Example Problem W-2

Aqueous Flow to a Well in a Confined Multi-Layer System

Abstract: *This test case illustrates flow to a well in a confined multi-layer system, where two identical aquifers (upper and lower) are separated by an aquitard. The well produces only from the lower aquifer, where it is fully penetrating. This problem is known in the literature as the leaky aquifer problem. The user is introduced to a two-dimensional domain, a cylindrical coordinate system, and Neumann boundary conditions.*

Problem Description

Two identical aquifers, confined, horizontal, homogeneous, isotropic, 5 m thick, with hydraulic conductivity of 2×10^{-5} m/s and a storage coefficient of 10^{-4} m^{-1}, are separated by an aquitard. The aquitard has a thickness of 10 m, a hydraulic conductivity of 10^{-8} m/s, and a storage coefficient of 8×10^{-4} m^{-1}. Figure 1 shows a sketch of the conceptual model. A pumping well penetrates through to the lower aquifer. The well is only open to the full thickness of the lower aquifer, and it is sealed to the aquitard and upper aquifer. In addition, it is assumed the well is of small diameter, and there are no head losses due to well construction. The well is pumped at a constant rate Q, for $t > 0$. Uniform head prevails over the entire domain at $t = 0$, i.e. zero drawdown, and no drawdown is allowed at a radial distance of 10,000 m for $t > 0$.
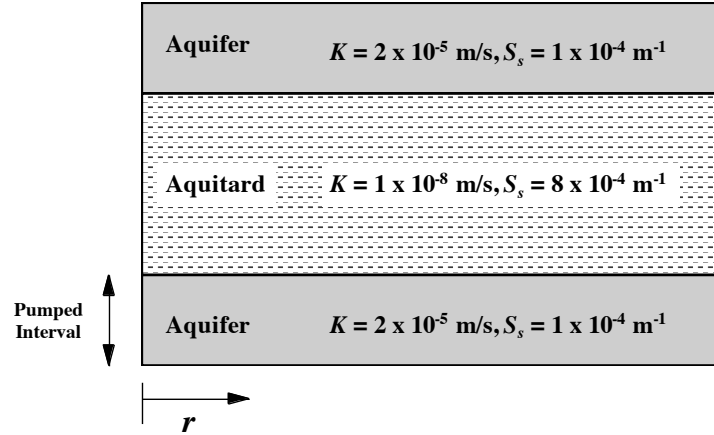
Figure 1. Schematic sketch of two-aquifer problem

The governing equation for transient flow in radial coordinates is

$$\frac{S_s}{K}\frac{\partial h}{\partial t} - \frac{\partial^2 h}{\partial r^2} - \frac{1}{r}\frac{\partial h}{\partial r} = 0 \tag{1}$$

where S_s, K, h, t, and r, are the specific storage coefficient, hydraulic conductivity, hydraulic head, time, and radial distance from the well, respectively.

A STOMP simulation with a duration of 100 years, an initial time-step of 36 s, and a time-step growth factor of 1.414 is shown in the input file. The simulation was started with uniform head initial conditions (hydrostatic pressure). Pumping at the well was implemented via a Neumann boundary condition, while a Dirichlet boundary condition was prescribed at r = 10,000 m. The *Boundary Conditions Card* reads:

```
#-------------------------------------------------------------------------------
~Boundary Conditions Card
#-------------------------------------------------------------------------------
2,
west,neumann,
1,1,1,1,1,5,1,
0,d,-5.02929620e-05,m/s,
east,hydraulic gradient,
13,13,1,1,1,20,1,
0,d,684036.8,Pa,
```

The initial conditions of the problem were such that a uniform aqueous phase gage head of 60 m (70.346 m absolute head) of water was imposed on the system with the reference elevation being the base of the domain. To obtain this initial distribution, the following *Initial Conditions Card* was imposed:

```
#-------------------------------------------------------------------------------
~Initial Conditions Card
#-------------------------------------------------------------------------------
Gas Pressure,Aqueous Pressure,
1,
Aqueous Pressure, 684036.8,Pa,,,,,-9793.5,1/m,1,13,1,1,1,20,
```

