Question 1

We say that a problem is *NP-hard*, if every problem in NP can be reduced in polynomial time to it, even though it may not be in NP [so, a problem is NP-complete if it is both in NP and NP-hard.]
Analogously, we say that a problem is *PSPACE-hard*, if every problem in PSPACE can be reduced in polynomial *time* to it, even though it may not be in PSPACE [so, a problem is PSPACE-complete if it is both in PSPACE and PSPACE -hard.]

Show that any PSPACE-hard language is also NP-hard.

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analogously: 类似地
terminal：终止符，终端

NP:在多项式时间内可被证明
PSPACE：由确定型图灵机在多项式空间可判定的语言。
PSPACE-hard：所有 PSPACE 中的问题都可以在多项式时间内归约到 PSPACE - Hard 问题。
NP ⊆ PSPACE（即所有 NP 类的问题都属于 PSPACE 类）。
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Provement

Let language $L \subseteq PSPACE - hard$.
We need to prove that it is $NP - hard$, which means that every NP problem can be reduced to it in poly-time.
According to the definition of PSPACE-hard, every $PSPACE$ problem can be redused to L in poly-time.
Since $NP \subseteq PSPACE$, e.g. every NP problem is $PSPACE$ problem and in turn can be reduced to L in poly-time，fitting the requirement we need for proving that $PSPACE - hard$ language is also $NP - hard$.

Question 2

Show that if every NP-hard language is also PSPACE-hard, then PSPACE = NP.

利用性质：NP的封闭性：若一个问题能多项式时间归约到 NP 中的问题，则它本身也属于NP
但是，由于NP-hard问题可能不是NP，所以能规约到NP-hard的问题不一定是NP

New! NP-complete是NP-hard和NP的交集，在证明题中可以用来沟通二者。
比如这题用NP-hard的性质证明了和NP-complete相关，
就直接转换为和NP的相关

证明包含关系的一般思路：一般是通过取A中实例，证明其在B中，来证明A包含于B

Provement

Let L be a NP-complete problem, which means that it is both NP and NP-hard.
Let L_{pspace} be a $PSPACE$ problem.
Base on the result from Question 1, we have proved that $PSPACE-hard$ language is also $NP-hard$.
Then, with the assumption proposed in Question 2, we have $PSPACE-hard$ = $NP-hard$.
By definition, PSPACE problem L_{pspace} can now be reduced to L.
Since L is alse NP and NP is closed, we have $L_{pspace} \in NP$, making $PSPACE \subseteq NP$.
As $NP \subseteq PSPACE$ has already be proved, NP is now equal to $PSPACE$.