.42, 5.75])
>>> weight_male = NormalDist.from_samples([180, 190, 170, 165])
>>> weight_female = NormalDist.from_samples([100, 150, 130, 150])
>>> foot_size_male = NormalDist.from_samples([12, 11, 12, 10])
>>> foot_size_female = NormalDist.from_samples([6, 8, 7, 9])
```

Next, we encounter a new person whose feature measurements are known but whose gender is unknown:

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 970)

Unknown directive type "doctest".

.. doctest::

>>> ht = 6.0  # height
>>> wt = 130  # weight
>>> fs = 8  # foot size
```

Starting with a 50% prior probability of being male or female, we compute the posterior as the prior times the product of likelihoods for the feature measurements given the gender:

```
System Message: ERROR/3 (p:\onboarding-resources\sample-onboarding-resources\cpython-
main\Doc\library\(cpython-main) (Doc) (library) statistics.rst, line 981)

Unknown directive type "doctest".

... doctest::

>>> prior_male = 0.5
>>> prior_female = 0.5
>>> posterior_male = (prior_male * height_male.pdf(ht) *
... weight_male.pdf(wt) * foot_size_male.pdf(fs))

>>> posterior_female = (prior_female * height_female.pdf(ht) *
... weight_female.pdf(wt) * foot_size_female.pdf(fs))
```

The final prediction goes to the largest posterior. This is known as the maximum a posterior or MAP:

```
System Message: ERROR/3 (D:\onboarding-resources\sample-onboarding-resources\cpython-main\Doc\library\(cpython-main\) (Doc) (library) statistics.rst, line 995)
```

Unknown directive type "doctest".

```
.. doctest::
>>> 'male' if posterior_male > posterior_female else 'female'
'female'
```