

Problem 1

Say that two Boolean formulas are equivalent if they have the same set of variables and are true on the same set of assignments to those variables (i.e., they describe the same Boolean function). A Boolean formula is minimal if no shorter Boolean formula is equivalent to it. Let MIN-FORMULA be the collection of minimal Boolean formulas.

Show that if $P = NP$, then $\text{MIN-FORMULA} \in P$.

boolean : 布尔

i.e. : 即

def of equivalent:

`same set of variable

`true to same set of assignment

P : poly-time可求解

NP: poly-time可证明

co-NP: NP 的补集

P对补集完全，因此若 $P=NP$ 可推出 $P=co-NP$

判断NP的方式：可以通俗的理解，用NTM猜一个多项式长度的结果，然后可以在多项式时间内验证这个结果，那么这个问题就是NP问题

重要思想：反过来想！

直接证明相等是很难的，但是证明不相等是比较容易的。

？其实二者是补集的关系

然后P对补集closed，对这个问题并没有非常明显的区别？

Provement

Based on the assumption that $P = NP$, we know that **judging whether a formula is equal to another can be done in poly-time**, since it is initially a $co - NP$ problem and P is closed to complement.

To judge whether F, which is a formula, is in $\text{MIN} - \text{FORMULA}$, it is equal to check whether there exists a formula F' equivalent to F and shorter than F as well.

Let a NTM working as follow:

Firstly, guess a formula F', whose length is shorter than F, which means that it is bounded by a

polynomial.

Then, test whether it is equal to F, which can be done in poly-time as proven above, making $MIN - FORMULA$ a NP problem.

Under the assumption that $N = NP$, we have $MIN - FORMULA \in P$.

Problem 2

Show that $MIN-FORMULA \in PSPACE$.

PSPACE: TM可在多项式空间内求解

$PSPACE = NPSPACE$

$NP \subseteq PSPACE$

As we have proved that $MIN - FORMULA$ is NP problem, and $NP \subseteq PSPACE$, we can easily get that $MIN - FORMULA \in PSPACE$.