Simulation of this problem requires a closely spaced grid near the well, in order to accurately represent the steep head gradient caused by pumping. Thus, a grid with progressively finer spacing close to the well is used in the radial direction. Uniform grid spacing is used to discretize the vertical direction. The resulting grid has 13 nodes in the radial direction and 20 nodes in the vertical direction. The radial symmetry of the problem allows the use of the cylindrical grid feature of STOMP. An 18° arc is used for the third dimension. Since the third dimension is not active, this angle is arbitrary. The *Grid Card* for this problem is:

```
#-------------------------------------------------------------------------------
~Grid Card
#-------------------------------------------------------------------------------
Cylindrical,
13,1,20,
1.0,m,1.195,m,2.365,m,3.955,m,7.285,m,12.715,m,22.885,m,40.315,m,72.085,m,127.915,m,
504.485,m,1495.515,m,4828.485,m,10000,m,
0.,deg,18.,deg,
0,m,20@1.0,m,
```

Exercises

1. Explain how the initial aqueous pressure was determined.
2. Compute the water production rate for the well in m^3/day.
3. Run the simulation using an appropriate executable and input file.
4. Compare the data in the *surface* file with the number computed in Exercise 2.
5. View the *output* file and plot the aqueous head and drawdown at the 4 reference nodes as a function of time. The location of the reference nodes are

shown in Figure 2. The aqueous head and drawdown are shown in Figure 3. Explain how the drawdown is computed.

6. Produce a plot of the final time plane that shows the head distribution to look similar to Figure 4.

7. Increase the hydraulic conductivity of the aquitard to equal that of the two aquifers. Run the simulation and make a plot of the final aqueous phase head distribution. Compare the results with the plot made in Exercise 6. Reset the hydraulic conductivity value in the *input* file.

8. Impose an anisotropy ratio of 10:1 in the aquitard only. Run the simulation and make a plot of the final aqueous phase head distribution. Compare the results with the plots made in Exercises 6 and 7. Reset the hydraulic conductivity value in the *input* file.

9. Instead of using the Neumann boundary condition, create a *Source Card* to obtain similar results. Use the online User Guide for guidance on syntax (https://stomp.pnnl.gov/user_guide/STOMP_guide.stm). Run the simulation and compare the results.
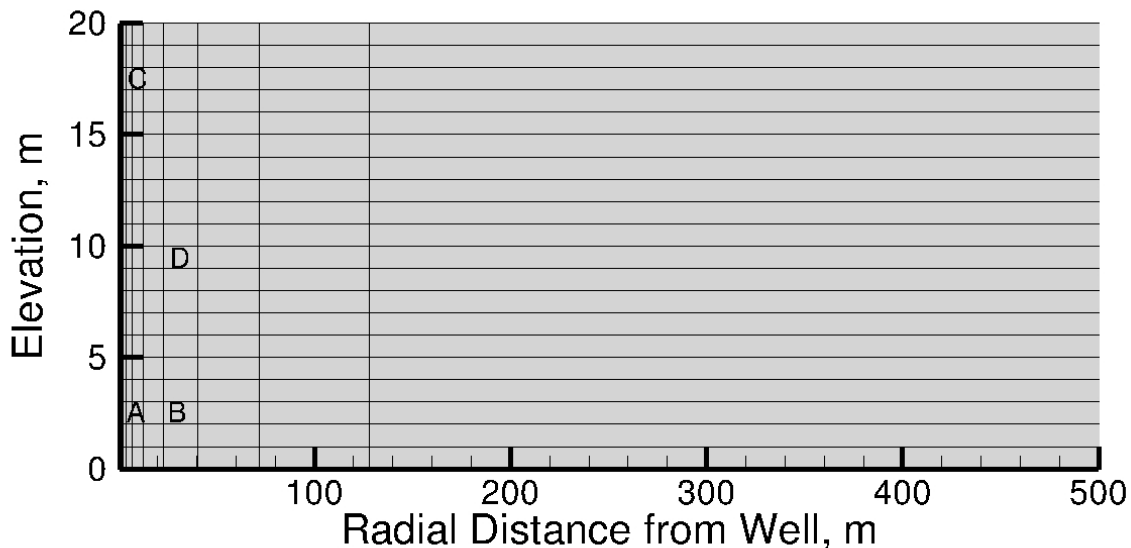

