

# Problem 1

Say that two Boolean formulas are equivalent if they have the same set of variables and are true on the same set of assignments to those variables (i.e., they describe the same Boolean function). A Boolean formula is minimal if no shorter Boolean formula is equivalent to it. Let MIN-FORMULA be the collection of minimal Boolean formulas.

Show that if  $P = NP$ , then  $\text{MIN-FORMULA} \in P$ .

boolean : 布尔

i.e. : 即

```
def of equivalent:  
`same set of variable  
`true to same set of assignment
```

$P$  : poly-time 可求解

$NP$ : poly-time 可证明

$co-NP$ :  $NP$  的补集

$P$  对补集完全，因此若  $P=NP$  可推出  $P=co-NP$

判断  $NP$  的方式：可以通俗的理解，用 NTM 猜一个多项式长度的结果，然后可以在多项式时间内验证这个结果，那么这个问题就是  $NP$  问题

重要思想：反过来想！

直接证明相等是很难的，但是证明不相等是比较容易的。

? 其实二者是补集的关系

然后  $P$  对补集 closed，对这个问题并没有非常明显的区别？

## Provement

Based on the assumption that  $P = NP$ , we know that **judging whether a formula is equal to another can be done in poly-time**, since it is initially a  $co - NP$  problem and  $P$  is closed to complement.

To judge whether  $F$ , which is a formula, is in  $\text{MIN} - \text{FORMULA}$ , it is equal to check whether there exists a formula  $F'$  equivalent to  $F$  and shorter than  $F$  as well.

**Let a NTM working as follow:**

Firstly, guess a formula  $F'$ , whose length is shorter than  $F$ , which means that it is bounded by a

polynomial.

Then, test whether it is equal to F, which can be done in poly-time as proven above, making  $\text{MIN} - \text{FORMULA}$  a  $NP$  problem.

Under the assumption that  $N = NP$ , we have  $\text{MIN} - \text{FORMULA} \in P$ .

## Problem 2

Show that  $\text{MIN-FORMULA} \in \text{PSPACE}$ .

PSPACE: TM可在多项式空间内求解

$\text{PSPACE} = \text{NPSPACE}$

$\$NP \subseteq \text{PSPACE}$

As we have proved that  $\text{MIN} - \text{FORMULA}$  is  $NP$  problem, and  $NP \subseteq \text{PSPACE}$ , we can easily get that  $\text{MIN} - \text{FORMULA} \in \text{PSPACE}$ .