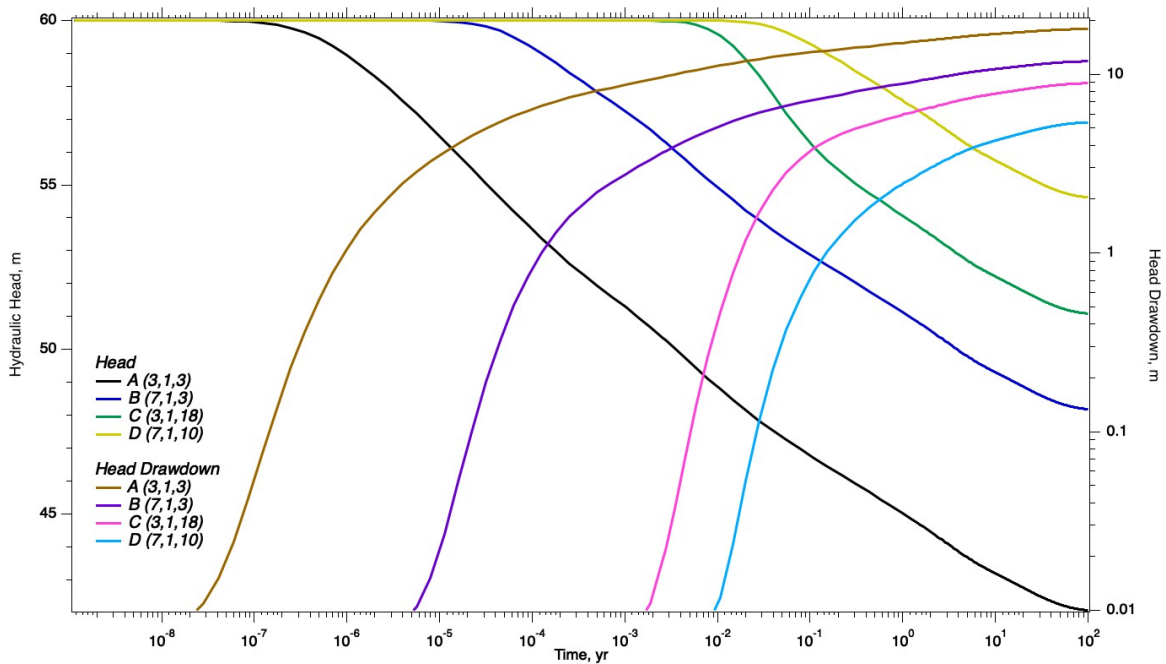**Figure 2.** Location of specified reference nodes and initial aqueous head distribution

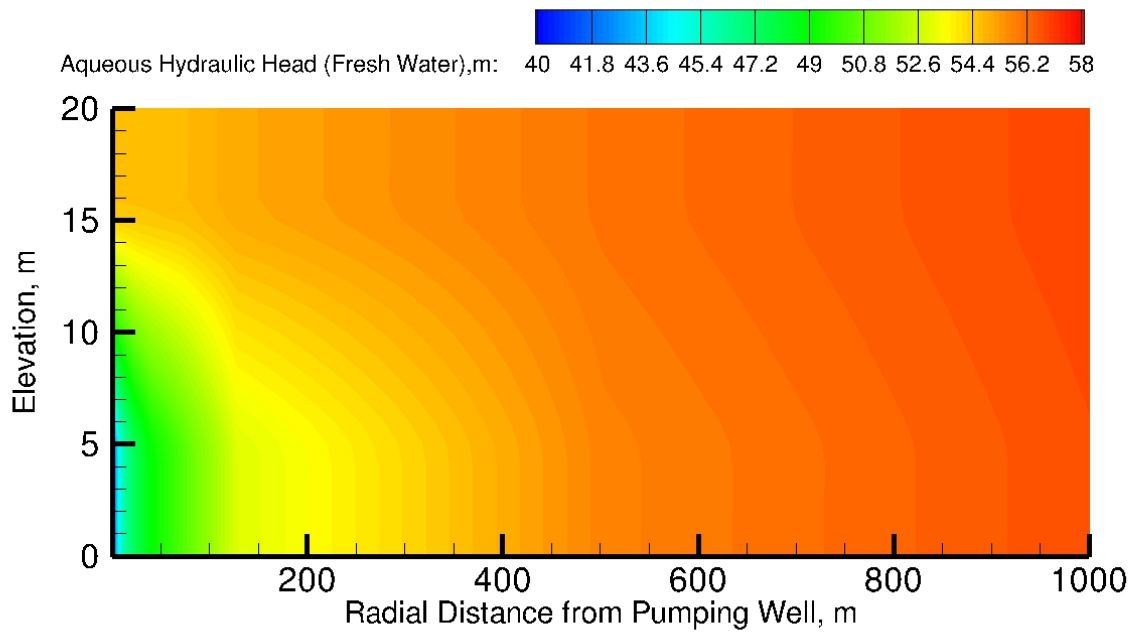**Figure 3**. Head and Drawdown at location A, B, C, D

Figure 4. Final aqueous head distribution

Input File

```
#------------------------------------------------------------------------------
~Simulation Title Card
#------------------------------------------------------------------------------
1,
STOMP Example Problem W-2,
Mart Oostrom,
PNNL,
June 03,
15:00,
4,
Two aquifer problem (Segol 1994, p. 423-432),
Two aquifers (5 m thick) separated by an aquitard (10 m thick),
Pumping well is screened in the lower aquifer only,
No drawdown allowed on other end,

#------------------------------------------------------------------------------
~Solution Control Card
#------------------------------------------------------------------------------
Normal,
STOMP-W,
1,
0,s,100,yr,36,s,50,d,1.414,24,1.e-6,
10000,
0,

#------------------------------------------------------------------------------
~Grid Card
#------------------------------------------------------------------------------
Cylindrical,
13,1,20,
1.0,m,1.195,m,2.365,m,3.955,m,7.285,m,12.715,m,22.885,m,40.315,m,72.085,m,127.915,m,
504.485,m,1495.515,m,4828.485,m,10000,m,
0.,deg,18.,deg,
0,m,20@1.0,m,

#------------------------------------------------------------------------------
~Rock/Soil Zonation Card
#------------------------------------------------------------------------------
3,
Aquifer1,1,13,1,1,16,20,
Aquitard,1,13,1,1,6,15,
Aquifer2,1,13,1,1,1,5,

#------------------------------------------------------------------------------
~Mechanical Properties Card
#------------------------------------------------------------------------------
Aquifer1,2650,kg/m^3,0.30,0.30,1.e-04,1/m,Millington and Quirk,
Aquitard,2650,kg/m^3,0.43,0.43,8.e-04,1/m,Millington and Quirk,
Aquifer2,2650,kg/m^3,0.30,0.30,1.e-04,1/m,Millington and Quirk,

#------------------------------------------------------------------------------
~Hydraulic Properties Card
#------------------------------------------------------------------------------
```

Aquifer1,2.e-05,hc m/s,,,2.e-05,hc m/s,
Aquitard,1.e-08,hc m/s,,,1.e-08,hc m/s,
Aquifer2,2.e-05,hc m/s,,,2.e-05,hc m/s,

```
#-------------------------------------------------------------------------------
~Saturation Function Card
#-------------------------------------------------------------------------------
```
Aquifer1,van Genuchten,0.133,1/cm,1.88,0.268,,
Aquitard,van Genuchten,0.133,1/cm,1.88,0.268,,
Aquifer2,van Genuchten,0.133,1/cm,1.88,0.268,,

```
#-------------------------------------------------------------------------------
~Aqueous Relative Permeability Card
#-------------------------------------------------------------------------------
```
Aquifer1,Mualem,,
Aquitard,Mualem,,
Aquifer2,Mualem,,

```
#-------------------------------------------------------------------------------
~Initial Conditions Card
#-------------------------------------------------------------------------------
```
Gas Pressure,Aqueous Pressure,
1,
Aqueous Pressure,684036.8,Pa,,,,,-9793.5,1/m,1,13,1,1,1,20,

```
#-------------------------------------------------------------------------------
~Boundary Conditions Card
#-------------------------------------------------------------------------------
```
2,
west,neumann,
1,1,1,1,1,5,1,
0,d,-5.02929620e-05,m/s,
east,hydraulic gradient,
13,13,1,1,1,20,1,
0,d,684036.8,Pa,

```
#-------------------------------------------------------------------------------
~Output Options Card
#-------------------------------------------------------------------------------
```
4,
3,1,3,
7,1,3,
3,1,18,
7,1,10,
1,1,yr,m,deg,6,6,6,
3,
aqueous saturation,,
aqueous pressure,Pa,
aqueous hydraulic head,m,
6,
0.0,d,
0.5,d,
1,d,
1,yr,
10,yr,

7

50,yr,
6,
no restart,,
aqueous hydraulic head,m,
aqueous pressure,Pa,
aqueous saturation,,
XNC aqueous volumetric flux,m/hr,
ZNC aqueous volumetric flux,m/hr,

```
#------------------------------------------------------------------------------
~Surface Flux Card
#------------------------------------------------------------------------------
2,
aqueous volumetric flux,m^3/day,m^3,west,1,1,1,1,1,5,
aqueous volumetric flux,m^3/day,m^3,east,13,13,1,1,1,20,
```

Solutions to Selected Exercises

Exercise 1

Initial conditions for aqueous pressure are set by specifying a pressure at the bottom node of the domain and the pressure gradient in the *Initial Conditions Card.* The problem specifies an absolute initial hydraulic head of 70.346 m (STOMP expects absolute pressures, not gage) defined to be at the bottom of the domain. To determine the corresponding pressure at the bottom of the domain, the following equation is used:

$$P = h \cdot \rho \cdot g$$

where P is the Pressure in Pa, h is the hydraulic head (70.346 m), ρ is the aqueous density of pure water (assumed to be 999.337 kg/m³), g is the acceleration of gravity (9.8 m/s), and ρg = 9793.5. This gives a pressure of 688933.55 Pa. Now we need to determine the pressure at the node which is half a cell above the bottom of the domain. The bottom cell has 1 m grid spacing. Therefore, we must calculate the pressure at 0.5 m above the bottom of the domain. Adding 0.5 m times the vertical pressure gradient of -9793.5 yields an initial pressure of 684036.8 Pa for the bottom node.

Exercise 2

The water production rate is 136.5 m³/day (1502.6 gal/hr), calculated using the specified boundary flux of 5.0292962 x 10⁻⁵ m/s and the well geometry, as

*5.0292962 x 10⁻⁵ m/s * 2π; * 1 m * 5 m = 1.58 x 10³ m³/s = 136.5 m³/day*

where, the screened interval area equals 31.416 m².

Exercises 3 and 4

The surface flux file reports a steady-flow flow rate into the well of 6.8256 m³/day, which can be converted to a well production rate, as

6.8256 m³/day * 360.0°/18.0° = 136.5 m³/day

which agrees with the prescribed water production rate for the well.

Exercise 5

The plot of hydraulic head at each location as a function of time, as shown in Figure 3, shows decreasing head with time. Also shown in Figure3 is the drawdown on the right-hand axis, calculated as the initial hydraulic head (i.e., 60 m) less the hydraulic head.

Exercise 7

Increasing the hydraulic conductivity of the aquitard to the value assigned to the aquifers reduces the overall flow resistance and homogenizes the aquifer. The lower resistance reduces the gradient in hydraulic head and the homogenization flattens the head contours, as shown in Figure 5.
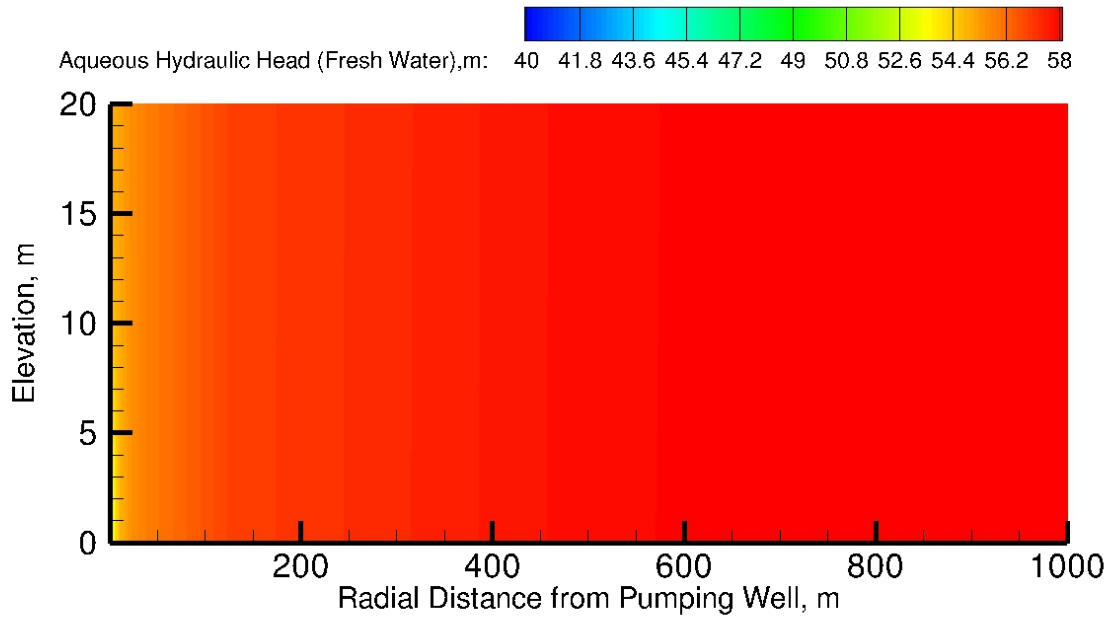
Figure 5. Contours of hydraulic head in response to pumping (homogeneous aquifer)

Exercise 8

Increasing the horizontal hydraulic conductivity of the aquitard by a factor of 10, reduces the horizontal flow resistance of the aquitard, but not enough to significantly change the response in hydraulic head, as shown in Figure 6.
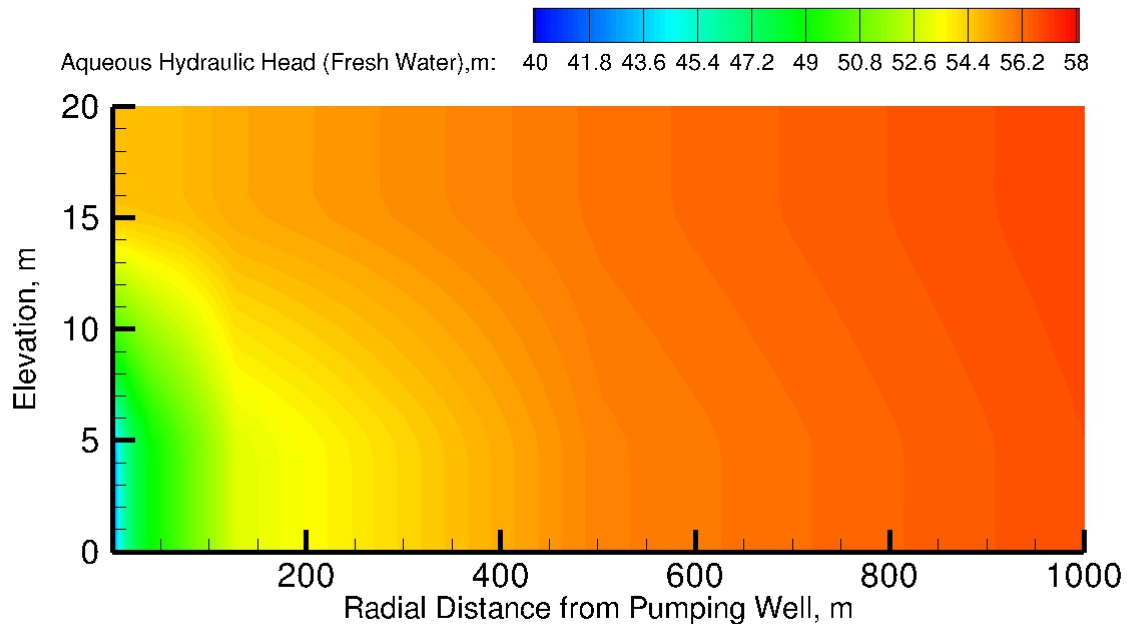

Figure 6. Contours of hydraulic head with pumping (10:1 anisotropy ratio)

10

Exercise 9

Specifying a source requires removing the west boundary condition and adding a *Source Card* as shown below:

```
#-----------------------------------------------------------------------------
~Source Card
#-----------------------------------------------------------------------------
1,
Aqueous Volumetric,1,1,1,1,1,5,1,
0,d,-1.58e-05,m^3/s,

#-----------------------------------------------------------------------------
~Boundary Conditions Card
#-----------------------------------------------------------------------------
1,
east,hydraulic gradient,
13,13,1,1,1,20,1,
0,d,684036.8,Pa,
```

Note that the source is specified as a negative number to indicate that it is a pumping well rather than an injection well. The resulting hydraulic head distribution (Figure 7) is the same as when using a boundary condition.
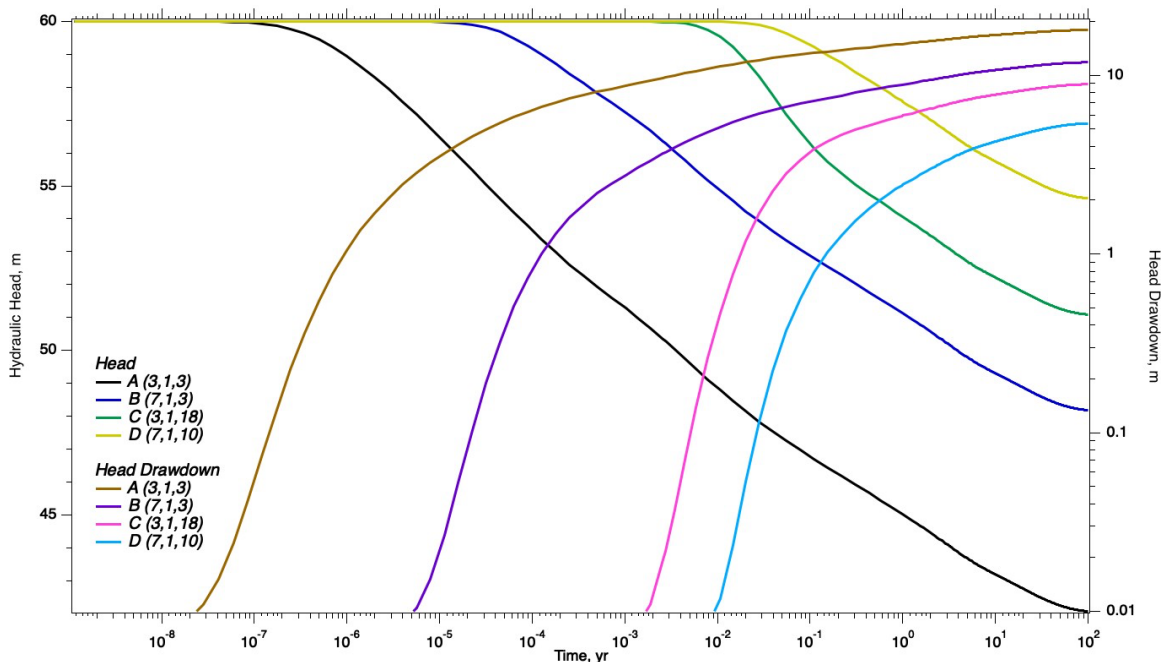

Figure 7. Hydraulic head and drawdown in response to pumping using a source term instead of a boundary condition.